# Model Implied Instrumental Variable (MIIV) Methods using MIIVsem: An R Package for Structural Equation Models (SEMs)

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#### System Wide Maximum Likelihood (ML)

- Pure, ideal ML estimator properties
  - Consistent
  - Asymptotic unbiased
  - Asymptotic efficient
  - Asymptotic normality
  - Asymptotic standard errors

#### System Wide Maximum Likelihood (ML)

- If your models are perfectly specified & your observed variables are from normal distributions, then
  - Go home, no need to attend workshop

#### System Wide Maximum Likelihood (ML)

- If your models are approximations & your observed variables are from nonnormal distributions, then
  - You've come to right place
  - Approximations undermine ML properties
  - Bias & inconsistent estimator likely
  - Efficiency & accurate standard errors no longer guaranteed

#### Other Issues with ML

- Underidentified models
  - Can prevent estimation & testing even if key equations in system are identified
- Nonconvergence
  - Prevent estimates from being obtained
  - Increasing iterations often does not help

#### Other Issues with ML

- Spread bias from bad parts of model to good parts even in well specified equations
- Global tests of fit often significant
  - Not always easy to find source of problem
  - Bad measurement model?
  - Bad latent variable model?
  - Both?

#### What do we need?

- 1. Estimator less likely to spread structural specification errors throughout system
- 2. Local estimates of equations
- 3. Local tests of equations
- 4. Ability to estimate identified equations, even if whole model not identified
- 5. Ideally a "distribution free" estimator
- 6. Noniterative without convergence problems

### Model Implied Instrumental Variables (MIIVs) addresses these:

- 1. MIIV-2SLS less likely to spread structural specification errors throughout system
- 2. Local estimates of equations
- 3. Local tests of equations
- 4. Ability to estimate identified equations, even if whole model not identified
- 5. A "distribution free" estimator
- 6. Noniterative without convergence problems

#### **Purposes**

- 1. Give an overview of MIIV-2SLS
- Show you how to download the MIIVsem R package
- 3. Present and illustrate the primary steps in using MIIVsem
- 4. Teach you how to use & interpret MIIVsem input & output
- 5. Provide empirical examples that are estimated and tested with MIIVsem

#### Installing MIIVsem

#### First steps

- Install MIIVsem Version 0.5.2
- Install lavaan
  - Will use for simulating data and estimating models later
- Type the following commands into the R console

```
install.packages("MIIVsem")
install.packages("lavaan")
```

#### Loading MIIVsem

Next, load MIIVsem

Type the following command:

```
library("MIIVsem")
```

Will load lavaan later when needed

#### Inputting Data File

First dataset is Bollen (1989)

- Comes with MIIVsem
- Accessed by typing bollen1989a in R console

To match the examples in the workshop we'll save the bollen1989a as data and rename the variables.

#### Inputting Data File

```
# save the political democracy
# dataset as data.
data <- bollen1989a
# rename the variables.
colnames(data) <- c("Z4", "Z5", "Z6",
                     "Z7","Z8","Z9",
                     "Z10", "Z11", "Z1",
                     "Z2", "Z3" )
```

- 1. Specify Model
- 2. Transform Latent to Observed (L2O) variable model
- 3. Find Model Implied Instrumental Variables (MIIVs, pronounced to rhyme with "gives")
- 4. Estimate with Two Stage Least Squares (2SLS)
- 5. Test each overidentified equation

#### 1. Specify Model

 Researcher lays out the latent variable and measurement models

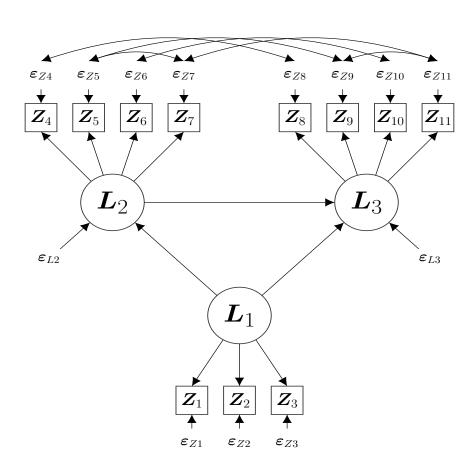
# Industrialization and Political Democracy Example

 $L_1$  = Industrialization at time 1

 $L_2$  = Political Democracy at time 1

 $L_3$  = Political Democracy at time 2

 $Z_1$  to  $Z_{11}$  are indicators of  $L_1$  to  $L_3$ 



# Industrialization and Political Democracy Example

#### **Latent Variable Model**

$$L_{1} = \varepsilon_{L_{1}}$$

$$L_{2} = \alpha_{L_{2}} + B_{21}L_{1} + \varepsilon_{L_{2}}$$

$$L_{3} = \alpha_{L_{3}} + B_{31}L_{1} + B_{32}L_{2} + \varepsilon_{L_{3}}$$

#### **Measurement Model**

$$\begin{split} Z_1 &= L_1 + \varepsilon_{z1} & Z_4 = L_2 + \varepsilon_{z4} & Z_8 = L_3 + \varepsilon_{z8} \\ Z_2 &= \alpha_{z2} + \Lambda_{21} L_1 + \varepsilon_{z2} & Z_5 = \alpha_{z5} + \Lambda_{52} L_2 + \varepsilon_{z5} & Z_9 = \alpha_{z9} + \Lambda_{93} L_3 + \varepsilon_{z9} \\ Z_3 &= \alpha_{z3} + \Lambda_{31} L_1 + \varepsilon_{z3} & Z_6 = \alpha_{z6} + \Lambda_{62} L_2 + \varepsilon_{z6} & Z_{10} = \alpha_{z10} + \Lambda_{10,3} L_3 + \varepsilon_{z10} \\ Z_7 &= \alpha_{z7} + \Lambda_{72} L_2 + \varepsilon_{z7} & Z_{11} = \alpha_{z11} + \Lambda_{11,3} L_3 + \varepsilon_{z11} \end{split}$$

#### MIIVsem: Model Syntax

Main ingredient of MIIVsem:

- Model
- To define, for search or estimation, MIIVsem uses lavaan (Rosseel, 2012) syntax

Major operators to define relationships in model:

- $=\sim$  "measured by" e.g., L1 = $\sim$  Z1
  - ~ "regressed on" e.g., L5 ~ L4
- ~~ "covaries with" e.g., L2~~L3
  - \* Assigns equality or numerical constraints

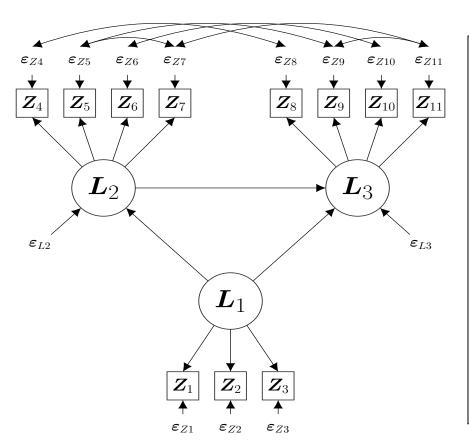
e.g., 
$$L4 \sim b*L3 + b*L2$$
 OR  $L1 = \sim 1*Z1 + .5*Z2 + Z3$ 

#### MIIVsem: Industrialization-Democracy Example

In **R**, <- is used for assignment. Below we specify the industrialization-democracy model using our model syntax operators, surrounding the equations in single quotes. This creates an object in our workspace named model.indem1

```
model.indem1 <- '</pre>
  L1 = ~Z1 + Z2 + Z3
  L2 = ~ Z4 + Z5 + Z6 + Z7
  L3 = ~Z8 + Z9 + Z10 + Z11
  L2 ~ L1
  L3 \sim L1 + L2
  74 ~~ Z8
  z_5 \sim z_7 + z_9
  Z6 ~~ Z10
  Z7 ~~ Z11
  79 ~~ 711 '
```

#### **EXERCISE**: In R, specify the model below.

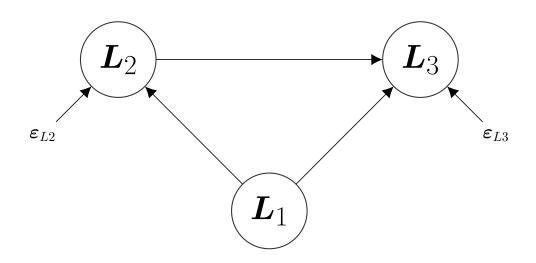


```
model.indem1 <- '</pre>
  L1 = ~Z1 + Z2 + Z3
  L2 = ~24 + 25 + 26 + 27
  L3 = ~Z8 + Z9 + Z10 + Z11
  L2 ~ L1
  L3 \sim L1 + L2
  Z4 ~~ Z8
  z_5 \sim z_7 + z_9
     ~~ Z10
  Z9 ~~ Z11
```

- Specify Model ✓
- 2. Transform Latent to Observed (L2O) variable model
- 3. Find Model Implied Instrumental Variables (MIIVs, pronounced to rhyme with "gives")
- 4. Estimate with Two Stage Least Squares (2SLS)
- 5. Test each overidentified equation

- Specify Model ✓
- 2. Transform Latent to Observed (L2O) variable model (Bollen, 1996)
  - MIIVsem does this automatically
  - Here we illustrate how MIIVsem does it

2. Transform Latent to Observed (L2O) variable model (Bollen, 1996)



## Transform Latent to Observed (L2O) variable model

$$L_{1} = \varepsilon_{L_{1}}$$

$$L_{2} = \alpha_{L_{2}} + B_{21}L_{1} + \varepsilon_{L_{2}}$$

$$L_{3} = \alpha_{L_{3}} + B_{31}L_{1} + B_{32}L_{2} + \varepsilon_{L_{3}}$$

$$Z_{1} = L_{1} + \varepsilon_{z1}$$

$$L_{1} = Z_{1} - \varepsilon_{z1}$$

$$Z_{4} = L_{2} + \varepsilon_{z4}$$

$$L_{2} = Z_{4} - \varepsilon_{z4}$$

$$L_{3} = Z_{8} - \varepsilon_{z8}$$

$$L_{3} = Z_{8} - \varepsilon_{z8}$$

## Transform Latent to Observed (L2O) variable model

Substitute scaling indicator minus error for each latent variable:

$$L_{2} = \alpha_{L2} + B_{21}L_{1} + \varepsilon_{L2} \implies$$

$$Z_{4} = \alpha_{L2} + B_{21}Z_{1} + u_{4} \text{ with } u_{4} = -B_{21}\varepsilon_{Z1} + \varepsilon_{Z4} + \varepsilon_{L2}$$

$$L_{3} = \alpha_{L3} + B_{31}L_{1} + B_{32}L_{2} + \varepsilon_{L3} \implies$$

$$Z_{8} = \alpha_{L3} + B_{31}Z_{1} + B_{32}Z_{4} + u_{8} \text{ with } u_{8} = -B_{31}\varepsilon_{Z1} - B_{32}\varepsilon_{Z4} + \varepsilon_{Z8} + \varepsilon_{L3}$$

Latent variable equations are transformed into observed variable equations with composite errors.

# Industrialization and Political Democracy Example

#### **Latent Variable Model**

$$L_{1} = \varepsilon_{L_{1}}$$

$$L_{2} = \alpha_{L_{2}} + B_{21}L_{1} + \varepsilon_{L_{2}}$$

$$L_{3} = \alpha_{L_{3}} + B_{31}L_{1} + B_{32}L_{2} + \varepsilon_{L_{3}}$$

#### **Measurement Model**

$$\begin{split} Z_1 &= L_1 + \varepsilon_{z1} & Z_4 = L_2 + \varepsilon_{z4} & Z_8 = L_3 + \varepsilon_{z8} \\ Z_2 &= \alpha_{z2} + \Lambda_{21} L_1 + \varepsilon_{z2} & Z_5 = \alpha_{z5} + \Lambda_{52} L_2 + \varepsilon_{z5} & Z_9 = \alpha_{z9} + \Lambda_{93} L_3 + \varepsilon_{z9} \\ Z_3 &= \alpha_{z3} + \Lambda_{31} L_1 + \varepsilon_{z3} & Z_6 = \alpha_{z6} + \Lambda_{62} L_2 + \varepsilon_{z6} & Z_{10} = \alpha_{z10} + \Lambda_{10,3} L_3 + \varepsilon_{z10} \\ Z_7 &= \alpha_{z7} + \Lambda_{72} L_2 + \varepsilon_{z7} & Z_{11} = \alpha_{z11} + \Lambda_{11,3} L_3 + \varepsilon_{z11} \end{split}$$

#### **EXERCISE**: Do the L2O transformation for $Z_5$ .

## Transform Latent to Observed (L2O) variable model Measurement Model

$$\begin{split} Z_1 &= L_1 + \varepsilon_{z1} & Z_4 = L_2 + \varepsilon_{z4} & Z_8 = L_3 + \varepsilon_{z8} \\ Z_2 &= \alpha_{z2} + \Lambda_{21} L_1 + \varepsilon_{z2} & Z_5 = \alpha_{z5} + \Lambda_{52} L_2 + \varepsilon_{z5} & Z_9 = \alpha_{z9} + \Lambda_{93} L_3 + \varepsilon_{z9} \\ Z_3 &= \alpha_{z3} + \Lambda_{31} L_1 + \varepsilon_{z3} & Z_6 = \alpha_{z6} + \Lambda_{62} L_2 + \varepsilon_{z6} & Z_{10} = \alpha_{z10} + \Lambda_{10,3} L_3 + \varepsilon_{z10} \\ Z_7 &= \alpha_{z7} + \Lambda_{72} L_2 + \varepsilon_{z7} & Z_{11} = \alpha_{z11} + \Lambda_{11,3} L_3 + \varepsilon_{z11} \end{split}$$

Find L2O Transformation for:  $Z_5 = \alpha_{z5} + \Lambda_{52}L_2 + \varepsilon_{z5}$ 

**SOLUTION**: Do the L2O transformation for  $Z_5$ .

Transform Latent to Observed (L2O) variable model

$$\begin{split} Z_4 &= L_2 + \varepsilon_{z4} \Rightarrow L_2 = Z_4 - \varepsilon_{z4} \\ \hline Z_5 &= \alpha_{z5} + \Lambda_{52} L_2 + \varepsilon_{z5} \\ \Rightarrow Z_5 &= \alpha_{z5} + \Lambda_{52} (Z_4 - \varepsilon_{z4}) + \varepsilon_{z5} \\ \hline Z_5 &= \alpha_{z5} + \Lambda_{52} Z_4 - \Lambda_{52} \varepsilon_{z4} + \varepsilon_{z5} = \alpha_{z5} + \Lambda_{52} Z_4 + u_{z5} \end{split}$$

#### **SOLUTION**: Do the L2O transformation for $Z_5$ .

## Transform Latent to Observed (L2O) variable model

$$Z_5 = \alpha_{z5} + \Lambda_{52} Z_4 + u_{z5}$$

Looks like simple regression with  $Z_5$  dependent variable and  $Z_4$  explanatory variable.

Why not use OLS?

Look at error,

$$Z_5 = \alpha_{z5} + \Lambda_{52} Z_4 - \Lambda_{52} \varepsilon_{z4} + \varepsilon_{z5}$$
$$C(Z_4, \varepsilon_{z4}) \neq 0$$

 $\Rightarrow$  *OLS* biased & inconsistent

## Transform Latent to Observed (L2O) variable model

Same problem for previous latent variable L2O transformation:

$$L_{2} = \alpha_{L2} + B_{21}L_{1} + \varepsilon_{L2} \implies$$

$$Z_{4} = \alpha_{L2} + B_{21}Z_{1} + u_{4} \text{ with } u_{4} = -B_{21}\varepsilon_{Z1} + \varepsilon_{Z4} + \varepsilon_{L2}$$

$$L_{3} = \alpha_{L3} + B_{31}L_{1} + B_{32}L_{2} + \varepsilon_{L3} \implies$$

$$Z_{8} = \alpha_{L3} + B_{31}Z_{1} + B_{32}Z_{4} + u_{8} \text{ with } u_{8} = -B_{31}\varepsilon_{Z1} - B_{32}\varepsilon_{Z4} + \varepsilon_{Z8} + \varepsilon_{L3}$$

## Transform Latent to Observed (L2O) variable model

$$Z_4 = \alpha_{L2} + B_{21}Z_1 + u_4 \text{ with } u_4 = -B_{21}\varepsilon_{Z1} + \varepsilon_{Z4} + \varepsilon_{L2}$$

$$Z_8 = \alpha_{L3} + B_{31}Z_1 + B_{32}Z_4 + u_8 \text{ with } u_8 = -B_{31}\varepsilon_{Z1} - B_{32}\varepsilon_{Z4} + \varepsilon_{Z8} + \varepsilon_{L3}$$

Problem: error correlates with Right Hand Side (RHS) Zs, OLS biased Instrumental variables can help.

- 1. Correlate with RHS Zs
- 2. Not correlate with composite errors
- 3. At least as many instruments as RHS Zs

Finding suitable instruments is the next step in MIIV-2SLS.

- Specify Model ✓
- Transform Latent to Observed (L2O) variable model ✓
- 3. Find Model Implied Instrumental Variables (MIIVs, pronounced to rhyme with "gives")
- 4. Estimate with Two Stage Least Squares (2SLS)
- 5. Tests each overidentified equation

- 3. Find Model Implied Instrumental Variables (MIIVs)
  - Instrumental variables help obtain consistent & asymptotic unbiased estimates of coefficients
    - Useful when equation error correlates with explanatory variables
      - Addresses situation where OLS does not work
  - Researchers typically search for instrumental variable external to model
  - My approach: MIIVs

## 3. Find Model Implied Instrumental Variables (MIIVs)

- Key property of instruments is that they are uncorrelated with equation error
- MIIV approach proposed in Bollen (1996) finds instruments among observed variables already part of model
  - If identified model, then MIIVs are generally part of model
  - No need to search outside of model
  - Structure of model implies which observed variables are uncorrelated with equation disturbance

## 3. Find Model Implied Instrumental Variables (MIIVs)

General algorithm to find MIIVs (Bollen, 1996)

- 1. Focus on single equation
- Find direct & indirect effects on the observed variables of each error in the composite error,
- 3. Eliminate the observed variables found in 2.,
- 4. Find the direct & indirect effects of any errors correlated with the composite error,
- 5. Eliminate the observed variables found in 4.,
- 6. Remaining observed variables are MIIVs.

- 3. Find Model Implied Instrumental Variables (MIIVs)
  - MIIVsem finds MIIVs automatically
    - R: MIIVsem (Fisher, Bollen, Gates & Rönkkö)
  - Useful to illustrate process with examples

# 3. Find Model Implied Instrumental Variables (MIIVs)

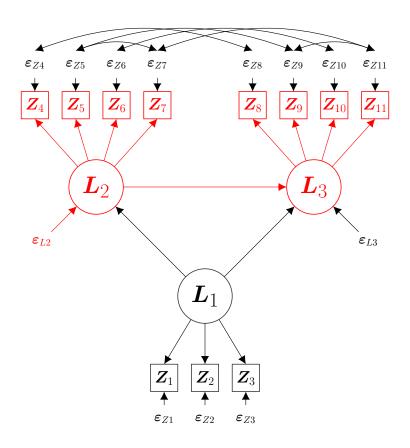
Consider first latent variable equation, latent political democracy  $(L_2)$  regressed on latent industrialization  $(L_1)$ :

$$L_2 = \alpha_{L2} + B_{21}L_1 + \varepsilon_{L2} \implies$$

$$Z_4 = \alpha_{L2} + B_{21}Z_1 + u_4$$
 with  $u_4 = -B_{21}\varepsilon_{Z1} + \varepsilon_{Z4} + \varepsilon_{L2}$ 

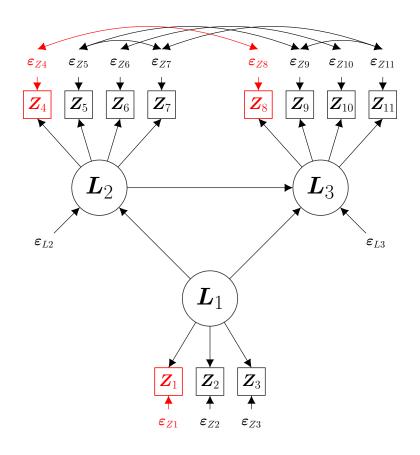
1. Find direct & indirect effects on observed variables of  $\varepsilon_{Z1}$ ,  $\varepsilon_{Z4}$ ,  $\varepsilon_{L2}$ . Let's start with  $\varepsilon_{L2}$  and return to path diagram of model.

Find direct & indirect effects of  $\varepsilon_{L2}$ 



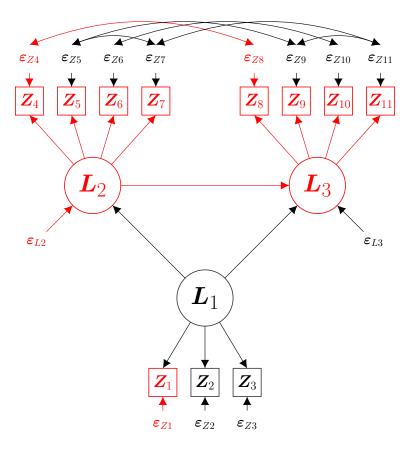
Only variables NOT eliminated by  $\varepsilon_{L2}$  are  $Z_1$ ,  $Z_2$ ,  $Z_3$ .

Find direct & indirect effects of  $\varepsilon_{z_1}, \varepsilon_{z_4}$ 



Eliminates  $Z_1$ ,  $Z_4$ , and  $Z_8$  as MIIVs.

Find direct & indirect effects of  $\varepsilon_{z_1}, \varepsilon_{z_4}, \varepsilon_{L_2}$ 



 $Z_2$ ,  $Z_3$  only MIIVs.

# **EXERCISE**: Find the MIIVs for the $Z_5$ equation.

# 3. Find Model Implied Instrumental Variables (MIIVs)

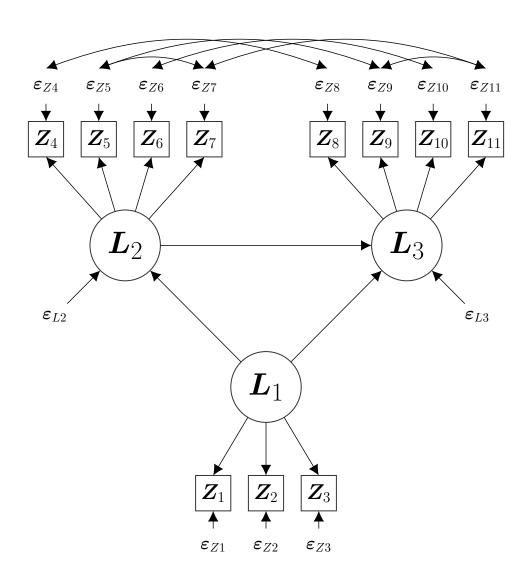
Return to previous L2O transformation

$$\begin{split} Z_4 &= L_2 + \varepsilon_{z4} \Longrightarrow L_2 = Z_4 - \varepsilon_{z4} \\ \hline Z_5 &= \alpha_{z5} + \Lambda_{52} L_2 + \varepsilon_{z5} \\ \Longrightarrow Z_5 &= \alpha_{z5} + \Lambda_{52} Z_4 - \Lambda_{52} \varepsilon_{z4} + \varepsilon_{z5} \end{split}$$

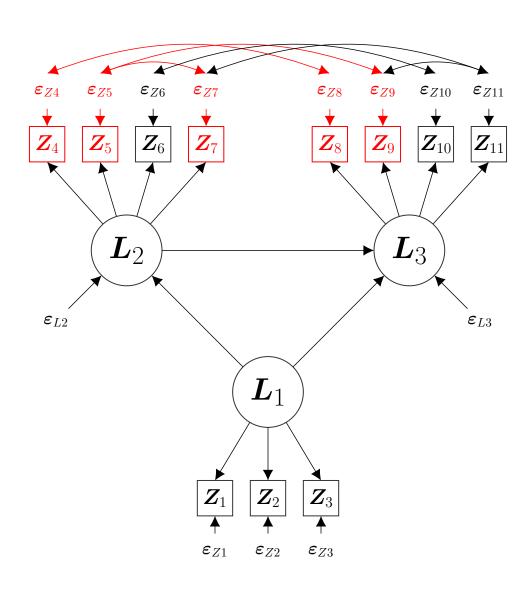
Find any variable directly or indirectly influenced by  $\varepsilon_{z4}$  or  $\varepsilon_{z5}$  and eliminate as MIIV.

Find any errors that correlate with  $\varepsilon_{z4}$  or  $\varepsilon_{z5}$  Eliminate as MIIVs any variable these influence.

# **EXERCISE**: Find the MIIVs for the $Z_5$ equation.



# **SOLUTION**: Find the MIIVs for the $Z_5$ equation.



**SOLUTION**: Find the MIIVs for the  $Z_5$  equation.

# Find Model Implied Instrumental Variables (MIIVs)

Return to previous L2O transformation

$$Z_5 = \alpha_{z5} + \Lambda_{52}Z_4 - \Lambda_{52}\varepsilon_{z4} + \varepsilon_{z5}$$

MIIVs are:

$$Z_1$$
 to  $Z_3$ ,  $Z_6$ ,  $Z_{10}$ ,  $Z_{11}$ 

The miivs(), or MIIV <u>search</u>, function in MIIVsem automatically performs the MIIV search.

The only argument needed to run miivs() is model.

```
model.indem1 <- '
       L1 = ~Z1 + Z2 + Z3
       L2 = ~24 + 25 + 26 + 27
       L3 = ~Z8 + Z9 + Z10 + Z11
       L2 ~ L1
       L3 \sim L1 + L2
       Z4 ~~ Z8
       Z5 \sim Z7 + Z9
       Z6 ~~ Z10
       Z7 ~~ Z11
       Z9 ~~ Z11 '
miivs(model.indem1)
```

#### **EXERCISE**: Use MIIVsem for the MIIV search.

The miivs(), or MIIV <u>search</u>, function in MIIVsem automatically performs the MIIV search.

```
model.indem1 <- '</pre>
   L1 = ~Z1 + Z2 + Z3
   L2 = ~24 + 25 + 26 + 27
   L3 = ~Z8 + Z9 + Z10 + Z11
   L2 ~ L1
   L3 \sim L1 + L2
   74 ~~ 78
   25 \sim 27 + 29
   Z6 ~~ Z10
   77 ~~ 711
   79 ~~ 711 '
miivs(model.indem1)
```

The only argument needed to run miivs() is model.

Using the earlier defined model for industrialization-democracy run the MIIV search and check your output to see if it matches the output on the following slide.

#### Output:

```
## Model Equation Information
##
## LHS RHS MIIVs
## Z2 Z1
             Z4, Z5, Z6, Z7, Z9, Z3, Z8, Z10, Z11
## Z3 Z1
             Z4, Z5, Z6, Z7, Z9, Z2, Z8, Z10, Z11
## Z5 Z4
             Z6, Z1, Z2, Z3, Z10, Z11
## Z6 Z4 Z5, Z7, Z9, Z1, Z2, Z3, Z11
## Z7 Z4 Z6, Z9, Z1, Z2, Z3, Z10
## Z9 Z8
             Z6, Z7, Z1, Z2, Z3, Z10
## Z10 Z8
             Z5, Z7, Z9, Z1, Z2, Z3, Z11
## Z11 Z8 Z5, Z6, Z1, Z2, Z3, Z10
## Z4 Z1 Z2, Z3
## Z8 Z1, Z4 Z5, Z6, Z7, Z2, Z3
```

Note: The summary method for miivs objects provides additional options for displaying the MIIV search information.

- Specify Model ✓
- Transform Latent to Observed (L2O) variable model ✓
- Find Model Implied Instrumental Variables (MIIVs) ✓
- 4. Estimate with Two Stage Least Squares (2SLS)
- 5. Tests each overidentified equation

## 4. Estimate with Two Stage Least Squares (2SLS)

In general,

 $\mathbf{Y}_{i}$  = vector containing values of *j*th dependent variable for L2O equation

 $\mathbf{Z}_{i}$  = matrix of explanatory variables on RHS of same *j*th L2O equation

 $V_i$  = matrix of MIIVs for same jth L2O equation

2SLS estimator of coefficients is  $(\hat{\mathbf{Z}}_{j}'\hat{\mathbf{Z}}_{j})^{-1}\hat{\mathbf{Z}}_{j}'\mathbf{Y}_{j}$ 

where 
$$\hat{\mathbf{Z}}_{j} = \mathbf{V}_{j} (\mathbf{V}_{j}' \mathbf{V}_{j})^{-1} \mathbf{V}' \mathbf{Z}_{j}$$

**Noniterative** 

No issues with convergence

## 4. Estimate with Two Stage Least Squares (2SLS)

Consider first latent variable equation, latent political democracy  $(L_2)$  regressed on latent industrialization  $(L_1)$ :

$$L_2 = \alpha_{L2} + B_{21}L_1 + \varepsilon_{L2} \implies \boxed{Z_4 = \alpha_{L2} + B_{21}Z_1 + u_4}$$
 MIIVs are:  $Z_2, Z_3$ 

$$\mathbf{Y}_{j} = \begin{bmatrix} Z_{41} \\ Z_{42} \\ \vdots \\ Z_{4N} \end{bmatrix} \qquad \mathbf{Z}_{j} = \begin{bmatrix} 1 & Z_{11} \\ 1 & Z_{12} \\ \vdots & \vdots \\ 1 & Z_{1N} \end{bmatrix} \qquad \mathbf{V}_{j} = \begin{bmatrix} 1 & Z_{21} & Z_{31} \\ 1 & Z_{22} & Z_{32} \\ \vdots & \vdots & \vdots \\ 1 & Z_{2N} & Z_{3N} \end{bmatrix}$$

2SLS estimator of coefficients is  $(\hat{\mathbf{Z}}_{j}'\hat{\mathbf{Z}}_{j})^{-1}\hat{\mathbf{Z}}_{j}'\mathbf{Y}_{j}$ 

where 
$$\hat{\mathbf{Z}}_j = \mathbf{V}_j \left( \mathbf{V}_j' \mathbf{V}_j \right)^{-1} \mathbf{V}' \mathbf{Z}_j$$

## 4. Estimate with Two Stage Least Squares (2SLS)

Comparison	MIIV-2SLS	ML
Consistency	✓	✓
Asymp. unbiased	✓	$\checkmark$
Asymp. normal	✓	✓
Asymp. efficient	✓*	$\checkmark$
Asymp. s.e.	✓	$\checkmark$
Noniterative	✓	-
Nonnormal robust	✓	_**
No SEM software needed	✓	-
Overidentification test	equation	model

<sup>\*2</sup>SLS efficient among limited information estimators.

<sup>\*\*</sup>Corrected significance tests available.

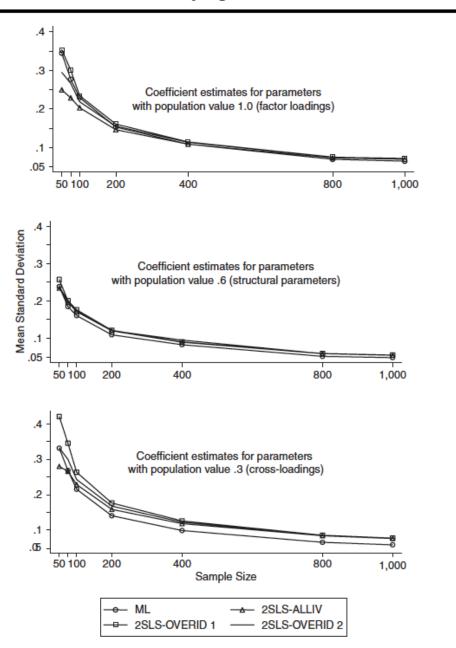
## 4. Estimate with Two Stage Least Squares (2SLS)

Illustration of ML and MIIV-2SLS simulation Bollen, Kirby, Curran, Paxton, & Chen (2007)

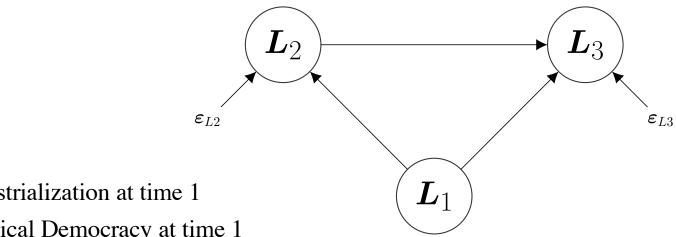
Graph on next page gives standard deviation of parameters under ideal conditions for ML:

Normality
Correct specification

#### Mean Standard Deviation of Estimates From Four Estimators by Sample Size for Parameter Estimates From Specification 1, the Correctly Specified Model



- 4. Estimate with Two Stage Least Squares (2SLS)
  - Return to latent variable model for example



 $L_1$  = Industrialization at time 1

 $L_2$  = Political Democracy at time 1

 $L_3$  = Political Democracy at time 2

#### miive()

- MIIV <u>e</u>stimation function in MIIVsem
- Use miive() to estimate the industrialization-democracy example using MIIV-2SLS.
- "data" is name assigned to the industrializationdemocracy example
- Next slide recaps all the steps for estimation for industrialization-democracy example.
  - Already loaded MIIVsem, renamed the data and specified the model,
  - Only step left is to run miive.

#### **EXERCISE**: Estimate the Industrialization-Democracy model.

```
model.indem1 <- '
      L1 = ~Z1 + Z2 + Z3
      L2 = ~Z4 + Z5 + Z6 + Z7
      T_{1}3 = ~78 + 79 + 710 + 711
      L2 ~ L1
       L3 \sim L1 + L2
       Z4 ~~ Z8
       Z5 \sim Z7 + Z9
       Z6 ~~ Z10
       Z7 ~~ Z11
       79 ~~ 711 '
data <- bollen1989a
colnames(data) <- c("Z4", "Z5", "Z6", "Z7",
                     "Z8", "Z9", "Z10", "Z11",
                     "Z1", "Z2", "Z3")
miive(model.indem1, data)
```

# MIIVsem: Header for estimation output.

**Number of observations**: the number of observations used for all equations in the system.

**Number of equations**: the total number of L2O equations estimated.

**Estimator**: the estimator used, either MIIV-2SLS or PIV.

# MIIVsem: Header for estimation output.

**Standard errors**: the method used to compute standard errors, this will be discussed again when we discuss the bootstrap.

**Missing**: listwise is the default. In our dataset there was no missing data, this can be verified by looking at the number of observations.

	Parameter 1	Estimates:						
##								
##	STRUCTURAL	COEFFICIENTS:						
##		Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
##	L1 =~							
##	$z_1$	1.000						
##	<b>Z</b> 2	2.078	0.128	16.171	0.000	8.301	8	0.405
##	<b>Z</b> 3	1.751	0.149	11.782	0.000	8.738	8	0.365
##	L2 =~							
##	<b>Z</b> 4	1.000						
##	<b>Z</b> 5	1.139	0.179	6.371	0.000	8.409	5	0.135
##	<b>Z</b> 6	0.969	0.140	6.924	0.000	5.874	6	0.437
##	<b>z</b> 7	1.210	0.139	8.713	0.000	4.276	5	0.510
##	L3 =~							
##	<b>Z</b> 8	1.000						
##	<b>Z</b> 9	1.051	0.165	6.377	0.000	8.712	5	0.121
##	Z10	1.180	0.151	7.814	0.000	9.538	6	0.146
##	Z11	1.203	0.154	7.798	0.000	2.795	5	0.731
##								
##	L2 ~							
##	L1	1.261	0.426	2.962	0.003	0.503	1	0.478
##	L3 ~							
##	L1	1.123	0.312	3.598	0.000	0.801	3	0.849
##	L2	0.724	0.101	7.140	0.000			

##	INTERCEPTS:					
##		Estimate	Std.Err	z-value	P(> z )	
##	L2	-0.909	2.170	-0.419	0.675	
##	L3	-4.499	1.424	-3.160	0.002	
##	<b>Z</b> 1	0.000				
##	<b>Z10</b>	0.135	0.830	0.163	0.870	
##	Z11	-2.137	0.853	-2.505	0.012	
##	<b>Z</b> 2	-5.711	0.654	-8.727	0.000	
##	<b>Z</b> 3	-5.292	0.758	-6.985	0.000	
##	Z 4	0.000				
##	<b>Z</b> 5	-1.969	1.044	-1.886	0.059	
##	<b>Z</b> 6	1.265	0.814	1.553	0.120	
##	<b>z</b> 7	-2.160	0.814	-2.654	0.008	
##	<b>Z</b> 8	0.000				
##	<b>Z</b> 9	-2.418	0.909	-2.659	0.008	

- Specify Model ✓
- Transform Latent to Observed (L2O) variable model ✓
- Find Model Implied Instrumental Variables (MIIVs) ✓
- 4. Estimate w/ Two Stage Least Squares (2SLS) ✓
- 5. Tests each overidentified equation

### 5. Tests each overidentified equation

$$\frac{\hat{\mathbf{u}}\mathbf{V}(\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'\hat{\mathbf{u}}}{\hat{\mathbf{u}}'\hat{\mathbf{u}}/\mathbf{N}} \stackrel{a}{\sim} \chi^2$$

where

 $\hat{\mathbf{u}} = 2SLS$  residuals

V = MIIVs

N = sample size

df = # MIIVs - # endogenous regressors

### 5. Tests each overidentified equation

#### Sargan Test:

H<sub>0</sub>: MIIVs uncorrelated with equation error

H<sub>a</sub>: At least 1 MIIV correlates with error

Reject H<sub>0</sub> is evidence against model because model led to MIIVs.

##	Parameter 1	Estimates:						
##								
##	STRUCTURAL	COEFFICIENTS:						
##		Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
##	L1 =~							
##	<b>Z</b> 1	1.000						
##	<b>Z</b> 2	2.078	0.128	16.171	0.000	8.301	8	0.405
##	<b>Z</b> 3	1.751	0.149	11.782	0.000	8.738	8	0.365
##	L2 =~							
##	$z_4$	1.000						
##	<b>Z</b> 5	1.139	0.179	6.371	0.000	8.409	5	0.135
##	<b>Z</b> 6	0.969	0.140	6.924	0.000	5.874	6	0.437
##	<b>z</b> 7	1.210	0.139	8.713	0.000	4.276	5	0.510
##	L3 =~							
##	<b>Z</b> 8	1.000						
##	<b>Z</b> 9	1.051	0.165	6.377	0.000	8.712	5	0.121
##	<b>Z10</b>	1.180	0.151	7.814	0.000	9.538	6	0.146
##	<b>Z11</b>	1.203	0.154	7.798	0.000	2.795	5	0.731
##								
##	L2 ~							
##	L1	1.261	0.426	2.962	0.003	0.503	1	0.478
##	L3 ~							
##	L1	1.123	0.312	3.598	0.000	0.801	3	0.849
##	L2	0.724	0.101	7.140	0.000			

#### **EXERCISE**: Adjust Sargan Test for multiple comparisons.

#### Multiple testing problem

- sarg.adjust argument of the miive()
  - p-value adjustment method for the Sargan test.

#### Reestimate the industrialization-democracy example

- Use Holm ("holm") correction for multiple comparisons
- Compare p-values to unadjusted counterparts

```
miive(model.indem1, data, sarg.adjust = "holm")
```

	Parameter 1	Estimates:						
##								
##	STRUCTURAL	COEFFICIENTS:						
##		Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
##	L1 =~							
##	<b>Z</b> 1	1.000						
##	<b>Z</b> 2	2.078	0.128	16.171	0.000	8.301	8	1.000
##	<b>Z</b> 3	1.751	0.149	11.782	0.000	8.738	8	1.000
##	L2 =~							
##	$z_4$	1.000						
##	<b>Z</b> 5	1.139	0.179	6.371	0.000	8.409	5	1.000
##	<b>Z</b> 6	0.969	0.140	6.924	0.000	5.874	6	1.000
##	<b>z</b> 7	1.210	0.139	8.713	0.000	4.276	5	1.000
##	L3 =~							
##	<b>Z</b> 8	1.000						
##	<b>Z</b> 9	1.051	0.165	6.377	0.000	8.712	5	1.000
##	Z10	1.180	0.151	7.814	0.000	9.538	6	1.000
##	Z11	1.203	0.154	7.798	0.000	2.795	5	1.000
##								
##	L2 ~							
##	L1	1.261	0.426	2.962	0.003	0.503	1	1.000
##	L3 ~							
##	L1	1.123	0.312	3.598	0.000	0.801	3	1.000
##	L2	0.724	0.101	7.140	0.000			

- 1. Specify Model
- 2. Transform Latent to Observed (L2O) variable model
- 3. Find Model Implied Instrumental Variables (MIIVs, pronounced to rhyme with "gives")
- 4. Estimate with Two Stage Least Squares (2SLS)
- 5. Test each overidentified equation

# ROBUSTNESS

#### 1. Distributional robustness

- Properties of MIIV-2SLS are "distribution-free"
- Asymptotic, but do not assume normal error or observed variables
- Bootstrap option in MIIVsem permits alternative way to estimate standard errors of parameter estimates

# MIIVsem: Bootstrap SEs and Cls.

MIIVsem bootstrap standard errors

- Requested by setting se = "boot"
- Default number of replications is 1000
- Adjusted using the bootstrap argument (e.g. bootstrap = 500)

Reestimate the industrialization-democracy model with bootstrap standard errors

• Also set the boot.ci argument to "perc" which requests a confidence interval from the empirical 2.5<sup>th</sup> and 97.5<sup>th</sup> percentiles of the bootstrap sample.

```
miive(model.indem1, data, se = "boot", boot.ci = "perc")
```

# MIIVsem: Bootstrap SEs and Cls.

#### Header information

- Shows boostrap standard errors requested
- Lists number of replications requested
- Includes number of successful replications
- Documents confidence interval method

MIIVsem (0.5.2) results	
Number of observations	75
Number of equations	10
Estimator	MIIV-2SLS
Standard Errors	bootstrap
Missing	listwise
Bootstrap reps requested	1000
Bootstrap reps successful	1000
Bootstrap intervals	Percentile

**Note**: Additional options for constructing bootstrap CIs are available. See the boot.ci argument of miive for more choices.

# MIIVsem: Bootstrap SEs and Cls.

STRUCTURAL C	OEFFICIENTS:							
	Estimate	Std.Err	Lower	Upper	Sargan	df	P(Chi)	
L1 =~								
<b>Z</b> 1	1.000							
Z2	2.078	0.127	1.787	2.300	8.301	8	0.405	
<b>Z</b> 3	1.751	0.132	1.461	1.972	8.738	8	0.365	
L2 =~								
Z 4	1.000							
<b>Z</b> 5	1.139	0.134	0.850	1.373	8.409	5	0.135	
<b>Z</b> 6	0.969	0.137	0.696	1.237	5.874	6	0.437	
<b>Z</b> 7	1.210	0.122	0.950	1.434	4.276	5	0.510	
L3 =~								
Z8	1.000							
<b>Z</b> 9	1.051	0.142	0.711	1.303	8.712	5	0.121	
Z10	1.180	0.148	0.858	1.452	9.538	6	0.146	
Z11	1.203	0.148	0.891	1.484	2.795	5	0.731	
L2 ~								
L1	1.261	0.422	0.406	2.059	0.503	1	0.478	
L3 ~								
L1	1.123	0.297	0.524	1.745	0.801	3	0.849	
L2	0.724	0.093	0.540	0.904				

# **EXERCISE**: Bootstrap SEs and Cls.

Reestimate the political democracy example with bootstrap standard errors. Try setting the number of replications using bootstrap to a different number and compare the results to those obtained from 1000 bootstrap replications.

For example, below we request 500 bootstrap replications.

```
miive(model.indem1, data, se = "boot",
  boot.ci = "perc", bootstrap = 500)
```

## **EXERCISE**: Bootstrap SEs and Cls.

Below is code for combining estimates from four fitted models for easy comparison.

```
# Fit Models
fit.standard <- miive(model.indem1, data)</pre>
fit.boot.250 <- miive(model.indem1, data, se = "boot", bootstrap = 250)</pre>
fit.boot.500 <- miive(model.indem1, data, se = "boot", bootstrap = 500)</pre>
fit.boot.1000 <- miive(model.indem1, data, se = "boot", bootstrap = 1000)
# Save Estimates
est.standard <- estimatesTable(fit.standard)[, c("lhs", "op", "rhs", "se")]
est.boot.250 <- estimatesTable(fit.boot.250)[, c("lhs", "op", "rhs", "se")]</pre>
est.boot.500 <- estimatesTable(fit.boot.500)[, c("lhs", "op", "rhs", "se")]</pre>
est.boot.1000 <- estimatesTable(fit.boot.1000)[,c("lhs", "op", "rhs", "se")]
            <- list(est.standard,est.boot.250,est.boot.500, est.boot.1000)
compare.se <- Reduce(function(...) merge(..., by= c("lhs", "op", "rhs")), list.est)</pre>
colnames(compare.se) <- c("","","", "standard", "boot.250", "boot.500", "boot.1000")</pre>
compare.se
```

# **SOLUTION**: Bootstrap SEs and CIs.

Comparison of standard errors.

```
boot, 250
                                boot.500
                                         boot.1000
          standard
         0.0000000 0.00000000 0.00000000
                                        0.0000000
L1
      Z2 0.1284989 0.13149889 0.12511317
                                        0.13265177
      Z3 0.1486077 0.13369027 0.12816840
                                        0.12758051
T.1
T_12 = \sim
      T_12 = \sim
         0.1788162 0.13971253 0.13728272 0.13531398
T_1 = -
         0.1400277 0.12746333 0.13526307
                                        0.13095981
         0.1388712 0.11816898 0.11031515
T_12 = \sim
                                        0.11557476
T<sub>1</sub>2
         0.4257016 0.40698842 0.41878643
                                        0.40293661
T<sub>1</sub>3 =~
     710 0.1510235 0.15270889 0.14275581 0.14965067
         0.1542894 0.15037637 0.13776855 0.15206394
L3
T_{1}3 = \sim
         0.1647411 0.14737877 0.14925931
T.3
                                        0.13995522
L3
         0.3121788 0.29706550 0.29238986 0.31008589
L3
      L2 0.1014416 0.09298976 0.09572984 0.09957574
```

- 2. Structural misspecification robustness
- omitted paths
- omitted variables
- wrong number of dimensions

Bollen (2001): Suppose that for the  $j^{th}$  equation in the correctly specified model, the model implied IVs are in a matrix  $\mathbf{V}_{j}$ . The 2SLS estimator of the coefficients is robust for any misspecification in other equations under two conditions:

- 1. The equation being estimated is correctly specified
- 2. The misspecifications in the other equations do not alter the variables in  $\mathbf{V}_{\mathrm{i}}$

- 2. Structural misspecification robustness
- omitted paths
- omitted variables
- wrong number of dimensions

After demonstrating the use of lavaan's simulateData() function we'll explore the consequences of structural misspecifications on the MIIV-2SLS estimates.

First, we'll walk through one full example together.

Measurement Model Robustness to Latent Variable Model Misspecification

#### **True Model:**

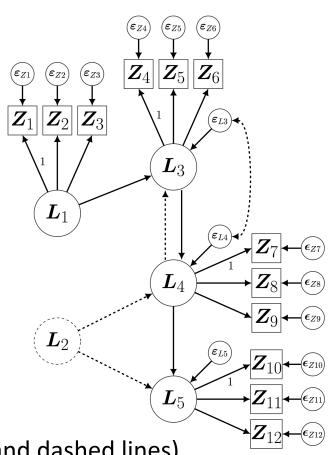
Solid and dashed lines

#### Misspecifications

- Omitted paths, omitted latent variables
- Omitted covariances of errors

#### Simulation

- Simulated data to true model
- 1,000 observations
- 3 indicators per latent variable
- Estimate measurement model 2x
  - 1. Latent variable model is correct (solid and dashed lines)
  - 2. Latent variable model is incorrect(omitting dashed lines)



Use lavaan to simulate data

simulateData function from lavaan.

Need to load lavaan

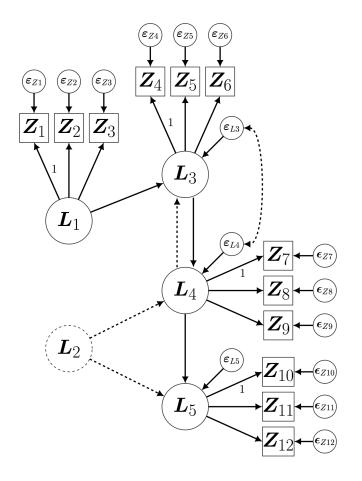
```
library("lavaan")
```

Specify the model from the previous slide

- Include population parameters
- Name this model: model.sim.1.

Here we specify the data generating model.

**Note**: Var(error) & Var(exogenous) =1 by default.



```
library("lavaan")
model.sim.1 <- '</pre>
  L1
      =\sim 1*Z1 + 1*Z2 + 1*Z3
  L2 = ~1*Z13 + 1*Z14 + 1*Z15
 L3 = ~1*Z4 + 1*Z5 + 1*Z6
  L4 = ~1*Z7 + 1*Z8 + 1*Z9
  L5 = ~1*Z10 + 1*Z11 + 1*Z12
  L3 \sim .3*L1 + .5*L4
 L4 \sim .5*L2 + .8*L3
 L5 \sim .3*L2 + .5*L4
  L3 ~~ .6*L4
```

#### Now use lavaan's simulateData function

- Set the model argument to model.sim.1
- Fix the number of observations to 1,000
- Choose random.seed of 123
  - Allows us to replicate our results exactly
- Save our dataset as data.sim.1

Below we recap the commands used to simulate our data set.

```
library("lavaan")
model.sim.1 <- '
  L1 = ~1*Z1 + 1*Z2 + 1*Z3
  L2 = ~1*Z13 + 1*Z14 + 1*Z15
  L3 = ~1*Z4 + 1*Z5 + 1*Z6
  L4 = ~1*Z7 + 1*Z8 + 1*Z9
  L5 = ~1*Z10 + 1*Z11 + 1*Z12
  L3 \sim .3*L1 + .5*L4
  L4 \sim .5*L2 + .8*L3
  L5 \sim .3*L2 + .5*L4
 L3 ~~ .6*L4
data.sim.1 <- simulateData(model = model.sim.1,</pre>
                            sample.nobs = 1000,
                            seed = 123)
```

Lastly, we need to define two estimating models:

- 1. model.correct.1
- 2. model.misspecified.1

model.correct.1 corresponds to the correct model (both solid and dashed lines).

```
model.correct.1 <- '

L1 =~ Z1 + Z2 + Z3

L2 =~ Z13 + Z14 + Z15

L3 =~ Z4 + Z5 + Z6

L4 =~ Z7 + Z8 + Z9

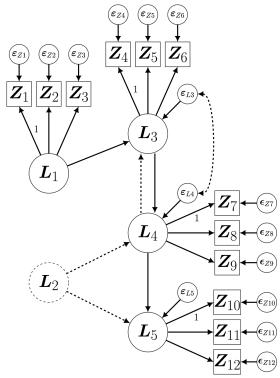
L5 =~ Z10 + Z11 + Z12

L3 ~ L1 + L4

L4 ~ L2 + L3

L5 ~ L2 + L4

L3 ~~ L4
```



Lastly, we need to define two estimating models:

- 1. model.correct.1
- 2. model.misspecified.1

model.misspecified.1 corresponds to the incorrect model (solid lines only).

```
model.misspecified.1 <- '

L1 =~ Z1 + Z2 + Z3

L2 =~ Z13 + Z14 + Z15

L3 =~ Z4 + Z5 + Z6

L4 =~ Z7 + Z8 + Z9

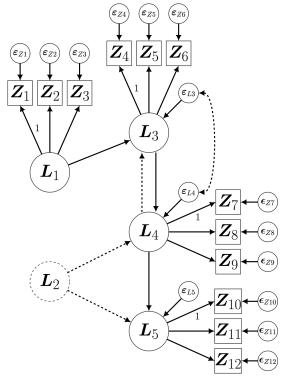
L5 =~ Z10 + Z11 + Z12

L3 ~ L1

L4 ~ L3

L5 ~ L4

'
```



At this point we should have defined both the correct and misspecified models.

```
model.correct.1 <- '

L1 =~ Z1 + Z2 + Z3

L2 =~ Z13 + Z14 + Z15

L3 =~ Z4 + Z5 + Z6

L4 =~ Z7 + Z8 + Z9

L5 =~ Z10 + Z11 + Z12

L3 ~ L1 + L4

L4 ~ L2 + L3

L5 ~ L2 + L4

L3 ~~ L4
```

```
model.misspecified.1 <- '

L1 =~ Z1 + Z2 + Z3

L2 =~ Z13 + Z14 + Z15

L3 =~ Z4 + Z5 + Z6

L4 =~ Z7 + Z8 + Z9

L5 =~ Z10 + Z11 + Z12

L3 ~ L1

L4 ~ L3

L5 ~ L4

'
```

Estimate correct & incorrect models in MIIVsem.

The estimatesTable() function

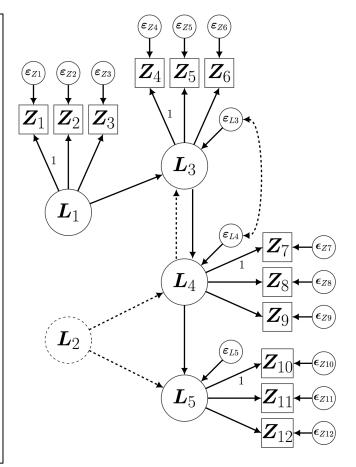
- Convenient way to store, view and manipulate the estimated parameters returned by miive()
- Save the fitted correct and misspecified models as fit.cor.1, and fit.mis.1, respectively.

```
fit.cor.1 <- miive(model.correct.1, data.sim.1)
fit.mis.1 <- miive(model.misspecified.1, data.sim.1)
estimatesTable(fit.cor.1)
estimatesTable(fit.mis.1)</pre>
```

# Simulation 1 Results

# Measurement Model Robustness to Latent Variable Model Misspecification

```
##
                 est.cor.1 est.mis.1
##
                 1.0000000 1.0000000
##
                 1.0873919
                            1.0873919
##
                 1.0302098 1.0302098
##
                 1.0000000
                            1,0000000
             713
##
                 1.0076301
                            1.0076301
##
                 0.9691372 0.9691372
             7.15
##
                 1.0000000
                            1.0000000
      T<sub>1</sub>3
##
                 0.9933688 0.9933688
##
                 0.9791256 0.9791256
                 1.0000000
                            1.0000000
                 1.0159817
                            1.0159817
              Z8
                 1.0228551
                            1.0228551
              Z9
                 1.0000000
             Z10
                            1.0000000
                 1.0019276
             7.11
                            1.0019276
             Z12 0.9667204 0.9667204
```



## **EXERCISE**: Simulation 2

Previously we examined the impact of a misspecified latent variable model on the measurement model estimates. We now consider another situation, the impact of a misspecified measurement model on the latent variable model estimates.

#### For this exercise you will:

- 1. Use lavaan's simulateData function to simulate 1,000 observations. In the data generating model set the regression coefficients and factor loadings to a value of 1, and the uniqueness covariances to a value of 0.3.
- 2. Fit the correct and misspecified models using MIIVsem.
- 3. Compare the estimates.

# **EXERCISE**: Simulation 2

**Latent Variable Model Robustness to Measurement** 

**Model Misspecification** 

#### True Model:

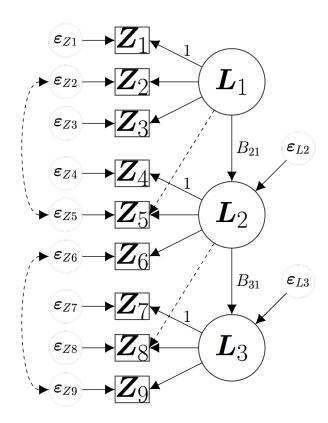
Solid and dashed lines

#### Misspecifications

- Omitted correlated errors of indicators
- Omitted cross-loadings

#### Simulation

- Simulated data to true model
- 1,000 observations
- 3 indicators per latent variable
- Estimate latent variable model 2x
  - 1. Measurement model is correct (solid and dashed lines)
  - 2. Measurement model is incorrect (omitting dashed lines)

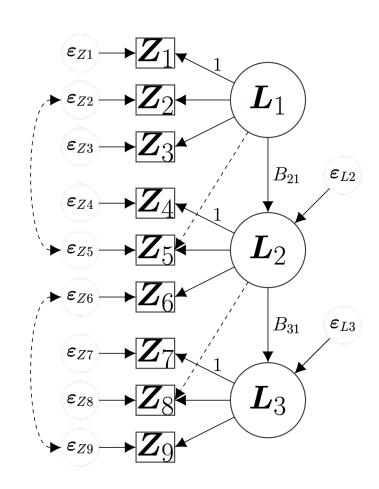


To simulate data for our example we will use the simulateData function from lavaan.

```
library("lavaan")
model.sim.2 <- '</pre>
  L1 = ~1*Z1 + 1*Z2 + 1*Z3 + 1*Z5
 L2 = ~1*Z4 + 1*Z5 + 1*Z6 + 1*Z8
  L3 = ~1*Z7 + 1*Z8 + 1*Z9
 L2 ~ 1*L1
 L3 ~ 1*L2
  72 ~~ .3*75
  Z6 ~~ .3*Z9'
data.sim.2 <- simulateData(model = model.sim.2,</pre>
                           sample.nobs = 1000,
                           seed = 123)
```

# **EXERCISE**: Simulation 2

- Now that we have simulated the data we'll have to specify the models.
- The correct model includes the solid and dashed lines. The misspecified model contains the dashed lines only.



Now, we can specify the correct (solid and dashed lines) and misspecified (solid lines only) models.

```
model.correct.2 <- '

L1 =~ Z1 + Z2 + Z3 + Z5

L2 =~ Z4 + Z5 + Z6 + Z8

L3 =~ Z7 + Z8 + Z9

L2 ~ L1

L3 ~ L2

Z2 ~~ Z5

Z6 ~~ Z9
'
```

```
model.misspecified.2 <- '

L1 =~ Z1 + Z2 + Z3

L2 =~ Z4 + Z5 + Z6

L3 =~ Z7 + Z8 + Z9

L2 ~ L1

L3 ~ L2

,
```

### Correctly Specified Model (Unadjusted Sargan Test)

STRUCTURAL	COEFFICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
L1 =~							
<b>Z</b> 1	1.000						
Z2	0.927	0.046	20.026	0.000	13.909	5	0.016
<b>Z</b> 3	0.928	0.044	20.871	0.000	4.656	6	0.589
<b>Z</b> 5	0.766	0.120	6.402	0.000	3.959	3	0.266
L2 =~							
Z 4	1.000						
<b>Z</b> 5	1.117	0.084	13.339	0.000	3.959	3	0.266
<b>Z</b> 6	0.969	0.034	28.863	0.000	9.882	5	0.079
<b>Z</b> 8	0.892	0.092	9.660	0.000	5.790	4	0.215
L3 =~							
<b>z</b> 7	1.000						
<b>Z</b> 8	1.125	0.083	13.533	0.000	5.790	4	0.215
Z 9	1.028	0.031	33.274	0.000	6.722	5	0.242
L2 ~							
L1	0.933	0.055	16.861	0.000	0.059	1	0.808
L3 ~	0.755	0.033	10.001	0.000	0.033	_	0.000
L2	0.922	0.041	22.242	0.000	8.924	4	0.063

#### **Correctly Specified Model** (Adjusted Sargan Test)

L1 =~  Z1	STRUCTURAL C	OEFFICIENTS:						
Table   Tabl		Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
Z2       0.927       0.046       20.026       0.000       13.909       5       0.13         Z3       0.928       0.044       20.871       0.000       4.656       6       1.00         Z5       0.766       0.120       6.402       0.000       3.959       3       1.00         L2 =~       Z4       1.000       1.000       3.959       3       1.00         Z5       1.117       0.084       13.339       0.000       3.959       3       1.00         Z6       0.969       0.034       28.863       0.000       9.882       5       0.47         Z8       0.892       0.092       9.660       0.000       5.790       4       1.00         Z8       1.125       0.083       13.533       0.000       5.790       4       1.00         Z9       1.028       0.031       33.274       0.000       6.722       5       1.00         L2 ~       L1       0.933       0.055       16.861       0.000       0.059       1       1.00	L1 =~							
Z3	<b>Z</b> 1	1.000						
Z5	Z2	0.927	0.046	20.026	0.000	13.909	5	0.130
L2 =~  Z4	<b>Z</b> 3	0.928	0.044	20.871	0.000	4.656	6	1.000
Z4       1.000         Z5       1.117       0.084       13.339       0.000       3.959       3       1.00         Z6       0.969       0.034       28.863       0.000       9.882       5       0.47         Z8       0.892       0.092       9.660       0.000       5.790       4       1.00         L3 =~       27       1.000       28       1.125       0.083       13.533       0.000       5.790       4       1.00         Z9       1.028       0.031       33.274       0.000       6.722       5       1.00         L2 ~       L1       0.933       0.055       16.861       0.000       0.059       1       1.00	<b>Z</b> 5	0.766	0.120	6.402	0.000	3.959	3	1.000
Z5	L2 =~							
Z6	Z 4	1.000						
Z8	<b>Z</b> 5	1.117	0.084	13.339	0.000	3.959	3	1.000
L3 =~  Z7	<b>Z</b> 6	0.969	0.034	28.863	0.000	9.882	5	0.472
Z7	Z8	0.892	0.092	9.660	0.000	5.790	4	1.000
Z8	L3 =~							
Z9 1.028 0.031 33.274 0.000 6.722 5 1.00  L2 ~  L1 0.933 0.055 16.861 0.000 0.059 1 1.00	<b>z</b> 7	1.000						
L2 ~ L1 0.933 0.055 16.861 0.000 0.059 1 1.00	Z8	1.125	0.083	13.533	0.000	5.790	4	1.000
L1 0.933 0.055 16.861 0.000 0.059 1 1.00	<b>Z</b> 9	1.028	0.031	33.274	0.000	6.722	5	1.000
L1 0.933 0.055 16.861 0.000 0.059 1 1.00								
	L2 ~							
L3 ~	L1	0.933	0.055	16.861	0.000	0.059	1	1.000
	L3 ~							
L2 0.922 0.041 22.242 0.000 8.924 4 0.44	L2	0.922	0.041	22.242	0.000	8.924	4	0.441

#### Misspecified Model (Unadjusted Sargan Test)

STRUCTURAL	COEFFICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
L1 =~							
<b>Z1</b>	1.000						
Z2	0.981	0.044	22.149	0.000	22.579	6	0.001
<b>Z</b> 3	0.928	0.044	20.871	0.000	4.656	6	0.589
L2 =~							
Z 4	1.000						
<b>Z</b> 5	1.602	0.047	33.958	0.000	104.349	6	0.000
<b>Z</b> 6	0.973	0.034	28.952	0.000	35.800	6	0.000
L3 =~							
<b>Z</b> 7	1.000						
Z8	1.849	0.050	37.256	0.000	64.561	6	0.000
<b>Z</b> 9	1.026	0.031	33.240	0.000	44.087	6	0.000
L2 ~							
L1	0.933	0.055	16.861	0.000	0.059	1	0.808
L3 ~							
L2	0.922	0.041	22.242	0.000	8.924	4	0.063

#### Misspecified Model (Adjusted Sargan Test)

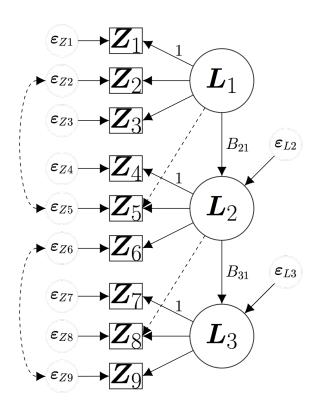
STRUCTURAL	COEFFICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
L1 =~							
<b>Z1</b>	1.000						
Z2	0.981	0.044	22.149	0.000	22.579	6	0.004
Z3	0.928	0.044	20.871	0.000	4.656	6	1.000
L2 =~							
Z 4	1.000						
<b>Z</b> 5	1.602	0.047	33.958	0.000	104.349	6	0.000
Z 6	0.973	0.034	28.952	0.000	35.800	6	0.000
L3 =~							
<b>Z</b> 7	1.000						
Z8	1.849	0.050	37.256	0.000	64.561	6	0.000
Z 9	1.026	0.031	33.240	0.000	44.087	6	0.000
L2 ~							
L1	0.933	0.055	16.861	0.000	0.059	1	1.000
L3 ~							
L2	0.922	0.041	22.242	0.000	8.924	4	0.189

Below is code for combining estimates from more than one fitted models for easy comparison .

```
# Save estimated models
fit.cor.2 <- miive(model.correct.2, data.sim.2)</pre>
fit.mis.2 <- miive(model.misspecified.2, data.sim.2)</pre>
# Save parameter tables
est.cor.2 <- estimatesTable(fit.cor.2)</pre>
est.mis.2 <- estimatesTable(fit.mis.2)</pre>
# Merge tables
compare.2 <- merge(</pre>
   est.cor.2[est.cor.2$op == "~", c("lhs", "op", "rhs", "est")],
   est.mis.2[est.mis.2$op == "~", c("lhs", "op", "rhs", "est")],
   by = c("lhs", "op", "rhs")
colnames(compare.2) <- c("","","","est.cor.2","est.mis.2")</pre>
compare.2
```

# Measurement Model Robustness to Latent Variable Model Misspecification

```
## est.cor.2 est.mis.2
## 1 L2 ~ L1 0.9332420 0.9332420
## 2 L3 ~ L2 0.9222349 0.9222349
```



## **EXERCISE**: Simulation 3

We now consider a final simulated data example. Here we make a different measurement model misspecification and ask you to answer the following three questions:

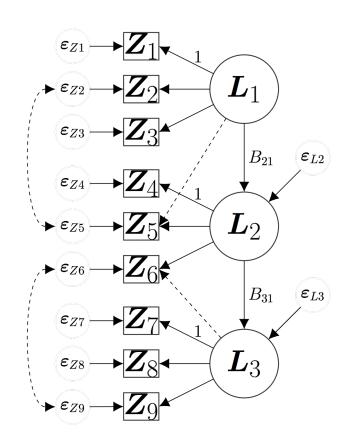
- 1. Do the robustness properties discussed earlier hold here?
- 2. If not, what is the reason for the change?
- 3. Is there any way of detecting this misspecification in practice?

Path diagram and questions presented on next slide.

# **EXERCISE**: Simulation 3

#### **Robustness Exercise**

- Simulated 1,000 observations according to true model (solid and dashed lines).
- Estimate two models:
  - 1. Model 1: True model
  - 2. Model 2: Misspecified model where the dashed paths are omitted.



#### **Questions**

- 1. Do the robustness properties discussed earlier hold here?
- 2. If not, what is the reason for the change?
- 3. Is there any way of detecting this misspecification in practice?

#### Correctly Specified Model (Adjusted Sargan)

STRUCTURAL	COEFFICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
L1 =~							
<b>Z</b> 1	1.000						
Z2	0.954	0.047	20.215	0.000	7.952	5	0.953
<b>Z</b> 3	0.921	0.044	20.718	0.000	3.205	6	1.000
<b>Z</b> 5	0.784	0.138	5.673	0.000	3.211	3	1.000
L2 =~							
Z 4	1.000						
<b>Z</b> 5	1.094	0.094	11.599	0.000	3.211	3	1.000
<b>Z</b> 6	0.930	0.091	10.179	0.000	2.893	3	1.000
L3 =~							
<b>Z</b> 7	1.000						
Z 6	1.001	0.078	12.757	0.000	2.893	3	1.000
Z8	0.960	0.027	35.206	0.000	8.561	6	0.999
<b>Z</b> 9	0.978	0.029	33.990	0.000	10.685	5	0.464
L2 ~							
L1	1.006	0.058	17.382	0.000	0.434	1	1.000
L3 ~							
L2	0.930	0.042	22.001	0.000	7.257	3	0.464

#### Misspecified Model (Adjusted Sargan)

STRUCTURAL	COEFFICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
L1 =~							
<b>Z</b> 1	1.000						
Z2	1.015	0.045	22.721	0.000	17.476	6	0.031
Z3	0.921	0.044	20.718	0.000	3.205	6	1.000
L2 =~							
Z 4	1.000						
<b>Z</b> 5	1.613	0.049	32.645	0.000	84.966	6	0.000
Z 6	2.034	0.061	33.073	0.000	113.670	6	0.000
L3 =~							
Z 7	1.000						
Z8	0.960	0.027	35.206	0.000	8.561	6	0.599
Z 9	1.012	0.028	36.543	0.000	22.987	6	0.004
L2 ~							
L1	1.006	0.058	17.382	0.000	0.434	1	1.000
L3 ~							
L2	1.073	0.042	25.418	0.000	109.497	4	0.000

- 2. Structural misspecification robustness
  - MIIV-2SLS is robust because the MIIVs are the same for all models
  - MIIV-2SLS depends on identification of equation, not identification of whole model
  - MIIV-2SLS is NOT robust to all structural misspecifications
    - E.g., the measurement model estimates are not robust to the different models illustrated.

In the next set of slides he show how MIIV-2SLS can be used to investigate dimensionality.

In doing so we demonstrate two features in MIIVsem:

- 1. How to add constraints to parameters in the lavaan style model syntax.
- 2. How to obtain Wald tests of these restrictions.

The data are included in MIIVsem (bollen1996) and come from a survey conducted in rural clusters of Tanzania to collect information on the perceived accessibility of a specific family planning facility that serviced each cluster (Bollen, 1996).

#### **Background Information:**

Six informants were chosen from each cluster: 3 female and 3 male. New informants were chosen for each cluster. Each informant was independently asked to rate the accessibility of the facility. More specifically the women informants were asked to rate how women of childbearing age perceived the accessibility of the clinic and the men informants were asked to rate how accessible men perceived the clinic to be.

The female informants' ratings are access1-3 and the male informants' ratings are access4-6.

#### Model 1

We will begin by fitting a confirmatory factor analysis model with a single accessibility latent variable for male and female information.

Since the informants change from cluster to cluster, the ordering of the informants is arbitrary and we have no reason to believe the factor loadings would differ in a systematic way.

In Model 1 we will constrain all the factor loadings to equality.

#### **Equality Constraints and Parameter Restrictions**

Identical to lavaan, labels can be used to specify equality constraints on parameters in the model syntax. Labels are prepended to the variable name using the `\*` operator. For numeric constrains one can specify a number instead.

```
model.1 <- '

accessibility =~ 1*access1 + 1*access2 + 1*access3 + 1*access4 + 1*access5 + 1*access6

'
```

By constraining each loading to 1 we constrain all the loadings to equality.

MIIVsem test statistics for constraints

Large-sample Wald test of constraints imposed on coefficient matrix

F and  $\chi^2$  distributed test statistics asymptotically equivalent

- Performance may differ in small samples
- See Greene (2003, pp. 346-347) for details Conduct Wald test by saving the miive object and using the summary() method request restrict.tests=TRUE

```
fit <- miive(model.1, bollen1996, sarg.adjust = "holm")
summary(fit, restrict.tests = TRUE)</pre>
```

```
STRUCTURAL COEFFICIENTS:
                  Estimate Std.Err z-value P(>|z|) Sargan
                                                             df P(Chi)
  accessibility =~
    access1
                    1.000
                                                       3.800
    access2
                    1.000
                                                                     0.568
                                                       1.107
                                                                    0.775
   access3
                    1.000
                                                      71.309
   access4
                    1.000
                                                                    0.000
                    1.000
                                                      82.893
                                                                    0.000
    access5
                                                      72.377
    access6
                    1.000
                                                                     0.000
MIIV-2SLS LINEAR HYPOTHESIS TESTS:
  access2 access1 = 1
  access3 access1 = 1
  access4 access1 = 1
  access5 access1 = 1
   access6 access1 = 1
                                             18,1306
  Wald Test (Chi<sup>2</sup>):
  Degrees of freedom:
  Pr(>Chi^2):
                                              0.0028
  Wald Test (F):
                                              3.6261
  Degrees of freedom:
                                                5, 5
                                              0.0029
  Pr(>F):
```

### **EXERCISE**: Dimensionality 1

#### Model 2

The informants change from cluster to cluster but we have evidence the factor loadings for the female informants and those for the male informants are not equal. Fit a model where females have the same loading, and males loadings are also constrained equal. Note males loadings can differ from females. What has changed across the two model?

### **SOLUTION**: Dimensionality 1

```
STRUCTURAL COEFFICIENTS:
                 Estimate Std.Err z-value P(>|z|) Sargan
                                                            df
                                                                 P(Chi)
  accessibility =~
                   1.000
   access1
                                                     3.800
                                                                  0.568
   access2
                   1.000
                                                     1.107
                   1.000
                                                                  0.775
   access3
                   0.727 0.065 11.249
                                             0.000 82.304
                                                                  0.000
   access4
   access5
                 0.727 0.065 11.249
                                             0.000 95.313
                                                                  0.000
                   0.727 0.065 11.249
                                             0.000 82.175
                                                                  0.000
   access6
MIIV-2SLS LINEAR HYPOTHESIS TESTS:
  access2 access1 = 1
  access3 access1 = 1
  access4 access1 - access5 access1 = 0
  access4 \ access1 - access6 \ access1 = 0
  Wald Test (Chi^2):
                                            0.3103
  Degrees of freedom:
  Pr(>Chi^2):
                                            0.9891
                                            0.0776
  Wald Test (F):
  Degrees of freedom:
                                             4, 4
  Pr(>F):
                                            0.9891
```

### **EXERCISE**: Dimensionality 2

#### Model 3

If women and men differ in their view of accessibility, then a two factor model could be more appropriate. Create a model that has Female Accessibility as the first factor and Male Accessibility as the second factor. Allow the two factors to correlate.

Scale the Female Accessibility latent variable to access 1 and the Male Accessibility latent variable to access 4. What happens to the overidentification tests?

# **SOLUTION**: Dimensionality 2

STRUCTURAL COEF	FICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
Female =~							
access1	1.000						
access2	1.041	0.104	10.039	0.000	3.499	3	0.963
access3	1.033	0.103	10.034	0.000	0.970	3	0.963
Male =~							
access4	1.000						
access5	1.026	0.073	14.039	0.000	6.623	3	0.340
access6	1.005	0.076	13.285	0.000	2.581	3	0.963

# CATEGORICAL ENDOGENOUS VARIABLES

#### Polychoric Instrumental Variable (PIV) Estimator

- Access indicators are ordinal
  - what happens if take account of ordinal nature?
- Polychoric correlations
  - assumes continuous underlying variable
  - ordinal variables are collapsed version
  - estimates of correlation between underlying variables
  - polychoric correlations then analyzed
- MIIVsem permits endogenous ordinal variables analyzed with polychoric correlations
  - Polychoric Instrumental Variable (PIV) estimator
  - See Bollen & Maydeu-Olivares (2007) for details

# CATEGORICAL ENDOGENOUS VARIABLES

Following the convention used in lavaan, we use the ordered argument to indicate which variables in the model syntax are categorical. This will be demonstrated in the following example.

In our previous Accessibility example, responses were recorded on a 1-5 scale. We will reestimate our final two-factor model to demonstrate the PIV estimator.

# CATEGORICAL ENDOGENOUS VARIABLES

#### Specify the model:

```
model <- '
femaleAccess =~ access1 + access2 + access3
maleAccess =~ access4 + access5 + access6
'</pre>
```

#### Declare the ordered categorical variables:

```
ordered <- c("access1", "access2", "access3", "access4", "access5", "access6")
```

#### Fit the model:

```
miive(model, bollen1996, ordered = ordered)
```

MIIVsem (0.5.2)	results						
Number of observ Number of equati Estimator Standard Errors Missing					MI	1	220 4 LS (PIV) standard listwise
Parameter Estima	tes:						
STRUCTURAL COEFF	ICIENTS:						
	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
Female =~							
access1	1.000						
access2	1.039	0.064	16.322	0.000			
access3	0.983	0.047	20.702	0.000			
Male =~							
access4	1.000						
access5	1.026	0.048	21.306	0.000			
access6	0.967	0.041	23.515	0.000			

Note: In the header, the estimator is now listed as MIIV-2SLS (PIV)

# Reisenzein (1986)

The final example uses data from an experiment conducted by Reisenzein (1986).

The data are not publically available but we have been given permission to use it for this workshop.

As such, we will have to download the data.

# Download Reisenzein (1986) Data

In R we can use the following code to save the Reisenzein data. The easiest way to get the data (across a number of platforms) is to install the readr package and run the following commands.

```
install.packages("readr")
library(readr)
address <- "http://bit.ly/2qSfrQ5"
reisenzein1986 <- read_csv(address)</pre>
```

Alternatively, you can download the CSV and read in directly from: https://raw.githubusercontent.com/zackfisher/M3/master/reisenzein1986.csv

### Reisenzein (1986)

Finally, using the path diagram below fit the models described by Reisenzein (1986). The first model contains solid lines only, the second contains both the solid and dashed lines.

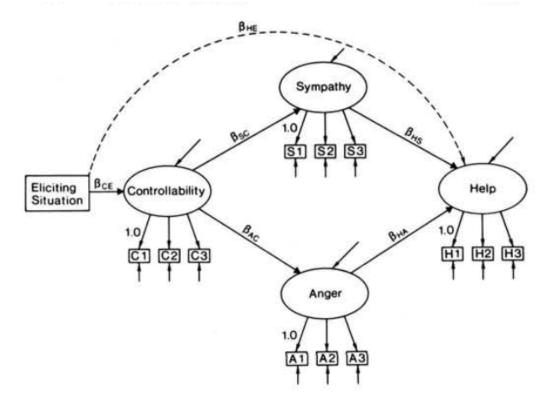


Figure 2 from Reisenzein (1986)

Model 1 Syntax: Figure 2 without dashed line

```
model.reisenzein.1 <- '</pre>
       sympathy =\sim S1 + S2 + S3
controllability =~ C1 + C2 + C3
          anger = \sim A1 + A2 + A3
            help = ~ H1 + H2 + H3
             help ~ sympathy + anger
        sympathy ~ controllability
            anger ~ controllability
 controllability ~ E
```

Fit Model 1 using the MIIV-2SLS estimator:

miive(model.reisenzein.1, reisenzein1986)

TS: Estimate	Std.Err	z-value	P(> z )	Sargan	a.e	- (al !)
	Std.Err	z-value	P(> z )	Caraan	-1 <b>-</b> 2	- (al !)
1 000			± (	Saryan	df	P(Chi)
1 000						
1.000						
0.895	0.075	12.009	0.000	5.281	10	1.000
0.885	0.071	12.474	0.000	9.269	10	1.000
1.000						
1.047	0.080	13.041	0.000	12.085	10	1.000
1.152	0.087	13.218	0.000	11.550	10	1.000
1.000						
1.098	0.059	18.746	0.000	7.632	10	1.000
0.428	0.034	12.686	0.000	11.850	10	1.000
1.000						
0.724	0.068	10.594	0.000	10.419	10	1.000
0.721	0.061	11.779	0.000	26.677	10	0.032
0.640	0.078	8.245	0.000	14.644	5	0.120
3.828	0.315	12.142	0.000			
	<del>-</del>		<del>-</del>			
0.430	0.082	5.270	0.000	16.016	6	0.123
-0.405	0.094	-4.320	0.000		-	-
			<del>-</del>			
-0.716	0.092	-7.780	0.000	10.436	5	0.510
	0.885  1.000 1.047 1.152  1.000 1.098 0.428  1.000 0.724 0.721  0.640  3.828  0.430 -0.405	0.885 0.071  1.000 1.047 0.080 1.152 0.087  1.000 1.098 0.059 0.428 0.034  1.000 0.724 0.068 0.721 0.061  0.640 0.078  3.828 0.315  0.430 0.082 -0.405 0.094	0.885       0.071       12.474         1.000       1.047       0.080       13.041         1.152       0.087       13.218         1.000       1.098       0.059       18.746         0.428       0.034       12.686         1.000       0.724       0.068       10.594         0.721       0.061       11.779         0.640       0.078       8.245         3.828       0.315       12.142         0.430       0.082       5.270         -0.405       0.094       -4.320	0.885       0.071       12.474       0.000         1.000       1.047       0.080       13.041       0.000         1.152       0.087       13.218       0.000         1.000       1.098       0.059       18.746       0.000         0.428       0.034       12.686       0.000         1.000       0.724       0.068       10.594       0.000         0.721       0.061       11.779       0.000         0.640       0.078       8.245       0.000         3.828       0.315       12.142       0.000         0.430       0.082       5.270       0.000         -0.405       0.094       -4.320       0.000	0.885       0.071       12.474       0.000       9.269         1.000       1.047       0.080       13.041       0.000       12.085         1.152       0.087       13.218       0.000       11.550         1.000       1.098       0.059       18.746       0.000       7.632         0.428       0.034       12.686       0.000       11.850         1.000       0.724       0.068       10.594       0.000       10.419         0.721       0.061       11.779       0.000       26.677         0.640       0.078       8.245       0.000       14.644         3.828       0.315       12.142       0.000         0.430       0.082       5.270       0.000       16.016         -0.405       0.094       -4.320       0.000	0.885       0.071       12.474       0.000       9.269       10         1.000       1.047       0.080       13.041       0.000       12.085       10         1.152       0.087       13.218       0.000       11.550       10         1.000       1.098       0.059       18.746       0.000       7.632       10         0.428       0.034       12.686       0.000       11.850       10         1.000       0.724       0.068       10.594       0.000       10.419       10         0.721       0.061       11.779       0.000       26.677       10         0.640       0.078       8.245       0.000       14.644       5         3.828       0.315       12.142       0.000         0.430       0.082       5.270       0.000       16.016       6         -0.405       0.094       -4.320       0.000

Model 2 Syntax: Figure 2 with dashed line

```
model.reisenzein.2 <- '</pre>
       sympathy =\sim S1 + S2 + S3
controllability =~ C1 + C2 + C3
          anger = \sim A1 + A2 + A3
            help = ~ H1 + H2 + H3
             help ~ sympathy + anger
        sympathy ~ controllability
            anger ~ controllability
 controllability ~ E
             help ~ E
```

Fit Model 2 using the MIIV-2SLS estimator:

miive(model.reisenzein.2, reisenzein1986)

	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
anger =~							
A1	1.000						
A2	0.895	0.075	12.009	0.000	5.281	10	1.000
A3	0.885	0.071	12.474	0.000	9.269	10	1.000
controllability =~							
C1	1.000						
C2	1.047	0.080	13.041	0.000	12.085	10	1.000
C3	1.152	0.087	13.218	0.000	11.550	10	1.000
help =~							
н1	1.000						
H2	1.098	0.059	18.746	0.000	7.632	10	1.000
Н3	0.428	0.034	12.686	0.000	11.850	10	1.000
sympathy =~							
S1	1.000						
S2	0.724	0.068	10.594	0.000	10.419	10	1.000
S3	0.721	0.061	11.779	0.000	26.677	10	0.032
anger ~							
controllabilty	0.640	0.078	8.245	0.000	14.644	5	0.108
controllability ~							
E	3.828	0.315	12.142	0.000			
help ~							
E	-0.712	0.462	-1.540	0.124	15.040	5	0.102
sympathy	0.376	0.086	4.370	0.000			
anger	-0.308	0.110	-2.812	0.005			
sympathy ~							
controllabilty	-0.716	0.092	-7.780	0.000	10.436	5	0.510

#### **Table 1 from Reisenzein (1986)**

#### Controllability

- (C1) How controllable, do you think, is the cause of the person's present condition? (1 = not at all under personal control, 9 = completely under personal control)
- (C2) How responsible, do you think, is that person for his present condition? (1 = not at all responsible, 9 = very much responsible)
- (C3) I would think that it was the person's own fault that he is in the present situation. (I = no. not at all, 9 = yes, absolutely so)
  Sympathy
  - (S1) How much sympathy would you feel for that person? (1 = none at all. 9 = very much)
  - (S2) I would feel pity for this person. (1 = none at all, 9 = very much)
  - (S3) How much concern would you feel for this person? (1 = none at all, 9 = very much)

#### Anger

- (A1) How angry would you feel at that person? (1 = not at all, 9 = very much)
- (A2) How irritated would you feel by that person? (1 = not at all, 9 = very much)
- (A3) I would feel aggravated by that person. (1 = not at all, 9 = very much so)

#### **Table 1 from Reisenzein (1986)**

#### Help

- (H1) How likely is it that you would help that person? (1 = definitely would not help, 9 = definitely would help)
- (H2) How certain would you feel that you would help the person?
  - $(1 = not \ at \ all \ certain, 9 = absolutely \ certain)$
- (H3) Which of the following actions would you most likely engage in?
  - I = not help at all. try to stay uninvolved (subway)/I would not lend my notes at all (class notes);
  - 2 = try to alert other bystanders, but stay uninvolved myself (subway)/I would agree to lend the notes if the person is willing to xerox them immediately and give them back (class notes);
  - 3 = try to inform the conductor or another official in charge (subway)/I would agree to lend the notes if the person promises to give them back the following day (class notes):
  - 4 = go over and help the person to a seat (subway)/I would lend him the notes without any special conditions (class notes);
  - 5 = help in any way that might be necessary, including if necessary first aid and/or accompanying the person to a hospital (subway only).

Table 3 from Reisenzein (1986)

EL-II BION	12/10/20 00/20/	15.5	
Model	$\chi^2 (N=138)$	df	p
Null model	1,463.94	78	<.001
Model 1	86.58	61	<.020
Model 2 (with $\beta_{HC}$ added)	85.95	60	<.016
Model 3 (with a correlation between the error terms of sympathy			
and anger added)	85.44	60	<.018
Model 4 (with both $\beta_{HC}$ and the correlation between the errors of			
sympathy and anger added)	84.97	59	<.016
Model 5 (with $\beta_{HE}$ added)	82.25	60	<.029
Model 5' (= 5 with 2 correlated error terms)	67.28	58	≅.189
Model comparisons			
1 versus 2	0.63	1	.50 > p > .25
1 versus 3	1.14	1	.50 > p > .25
1 versus 4	1.61	2	.50 > p > .25
1 versus 5	4.33	1	<.05
1 versus null	1,377.36	17	<.001
5 versus null	1.69	18	<.001
5 versus 5'	14.97	2	<.001

### Cross-loading Model 1 Syntax:

```
model.cross.1 <- '</pre>
       sympathy =\sim S1 + S2 + S3
controllability =~ C1 + C2 + C3
           anger = \sim A1 + A2 + A3
            help = ~H1 + H2 + H3 + S3
             help ~ sympathy + anger
        sympathy ~ controllability
            anger ~ controllability
 controllability ~ E
```

Fit Cross-Loading Model 1:

```
miive(model.cross.1, reisenzein1986)
```

	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
anger =~							
A1	1.000						
A2	0.895	0.075	12.009	0.000	5.281	10	1.000
A3	0.885	0.071	12.474	0.000	9.269	10	1.000
controllability =~							
C1	1.000						
C2	1.047	0.080	13.041	0.000	12.085	10	1.000
C3	1.152	0.087	13.218	0.000	11.550	10	1.000
help =~							
_ н1	1.000						
S3	0.388	0.079	4.906	0.000	6.209	8	1.000
Н2	1.098	0.059	18.746	0.000	7.632	10	1.000
н3	0.428	0.034	12.686	0.000	11.850	10	1.000
sympathy =~							
S1	1.000						
S2	0.724	0.068	10.594	0.000	10.419	10	1.000
S3	0.473	0.073	6.501	0.000	6.209	8	1.000
anger ~							
controllabilty	0.651	0.078	8.329	0.000	11.951	4	0.195
controllability ~							
E	3.828	0.315	12.142	0.000			
help ~							
sympathy	0.352	0.086	4.074	0.000	9.424	5	0.840
anger	-0.440	0.094	-4.688	0.000			
sympathy ~							
controllabilty	-0.716	0.092	-7.780	0.000	10.436	5	0.638

### Cross-loading Model 2 Syntax:

```
model.cross.2 <- '
       sympathy =\sim S1 + S2 + S3
controllability =~ C1 + C2 + C3
          anger = \sim A1 + A2 + A3
           help = ~H1 + H2 + H3 + S3
            help ~ sympathy + anger
        sympathy ~ controllability
           anger ~ controllability
 controllability ~ E
            help ~ E
```

Fit Cross-Loading Model 2:

```
miive(model.cross.2, reisenzein1986)
```

	Estimate	Std.Err	z-value	P(> z )	Sargan	df	P(Chi)
anger =~							
A1	1.000						
A2	0.895	0.075	12.009	0.000	5.281	10	1.000
A3	0.885	0.071	12.474	0.000	9.269	10	1.000
<pre>controllability =~</pre>							
C1	1.000						
C2	1.047	0.080	13.041	0.000	12.085	10	1.000
C3	1.152	0.087	13.218	0.000	11.550	10	1.000
help =~							
H1	1.000						
S3	0.388	0.079	4.906	0.000	6.209	8	1.000
Н2	1.098	0.059	18.746	0.000	7.632	10	1.000
н3	0.428	0.034	12.686	0.000	11.850	10	1.000
sympathy =~							
S1	1.000						
S2	0.724	0.068	10.594	0.000	10.419	10	1.000
S3	0.473	0.073	6.501	0.000	6.209	8	1.000
anger ~							
<pre>controllability controllability ~</pre>	0.651	0.078	8.329	0.000	11.951	4	0.195
E	3.828	0.315	12.142	0.000			
help ~							
E	-1.059	0.465	-2.274	0.023	5.330	4	1.000
sympathy	0.255	0.092	2.769	0.006			
anger	-0.303	0.107	-2.827	0.005			
sympathy ~							
controllabilty	-0.716	0.092	-7.780	0.000	10.436	5	0.638

Model	$\chi^2 (N=138)$	df	p
Null model	1,463.94	78	<.001
Model 1	86.58	61	<.020
Model 2 (with $\beta_{HC}$ added)	85.95	60	<.016
Model 3 (with a correlation between the error terms of sympathy			
and anger added)	85.44	60	<.018
Model 4 (with both $\beta_{HC}$ and the correlation between the errors of			
sympathy and anger added)	84.97	59	<.016
Model 5 (with $\beta_{HE}$ added)	82.25	60	<.029
Model 5' (= 5 with 2 correlated error terms)	67.28	58	≅.189
Model comparisons			
1 versus 2	0.63	1	.50 > p > .25
1 versus 3	1.14	1	.50 > p > .25
1 versus 4	1.61	2	.50 > p > .25
1 versus 5	4.33	1	<.05
l versus null	1,377.36	17	<.001
5 versus null	1.69	18	<.001
5 versus 5'	14.97	2	<.001

Cross-loading Model 1:  $\chi^2$  (60, N = 138) = 67.244

Cross-loading Model 2:  $\chi^2(59, N = 138) = 60.143$ 

### **EXTENSIONS**

- Categorical endogenous variables
  - Bollen & Maydeu-Oliveres (2007)
  - Nestler (2012)
  - Jin, Luo, & Yang-Wallentin (2016)
- Interactions of latent variables
  - Bollen (1995)
  - Bollen & Paxton (1998)
- 2<sup>nd</sup> Order growth curve models
  - Nestler (2014)

### **EXTENSIONS**

- Higher order factor analysis
  - Bollen & Biesanz (2002)
- Specification error tests for nonlinearity and interactions
  - Nestler (2015)
- Testing dimensionality of measures
  - Bollen (2011)
- General Method of Moments estimator
  - Bollen, Kolenikov, & Bauldry (2014)

### Resources

#### lavaan Resources:

http://lavaan.ugent.be/

#### MIIVsem Github

- https://github.com/zackfisher/MIIVsem
- readme file with examples
- vignette coming soon

### MIIVsem Planned Developments

- Missing data (normal or nonnormal distributions)
- General Method of Moments estimator
  - Permits subsets of equations to estimate
- Additional diagnostic tests of MIIVs
- Weak MIIV diagnostics
- MIIV selection methods (e.g. LASSO)

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