

# Essentials of Applied Data Analysis

## IPSA-USP Summer School 2017

### The Basics of Probability Theory - Compound events

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### Introduction to Probability - Part II

Basic Notions of probability, part II.

#### Single or Compound events? Independence and Exclusivity

An event can be simple (a single outcome) or compound (two or more single events).

The relation between the events that form a compound event can be defined as:

- *Independent*: two events are independent if the probability that one occurs does not change as a consequence of the other event's occurring.
- *Mutual exclusivity*: two events are mutually exclusive when one cannot occur if the other has occurred.
- *Collective exhaustivity*: the set of collective exhaustive events is the whole sample space.

## Axioms and theorems of probability (2)

- If  $A$  and  $B$  are mutually exclusive, then:

$$P(A \cup B) = P(A) + P(B)$$

- If  $A_1, A_2, \dots$  is a sequence of mutually exclusive events, then:

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$

## Dices and mutually exclusive events

Roll a 6-side dice.

What's is the probability of getting a 5 **OR** a 6?

$$P(5 \text{ or } 6) = P(5 \cup 6) = ?$$

What's is the probability of getting a prime **OR** an odd number?

$$P(\text{prime or odd}) = P(\text{prime} \cup \text{odd}) = ?$$

## Dices and mutually exclusive events - answers

Roll a 6-side dice.

What's is the probability of getting a 5 **OR** a 6?

$$P(5 \text{ or } 6) = P(5 \cup 6) = \frac{\#\{5, 6\}}{6} = \frac{2}{6} = \frac{1}{3} = \frac{1}{6} + \frac{1}{6} = P(5) + P(6)$$

since 5 and 6 are mutually exclusive events.

What's is the probability of getting a prime **OR** an odd number?

**Beware!!!** This is not true for events that are not mutually exclusive (note that 3 and 5 are both prime and odd).

$$P(\text{prime}) + P(\text{odd}) = \frac{\#\{2, 3, 5\}}{6} + \frac{\#\{1, 3, 5\}}{6} = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

$$P(\text{prime or odd}) = P(\text{prime} \cup \text{odd}) = \frac{\#\{1, 2, 3, 5\}}{6} = \frac{4}{6} \neq P(\text{prime}) + P(\text{odd})$$

### Mutually exclusive events - random legislator

Let's go back to the Legislative House example.

What's is the probability of getting a legislator from Party A **OR** B?

$$P(A \text{ or } B) = P(A \cup B) = ?$$

What's is the probability of getting a legislator from Party A **OR** a woman (W)?

$$P(A \text{ or } W) = P(A \cup W) = ?$$

### Mutually exclusive events - random legislator - answers

Let's go back to the Legislative House example.

What's is the probability of getting a legislator from Party A **OR** B?

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

What's is the probability of getting a legislator from Party A **OR** a woman (W)?

If there is at least a woman on party A, the events are not mutually exclusive and

$$P(A \text{ or } W) = P(A \cup W) \neq P(A) + P(W)$$

If it is a all-men party, being a woman and belonging to party A are mutually exclusive and

$$P(A \cup W) = P(A) + P(W)$$

### Axioms and theorems of probability (3)

$A^c$  is the complementary event of A.

- $P(A^c) = 1 - P(A)$
- $P(A \cup A^c) = P(A) + P(A^c) = 1$  (because they are mutually exclusive)

### Complementary event (not A, A, A' or $A^c$ )

What is the probability of **NOT** getting a 5 on a 6-side dice?

$$P(\text{not } 5) = P(\{1,2,3,4,6\}) = \frac{\#\{1,2,3,4,6\}}{6} = 5/6$$

Or, more elegantly:

$$P(\text{not } 5) = 1 - P(5) = 1 - \frac{1}{6} = \frac{5}{6}$$

### Axioms and theorems of probability (4)

- $P(A \cap B)$  is the the probability of A **AND** B happening at the same time.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

### Not mutually exclusive events - dice and legislators

Going back to our problem, what's is the probability of getting a prime **OR** an odd number?

$$P(\text{prime} \cup \text{odd}) = P(\text{prime}) + P(\text{odd}) - P(\text{prime} \cap \text{even}) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$

What's is the probability of getting a legislator from Party A **OR** a woman (W)?

If there is at least a W on party A, then

$$P(A \cup W) = P(A) + P(W) - P(A \cap W)$$