W271 Lab 3

April 17, 2016

Part 1

Load data and display some basic statistics:

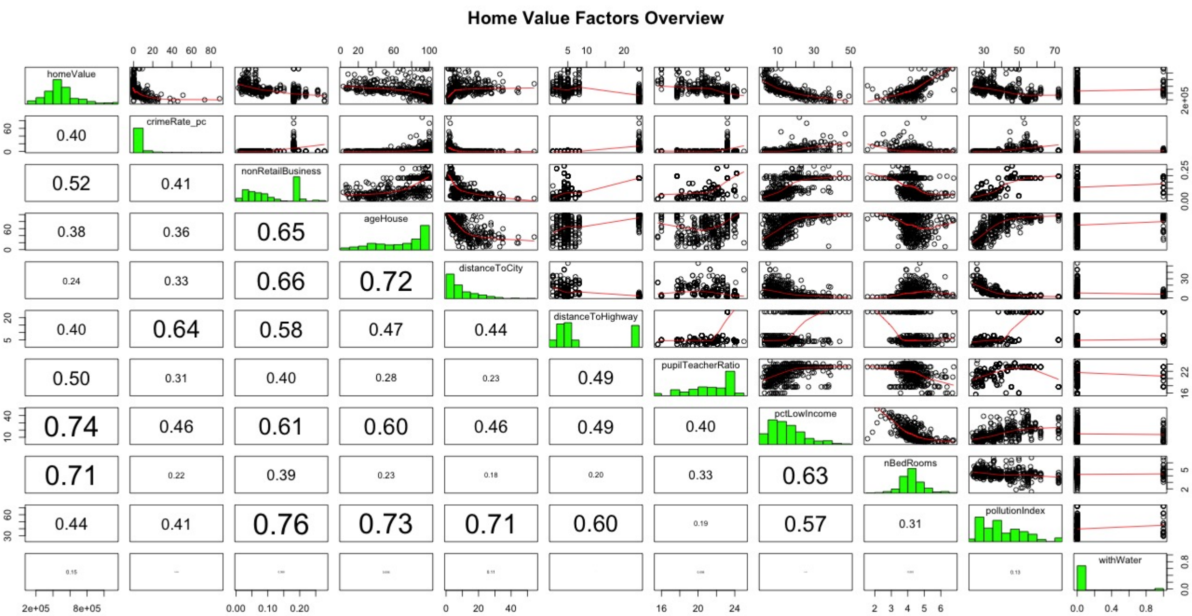
```
## Loading required package: zoo
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
##
## Loading required package: survival
## Loading required package: splines
## Loading required package: timeDate
## Loading required package: timeSeries
##
## Attaching package: 'timeSeries'
##
## The following object is masked from 'package:zoo':
##
##     time<-
##
## Loading required package: fBasics
##
##
## Rmetrics Package fBasics
## Analysing Markets and calculating Basic Statistics
## Copyright (C) 2005-2014 Rmetrics Association Zurich
## Educational Software for Financial Engineering and Computational Science
## Rmetrics is free software and comes with ABSOLUTELY NO WARRANTY.
## https://www.rmetrics.org --- Mail to: info@rmetrics.org
##
## Attaching package: 'fBasics'
##
## The following object is masked from 'package:car':
##
##     densityPlot
##
##
## Please cite as:
##
##  Hlavac, Marek (2015). stargazer: Well-Formatted Regression and Summary Statistics Tables.
##  R package version 5.2. http://CRAN.R-project.org/package=stargazer
##
## Loading required package: forecast
## This is forecast 6.2
```

```
## 'data.frame':    400 obs. of  11 variables:
```

```
## $ crimeRate_pc     : num  37.6619 0.5783 0.0429 22.5971 0.0664 ...
## $ nonRetailBusiness: num  0.181 0.0397 0.1504 0.181 0.0405 ...
## $ withWater        : int  0 0 0 0 0 0 0 0 0 0 ...
## $ ageHouse         : num  78.7 67 77.3 89.5 74.4 71.3 68.2 97.3 92.2 96.2 ...
## $ distanceToCity   : num  2.71 4.12 7.82 1.95 5.54 ...
## $ distanceToHighway: int  24 5 4 24 5 5 5 5 3 5 ...
## $ pupilTeacherRatio: num  23.2 16 21.2 23.2 19.6 23.9 22.2 17.7 20.8 17.7 ...
## $ pctLowIncome     : int  18 9 13 41 8 9 12 18 5 4 ...
## $ homeValue        : int  245250 1125000 463500 166500 672750 596250 425250 483750 852750 1125000 .
## $ pollutionIndex   : num  52.9 42.5 31.4 55 36 37 34.9 72.1 33.8 45.5 ...
## $ nBedRooms        : num  4.2 6.3 4.25 3 4.86 ...
```

```
##   crimeRate_pc      nonRetailBusiness   withWater         ageHouse
##  Min.   : 0.00632   Min.   :0.0074    Min.   :0.0000   Min.   :  2.90
##  1st Qu.: 0.08260   1st Qu.:0.0513    1st Qu.:0.0000   1st Qu.: 45.67
##  Median : 0.26600   Median :0.0969    Median :0.0000   Median : 77.95
##  Mean   : 3.76256   Mean   :0.1115    Mean   :0.0675   Mean   : 68.93
##  3rd Qu.: 3.67481   3rd Qu.:0.1810    3rd Qu.:0.0000   3rd Qu.: 94.15
##  Max.   :88.97620   Max.   :0.2774    Max.   :1.0000   Max.   :100.00
##  distanceToCity   distanceToHighway pupilTeacherRatio  pctLowIncome
##  Min.   : 1.228   Min.   : 1.000    Min.   :15.60     Min.   : 2.00
##  1st Qu.: 3.240   1st Qu.: 4.000    1st Qu.:19.90     1st Qu.: 8.00
##  Median : 6.115   Median : 5.000    Median :21.90     Median :14.00
##  Mean   : 9.638   Mean   : 9.582    Mean   :21.39     Mean   :15.79
##  3rd Qu.:13.628   3rd Qu.:24.000    3rd Qu.:23.20     3rd Qu.:21.00
##  Max.   :54.197   Max.   :24.000    Max.   :25.00     Max.   :49.00
##    homeValue        pollutionIndex    nBedRooms
##  Min.   : 112500   Min.   :23.50   Min.   :1.561
##  1st Qu.: 384188   1st Qu.:29.88   1st Qu.:3.883
##  Median : 477000   Median :38.80   Median :4.193
##  Mean   : 499584   Mean   :40.61   Mean   :4.266
##  3rd Qu.: 558000   3rd Qu.:47.58   3rd Qu.:4.582
##  Max.   :1125000   Max.   :72.10   Max.   :6.780
```

We first generate the matrix plot to have an overview of all variables.



Upon first galance, two things stands out: no highly-correlated pair of variables, thus collinearity won't be a concern of our analysis, in addition, the majority of the distributions are skewed and non-normal. More specifically:

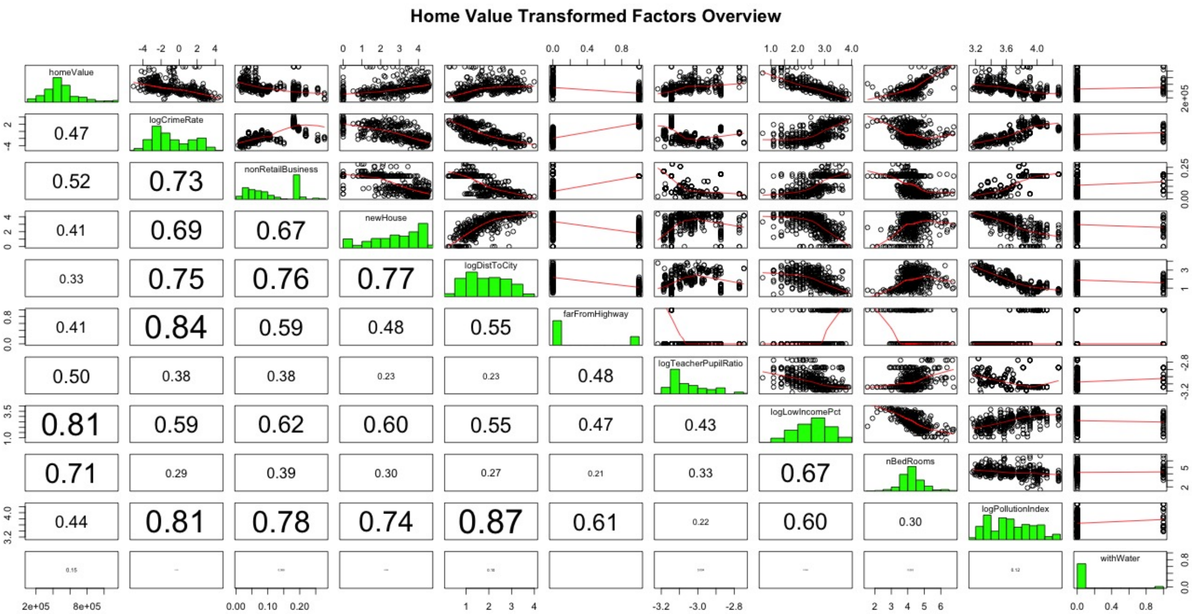
- crime rate, distance to city, low income percentage, and pollution index are negatively skewed.
- age of house, pupil teacher ratio are positively skewed
- non retail business, and distance to highway have bi-modal distribution
- home value, number of bedroom are approximately normal

we then do some transformation on the variables:

- take log of the negatively skewed variables
- convert distance to highway to a binary variable, **farFromHighway**, if it's bigger than 10
- for positive skewness, we "reverse" the variable first then take log, and the interpretation of coefficients in the model need to adjust accordingly. Specifically:

a. take the reciprocal of pupilTeacherRatio, it becomes teacherPupilRatio
b. take 100 - ageHouse, it becomes proportion of house built **after** 1950

Let's evaluate matrix plot again with the transformed variables:



Based on correlation coefficients, we propose a hypothesis of house value:

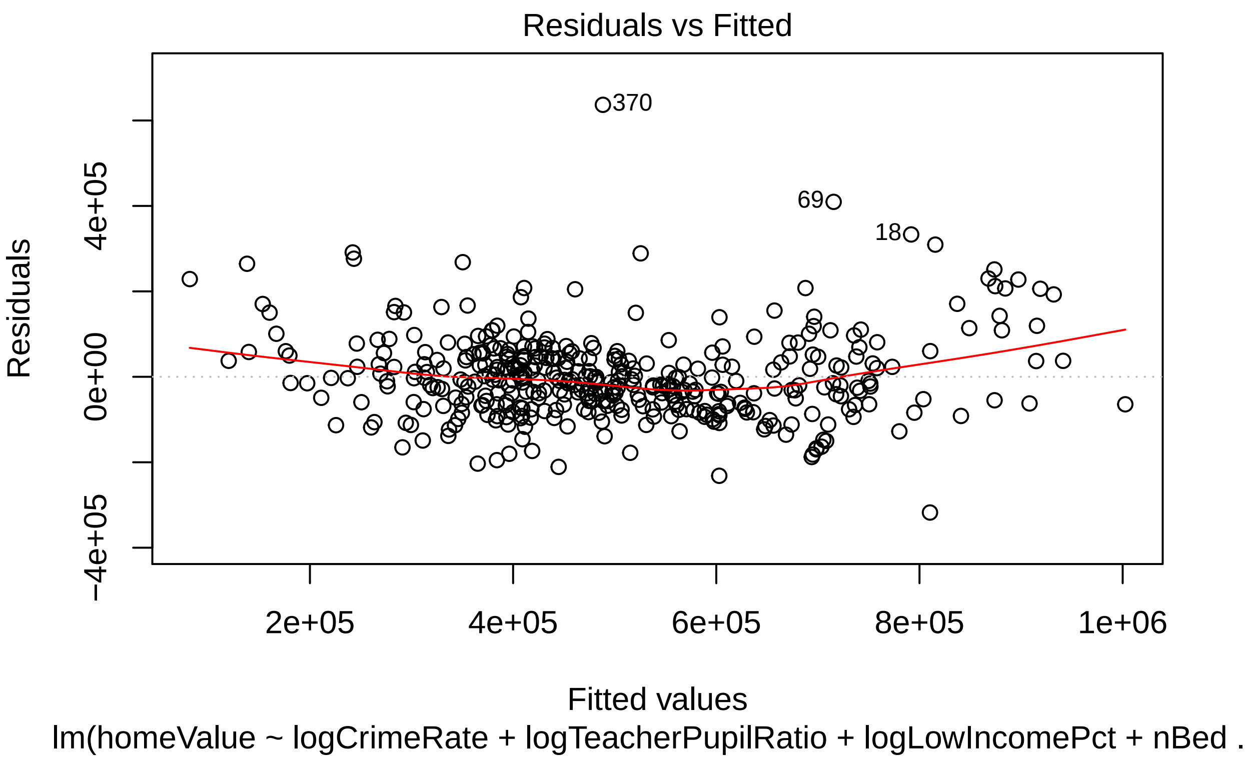
House value is significantly affected by factors from crime rate, education quality (represented by teacher pupil ratio), low income percentage, bedroom nuber, and pollution index.

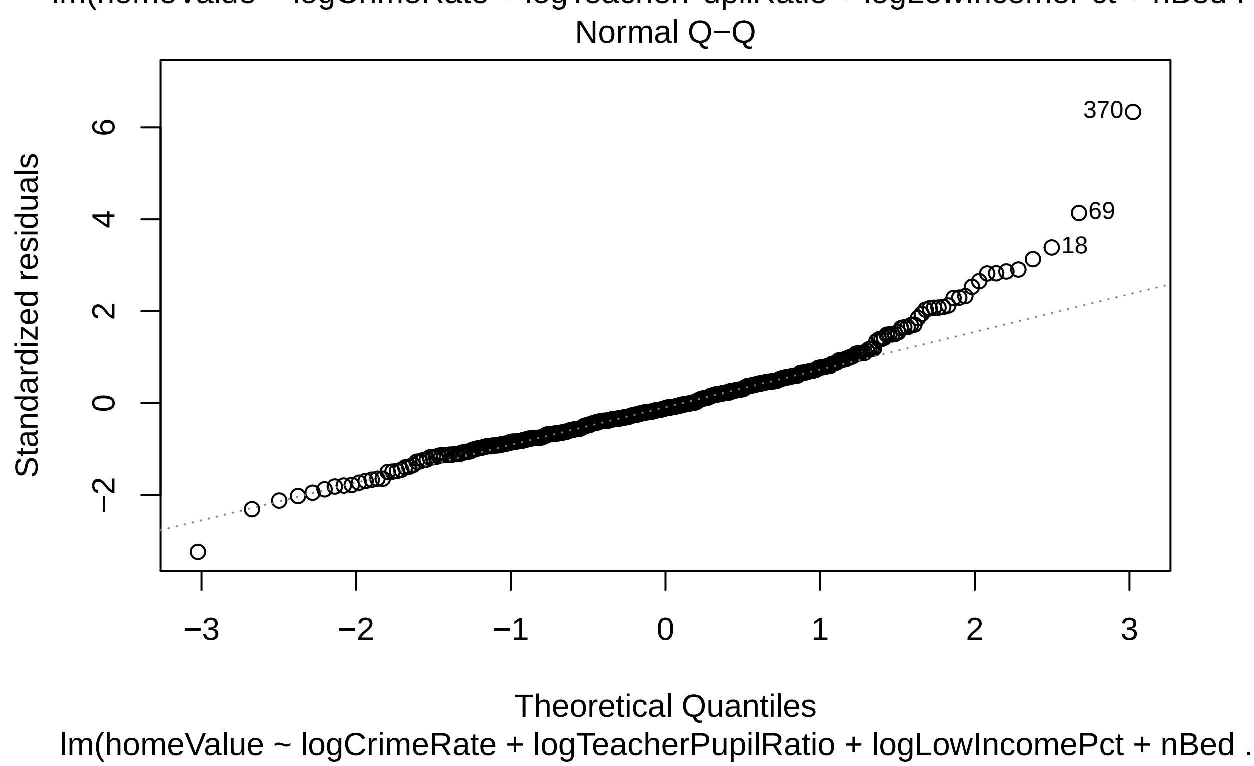
We build a linear model first with those variables:

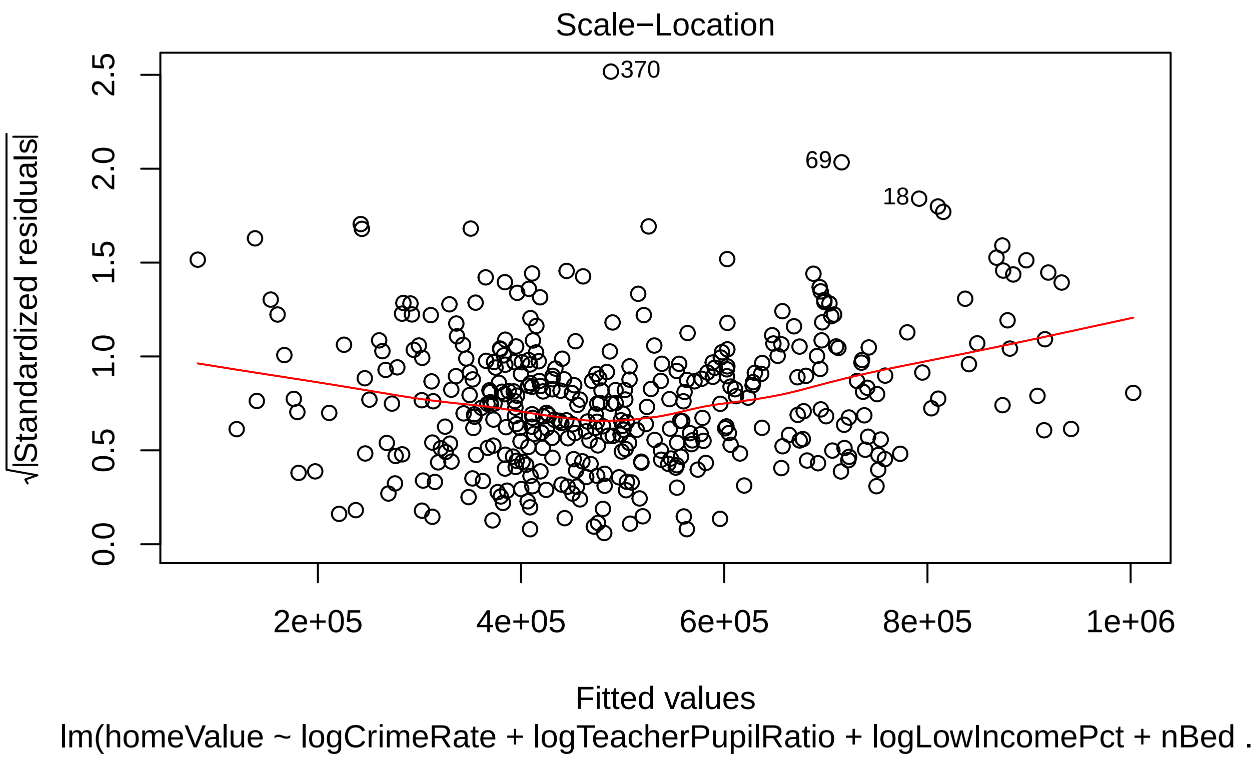
```
##
## Call:
## lm(formula = homeValue ~ logCrimeRate + logTeacherPupilRatio +
##     logLowIncomePct + nBedRooms + logPollutionIndex, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -317589  -64311  -10894   46801  636599
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          1554893.2   234001.9   6.645 1.01e-10 ***
## logCrimeRate             961.9     4320.6   0.223    0.824
## logTeacherPupilRatio  314867.8    55323.6   5.691 2.46e-08 ***
## logLowIncomePct      -173745.0    13708.0 -12.675  < 2e-16 ***
## nBedRooms              78593.6     9713.1   8.092 7.38e-15 ***
## logPollutionIndex       5615.7    32473.7   0.173    0.863
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 101100 on 394 degrees of freedom
## Multiple R-squared:  0.7374, Adjusted R-squared:  0.7341
## F-statistic: 221.3 on 5 and 394 DF,  p-value: < 2.2e-16
```

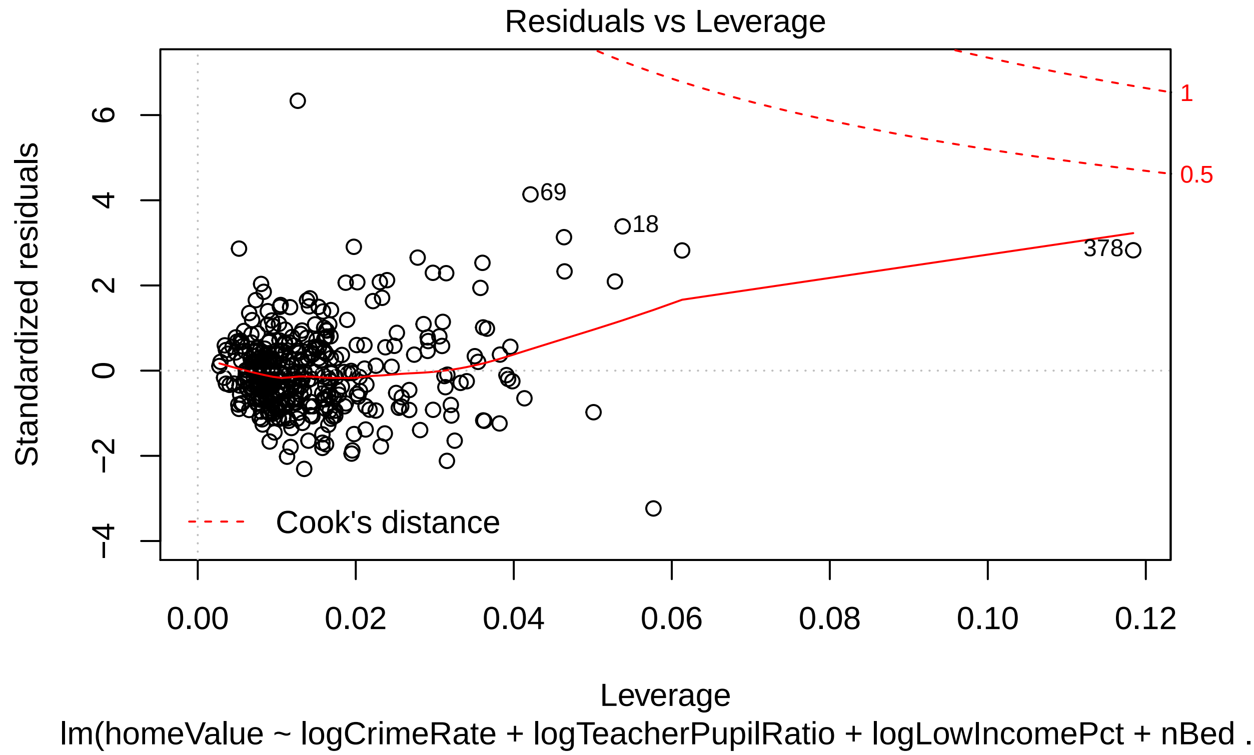
We can see that education quality, low income percentage, and number of bedrooms have significant impact on house value. On average, one more bedroom will increase the value by $78.6k, one percent increase in the low income percentage will reduce house value by $173.7k, and one percent increase in teacher pupil ratio will increase house value by $314.9k. Surprisingly here crime rate is not a significant factor.

Next, we do model diagnostics:









From the chart we can see, the model doesn't violate homoscedasticity assumption, and there is no concern of outliers in the data. However, the normality and zero-conditional mean assumptions are questionable towards the high value house.

We now add the omitted variables to our model and compare the results:

We can see that in model 3 pollution index becomes significant. In addition, distance to city and water

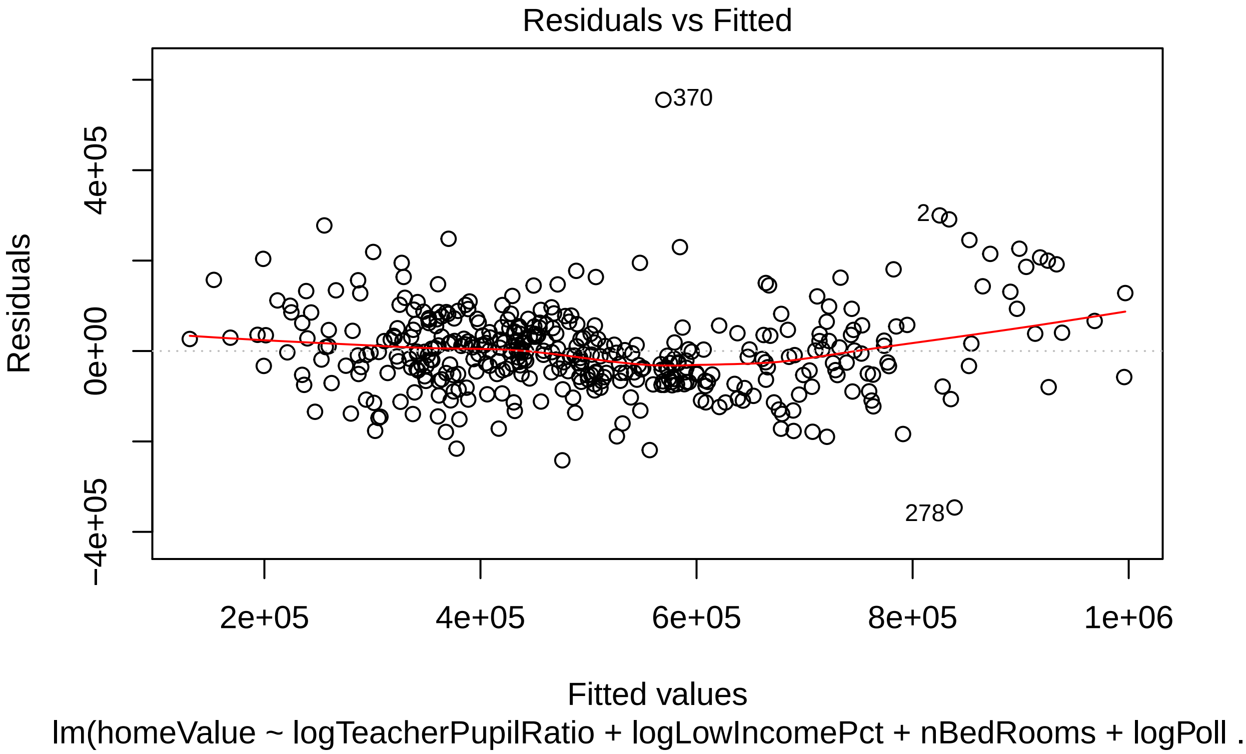
Table 1: House Value Model Summary

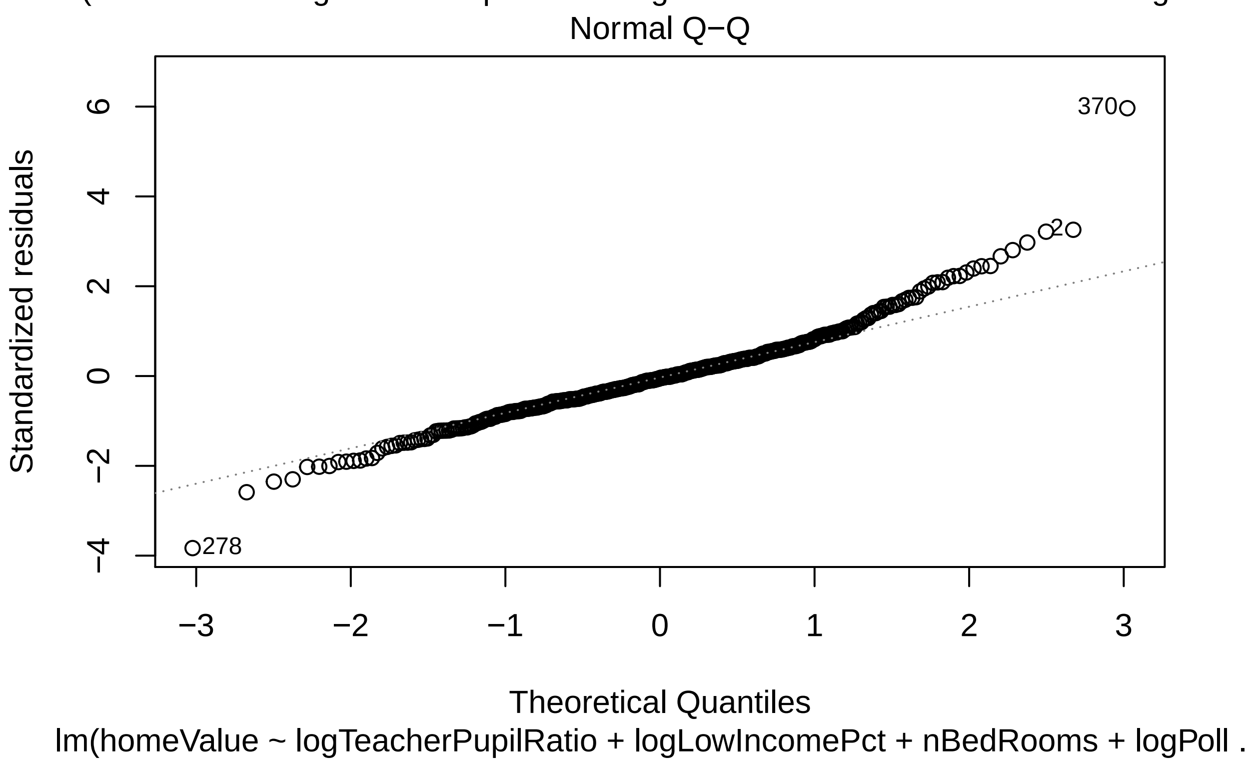
	Dependent variable:		
	House Value		
	(1)	(2)	(3)
logCrimeRate	961.901	8,156.263	1,666.250
	(−7,506.387, 9,430.188)	(−3,607.645, 19,920.170)	(−9,613.651, 12,946.150)
logTeacherPupilRatio	314,867.800***	276,554.600***	274,726.300***
	(206,435.600, 423,300.000)	(163,014.900, 390,094.300)	(165,000.700, 384,451.900)
logLowIncomePct	−173,745.000***	−172,090.700***	−181,403.400***
	(−200,612.200, −146,877.800)	(−198,877.200, −145,304.200)	(−208,434.500, −154,372.300)
nBedRooms	78,593.580***	78,980.880***	69,215.170***
	(59,556.310, 97,630.850)	(60,093.840, 97,867.920)	(50,660.260, 87,770.080)
logPollutionIndex	5,615.722	−14,073.550	−182,025.200***
	(−58,031.470, 69,262.920)	(−78,298.340, 50,151.240)	(−264,518.800, −99,531.650)
farFromHighway		−37,459.410	−14,017.560
		(−82,239.580, 7,320.766)	(−57,147.040, 29,111.930)
withWater		53,643.820***	54,161.730***
		(13,550.510, 93,737.120)	(16,438.880, 91,884.590)
nonRetailBusiness			−297,234.800**
			(−540,375.300, −54,094.240)
ageHouse			393.526
			(−237.781, 1,024.833)
logDistToCity			−81,172.700***
			(−105,548.500, −56,796.860)
Constant	1,554,893.000***	1,515,577.000***	2,339,513.000***
	(1,096,258.000, 2,013,528.000)	(1,056,843.000, 1,974,311.000)	(1,827,152.000, 2,851,874.000)
Observations	400	400	400
R^2	0.737	0.744	0.777
Adjusted R^2	0.734	0.739	0.771
Residual Std. Error	101,125.200	100,125.200	93,770.050
F Statistic	221.330***	162.682***	135.630***
Note:			*p<0.1; **p<0.05; ***p<0.01

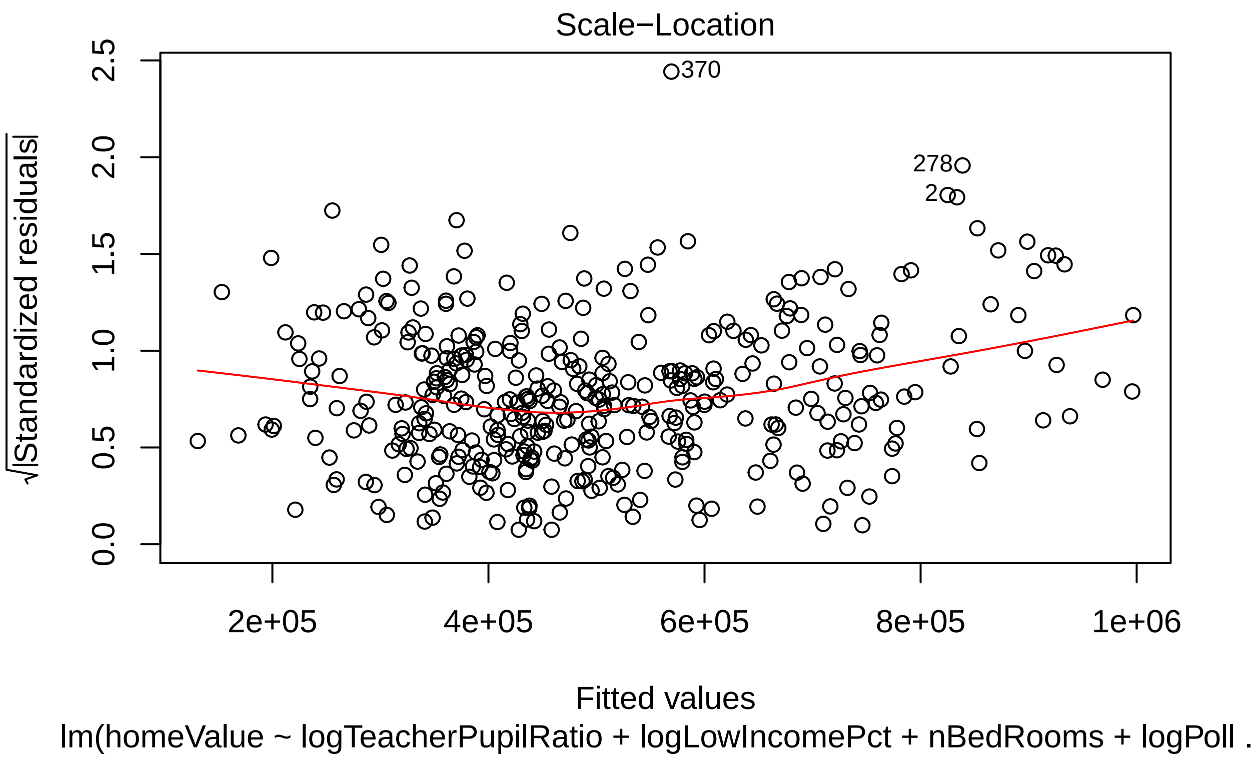
proximity are also significantly affecting house value. Finally, we build the linear model with the significant predictors identified above:

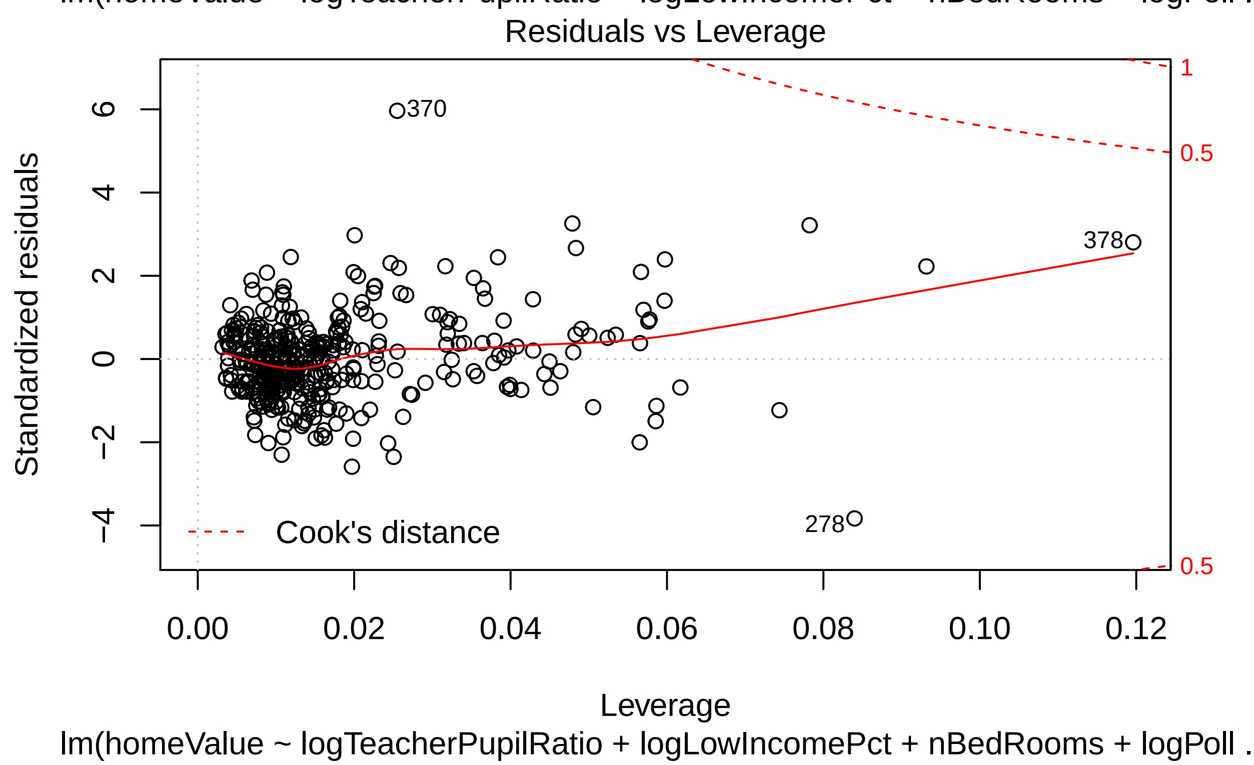
```
##
## Call:
## lm(formula = homeValue ~ logTeacherPupilRatio + logLowIncomePct +
##     nBedRooms + logPollutionIndex + withWater + logDistToCity,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -346067  -53036   -4417   46708  555679
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           2517544     216732  11.616  < 2e-16 ***
## logTeacherPupilRatio   318860      49288   6.469 2.93e-10 ***
## logLowIncomePct       -178453      12737 -14.011  < 2e-16 ***
## nBedRooms               73823       9028   8.177 4.06e-15 ***
## logPollutionIndex     -204869      35696  -5.739 1.90e-08 ***
## withWater               52940      19262   2.749  0.00626 **
## logDistToCity          -79854      11138  -7.169 3.77e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 94370 on 393 degrees of freedom
## Multiple R-squared:  0.7719, Adjusted R-squared:  0.7685
## F-statistic: 221.7 on 6 and 393 DF,  p-value: < 2.2e-16
```

we see that being further away from city will reduce house value, while having a body of water closeby will increase the value. Finally we diagnose this model


Residuals vs Fitted
Residuals
Fitted values
lm(homeValue ~ logTeacherPupilRatio + logLowIncomePct + nBedRooms + logPoll .
370
2
278
4e+05
0e+00
−4e+05
2e+05
4e+05
6e+05
8e+05
1e+06


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(homeValue ~ logTeacherPupilRatio + logLowIncomePct + nBedRooms + logPoll .
370
2
278
6
4
2
0
−2
−4
−3
−2
−1
0
1
2
3





Similarly, the normality and zero-conditional mean assumption are questionable as price increases. Therefore we will use robust error to compensate:

```
##
## Call:
## lm(formula = homeValue ~ logTeacherPupilRatio + logLowIncomePct +
```

```
##     nBedRooms + logPollutionIndex + withWater + logDistToCity,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -346067  -53036   -4417   46708  555679
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           2517544     216732  11.616  < 2e-16 ***
## logTeacherPupilRatio   318860      49288   6.469 2.93e-10 ***
## logLowIncomePct       -178453      12737 -14.011  < 2e-16 ***
## nBedRooms               73823       9028   8.177 4.06e-15 ***
## logPollutionIndex     -204869      35696  -5.739 1.90e-08 ***
## withWater               52940      19262   2.749  0.00626 **
## logDistToCity          -79854      11138  -7.169 3.77e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 94370 on 393 degrees of freedom
## Multiple R-squared:  0.7719, Adjusted R-squared:  0.7685
## F-statistic: 221.7 on 6 and 393 DF,  p-value: < 2.2e-16
```

```
## [1] "Robust Standard Errors"
```

```
##          (Intercept) logTeacherPupilRatio      logLowIncomePct
##            231450.28             55184.01             18941.97
##            nBedRooms    logPollutionIndex            withWater
##             15365.45             36594.87             23188.05
##        logDistToCity
##             14596.04
```

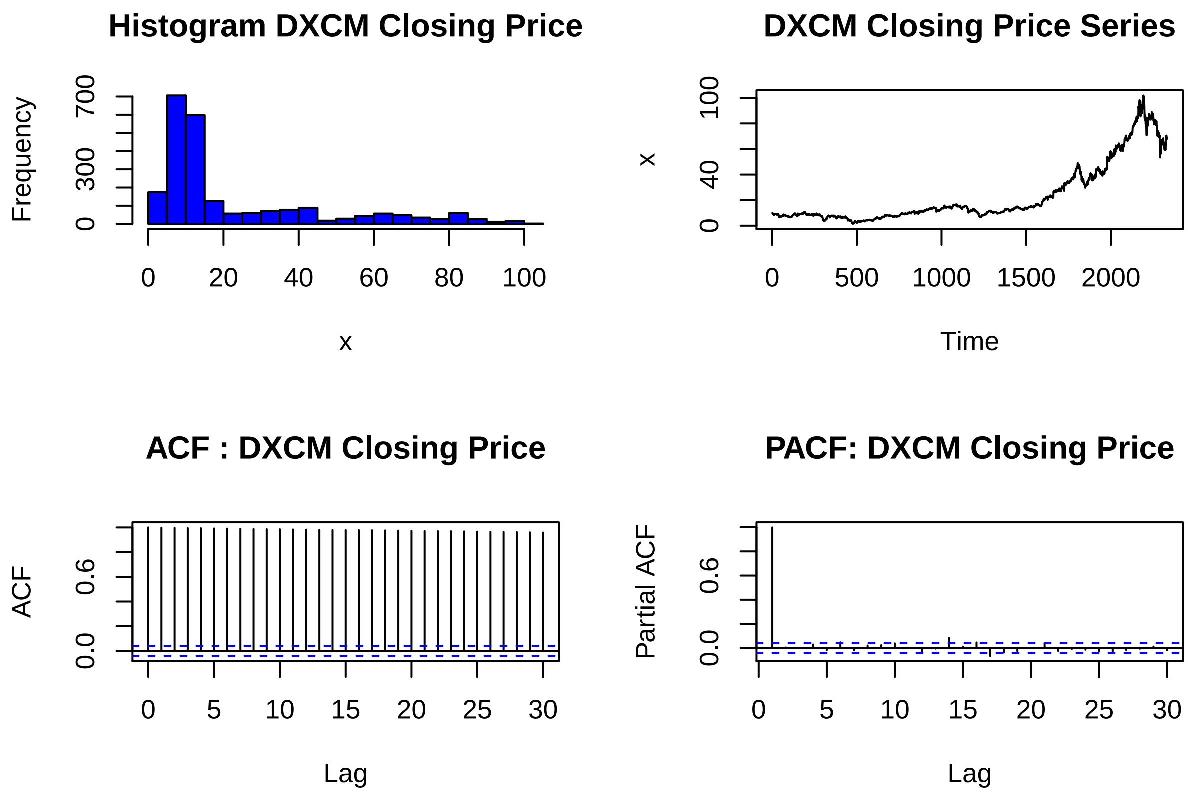
Part 2

Load data, package, and show descriptive statistics:

```
## 'data.frame':    2332 obs. of  2 variables:
##  $ X         : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ DXCM.Close: num  9.88 9.79 9.68 9.64 9.42 9.47 9.16 8.99 8.6 8.81 ...
```

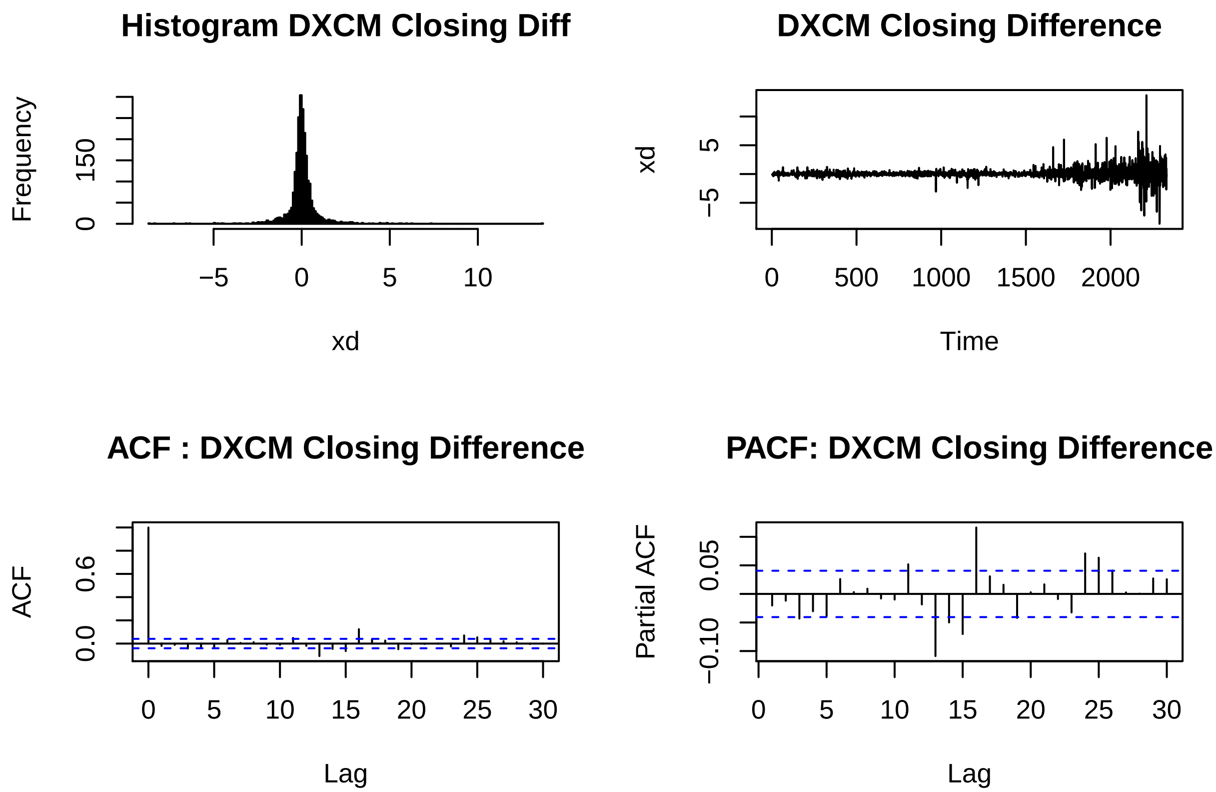
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.390   8.188  12.360  23.210  32.560 101.900
```

Let's evaluate the time series plot, histogram, ACF and PACF of the data:




```
##
##  Box-Ljung test
##
## data:  x
## X-squared = 2327.388, df = 1, p-value < 2.2e-16
```

The Box test indicates that our original series x is **not** a stationary series, and we can observe a upward trend, thus simple ARMA model won't be adequate and we further evaluate the difference of the x, x_d:




```
##
##  Box-Ljung test
##
## data:  xd
## X-squared = 0.9615, df = 1, p-value = 0.3268
```

Box test now indicates x_d is stationary, however we can see that the variance of x_d is time-varying, as such, we **cannot** apply ARIMA alone. To address that, we use ARIMA to fit the original series x, then apply GARCH model on the ARIMA residue to estimate conditional variance and obtain the final prediction by integrating GARCH results into the ARIMA prediction. To obtain the best ARIMA model, we run the following procedure to identify the optimal order (p, d, q):

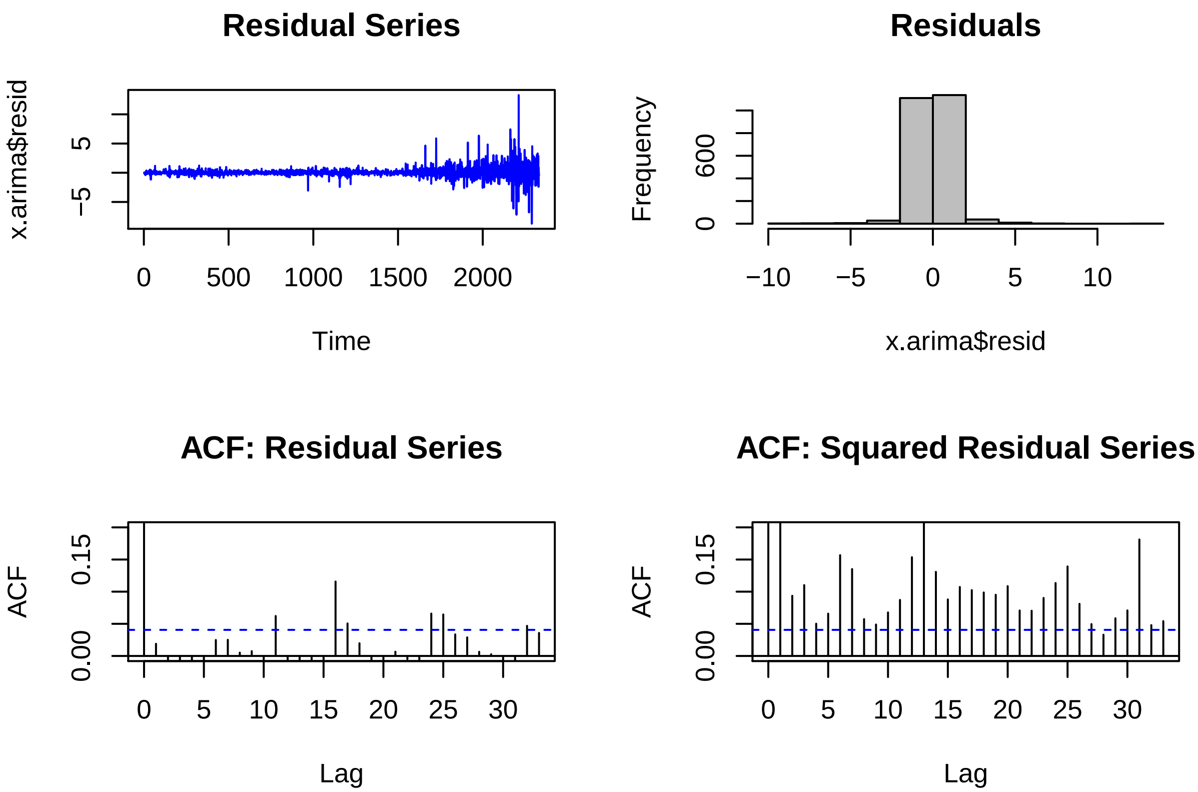
```
# procedure to get best ARIMA order
get.best.arima <- function(x.ts, maxord = c(1,1,1))  # don't change any of this code
{
  best.aic <- 1e8
  n <- length(x.ts)
  for (p in 0:maxord[1]) for(d in 0:maxord[2]) for(q in 0:maxord[3])
  {
    fit <- arima(x.ts, order = c(p,d,q))
    fit.aic <- -2 * fit$loglik + (log(n) + 1) * length(fit$coef)
    if (fit.aic < best.aic)
    {
      best.aic <- fit.aic
      best.fit <- fit
      best.model <- c(p,d,q)
    }
  }
```

```r
  list(best.aic, best.fit, best.model)
}
# model selection
#x.best = get.best.arima(x, maxord = c(2,2,2))[[3]]
```

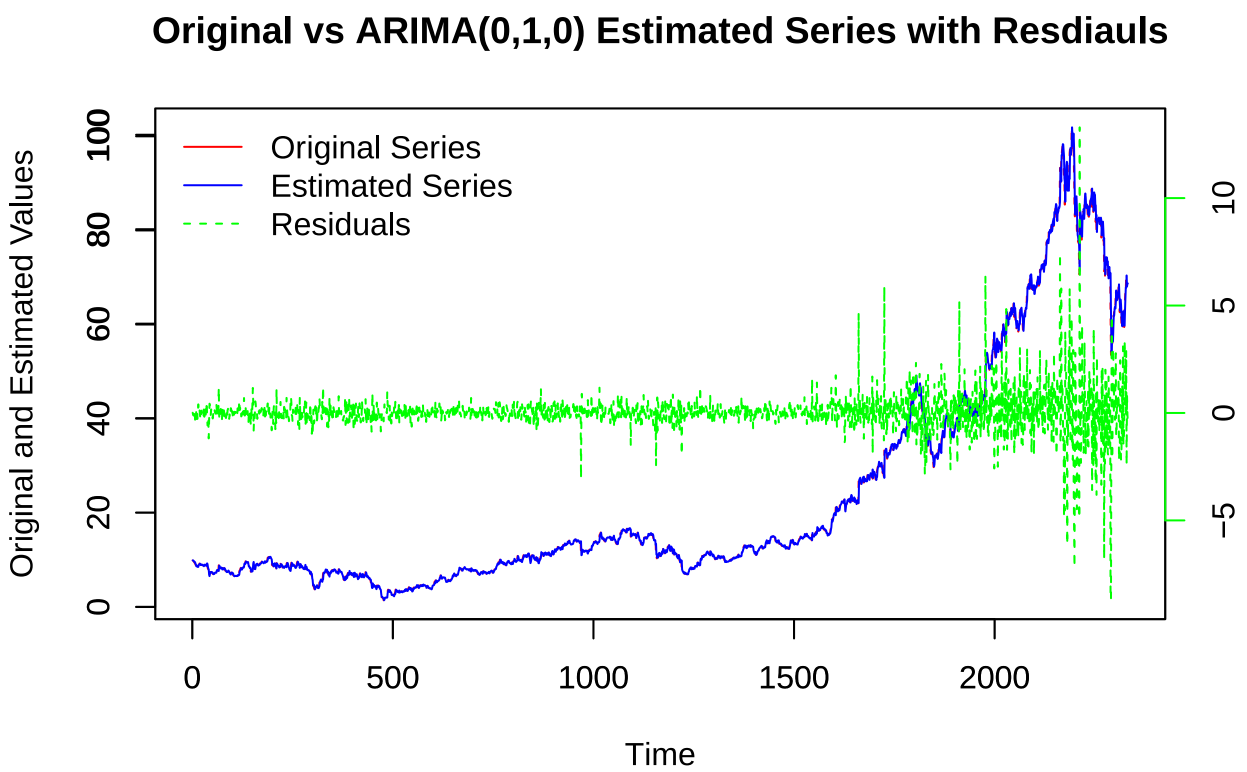
the best order of ARIMA is $(0, 1, 0)$:

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -8.716000 -0.211900  0.003836  0.028970  0.256700 13.290000
```

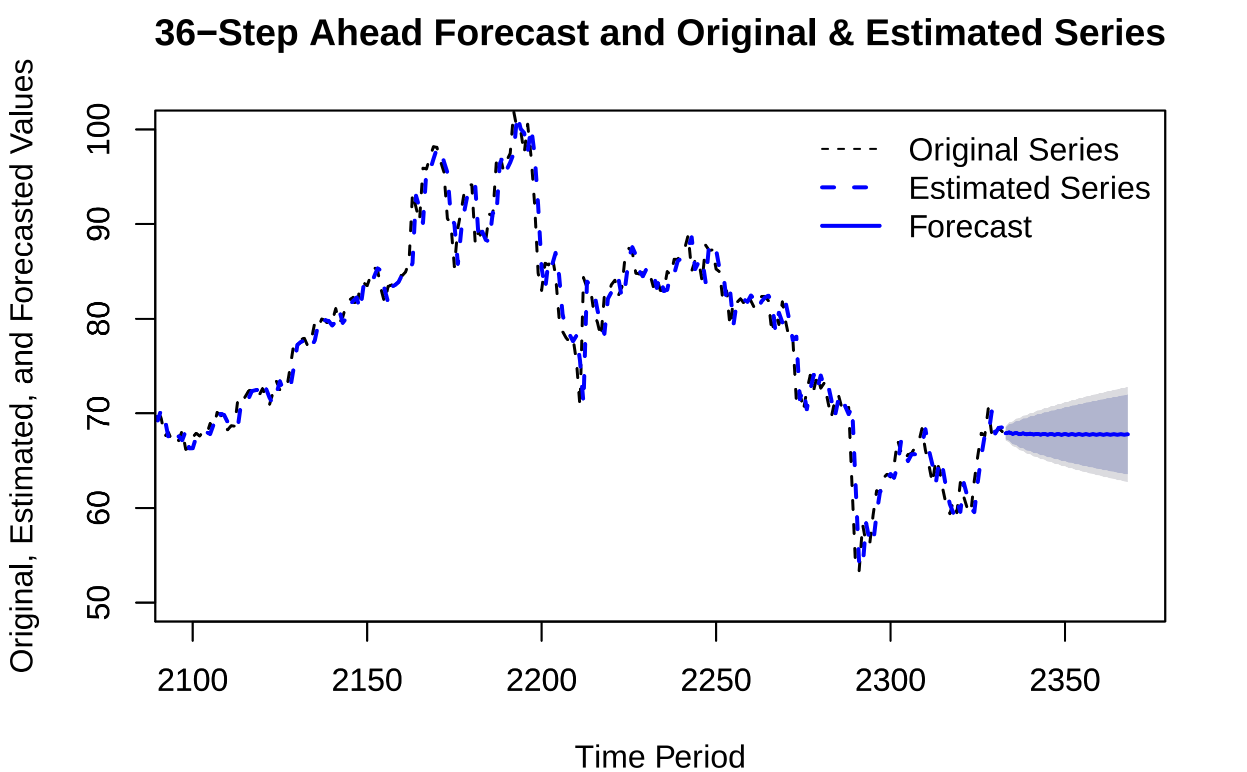
```
##
##  Box-Ljung test
##
## data:  x.arima$resid
## X-squared = 0.8374, df = 1, p-value = 0.3601
```



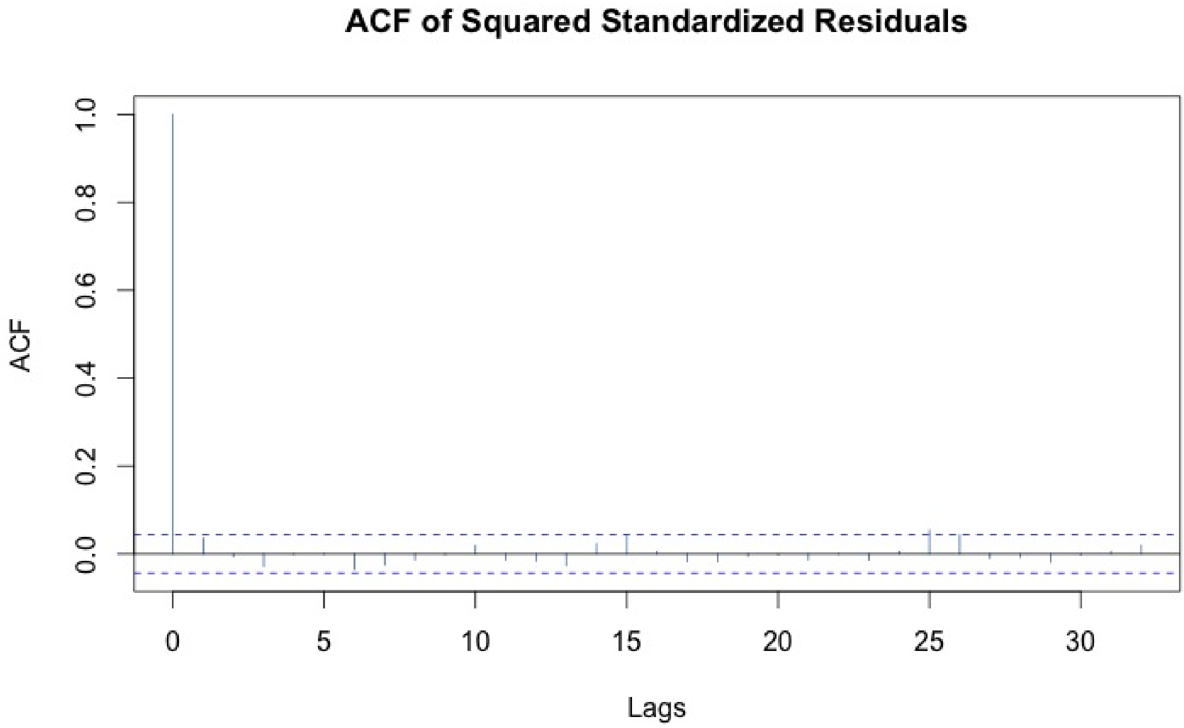
we can see although the ACF of residual indicates insignificant autocorrelation, ACF of squared residual showed otherwise. Let further check the in-sample fit of the model:



the in-sample fit closely follow the original series. However the residuals demonstrate varying variance, we apply GARCH model on the residuals to correct the prediction uncertainty:



The prediction of $ARIMA(0,1,0)$ gives a flat prediction, due to the fact that the model doesn't have AR and MA coefficient. Thus the prediction is equivalent to a random walk without noise, which has become a constant of the last observation. However, after correct the prediction variance, we find the confidence interval is reduced compared with that given by ARIMA model. Finally, the ACF of squared residuals of GARCH model shows the variance of residual is no long varying by time.

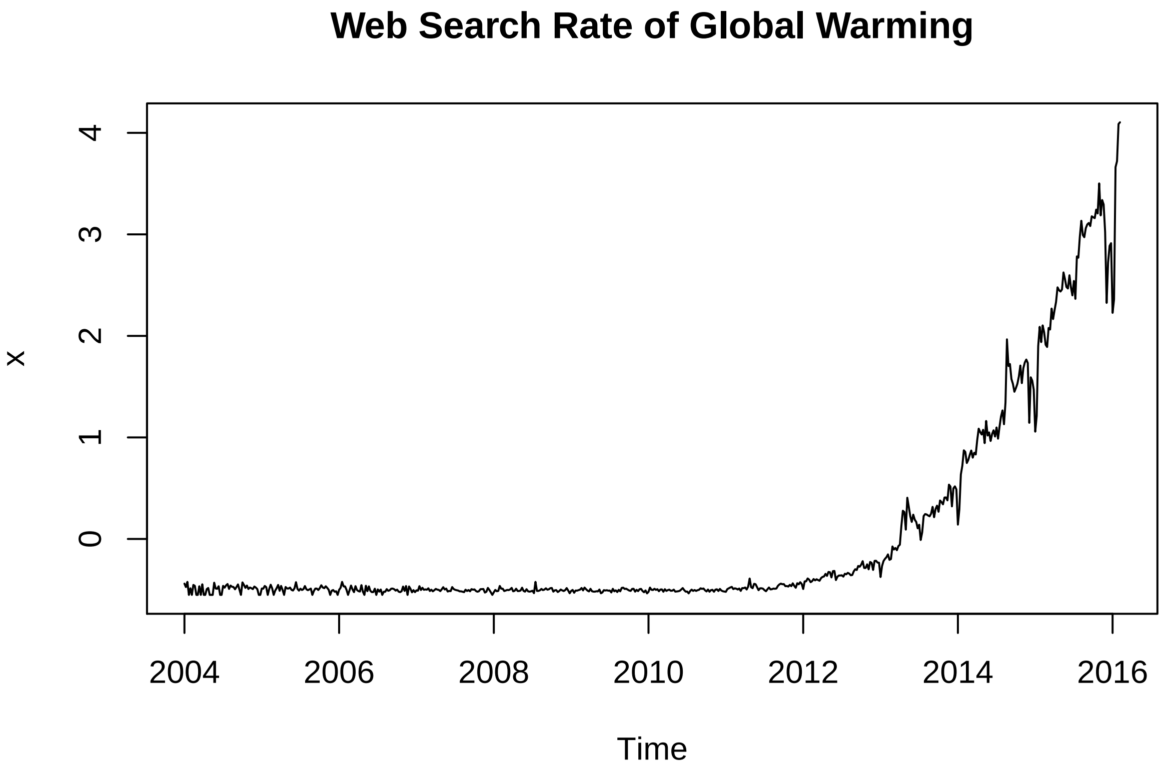


Part 3

Load data and show descriptive statistics:

```
## 'data.frame':    630 obs. of  2 variables:
##  $ Date        : Factor w/ 630 levels "1/1/06","1/1/12",..: 47 5 18 33 215 260 226 239 251 311 ...
##  $ data.science: num  -0.44 -0.474 -0.423 -0.551 -0.486 -0.551 -0.453 -0.462 -0.551 -0.551 ...
```

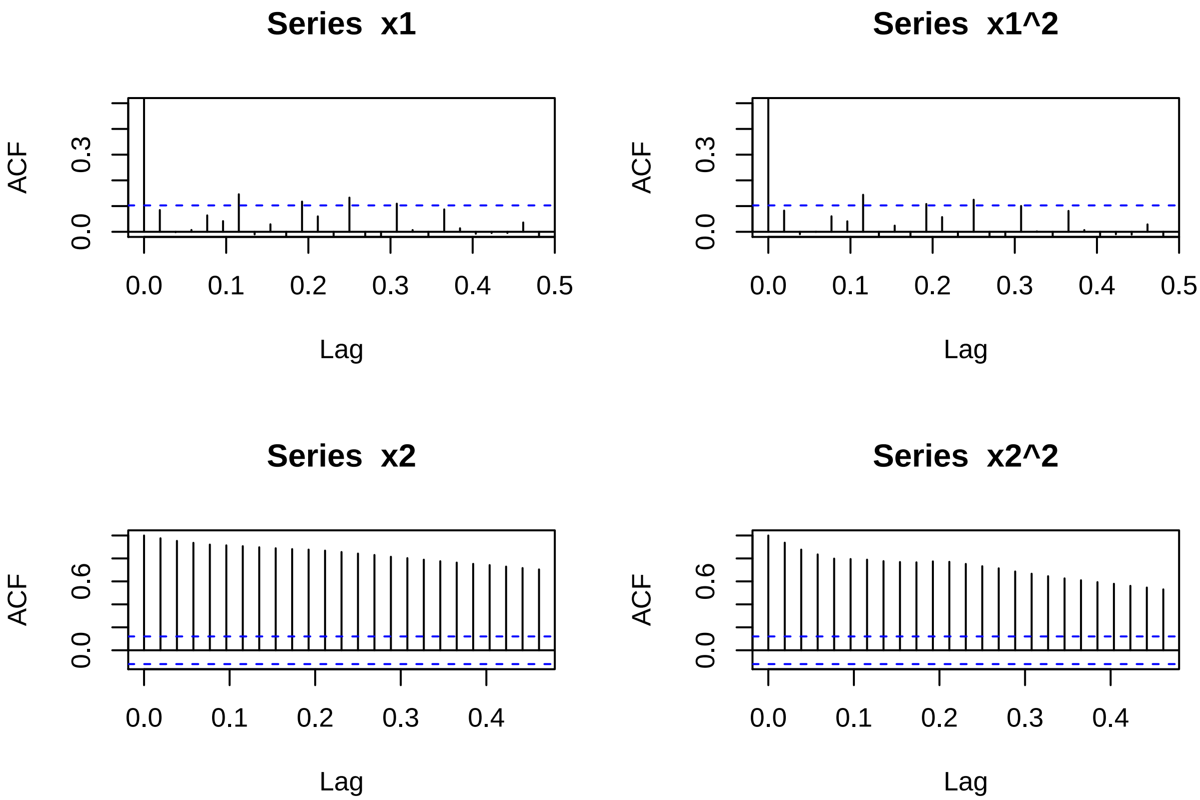
```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.551000 -0.506000 -0.485000  0.000038 -0.200000  4.104000
```



upon close look, it appears the majority of the dynamics of the series appears after 2011, which prior to that it's mostly flat without too much changes. Let's then cut the series into two: before and after 2011

```
cutoff = 365
x1 <- ts(data$data.science[1:cutoff], start=c(2004,1,4), frequency = 52)
x2 <- ts(data$data.science[(cutoff+1):630], start=c(2011,1,2), frequency = 52)
```

we then evaluate ACF of both series, and conduct Box test:

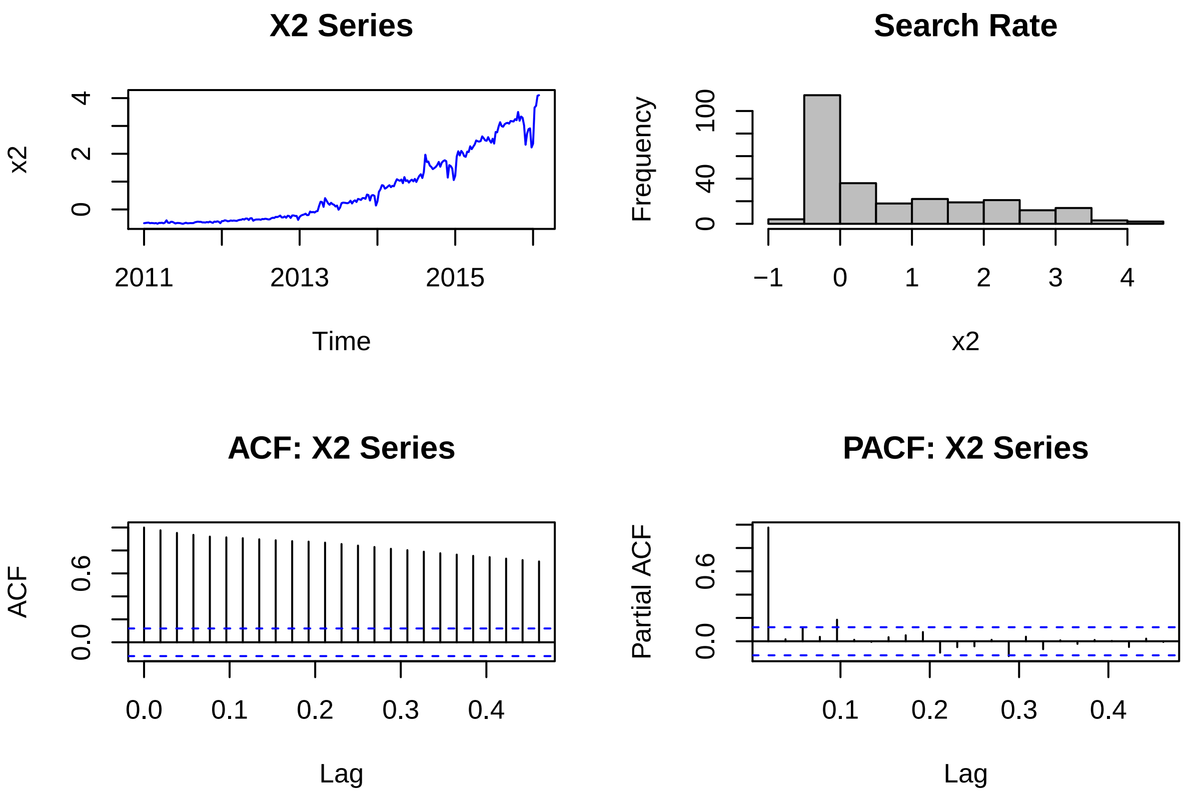



```
##
##  Box-Ljung test
##
## data:  x1
## X-squared = 2.6322, df = 1, p-value = 0.1047
```

```
##
##  Box-Ljung test
##
## data:  x2
## X-squared = 255.178, df = 1, p-value < 2.2e-16
```

For series x_1, ACF of both original and squared value don't show significant autocorrelation, while x_2 shows strong autocorrelation for both. In addition, the Box test indicates that we can't reject the null hypothesis that x_1 is white-noise, white x_2 is significantly autocorrelated.

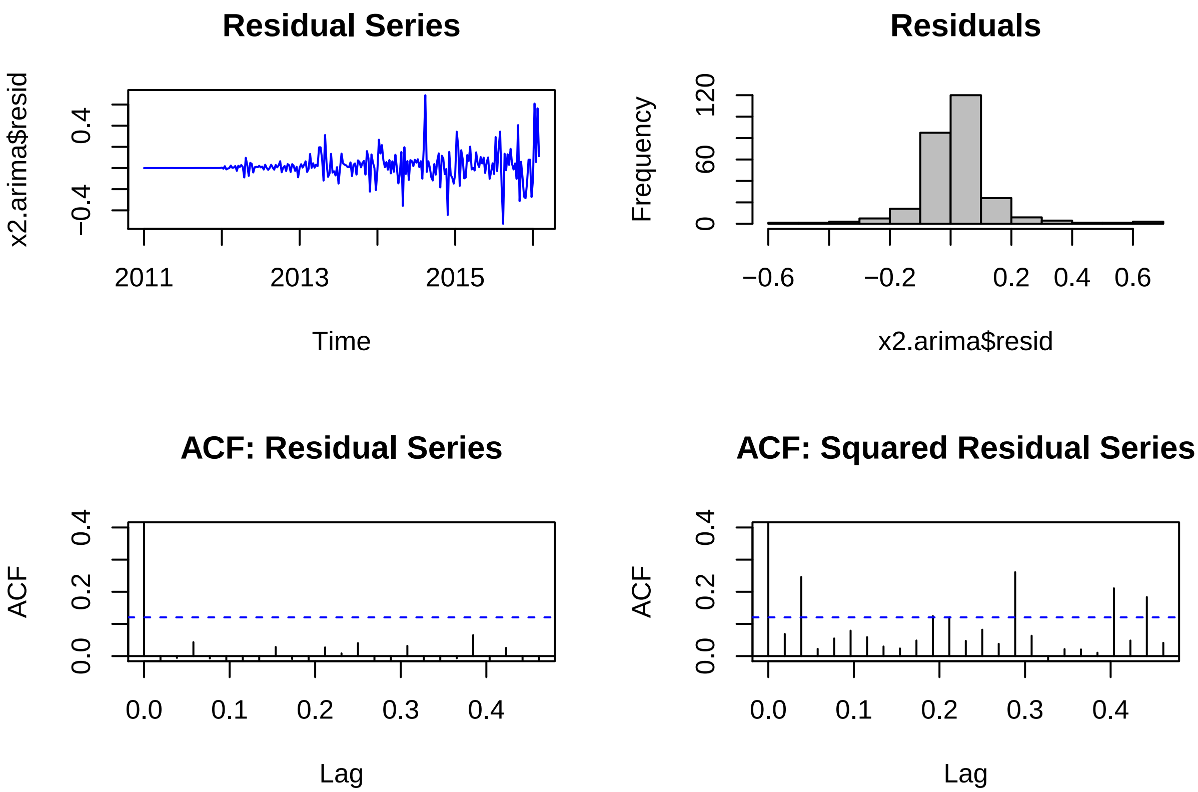
Thus we will focus on modeling x_2 to predict the search rate of Gobal Warming.



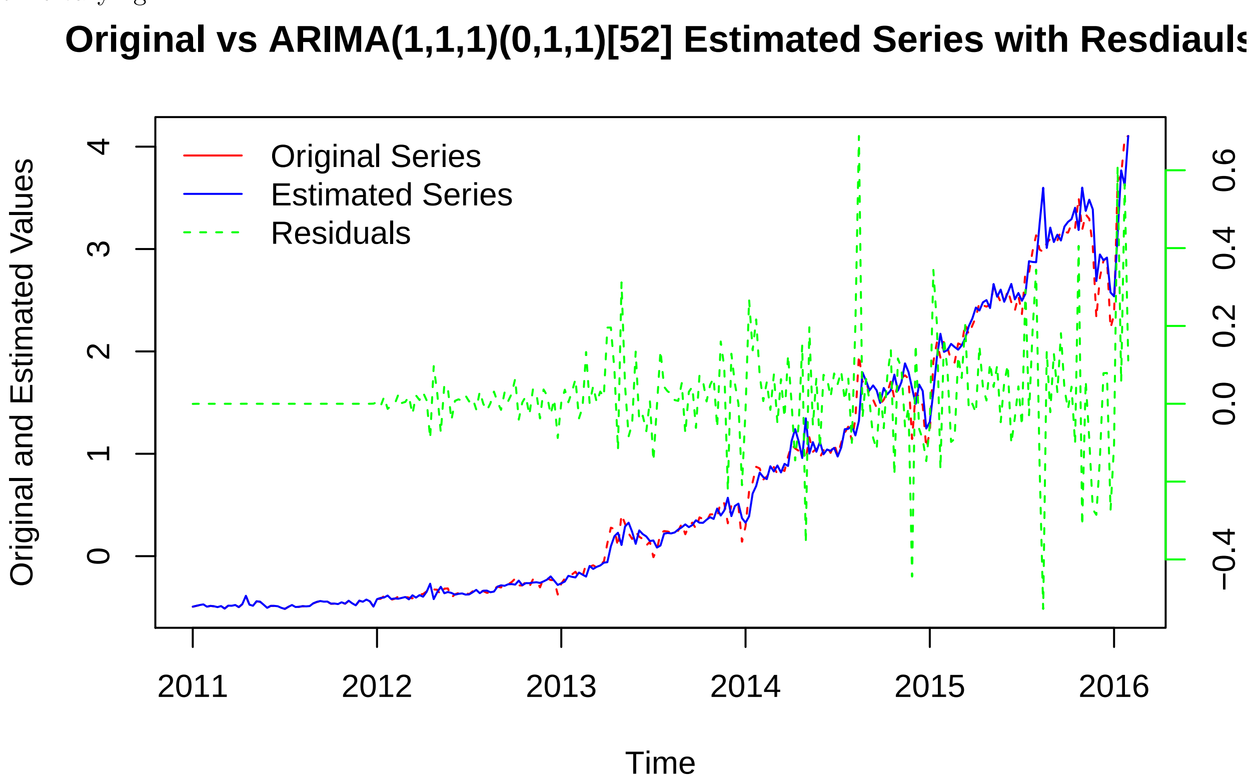
From ACF we see that it's a non-staionary series, which show strong persistent upward trend. We then use ARIMA model to fit it.

```
## Series: x2
## ARIMA(1,1,1)(0,1,1)[52]
##
## Coefficients:
##          ar1      ma1     sma1
##       0.4550  -0.7844  -0.1681
## s.e.  0.1423   0.1072   0.0723
##
## sigma^2 estimated as 0.01957:  log likelihood=115.26
## AIC=-222.52   AICc=-222.32   BIC=-209.09
```

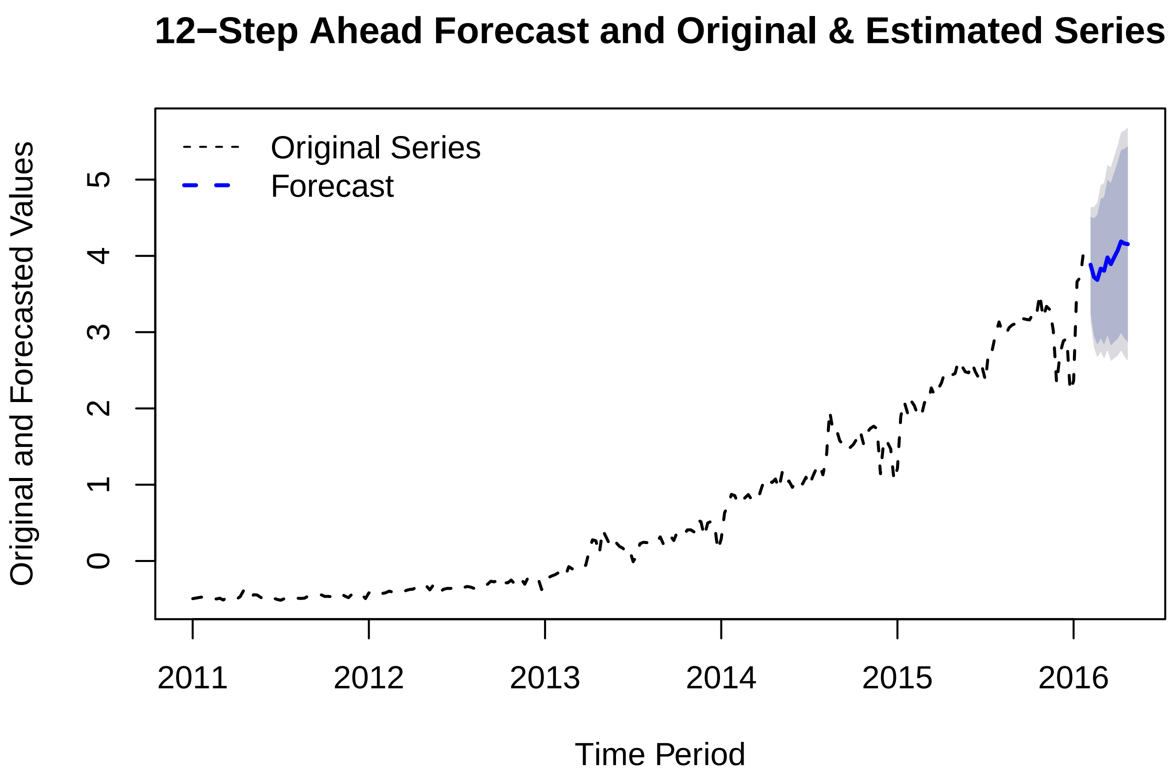
```
##
##  Box-Ljung test
##
## data:  x2.arima$resid
## X-squared = 0.4348, df = 1, p-value = 0.5096
```



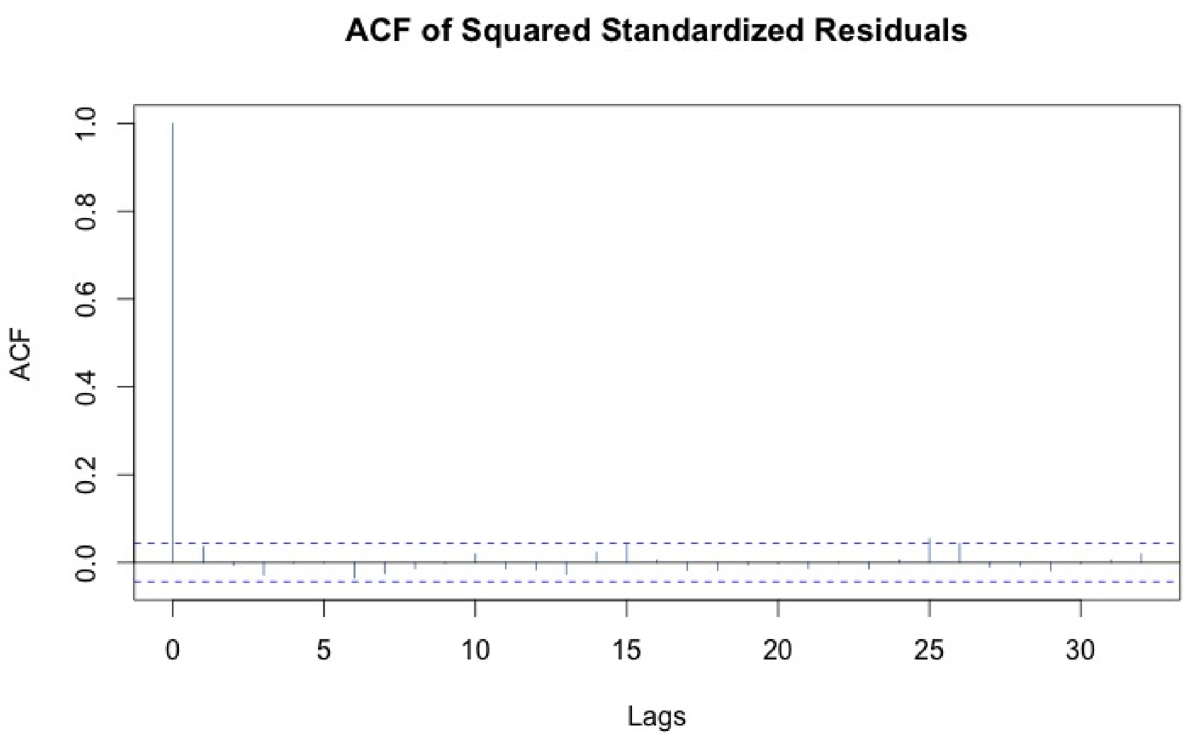
The best model is ARIMA(1,1,1)(0,1,1)[52], and the ACF of squared residual also indicate its variation is time-varying.



Similar with part 2, we fit a GARCH model on the residuals, and integrate the conditional variance from GARCH into the prediction of ARIMA model:



Finally, the ACF of squared residuals of GARCH model shows the variance of residual is no long varying by time.

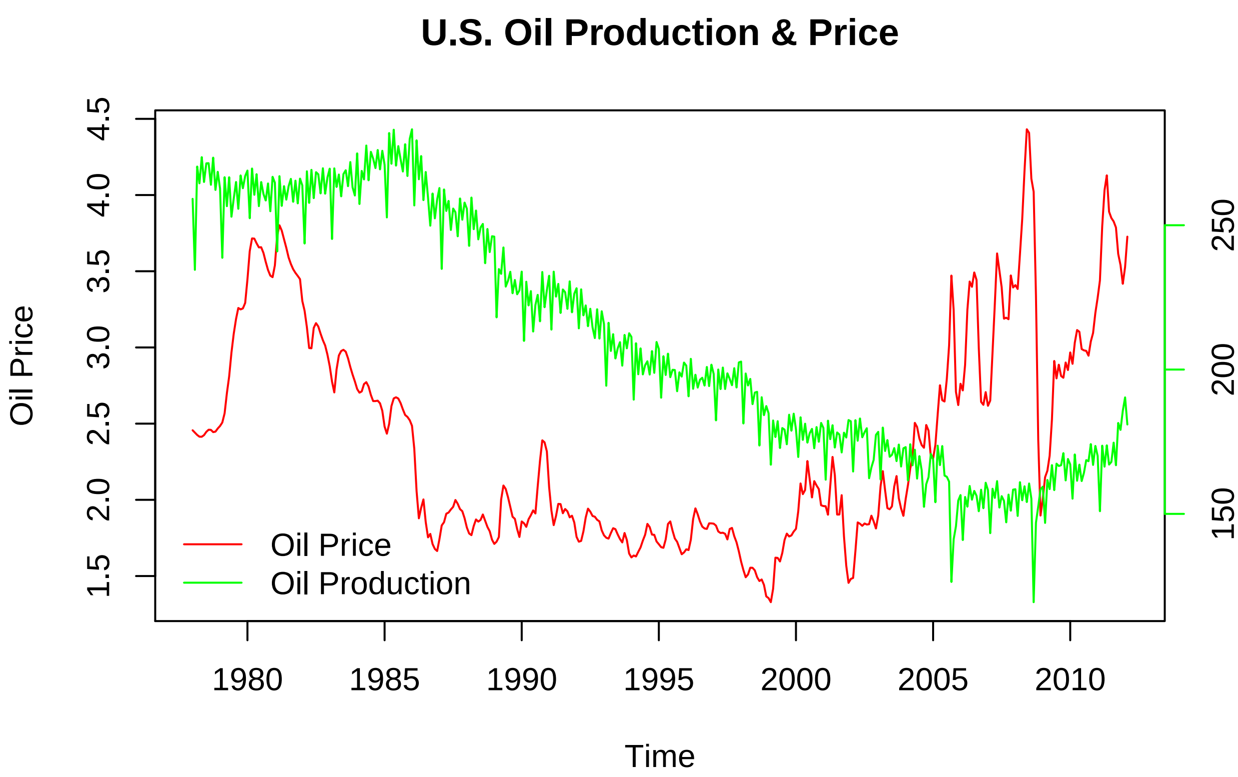


Part 4

Load data and show descriptive statistics:

```
## 'data.frame':    410 obs. of  3 variables:
##  $ Date      : chr  "1978-01-01" "1978-02-01" "1978-03-01" "1978-04-01" ...
##  $ Production: num  259 235 270 265 274 ...
##  $ Price     : num  2.46 2.44 2.43 2.41 2.41 ...
```

```
##      Date             Production        Price
##  Length:410         Min.   :119.4   Min.   :1.329
##  Class :character   1st Qu.:173.0   1st Qu.:1.823
##  Mode  :character   Median :201.4   Median :2.096
##                     Mean   :210.0   Mean   :2.391
##                     3rd Qu.:255.8   3rd Qu.:2.909
##                     Max.   :283.2   Max.   :4.432
```



Task 1 - Reproduce AP analysis

One naive way to analyze the data is to build a simple regression model between oil production and pric, and check the significance of the regression coefficient. For example:

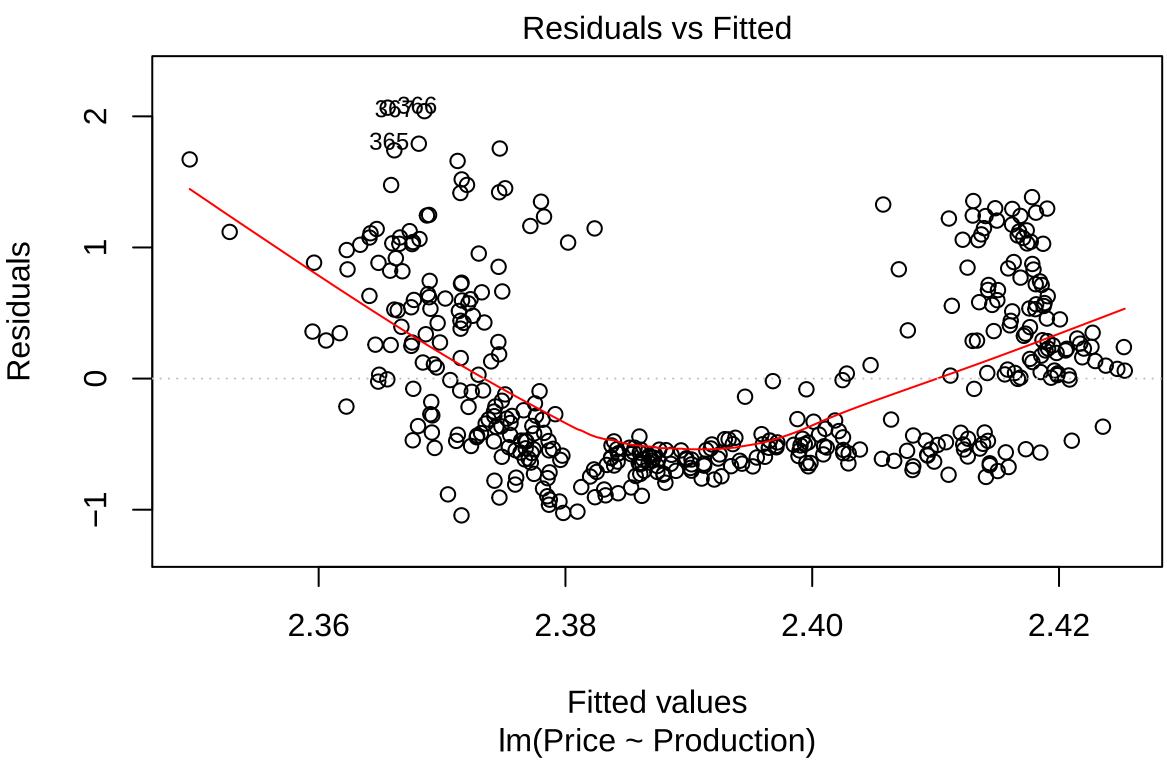
```r
m0 <- lm(Price~Production, data=gasOil)
summary(m0)
```

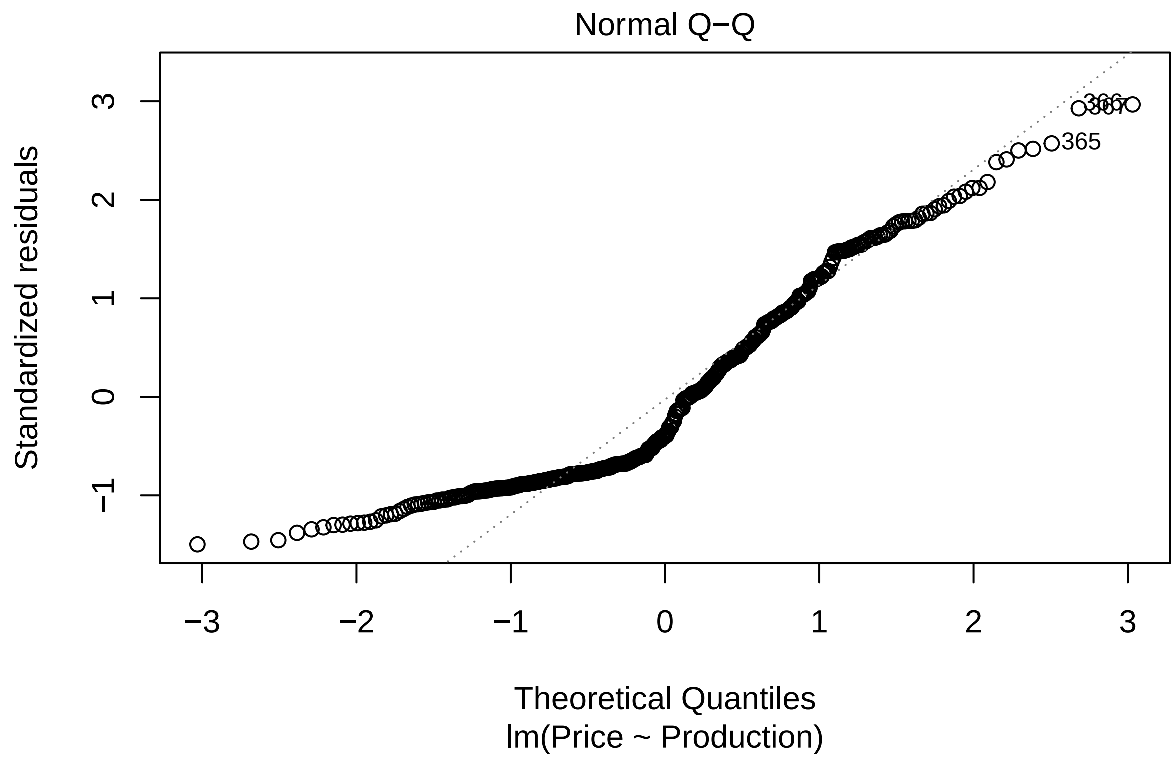
```
##
## Call:
```

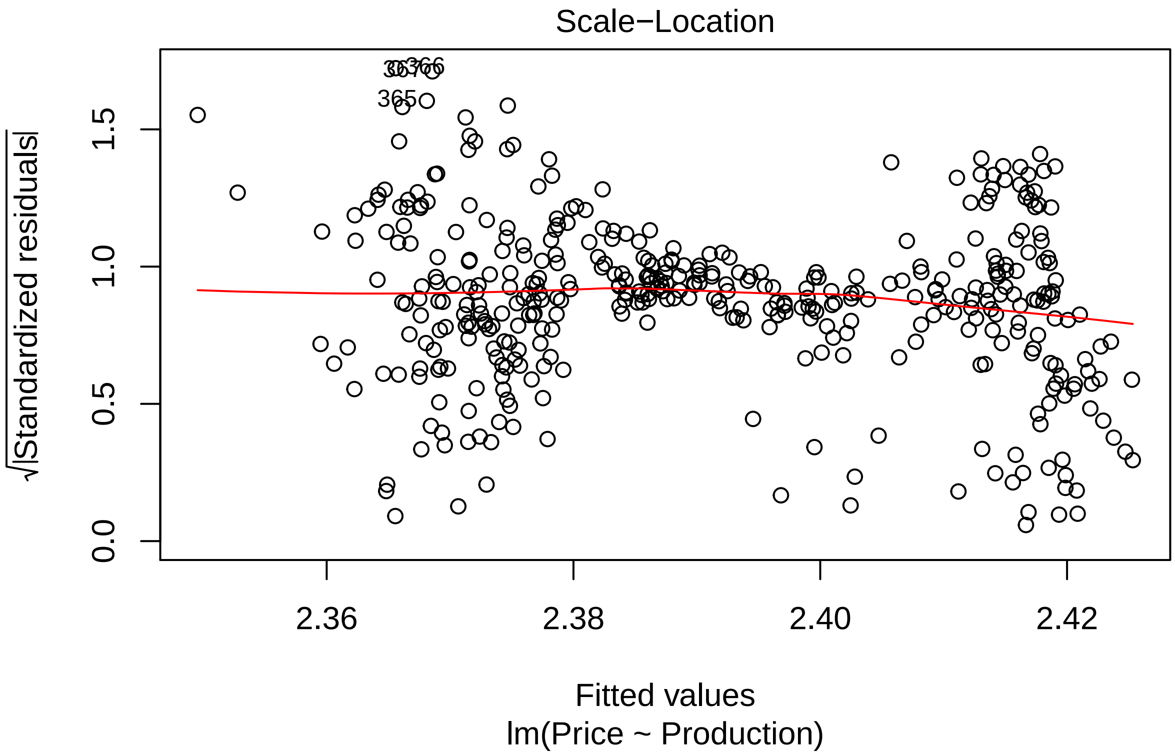
```
## lm(formula = Price ~ Production, data = gasOil)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.0430 -0.5683 -0.2762  0.5287  2.0660
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.2943109  0.1765964  12.992   <2e-16 ***
## Production  0.0004626  0.0008247   0.561    0.575
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6984 on 408 degrees of freedom
## Multiple R-squared:  0.0007705,  Adjusted R-squared:  -0.001679
## F-statistic: 0.3146 on 1 and 408 DF,  p-value: 0.5752
```

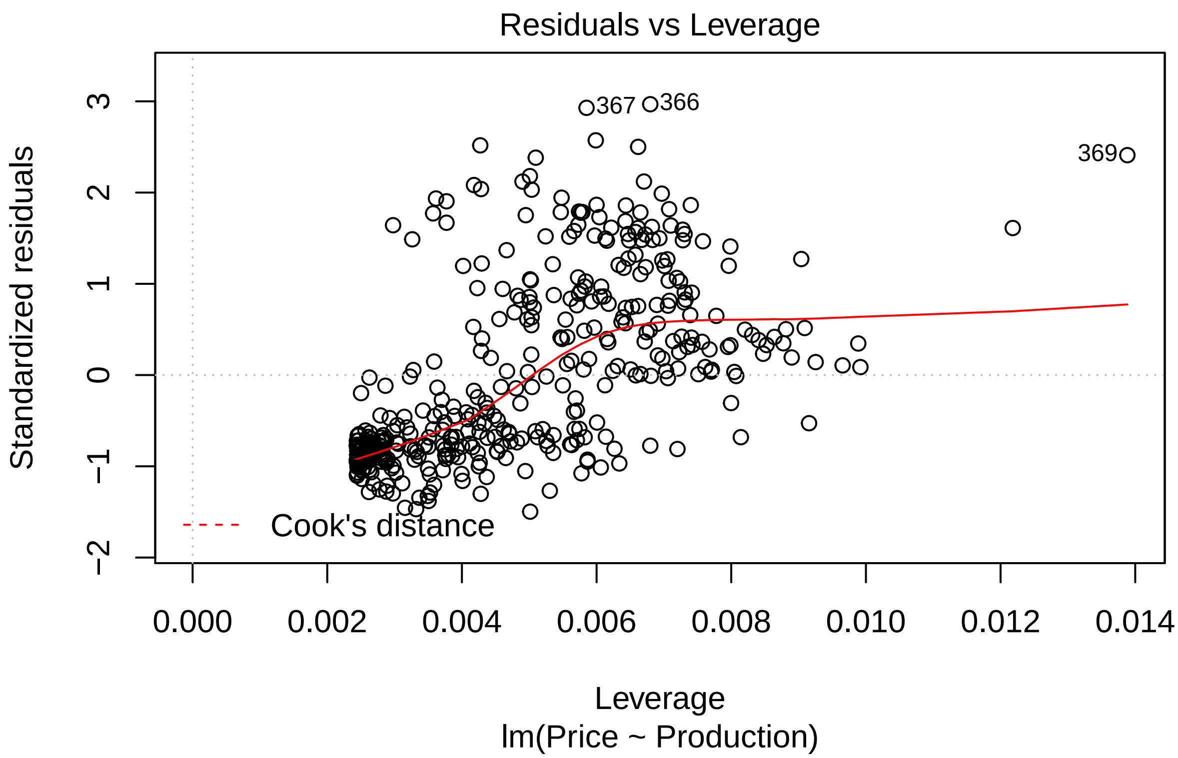
we can see the p-value for the effect of oil production on price here is 0.575, which is insignificant. And with this number we can claim that there is "evidence of no statistical correlation" between oil production and gas prices.

However, with further model diagnostics, we can see this model violate basically all assumptions of linear regression:




Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(Price ~ Production)
365
366
367


Scale−Location
√|Standardized residuals|
Fitted values
lm(Price ~ Production)
365
366
367



we can see that the residue is heterskodastic, non-normal, and the dependent variable does not have zero-conditional mean. In fact, if we add a nonlinear term in the model, we can have significant result:

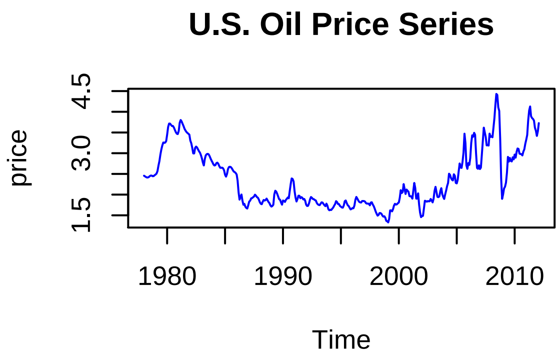
```
gasOil$Production2 <- gasOil$Production^2
m0 <- lm(Price~Production+Production2, data=gasOil)
summary(m0)
```

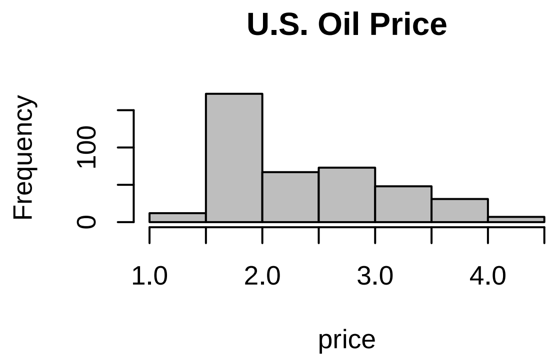
```
##
## Call:
## lm(formula = Price ~ Production + Production2, data = gasOil)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.1704 -0.3518 -0.1100  0.2580  1.8083
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.598e+01  9.020e-01   17.72   <2e-16 ***
## Production  -1.328e-01  8.701e-03  -15.27   <2e-16 ***
## Production2  3.120e-04  2.031e-05   15.36   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5563 on 407 degrees of freedom
## Multiple R-squared:  0.3675, Adjusted R-squared:  0.3644
## F-statistic: 118.3 on 2 and 407 DF,  p-value: < 2.2e-16
```

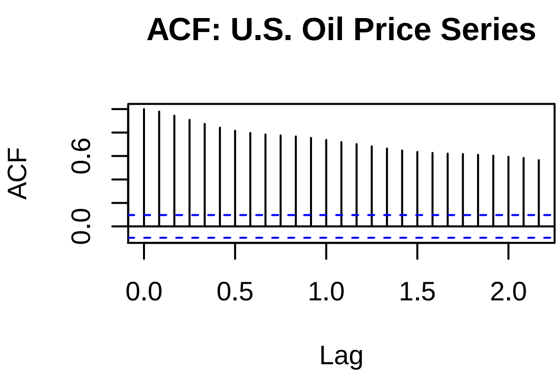
here the model indicates that when oil production increases the gas price will drop and the effect is statistically significant.

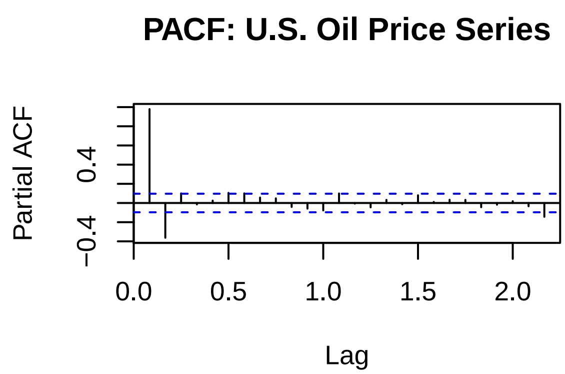
To sumarize, the production series is very volatile, for regression analysis it's better to smooth it with moving average window. In addition, we can see before ~1985, when the oil production went up gas price is decreasing. After that, oil production keeps decreasing and gas price is slightly decreasing as well before ~2000. Then the price start trending up again. Therefore it is unconvincing to simply say the two are not correlated. The analysis needs to be put under certain context and time period. One way to investigate is through instrumental variable, with some variable that is irreleavant with gas price, but is highly correlated with oil production.

Task 2 - Forecast the inflation-adjusted gas prices from 2012 to 2016



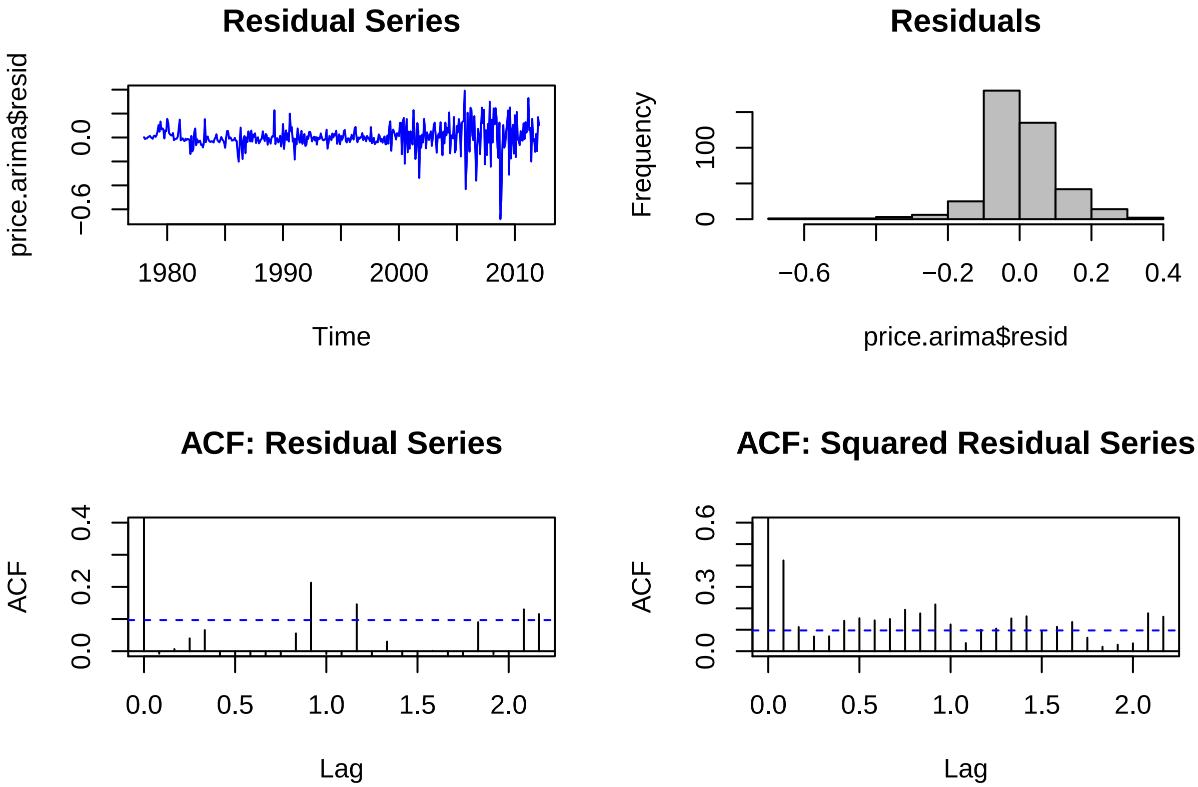




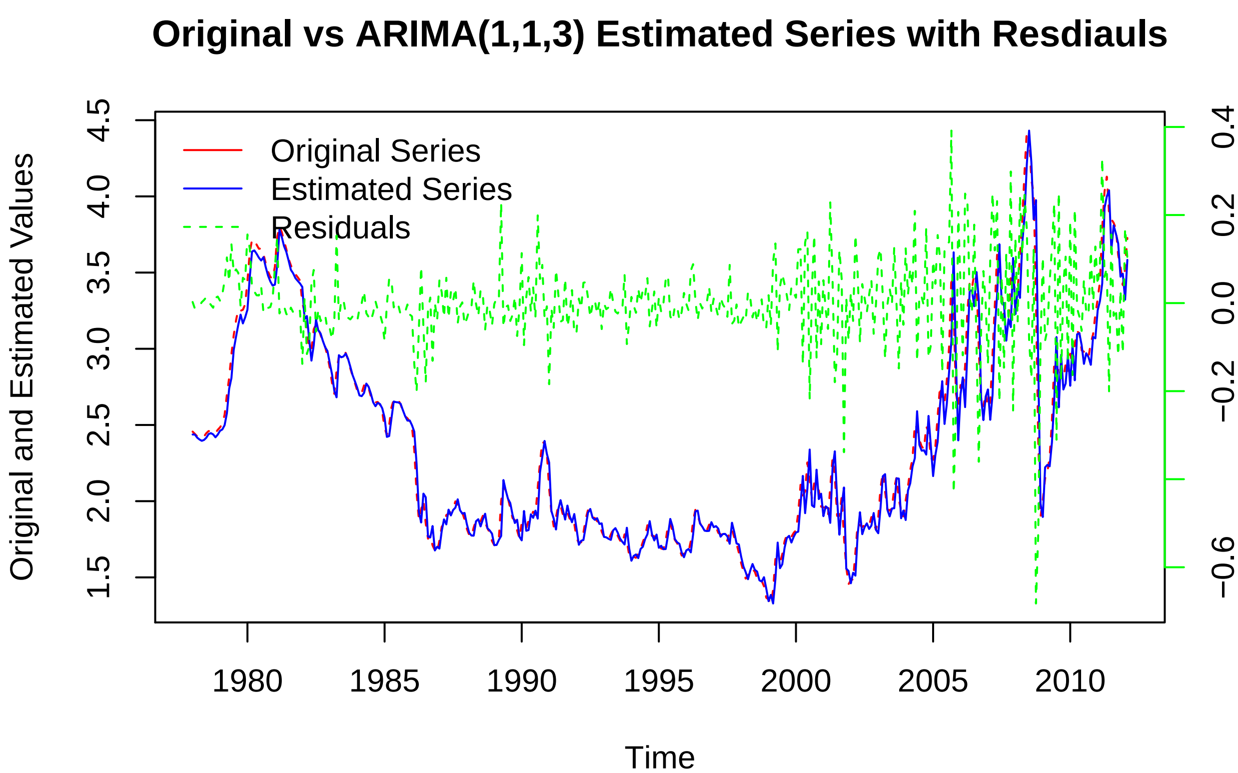


From ACF we see that it's a non-staionary series, which show strong persistent upward trend. We then use ARIMA model to fit it.

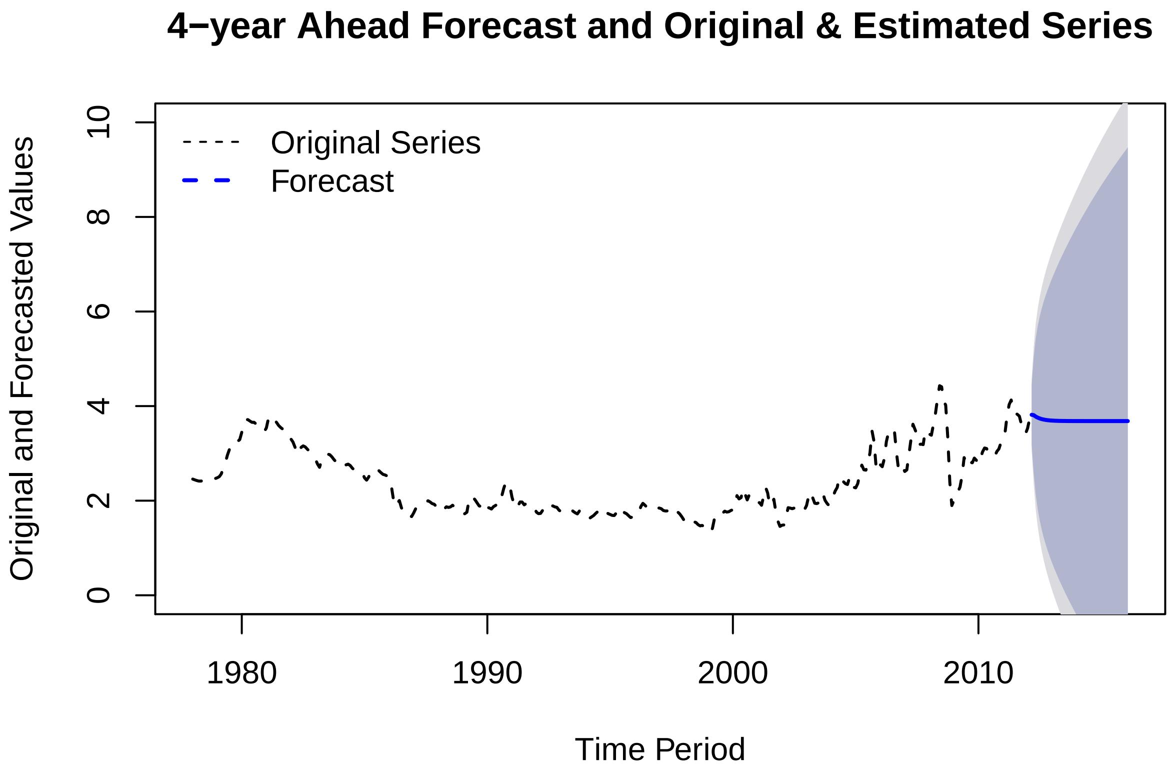
```
##
##  Box-Ljung test
##
## data:  price.arima$resid
## X-squared = 0.0245, df = 1, p-value = 0.8755
```



The best model is ARIMA(1,1,3), and the ACF of squared residual also indicate its variation is time-varying.



Similar with part 2, we fit a GARCH model on the residuals, and integrate the conditional variance from GARCH into the prediction of ARIMA model:



Finally, the ACF of squared residuals of GARCH model shows the variance of residual is no long varying by time.