Esssentials of Applied Data Analysis IPSA-USP Summer School 2018

The Basics of Probability Theory - Compound events

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jan/19

Introduction to Probability - Part II

Basic Notions of probability, part II.

Single or Compound events? Independence and Exclusivity

An event can be simple (a single outcome) or compound (two or more single events).

The relation between the events that form a compound event can be defined as:

- *Independent*: two events are independent if the probability that one occurs does not change as a consequence of the other event's occurring.
- *Mutual exclusivity*: two events are mutually exclusive when one cannot occur if the other has occurred.
- *Collective exhaustivity*: the set of collective exhaustive events is the whole sample space.

Axioms and theorems of probability (2)

- If A and B are mutually exclusive, then:

$$P(A \cup B) = P(A) + P(B)$$

- If A_1, A_2, ... is a sequence of mutually exclusive events, then:

$$P(A_1 \cup A_2 \cup ... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$$

Dices and mutually exclusive events

Roll a 6-side dice.

What's is the probability of getting a 5 **OR** a 6?

$$P(5 \text{ or } 6) = P(5 \cup 6) = \frac{\#\{5,6\}}{\#\{1,2,3,4,5,6\}} = \frac{2}{6} = \frac{1}{3} = \frac{1}{6} + \frac{1}{6} = P(5) + P(6)$$

since 5 and 6 are mutually exclusive events.

What's is the probability of getting a prime **OR** an odd number?

Could it be $P(\text{prime or odd}) = P(\text{prime}) + P(\text{odd})$?

No! **Beware!!!** This is not true for events that are not mutually exclusive (note that 3 and 5 are both prime and odd).

$$P(\text{prime or odd}) = P(\text{prime } \cup \text{ odd}) = \frac{\#\{1,2,3,5\}}{6} = \frac{4}{6} \neq$$

$$\neq P(\text{prime}) + P(\text{odd}) = \frac{\#\{2,3,5\}}{6} + \frac{\#\{1,3,5\}}{6} = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

Mutually exclusive events - random legislator

Let's go back to the Legislative House example.

What's is the probability of getting a legislator from Party A **OR** B?

$$P(\text{A or B}) = P(A \cup B) = P(A) + P(B)$$

since A and B are independent events.

What's is the probability of getting a legislator from Party A **OR** a woman (W)?

If there is at leat a woman on party A, the events are not mutually exclusive and

$$P(\text{A or W}) = P(A \cup W) \neq P(A) + P(W)$$

If A is a all-men party, being a woman and beloging to party A are mutually exclusive and

$$P(A \cup W) = P(A) + P(W)$$

Axioms and theorems of probability (3)

A^c is the complementary event of A.

- $P(A^c) = 1 - P(A)$
- $P(A \cup A^c) = P(A) + P(A^c) = 1$ (because they are mutually exclusive)

Complementary event (not A, A, A' or A^c)

What is the probability of **NOT** getting a 5 on a 6-side dice?

$$P(\text{not } 5) = P(\{1,2,3,4,6\}) = \frac{\#\{1,2,3,4,6\}}{6} = 5/6$$

Or, more elegantly:

$$P(\text{not } 5) = 1 - P(5) = 1 - \frac{1}{6} = \frac{5}{6}$$

Axioms and theorems of probability (4)

- $P(A \cap B)$ is the the probability of A **AND** B happening at the same time.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Not mutually exclusive events - dice and legislators

Going back to our problem, what's is the probability of getting a prime **OR** an odd number?

$P(\text{prime} \cup \text{odd}) = P(\text{prime}) + P(\text{odd}) - P(\text{prime} \cap \text{odd}) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

What's is the probability of getting a legislator from Party A **OR** a woman (W)?

If there is at leat a W on party A, then

$$P(A \cup W) = P(A) + P(W) - P(A \cap W)$$