Maximum Likelihood, Logistic Regression, Generalized Linear Models

Maximum Likelihood

$$L = \prod_i P(X_i)$$

$$\ell = log(L) = \sum_i log(P(X_i))$$

Maximum Likelihood: Binomial

$$P(X_i) = C \times p^{y_i} \times (1-p)^{1-y_i}$$
$$\log(P(X_i)) = c + y_i \log(p) + (1-y_i)\log(1-p)$$

Maximum Likelihood: Binomial

$$P(X_i) = C \times p^{y_i} \times (1-p)^{1-y_i}$$
$$\log(P(X_i)) = c + y_i \log(p) + (1-y_i)\log(1-p)$$

Logistic Regression: make a function of *p* (the parameter of interest) a linear combination of your features.

Log odds:

$$P(X_i) = C \times p^{y_i} \times (1-p)^{1-y_i}$$

$$\log(P(X_i)) = c + y_i \log(p) + (1-y_i)\log(1-p)$$

$$\ell odds = \log \frac{p}{1-p} = x_1\beta_1 + \ldots + x_m\beta_m$$

Log odds:

$$P(X_i) = C \times p^{y_i} \times (1-p)^{1-y_i}$$

$$\log(P(X_i)) = c + y_i \log(p) + (1-y_i)\log(1-p)$$

$$\ell odds = \log \boxed{\frac{p}{1-p}} = x_1\beta_1 + \ldots + x_m\beta_m$$

Q: What values can the odds take?

Log odds:

$$P(X_i) = C \times p^{y_i} \times (1-p)^{1-y_i}$$
$$\log(P(X_i)) = c + y_i \log(p) + (1-y_i)\log(1-p)$$

$$\ell odds = \boxed{\log \frac{p}{1-p}} = x_1\beta_1 + \ldots + x_m\beta_m$$

Q: What values can the log-odds take?

Log odds:

$$P(X_i) = C \times p^{y_i} \times (1-p)^{1-y_i}$$

$$\log(P(X_i)) = c + y_i \log(p) + (1-y_i)\log(1-p)$$

$$\ell odds = \log \frac{p}{1-p} = x_1\beta_1 + \ldots + x_m\beta_m$$

$$\implies p = \exp(\ell odds)/(1+\exp(\ell odds)))$$

Maximum likelihood: we plug the last line into the 2nd line

Log odds: interpreting coefficients

$$\ell odds = \log \frac{p}{1-p} = x_1\beta_1 + \ldots + x_m\beta_m$$

$$\implies p = \exp(\ell odds)/(1 + \exp(\ell odds)))$$

What does an intercept mean?

What does a linear coefficient mean?

Log odds: interpreting coefficients

$$\ell odds = \log \frac{p}{1-p} = x_1\beta_1 + \ldots + x_m\beta_m$$

$$\implies p = \exp(\ell odds)/(1 + \exp(\ell odds)))$$

What does an intercept mean?

The base log odds ratio.

0 => log(odds) = 0 => odds = 1 => p=½;

log(2) => log(odds) = log(2) => odds = 2 => p=⅔

Log odds: interpreting coefficients

$$\ell odds = \log \frac{p}{1-p} = x_1\beta_1 + \ldots + x_m\beta_m$$

$$\implies p = \exp(\ell odds)/(1 + \exp(\ell odds)))$$

What does a linear coefficient mean?

*When **exponentiated**, an odds ratio multiplier.*

Let slope=0, thus a unit increase in predictor of 1 implies:

log(new_odds) = log(odds) + 0

*new_odds = exp(log(odds) + 0) = exp(log(odds)) * exp(0) = odds * 1 = odds*

Log odds: interpreting coefficients

$$\ell odds = \log \frac{p}{1-p} = x_1\beta_1 + \ldots + x_m\beta_m$$

$$\implies p = \exp(\ell odds)/(1 + \exp(\ell odds)))$$

What does a linear coefficient mean?

*When **exponentiated**, an odds ratio multiplier.*

Let slope=log(2) unit increase in predictor of 1 =>

log(new_odds) = log(odds) + log(2)

*new_odds = exp(log(odds) + log(2)) = exp(log(odds)) * exp(log(2)) = odds * 2*

Generalized linear models

What did we do with logistic regression?

1. used the appropriate probability model
2. transformed the parameter to an unbounded range
3. set that equal to a linear model
4. work backwards to get the likelihood

The above is the basic process for GLMs. See:
http://en.wikipedia.org/wiki/Generalized_linear_model#Link_function