

Functions, Parameters, and Fitting for Teaching Calculus

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Date: Tuesday, January 8th, 2013; 9:00am - 4:00pm

Location: **Marriott Marquis San Diego** in the **Carlsbad/Cardiff Rooms**: South Tower — Level 3

Description

This one-day workshop is designed to help instructors move toward teaching introductory calculus in a modeling- and data-oriented way. The parameters of basic functions are often treated casually in introductory calculus, as if they were a nuisance or merely fodder for the chain rule. In the first half of the workshop, we'll show how to interpret them physically, how to relate them to data, and how to use them for model building. The second half emphasizes polynomial approximation, the relationship between Taylor series and model fitting, and model building using information about derivatives. A small, general purpose mathematical toolkit suffices to build and interpret compelling models in a very wide range of settings spanning economics, biology, physics, etc. We'll provide in-class activities, assignments, student project topics, and example exam questions. Both paper-and-pencil as well as computer-oriented approaches (using R, Sage, or Mathematica) will be featured, drawing on the materials developed through the NSF-supported Project MOSAIC, which aims to construct strong connections in teaching modeling, calculus, statistics, and computation. Participants are encouraged to bring a laptop. If you don't currently use software for teaching calculus, we'll set you up with the free R system running through a web browser.

Workshop Leaders:

- Danny Kaplan, Macalester College
- Randall Pruim, Calvin College

Support:

- A grant to **Project MOSAIC** from the US National Science Foundation, NSF DUE-0920350

- **CAUSE**: The Consortium for the Advancement of Undergraduate Statistics Education
- MAA: The Mathematical Association of America

Handouts:

- **R/mosaic Cheat Sheet**

Other Resources:

- Ben Baumer's activity on fitting power laws using **Tukey's Power Rule and the Ladder of Powers**

Schedule and Outline

9am: Functions and Parameters **Introduction**

Introductory activity (15 min)

Interactive: Technology: Accessing the web server and RStudio (15 min)

- Introduction to enough R to let them load and plot data
 - `fetchData()`
 - `plotPoints()`

Compiling the list of functions and parameterizations (15 min) At this point, just enter the forms, without much discussion.

ACTIVITY: Data and eyeballing parameters. (20 min)

- Hand out a document with these graphed out. **TO DO**
- Load in the data files and find the parameters. Data files illustrate several of the function types:
 - Exponentials
 - Income-housing for exponential. **exercise**
 - `stan-data.csv` for exponential. **Is cooling exponential?**
 - Trigonometric
 - tides for sine wave **from PREP2012**

- Power Law
- atomic bomb radius versus time
- S-curves
- gaussian and its cumulative as an S-curve
- logistic function
- **Hill function**
- Gaussian bump
 - * Ben Baumer's shortstop example
- Quadratic
 - * Kepler's orbit

Some Statistics

- The criteria for `fitModel`
- Sampling and resampling distributions
- The standard error

Document: **How to get a quick estimate of sampling variation**

Noon Lunch at the Marina Kitchen in the Marriott. Free to participants.

Summary Create and display a document summarizing the findings.

- A template document on **Calculus Functions and their Parameters**

10:15 break

10:30 Theme Fitting

Calculus Operations ACTIVITY

- Write a function that returns a function that is a constant, linear, or quadratic approximation to a given function at any point.
- (Maybe for PREP 2013). Find the curvature of a function at any point, and plot out the associated circle along the graph. (A nice trigonometric exercise.)
 - Add a `plotPath()` function to mosaic: `plotPath(c(x=cos(t),y=sin(t))~`

Unknown Function Forms

- Flow of orange juice via spline **from PREP 2012**
- Traffic light capacity (integrating a hill function). **from PREP 2012**
- **Maximum heart rate versus age.** Get the data. **Wikipedia article**

Manifest versus latent function forms:

- Income-Housing (manifest form)
- Space-Shuttle

Lecture: Motivation for some functional forms (15 min) Approximation

- Linearization
 - Hooke's Law
 - Newton's Law of cooling
 - Bi-linear: Idea gas law, law of mass action
- Polynomials, Taylor series and least squares
- Sine waves and orthogonality: our emphasis on polynomials pre-dates the development of wave- and oscillation-based technology.

Activity:

- `mTalyor` applet.

Task: Use this to create a lesson plan on the quality of an approximation.

Lecture: Power laws and where they come from: (15 min)

- Units & Dimensional analysis
 - Analysis of atomic bomb
- Allometrics
 - Relationships in the internal combustion engine
 - Species-Area data "<http://www.math.smith.edu/~>

2:15 break

2:30 Functions in two variables

- The Ideal Gas Law and the Law of Mass Action **PREP 2012**
- **Cobb-Douglas**
- The landscape functions: **mountain**, valley, **hillside**, amphitheater

The modeling polynomial: 2nd order in two variables

- **picking model terms**
- Fitting and interpreting
- Used car prices

Starting partial derivatives

- **Celebrity Divorce**: is it a reasonable model?
- **Eyeballing interactions**

Optimization

- **Health policy example**

4:00 Adjourn

Lunch Menu

Choice of:

- Grilled Flat Iron Steak Salad with Butter Lettuce, Arugula, Roasted Tomato, Beef Bacon and Balsamic Blue Cheese Dressing
- Chicken Mozzarella Sandwich served Warm with Arugula Sun Dried Tomato Tapenade - \$15.00 per person
- Grilled Portobello Mushroom Sandwich with Zucchini and Goat Cheese - \$15.00 per person

Desert: Mini Slice of Daily New York Style Cheesecake - \$4.00 per person

Coffee, tea, iced tea and soft drinks included with meal.