Esssentials of Applied Data Analysis
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The Basics of Set Theory

Leonardo Sangali Barone
leonardo.barone@usp.br

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Set Theory

Basic notions and notation of set theory.

First concepts and notation

- Sets are a list or collection of objects.
- These objects are elements.
- $\varnothing$ is the empty set (or null set).
- $p \in A$: is p is an element in the set A.
- $A \subseteq B$: A is a subset of B
- $A \subset B$: A is a proper subset of B and B has at least one element that A does not

Common Sets

Notation	Meaning	Examples	How it relate to other sets
$\mathbb{N}$	Natural numbers	$(0, 1, 2, ...)$	
$\mathbb{Z}$	Integers	$(..., -2, -1, 0, 1, 2, ...)$	all $\mathbb{N}$ are $\mathbb{Z}$
$\mathbb{Z}^-$	Negative Integers	$(..., -2, -1)$	$\mathbb{Z}^+$ is a subset of $\mathbb{Z}$
$\mathbb{Q}$	Rational numbers	$(..., \frac{-42}{13}, -1, \frac{-1}{2}, 0, \frac{1}{2}, \frac{17}{13}, 100, ...)$	all $\mathbb{Z}$ are $\mathbb{Q}$
$\mathbb{R}$	Real numbers	$(..., \frac{-42}{13}, -1, 0, \sqrt{2}, \pi, 100, ...)$	all $\mathbb{Q}$ are $\mathbb{R}$
$\mathbb{C}$	Complex numbers	$(1 + 2i,\ 42 - 3i)$, where $i = \sqrt{-1}$	

Properties of Sets

Property	Definition	Examples
Finite	sets with finite number of elements	$\{S, M, G\}$; $(1, ..., 10)$
Infinite	sets with number of elements	$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$
Countable	number of elements can be counted	$\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$
Uncountable	not countable	$\mathbb{R}$ and $\mathbb{C}$
Bounded	finite size or shape (even if infinite)	x $\in \mathbb{R} : 0 \leq x \leq 1$
Unbounded	infinite size	x $\in \mathbb{R} : x \geq 42$
Ordered	$a, b, c \neq b, a, c$	
Unordered	$a, b, c = b, a, c$	

Set Theory - operations

- $A \cup B$: union of A and B.
  - $p \in (A \cup B)$: p is an element of A **OR** B.
- $A \cap B$: intersection of A an B.
  - $p \in (A \cup B)$: p is an element of A **AND** B.
- If $A \cap B$ is equal to $\varnothing$, then A and B are **disjoint** sets.
- A^c (A', $\tilde{}A$ or simply *not* A) is the set of all elements that does not belong to A. A^c is the complement of A.
- $A \setminus B$ is the set of all elements of set A that does not belong to B (difference).

Venn Diagramas

We can represent sets with diagrams. These are called "Venn Diagrams". See Figure 1 and locate the following sets as a quick exercise:

1) $A \cup B$	5) $(A \cup B) \cup C$	9) A^c	13) $((A \cap B) \cap C)^c$
2) $A \cap B$	6) $(A \cap B) \cap C$	10) $(A \cap B)^c$	14) $((A \cup B) \cap C)^c$
3) $A \cup C$	7) $(A \cup B) \cap C$	11) $(A \cup C)^c$	15) $A \setminus B$
4) $A \cap C$	8) $(A \cap B) \cup C$	12) $((A \cup B) \cup C)^c$	15) $A \setminus C$

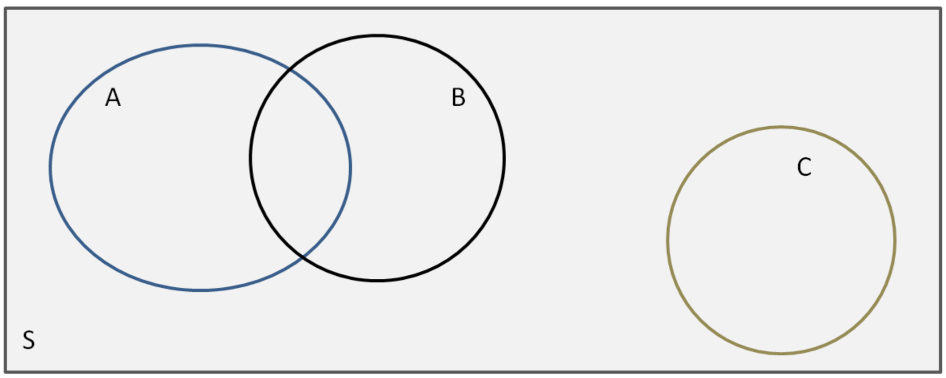

Figure 1: Venn Diagrams