Introduction to Statistical Analysis (using Shiny Apps) CRUK:- Monday 28th November 2016

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www.tiny.cc/crukStats

Acknowledgements: Sarah Vowler, Sarah Dawson, Liz Merrell, Deepak Parashar, Rob Nicholls

Tests for continuous variables non-parametric methods

		RESPONSE				
NO OF SAMPLES		NOMINAL	ORDINAL OR NON- NORMAL	NORMALLY DISTRIBUTED		
	ONE MPLE	χ²-test, Z-test	Kolmogorov-Smirnov Sign test	t-test		
TWO	INDEPENDENT	χ²-test (r x c), Fisher's exact test	Mann-Whitney U Median test	Unpaired t-test		
SAMPLE	PAIRED	McNemar's test Stuart-Maxwell test	Wilcoxon signed rank Sign test	Paired t-test		
MULTIPLE SAMPLES INDEPENDENT		χ²-test (r x k) Fisher-Freeman-Halton	Kruskal-Wallis test Median Test Jonckheere-Terpstra test	Analysis of variance (ANOVA)		
(K>2)	PAIRED	Cochran Q test	Friedman test Page test Quade test	Repeated measures ANOVA		
ASSOCIATION BETWEEN TWO VARIABLES		Contingency coefficient Phi, rø Cramér, C	Spearman's rank Kendall's tau	Pearson product moment correlation		
AGREEMENT BETWEEN TWO VARIABLES		Simple kappa	Weighted kappa	Limits of agreement		

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Sign Test

- A very simple non-parametric test
 - based on the Binomial distribution

Uses directions of differences

- One-sample case: compares to proposed value
- Paired two-sample case: compares medians

• Assumptions:

- Order in coding system (minimally requires ordinal data)
- Randomly selected observations (independent)

Hypotheses:

- $-H_0$: median is equal to a specific value
- H_{\(\time\)}: median is not equal to that specific value

Method:

Compare values to a specific value:

+ : if bigger

- : if smaller

= : if equal

Count the number of +'s and -'s, and calculate:

x = smallest of the positives and negatives

n = number of non-ties

Compare to binomial tables

With p = 0.5 (binomial success probability, not p-value)

- General health section of SF-36 collected in a breast cancer study
- Expected value in general population: 72

 H_0 : median value in sample is equal to 72

GH Value

GH Value	Sign
60	-
55	-
75	+
100	+
55	-
60	-
50	-
60	-
72	=
40	-
90	+
75	+
70	-
75	+
55	-

9: observations <72

5 : observations >72

1 : observation =72

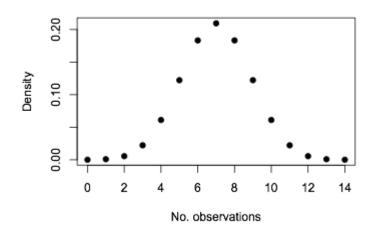
GH Value	Sign
60	-
55	-
75	+
100	+
55	-
60	-
50	-
60	-
72	=
40	-
90	+
75	+
70	-
75	+
55	-

9 : observations <72

5 : observations >72

1: observation =72

Binomial tables: n=14, p=0.5, x=5



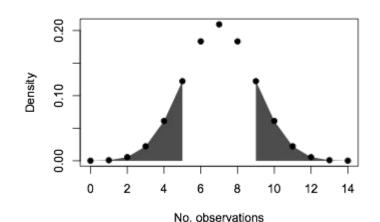
GH Value	Sign
60	-
55	-
75	+
100	+
55	-
60	-
50	-
60	-
72	=
40	-
90	+
75	+
70	-
75	+
55	-

9: observations <72

5 : observations >72

1 : observation =72

Binomial tables: n=14, p=0.5, x=5



P-value = 0.42

 H_0 : median value in sample is equal to 72

- Sign test p-value = 0.42
- Insufficient evidence to reject H₀

Conclusion: insufficient evidence to suggest that the median value is different from 72

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Method:

- Compare paired values between the two samples:
 - + : if the value in sample 1 is bigger
 - if the value in sample 1 is smaller
 - = : if the value in the two samples is equal
- Count the number of +'s and -'s, and calculate:

```
x = smallest of the positives and negatives
```

n = number of non-ties

Compare to binomial tables

```
With p = 0.5 (binomial success probability, not p-value)
```

- General health section of SF-36 collected in a breast cancer study
- Data collected at two time points
- Is there a difference between the time points?

 H_0 : medians of the two samples are the same

GH Value 1	GH Value 2
60	70
55	65
75	100
100	50
55	70
60	95
50	95
60	65
72	85
40	55
90	95
75	45
70	75

Data are paired

GH Va	lue 1	GH Va	ue 2	Difference

Data are paired

GH Value 1	GH Value 2	Difference	Sign
60	70	-10	-
55	65	-10	-
75	100	-25	-
100	50	50	+
55	70	-15	-
60	95	-35	-
50	95	-45	-
60	65	-5	-
72	85	-13	-
40	55	-15	-
90	95	-5	
75	45	30	+
70	75	-5	-
75	65	10	+
55	60	-5	

Data are paired

Negative signs: 12

Positive signs: 3

	GH	Value	1 GH	l Value	2 Diff	ference	Sign
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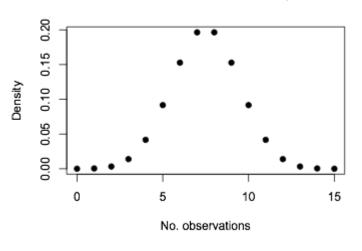
60	70	-10	-
55	65	-10	-
75	100	-25	-
100	50	50	+
55	70	-15	-
60	95	-35	-
50	95	-45	-
60	65	-5	
72	85	-13	-
40	55	-15	-
90	95	-5	-
75	45	30	+
70	75	-5	-
75	65	10	+
55	60	-5	-

Data are paired

Negative signs: 12

Positive signs: 3

Binomial tables: n=15, p=0.5, x=3



60	70	-10	-
55	65	-10	1.7
75	100	-25	-
100	50	50	+
55	70	-15	-
60	95	-35	-
50	95	-45	-
60	65	-5	-
72	85	-13	-
40	55	-15	
90	95	-5	-

45

75

65

60

30

-5

10

-5

75

70

75

55

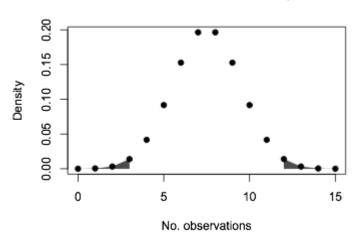
GH Value 1 GH Value 2 Difference Sign

Data are paired

Negative signs: 12

Positive signs: 3

Binomial tables: n=15, p=0.5, x=3



P-value = 0.035

 H_0 : medians of the two samples are the same

- Sign test p-value = 0.035
- Reject the null hypothesis

Conclusion: there is a difference in general health between the two time points

Presentation of the Results

One-sample case:

"There is insufficient evidence to suggest a significant difference between the median general health value (60) observed in this sample and the value (72) observed in the general population (p=0.42, sign test)."

Two-sample case:

"The median general health value observed at the second time point (70) was found to be significantly higher than the median (60) observed at the first time point (p=0.035, sign test)."

Sign Test - Advantages & Limitations

- Simple few assumptions thus widely applicable
- Significance threshold can be adjusted
- Less powerful than other tests
 - Does not consider magnitude of differences
 - May fail to reject null hypothesis when other tests would achieve significance.
- Can be used for quick assessment of direction

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- Alternative to sign test
- Assumptions:
 - Paired data (e.g. matched samples, repeated measurements)
 - Each pair is independent
 - Continuous or ordinal data (Normality not assumed)
 - Symmetry of difference scores about true median difference (test by looking at histogram/boxplot)
- Hypothesis:
 - H_0 : sum of positive ranks equals sum of negative ranks
 - H_A: sum of positive ranks not equal to sum of negative ranks

Method:

- Calculate differences for each pair
- Rank the paired differences by magnitude
- Split the ranks into two groups:
 - positive and negative signed differences
- Calculate sum of positive ranks: W⁺
- Calculate sum of negative ranks: W⁻
- Compare smaller of and W⁺ and W⁻ to the critical value from the tables

- General health section of SF-36 collected in a breast cancer study
- Data collected at two time points
- Is there a difference between the time points?

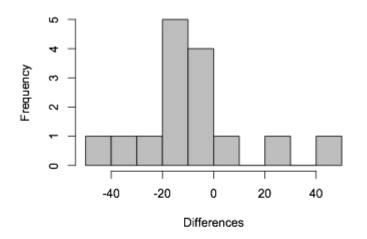
 H_0 : medians of the two samples are the same

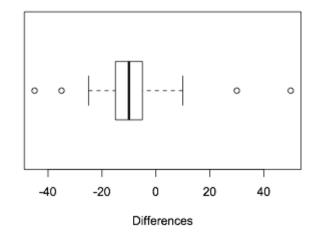
- General health section of SF-36 collected in a breast cancer study
- Data collected at two time points
- Is there a difference between the time points?

H_o: medians of the two samples are the same

H₀: distribution of paired differences is symmetric about zero

GH Value 1	GH Value 2	Difference	Sign
60	70	-10	-
55	65	-10	-
75	100	-25	-
100	50	50	+
55	70	-15	-
60	95	-35	-
50	95	-45	-
60	65	-5	-
72	85	-13	-
40	55	-15	
90	95	-5	-
75	45	30	+
70	75	-5	-
75	65	10	+
55	60	-5	-





GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.
60	70	-10	-	10
55	65	-10	-	10
75	100	-25	-	25
100	50	50	+	50
55	70	-15	-	15
60	95	-35	-	35
50	95	-45	-	45
60	65	-5	-	5
72	85	-13	-	13
40	55	-15		15
90	95	-5	-	5
75	45	30	+	30
70	75	-5	-	5
75	65	10	+	10
55	60	-5	-	5

GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.
60	65	-5	-	5
90	95	-5	-	5
70	75	-5	-	5
55	60	-5	-	5
60	70	-10	-	10
55	65	-10	-	10
75	65	10	+	10
72	85	-13	-	13
55	70	-15	-	15
40	55	-15	-	15
75	100	-25	-	25
75	45	30	+	30
60	95	-35	-	35
50	95	-45	-	45
100	50	50	+	50

GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.	Rank
60	65	-5	-	5	2.5
90	95	-5	-	5	2.5
70	75	-5	-	5	2.5
55	60	-5	-	5	2.5
60	70	-10	-	10	6
55	65	-10	-	10	6
75	65	10	+	10	6
72	85	-13	-	13	8
55	70	-15	-	15	9.5
40	55	-15	-	15	9.5
75	100	-25	-	25	11
75	45	30	+	30	12
60	95	-35	-	35	13
50	95	-45	-	45	14
100	50	50	+	50	15

GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.	Rank	Signed-Rank
60	65	-5	-	5	2.5	-2.5
90	95	-5	-	5	2.5	-2.5
70	75	-5	-	5	2.5	-2.5
55	60	-5	-	5	2.5	-2.5
60	70	-10		10	6	-6
55	65	-10	-	10	6	-6
75	65	10	+	10	6	6
72	85	-13	-	13	8	-8
55	70	-15	-	15	9.5	-9.5
40	55	-15	-	15	9.5	-9.5
75	100	-25	-	25	11	-11
75	45	30	+	30	12	12
60	95	-35	-	35	13	-13
50	95	-45	-	45	14	-14
100	50	50	+	50	15	15

GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.	Rank	Signed-Rank	
60	65	-5	-	5	2.5	-2.5	Davide Compan
90	95	-5	-	5	2.5	-2.5	Rank-Sums:
70	75	-5	-	5	2.5	-2.5	$W^{+} = 33$
55	60	-5	-	5	2.5	-2.5	W ⁻ = 87
60	70	-10	-	10	6	-6	VV - 07
55	65	-10	-	10	6	-6	
75	65	10	+	10	6	6	
72	85	-13	-	13	8	-8	
55	70	-15	-	15	9.5	-9.5	
40	55	-15	-	15	9.5	-9.5	
75	100	-25	-	25	11	-11	
75	45	30	+	30	12	12	
60	95	-35	-	35	13	-13	
50	95	-45	-	45	14	-14	
100	50	50	+	50	15	15	

GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.	Rank	Signed-Rank	
60	65	-5	-	5	2.5	-2.5	Davids Courses
90	95	-5	-	5	2.5	-2.5	Rank-Sums:
70	75	-5	-	5	2.5	-2.5	$W^{+} = 33$
55	60	-5	-	5	2.5	-2.5	\A/ 07
60	70	-10	-	10	6	-6	$W^{-} = 87$
55	65	-10	-	10	6	-6	
75	65	10	+	10	6	6	
72	85	-13	-	13	8	-8	
55	70	-15	-	15	9.5	-9.5	
40	55	-15	-	15	9.5	-9.5	
75	100	-25	-	25	11	-11	0 20 40 60 80 100 120
75	45	30	+	30	12	12	Test Statistic
60	95	-35	-	35	13	-13	
50	95	-45	-	45	14	-14	
100	50	50	+	50	15	15	

GH Value 1	GH Value 2	Difference	Sign	Abs.Diff.	Rank	Signed-Rank	•
60	65	-5	-	5	2.5	-2.5	Davids Comme
90	95	-5	-	5	2.5	-2.5	Rank-Sums:
70	75	-5	-	5	2.5	-2.5	$W^{+} = 33$
55	60	-5	-	5	2.5	-2.5	W⁻ = 87
60	70	-10	-	10	6	-6	VV = 87
55	65	-10	-	10	6	-6	
75	65	10	+	10	6	6	
72	85	-13	-	13	8	-8	
55	70	-15		15	9.5	-9.5	
40	55	-15	-	15	9.5	-9.5	
75	100	-25	-	25	11	-11	0 20 40 60 80 100 120
75	45	30	+	30	12	12	Test Statistic
60	95	-35	-	35	13	-13	P-value = 0.12
50	95	-45	-	45	14	-14	
100	50	50	+	50	15	15	

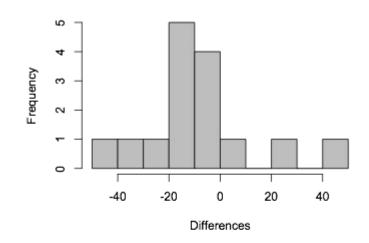
H₀: distribution of paired differences is symmetric about zero

- Wilcoxon test p-value = 0.12
- Insufficient evidence to reject the null hypothesis

Conclusion: Insufficient evidence to conclude that there is a difference in general health between the two time points

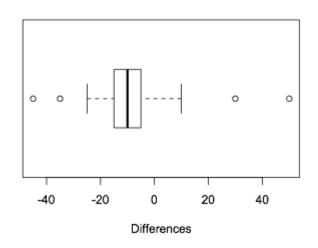
Note:

Validity of assumptions may affect results.



H_o: medians of the two samples are the same

 ${\rm H_0}$: distribution of paired differences is symmetric about zero



Advantages and Limitations

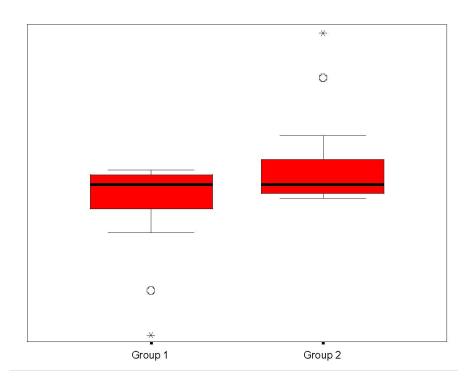
- Easy to apply
- Powerful
 - Utilises more information than the Sign test (but less than the paired t-test)
- Sometimes misinterpreted
 - Assumes symmetry of difference scores about the true median difference

When to use which test

NO OF SAMPLES		NOMINAL	ORDINAL OR NON- NORMAL	NORMALLY DISTRIBUTED
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TWO	INDEPENDENT	χ²-test (r x c , Fisher's exact test	Mann-Whitney U Median test	Unpaired t-test
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- Also called the Wilcoxon Rank Sum test
- Assumptions:
 - Two independent groups
 - At least ordinal dependent variable
 - Randomly selected observations
 - Population distributions same shape
- Hypotheses:
 - $-H_{n}$: populations have the same median
 - $-H_0$: populations have the same spread and shape

Misunderstood test



Statistics	Group 1	Group 2	
Minimum	9.03	0.40	
Median	9.94	9.94	
Maximum	19.48	10.85	
Mann-Whitney U	U=303, p=0.03		

Method:

- Pool the whole sample
- Rank observations from smallest to largest (assign average rank to ties)
- Calculate sum of ranks for each group
- Calculate U test statistic
- Compare U to critical value in the tables

- Coronary artery surgery study (Fisher's book)
- Exercise times in seconds for 2 groups:
 - Control and 3-Vessel Disease group
- Is there a difference in exercise times between the two groups?

H₀: distributions of both populations are equal

Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

Data: 1014 684 810 990 840 978 1002 1110 864 636 638 708 786 600 1320 750 594 750

Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

Data: 1014 684 810 990 840 978 1002 1110 864 636 638 708 786 600 1320 750 594 750

Sorted: 594 600 636 638 684 708 750 750 786 810 840 864 978 990 1002 1014 1110 1320

Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

Data: 1014 684 810 990 840 978 1002 1110 864 636 638 708 786 600 1320 750 594 750

Sorted: 594 600 636 638 684 708 750 750 786 810 840 864 978 990 1002 1014 1110 1320

Ranks: 1 2 3 4 5 6 7.5 7.5 9 10 11 12 13 14 15 16 17 18

Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

Data: 1014 684 810 990 840 978 1002 1110 864 636 638 708 786 600 1320 750 594 750

Sorted: 594 600 636 638 684 708 750 750 786 810 840 864 978 990 1002 1014 1110 1320

Ranks: 1 2 3 4 5 6 7.5 7.5 9 10 11 12 13 14 15 16 17 18

Rank-sums: Test statistics:

Control: 101 U₁: 65

3-Vessel: 70 U₂: 15

Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

Data: 1014 684 810 990 840 978 1002 1110 864 636 638 708 786 600 1320 750 594 750

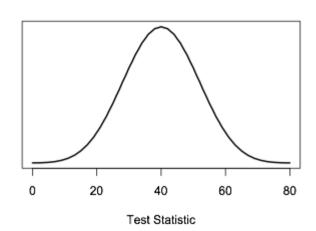
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Ranks: 1 2 3 4 5 6 7.5 7.5 9 10 11 12 13 14 15 16 17 18

Rank-sums: Test statistics:

Control: 101 U_1 : 65

3-Vessel: 70 U₂: 15



Control: 1014 684 810 990 840 978 1002 1110

3-Vessel: 864 636 638 708 786 600 1320 750 594 750

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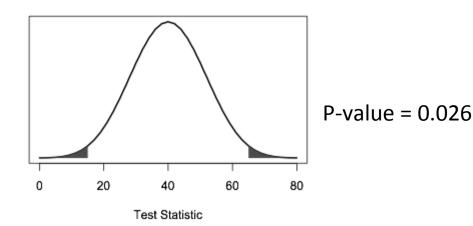
Sorted: 594 600 636 638 684 708 750 750 786 810 840 864 978 990 1002 1014 1110 1320

Ranks: 1 2 3 4 5 6 7.5 7.5 9 10 11 12 13 14 15 16 17 18

Rank-sums: Test statistics:

Control: 101 U₁: 65

3-Vessel: 70 U₂: 15



Advantages and limitations

- Almost as powerful as t-test
 - almost as likely as t-test to reject H₀ if false

Sensitive to central tendencies of scores

- Often misinterpreted:
 - Difference in medians if same shape distributions
 - Otherwise tests for a combination of differences
 between the distributions, including spread and shape

Summary: One Sample

One-sample t-test:

Compares mean to a proposed value, providing the data can be assumed to be Normally distributed.

One-sample Sign test:

Compares the median to a proposed value.

Summary: Two Independent Samples

Two-sample t-test:

Compares means, providing the data can be assumed to be Normally distributed.

Mann-Whitney U test (Wilcoxon Rank Sum test):

Compares medians in two independent groups, without assuming Normality. However, does assume similarity of distributions. Otherwise, compares the shape and spread of the two groups, leading to potential misinterpretation of results.

Summary: Paired Groups

Paired t-test:

Compares means, providing paired differences can be assumed to be Normally distributed.

Wilcoxon Signed Rank test:

Compares means, providing the distribution of differences is symmetric.

Two-sample Sign test:

Compares the medians between matched pairs.