Esssentials of Applied Data Analysis
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Axioms and theorems of probability - summary

(Also know as "rules of probability")

1. For every event A, $0 \leq P(A) \leq 1$
2. $P(S) = 1$
3. $P(\varnothing) = 0$
4. If A and B are mutually exclusive, then:

$$P(A \cup B) = P(A) + P(B)$$

5. If A_1, A_2, ... is a sequence of mutually exclusive events, then:

$$P(A_1 \cup A_2 \cup ... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$$

6. $P(A^c) = 1 - P(A)$
7. $P(A \cup A^c) = P(A) + P(A^c) = 1$ (because they are mutually exclusive)
8. $P(A \cap B)$ is the the probability of A **AND** B happening at the same time.
9. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
10. If A and B are independent events, then $P(A \cap B) = P(A) * P(B)$

11. If A and B are not independent events, then the conditional probability of A given B, $P(A|B)$, is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

12. If A and B are independent events, then $P(A|B) = P(A)$ and the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

can be rewritten as (as previously seen)

$$P(A \cap B) = P(A) * P(B)$$

13. If A and B are mutually exclusive, then

$$P(A \cap B) = 0$$

hence

$$P(A|B) = P(B|A) = 0$$

14. Law of Total Probability note that we can write the probability of A and B ($A \cap B$ in two different forms:

$$P(A \cap B) = P(A|B) * P(B) = P(B|A) * P(A)$$

From that we can derive the law of total probability which states that the probability of A can be separated into two parts: the conditional probability of A on B times the probability of B and the conditional probability of A on the complement of B (B^c) times the probability of B^C

$$P(A) = P(A|B) * P(B) + P(A|B^c) * P(B^c)$$

15. Suppose there are A_1, A_2,..., A_n events and $A_1 \cup A_2 \cup ... \cup A_n$ = S. A_i events are mutually exclusive. Suppose there is another event B. For each A_i, B has a different joint conditional probability $P(A_i|B)$. B can be described as:

$$P(B) = P(A_1) * P(B|A_1) + P(A_2) * P(B|A_2) + ... + P(A_n) * P(B|A_n)$$

The probability that an specific A_i will occur given B is defined as:

$$P(A_i|B) = \frac{P(A_i) * P(B|A_i)}{P(A_1) * P(B|A_1) + P(A_2) * P(B|A_2) + ... + P(A_n) * P(B|A_n)}$$