MIDS W271-4 Lab 3

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Part 1 - Modeling House Values

Exploratory Data Analysis

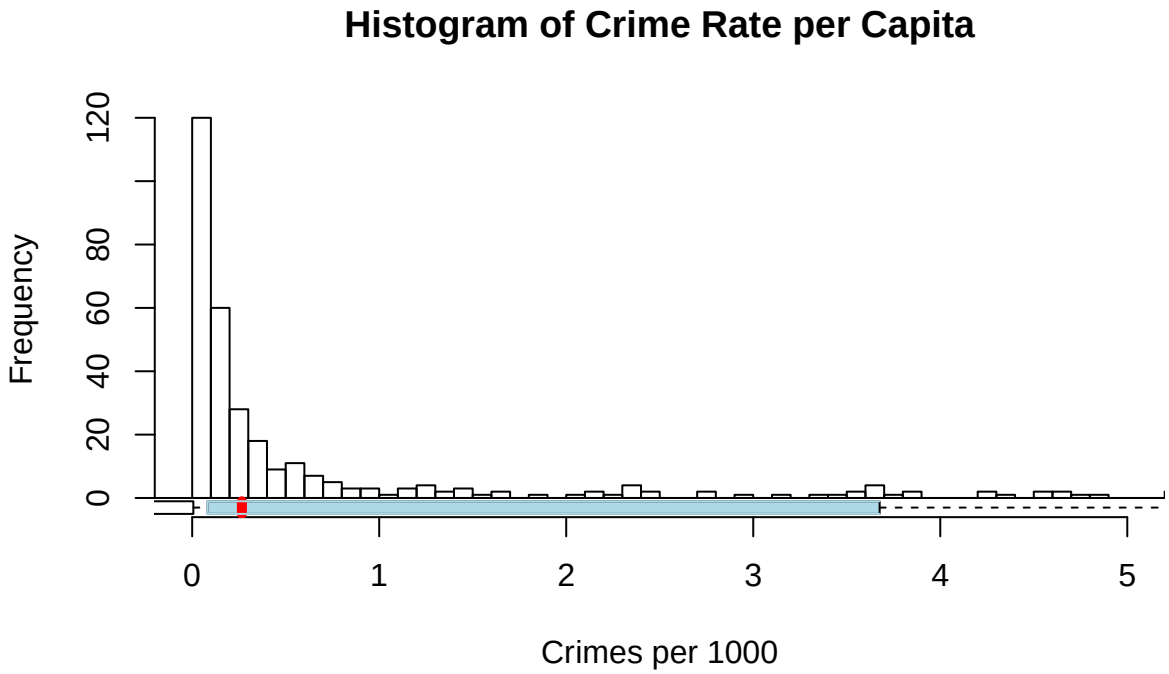
An examination of the provided data set reveals 11 variables of which *withWater* is binary and rest are continuous. There are no NA's in the data set, however the *distanceToHighway* variable appears to have a coding issue. We'll examine this variable in more detail a bit later, but first we summarize the variables in the following table.

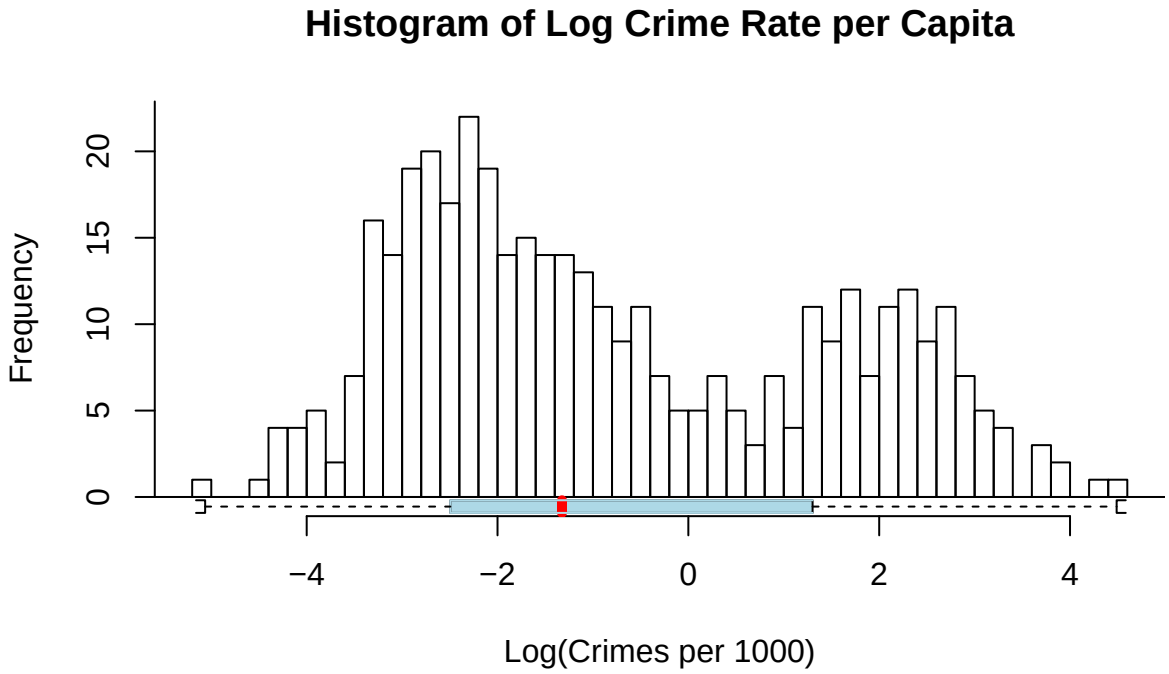
Table 1: Summary of Data

Statistic	N	Mean	St. Dev.	Min	Max
crimeRate_pc	400	3.763	8.872	0.006	88.976
nonRetailBusiness	400	0.112	0.070	0.007	0.277
withWater	400	0.068	0.251	0	1
ageHouse	400	68.932	27.977	2.900	100.000
distanceToCity	400	9.638	8.786	1.228	54.197
distanceToHighway	400	9.582	8.672	1	24
pupilTeacherRatio	400	21.391	2.168	15.600	25.000
pctLowIncome	400	15.795	9.341	2	49
homeValue	400	499,584.400	196,115.700	112,500	1,125,000
pollutionIndex	400	40.615	11.825	23.500	72.100
nBedRooms	400	4.266	0.719	1.561	6.780

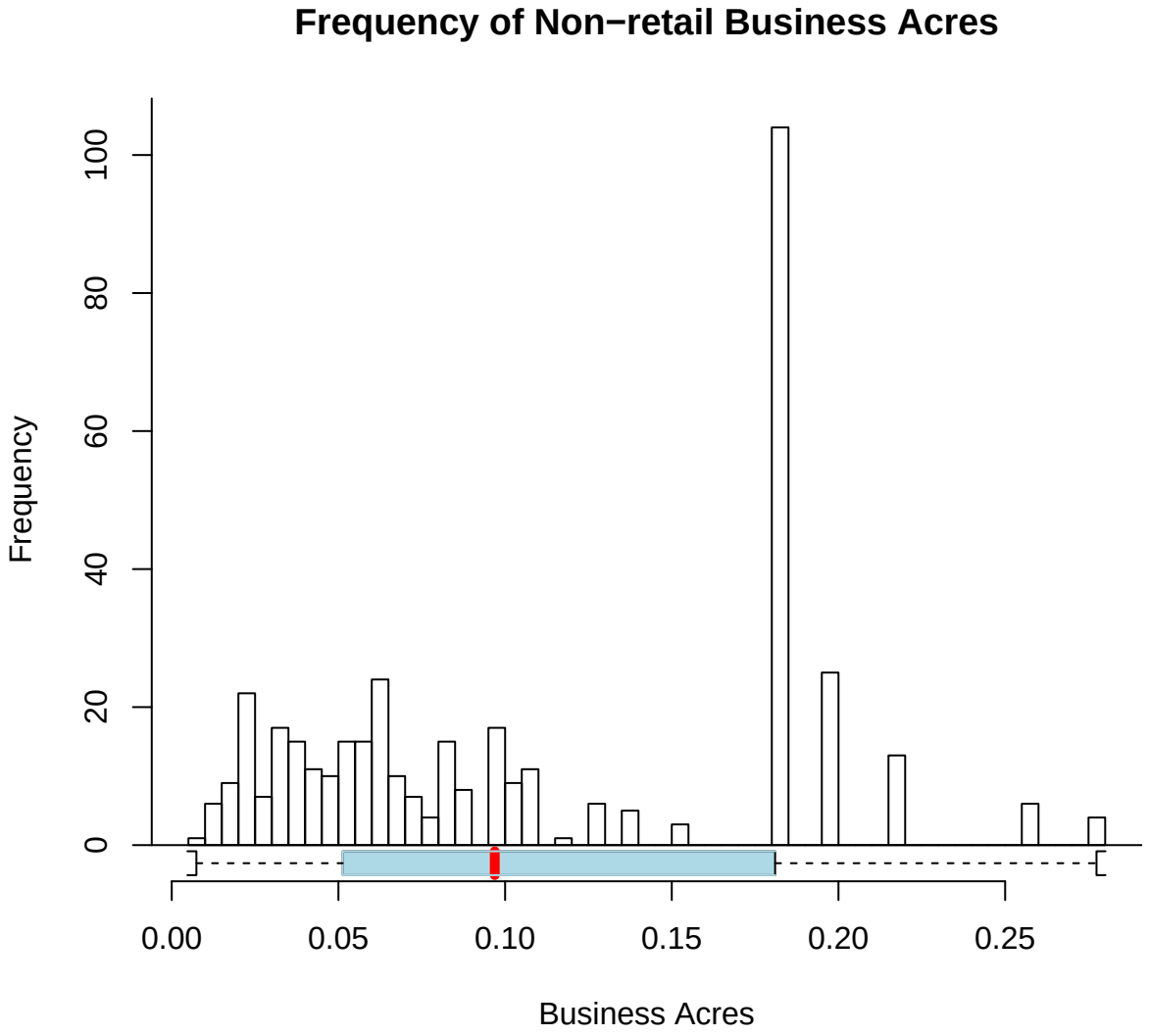
For the purposes of this analysis we categorize the variables *crimeRate_pc*, *nonRetailBusiness*, *withWater*, *distanceToCity*, *distanceToHighway*, *pollutionIndex*, *pupilTeacherRation* and *pctLowHousing* to be environmental variables. The variables *ageHouse* and *nBedRooms* are attributes of the house. The variable *homeValue* is the dependent variable we would like to explain in terms of primarily the environment variables but we will compare to explanations in terms of house attributes as well.

In the next several pages we examine the distribution of each of the variables and, where indicated, the distribution of the transformed as well.

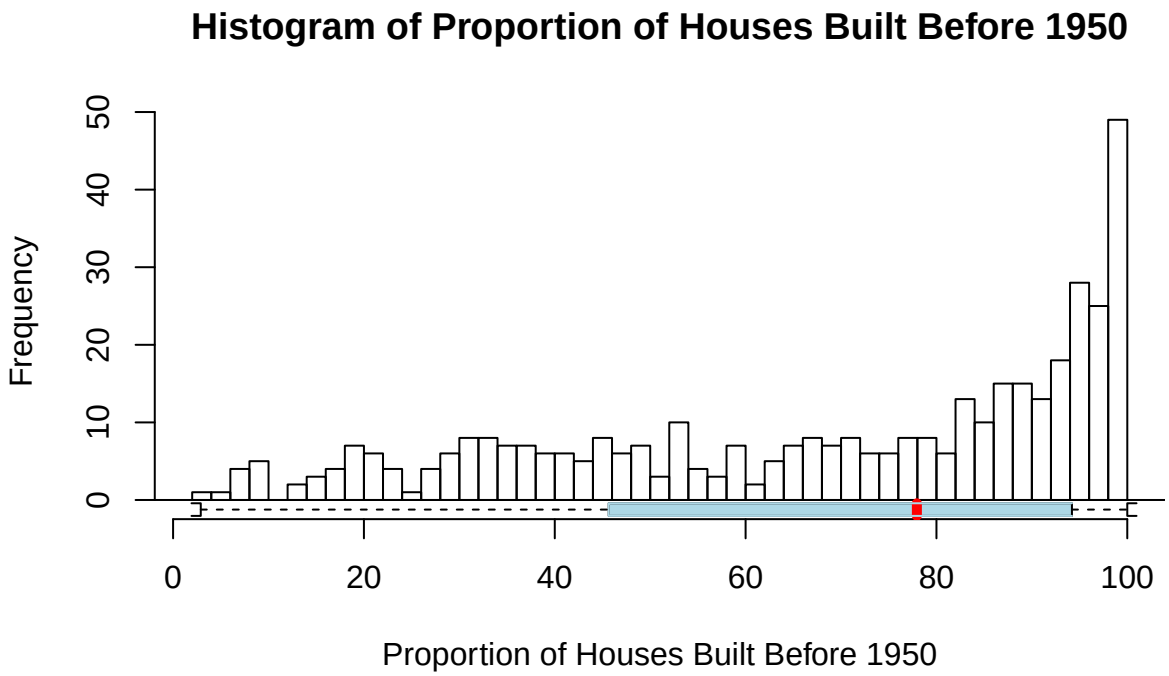


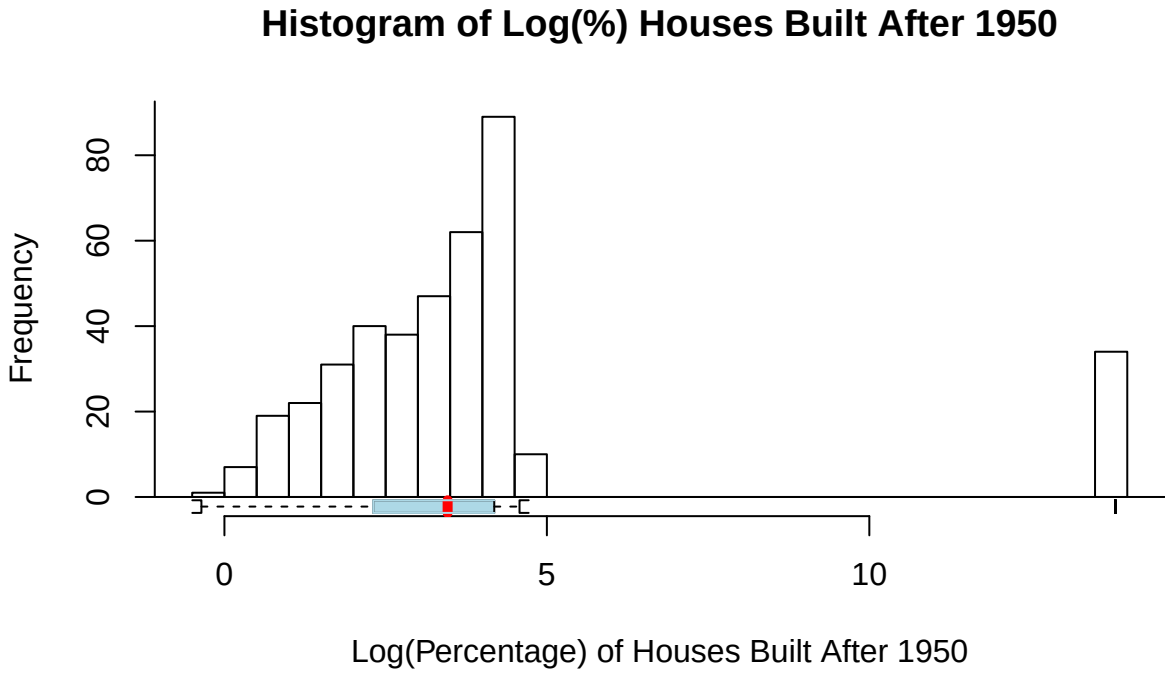


The distribution of *crimeRate_pc* is highly right-skewed with a very long tail. This makes sense as most neighborhoods are very low crime neighborhoods. The distribution of *log(crimeRate_pc)* appears almost bi-modal; however the analysis of skew and kurtosis show a significant improvement of each with a log transformation.

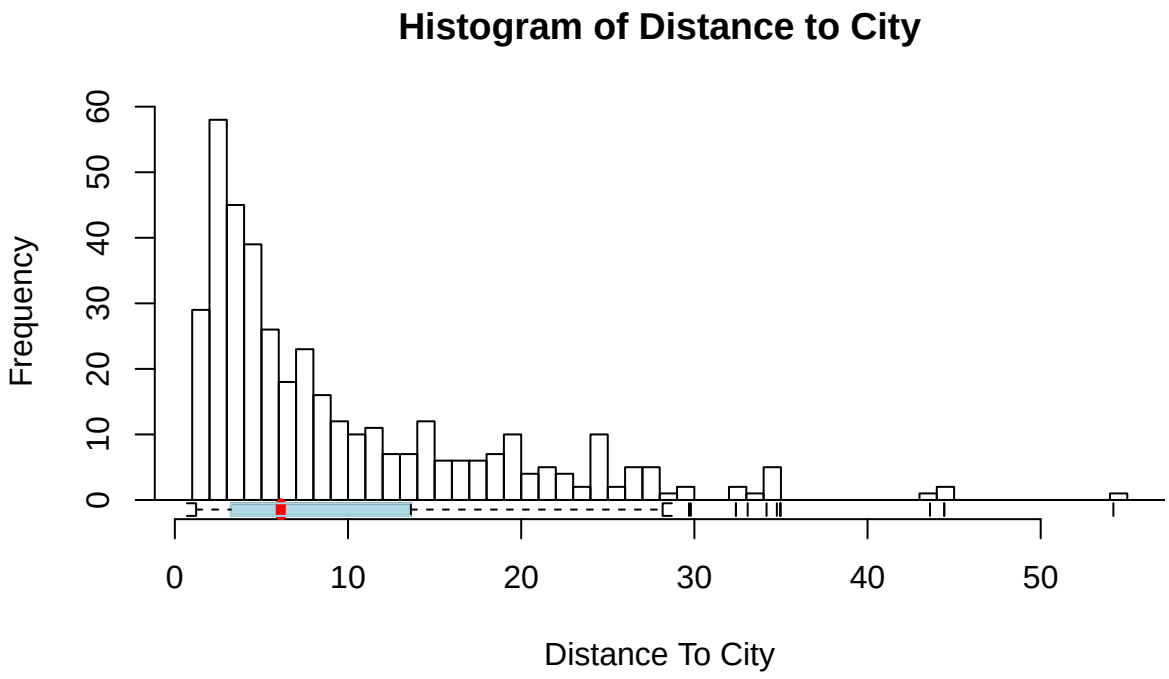


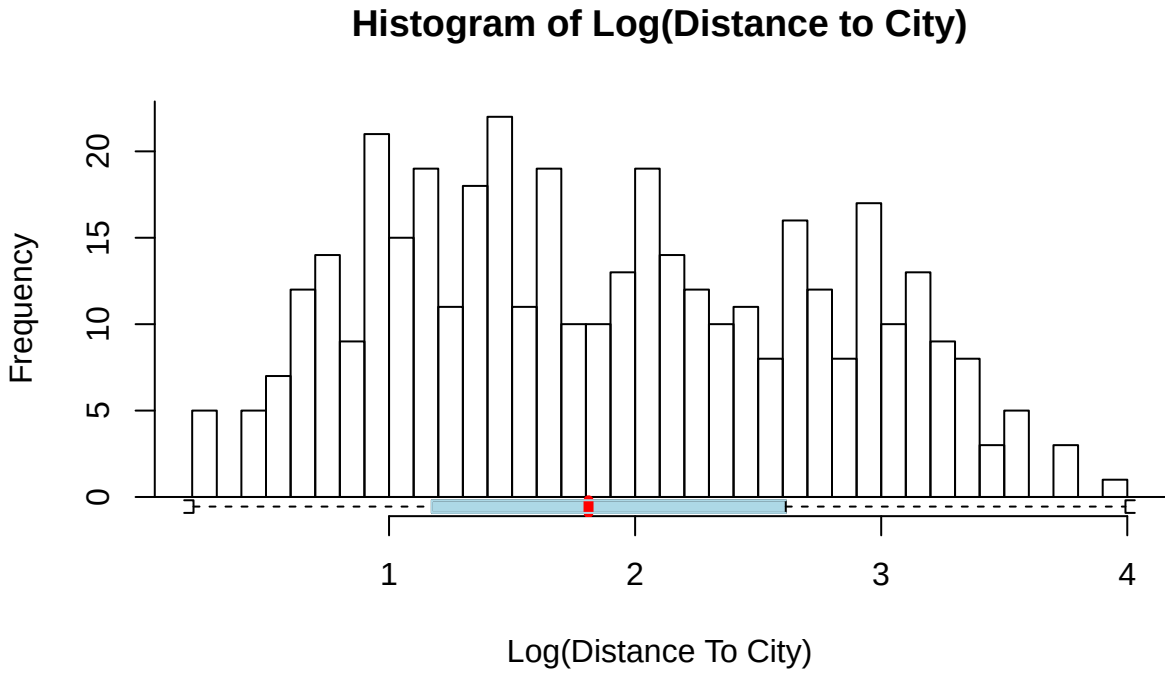
The variable *nonRetailBusiness* is a measure of the footprint of industy in a neaghborhood. This may range from light industrial to manufacturing but that information is not given. The distribution of *nonRetailBusiness* shows a spike at 0.18 business-acres but is otherwise somewhat uniform. There was no transform that improved skew or kurtosis for this variable.



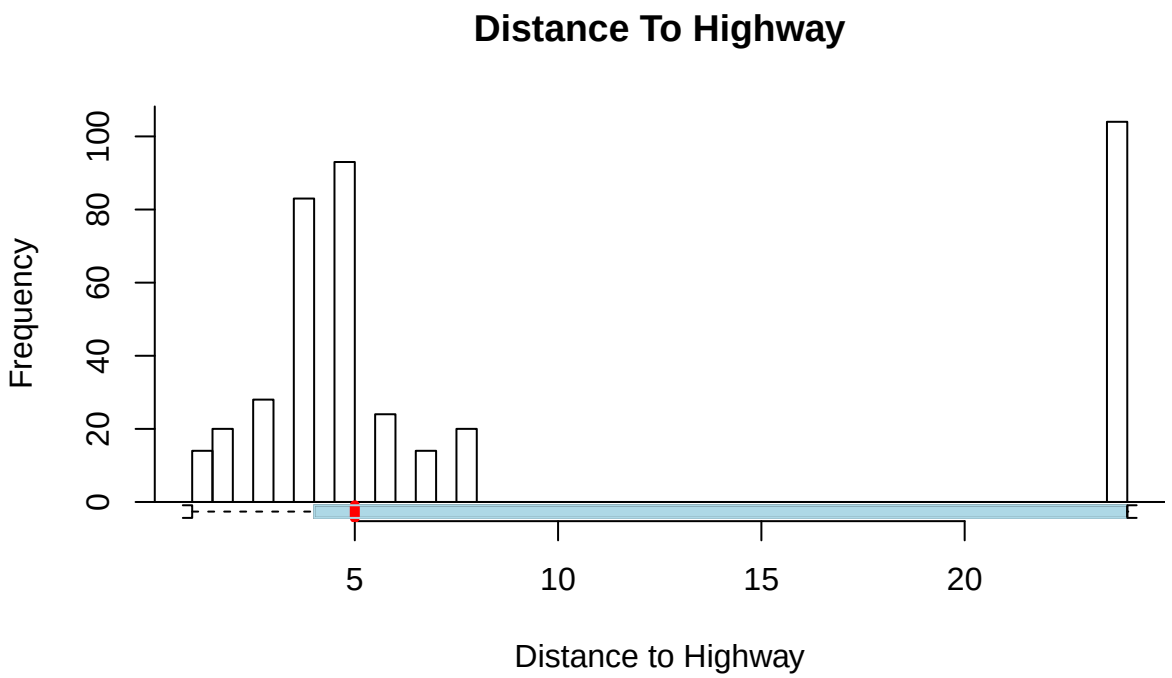


The variable *ageHouse* is the percentage of houses in a neighborhood built before 1950 and shows a significant left-skew with a long tail to the left. Subtracting the variable from 100% transforms it from percentage of houses built before 1950 to percentage of houses built after 1950, giving a left skewed distribution. Taking the log of the distribution helps remove the skew.



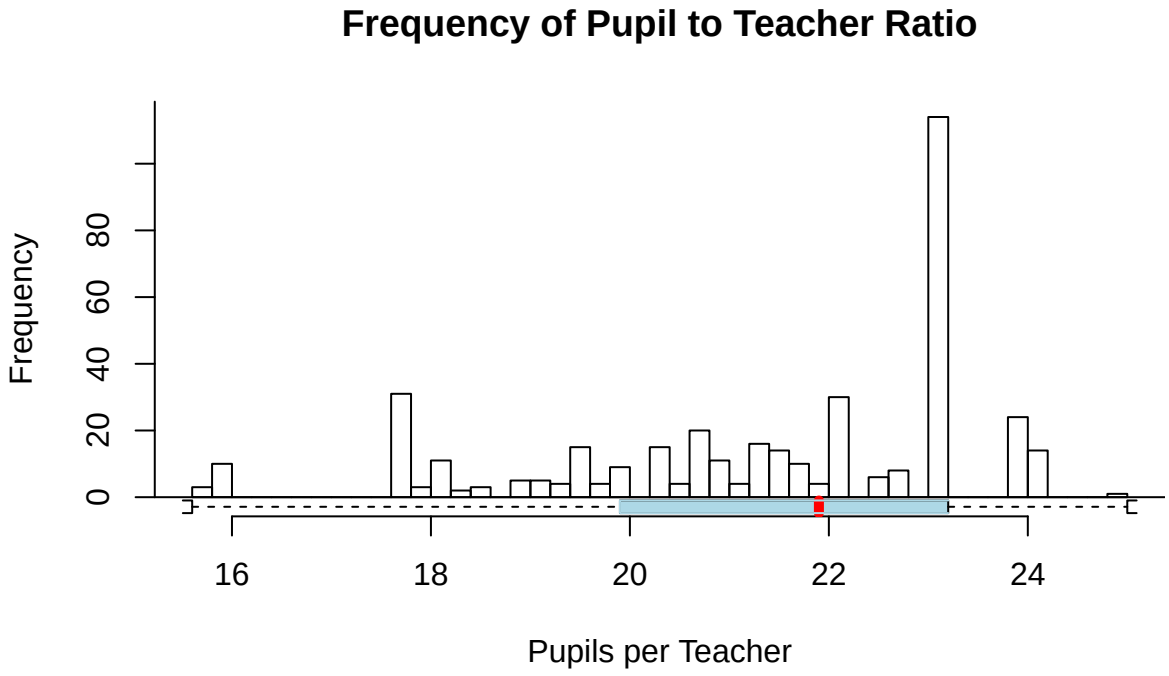


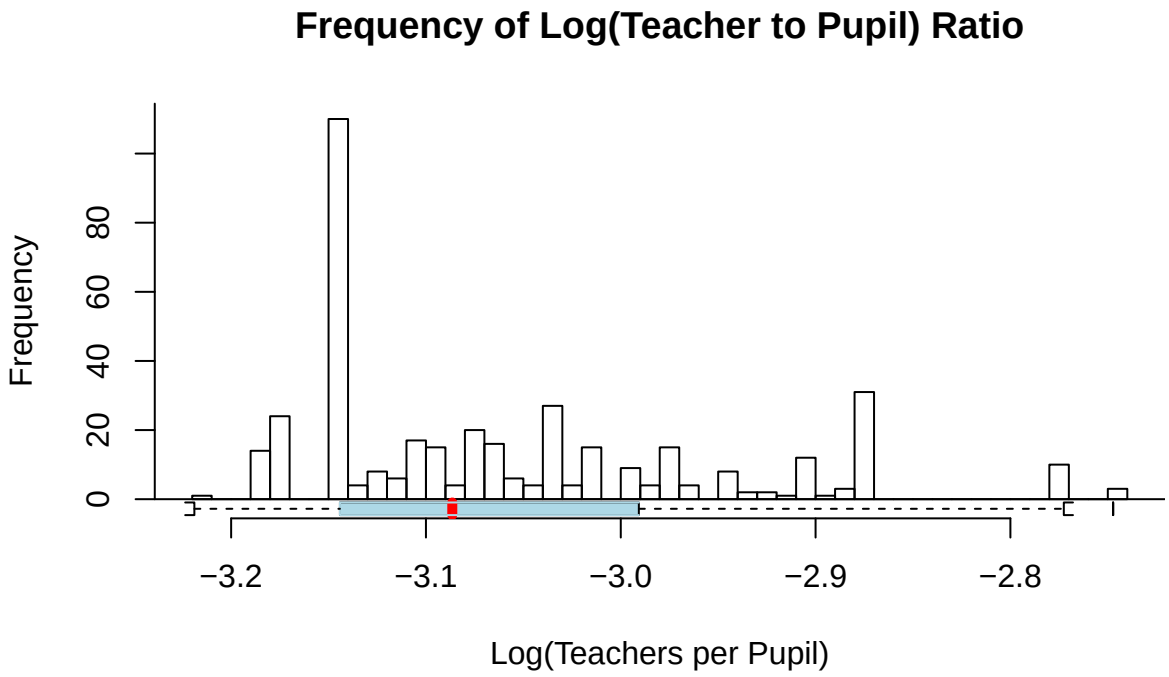
The *distanceToCity* variable shows a right-skewed distrubution that is much improved by a log transformation.



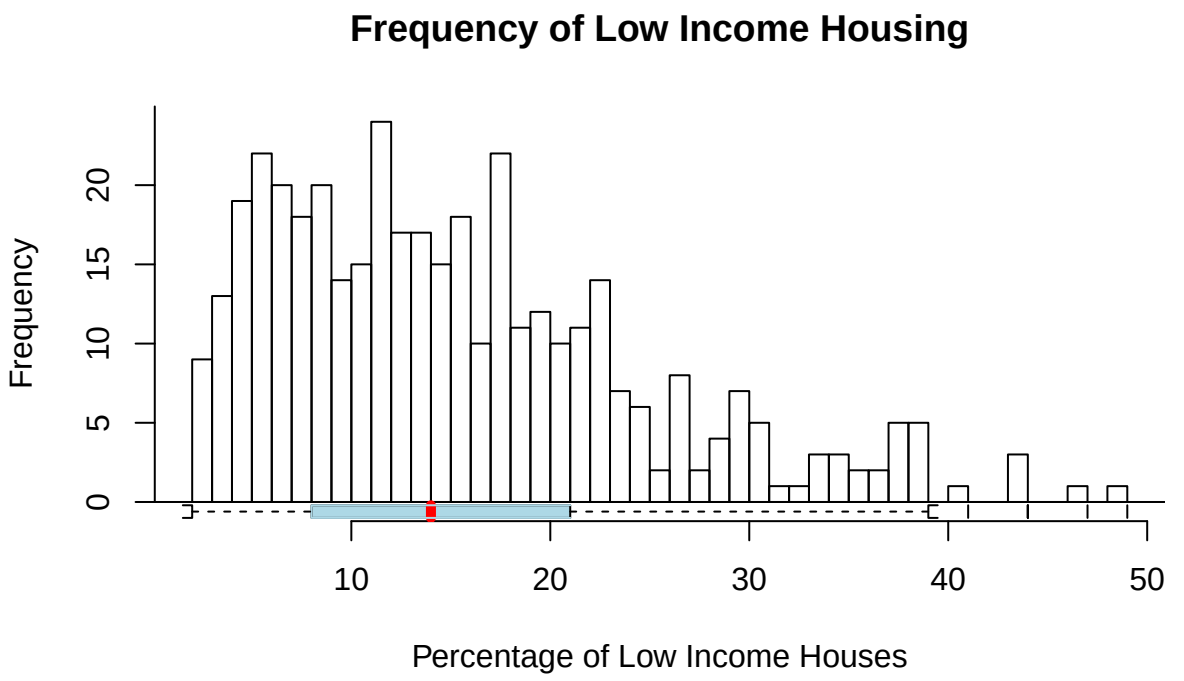
The *distanceToHighway* variable shows the concern with coding error in this histogram as there is a large occurance of the value *24*. About 25% of the dataset have this value, some of which may be correct but it seems unlikely that the *distanceToHighway* variable would be much greater than the *distanceToCity* variable.

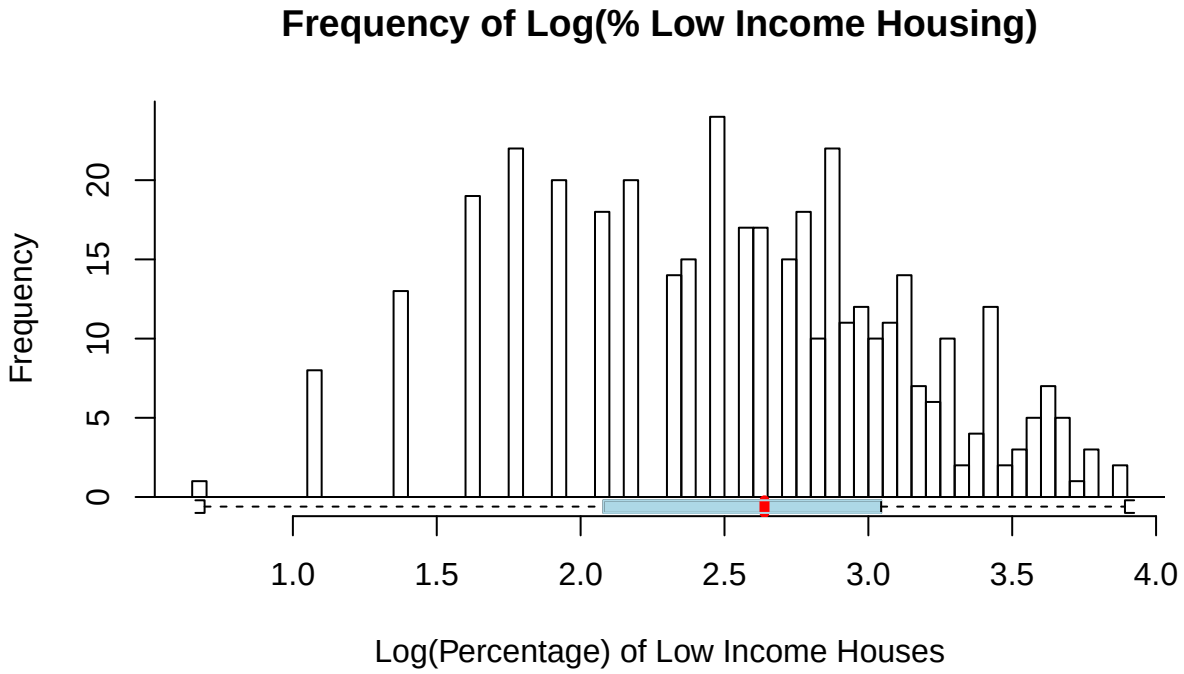
The *pupilTeacherRatio* variable shows a roughly uniform distribution except for a large number of occurances of the value *23*, which must be a more common classroom size.



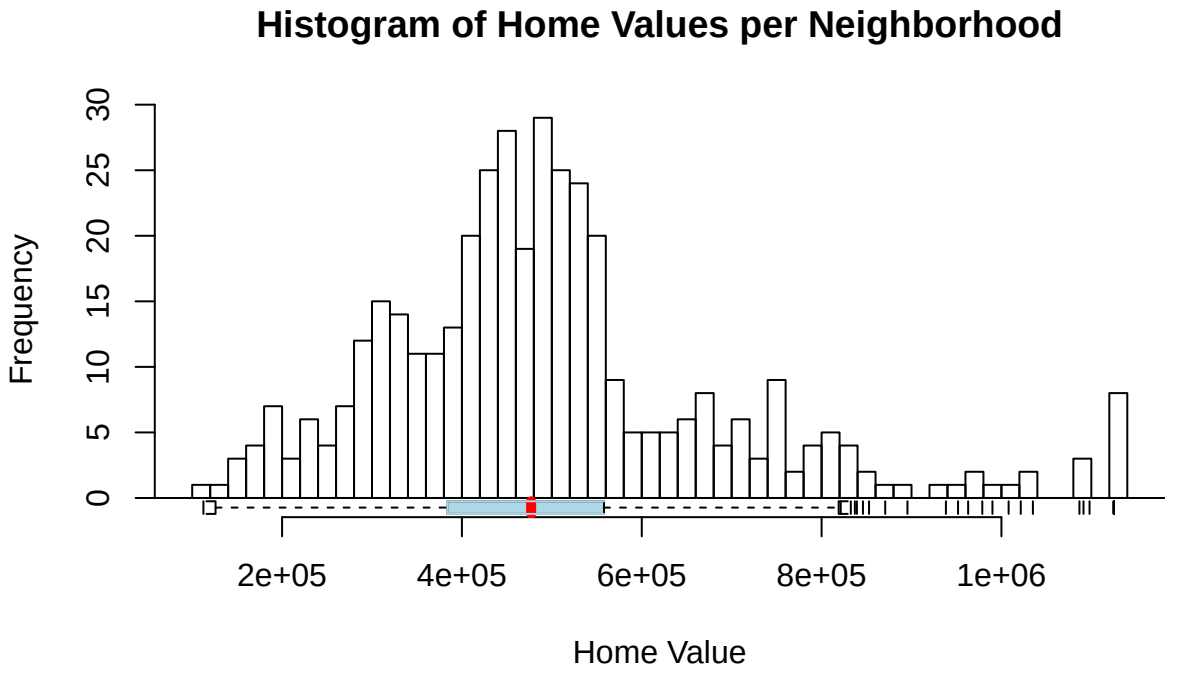


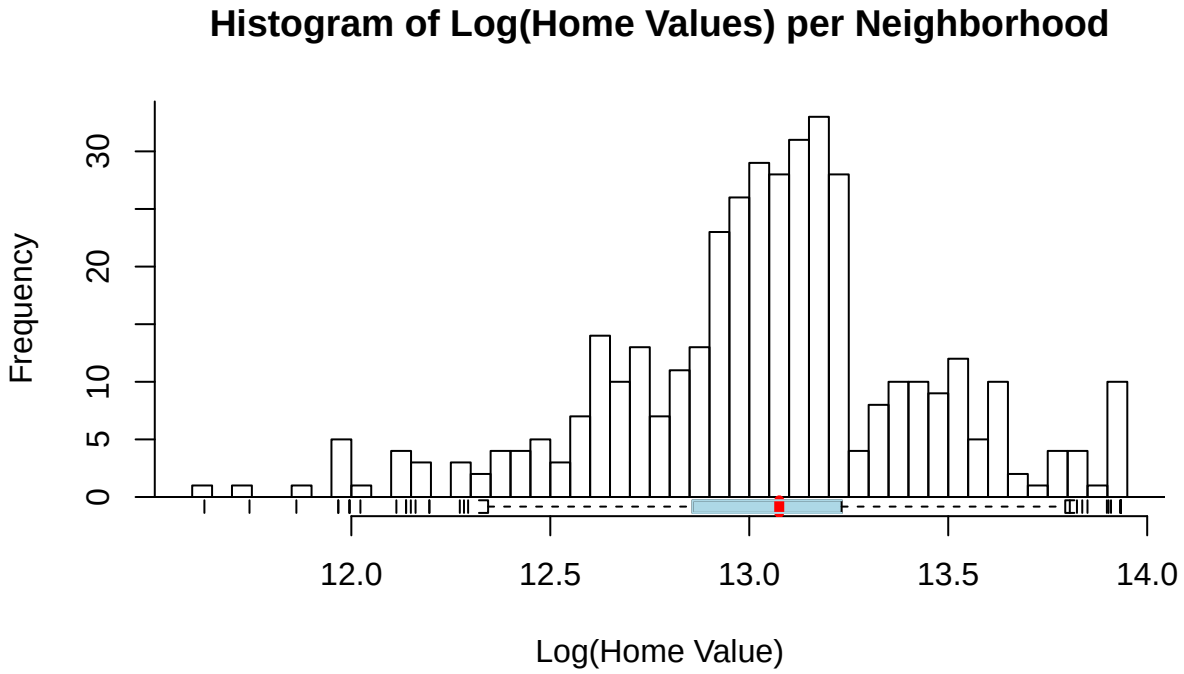
The series is transformed by inverting the ratio to become a Teacher to Pupil ratio, then taking the log.



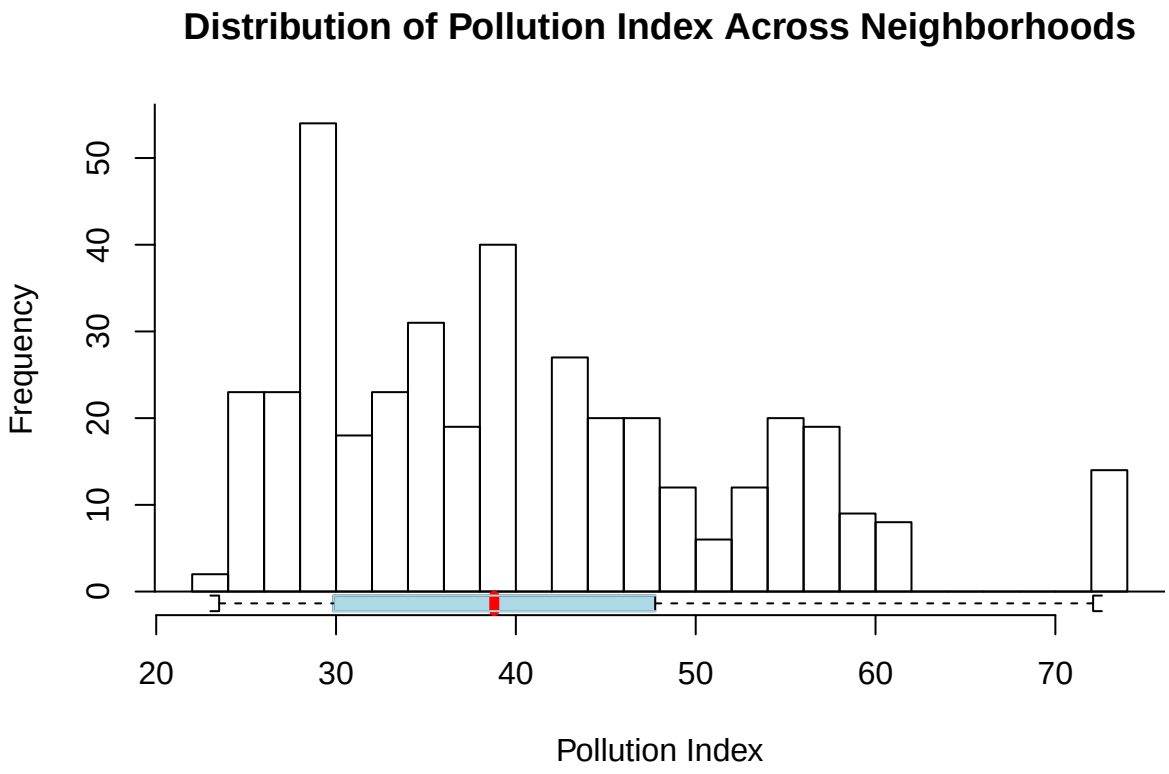


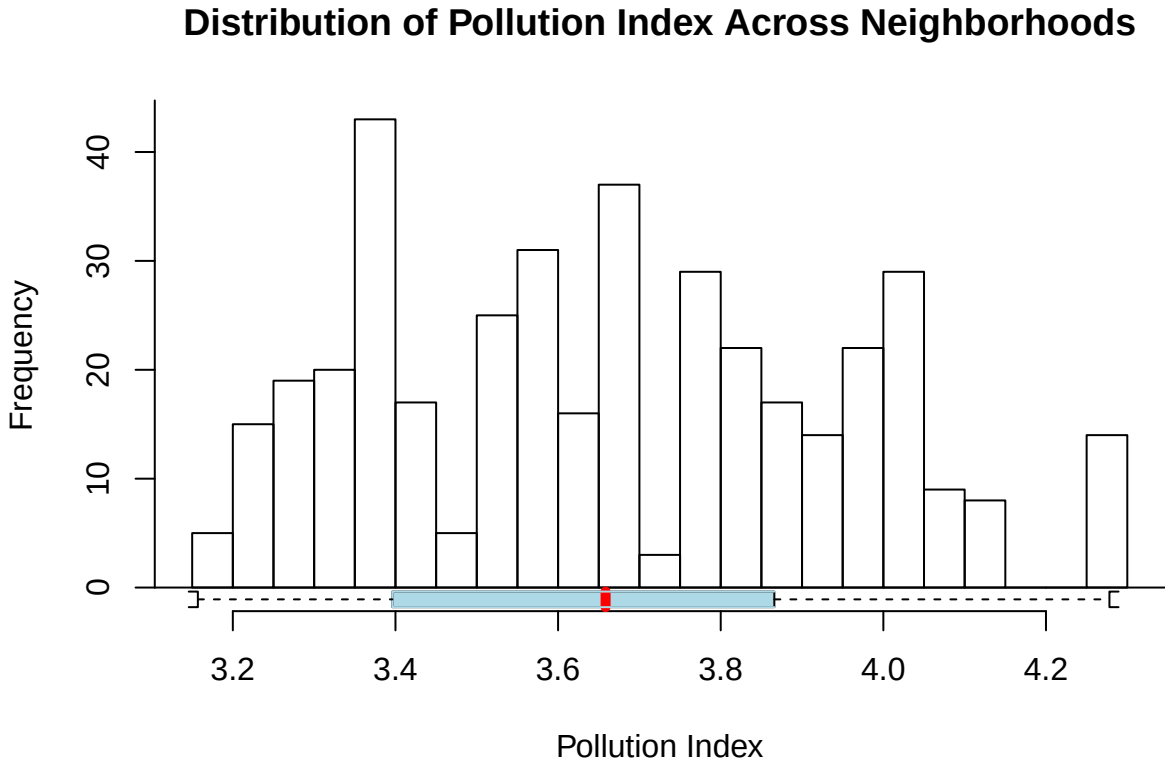
The *pctLowIncome* variable has a right-skewed distribution that tapers off to the right relatively quickly. A log transformation greatly improves the skew and kurtosis of the distribution.



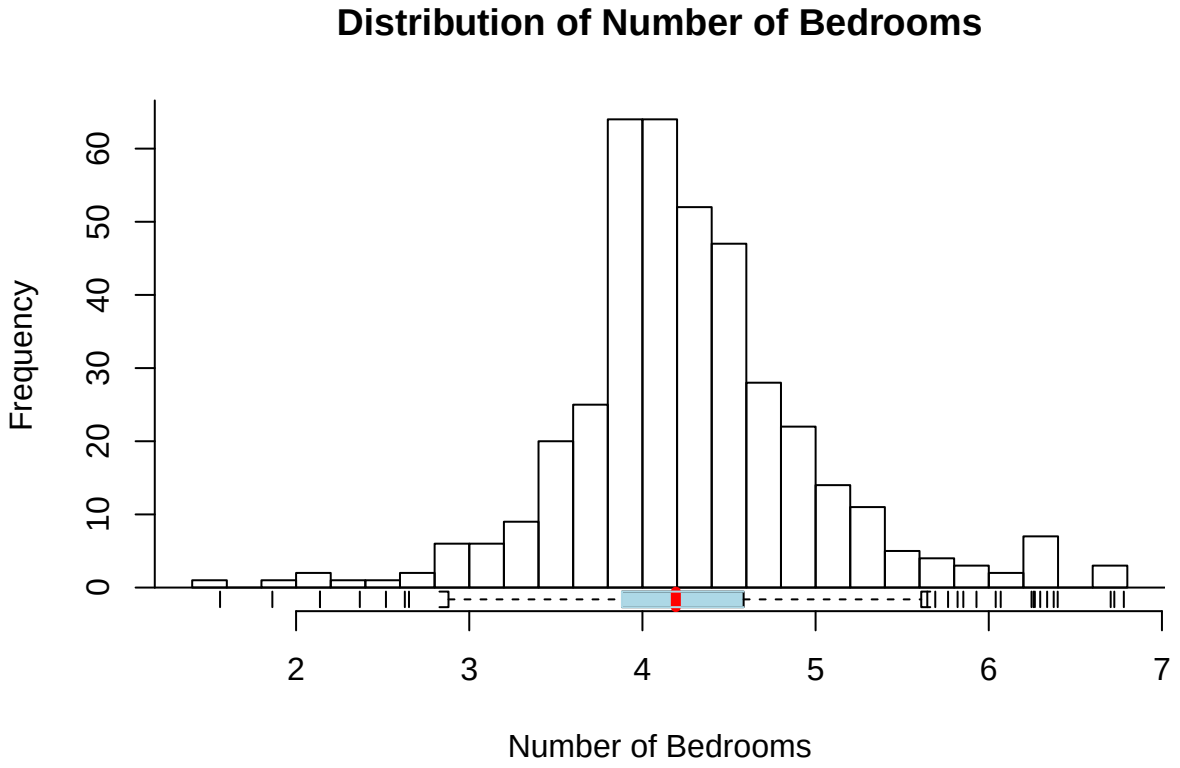


The *homeValue* variable shows a slight right-skew and a log transformation is used to help with this. It also allows discussion in terms of percentage change of home value when controlling for other variables.





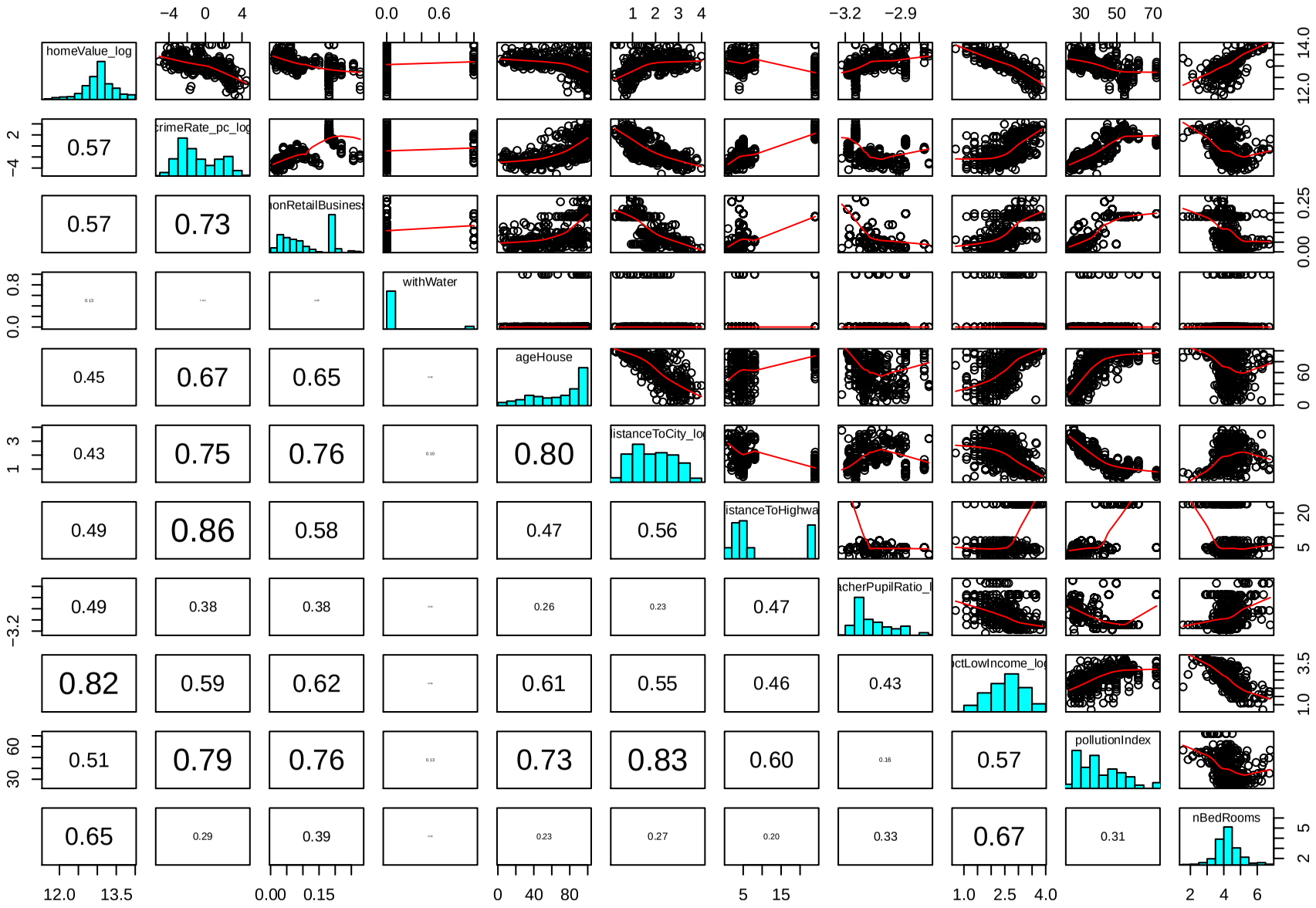
The *pollutionIndex* variable shows a right-skewed distribution upon which we perform a log transform.



The *nBedRooms* variable appears amazingly normal-like in its distribution.

The next page brings all the variables into a single matrix for comparison and to get a first look at correlations to explore further.

All Variables Scatterplot Matrix



DistanceToHighway Variable Detailed Examination

We saw previously that the *distanceToHighway* variable looked suspicious so in this section we look at how to address a possible coding issue. The number of rows in the dataset that have the *distanceToHighway* variable as 24 is 25% of the dataset. Removing these rows would remove a significant amount of data, reducing *N=400* to *N=296*. We examine two strategies and compare them to the row-removal option: replacing values of 24 with the mean of the filtered values or with the value of *distanceToCity*.

The following two tables compare the summaries of the filtered dataset with the summaries of the dataset with transformed values. Comparing the *distanceToHighway_meanMod* and *distanceToHighway_cityMod* shows that the latter is much closer to the values of the filtered dataset. This indicates that replacing the value of 24 with the value of *distanceToCity* is a reasonable transformation to deal with the coding issue. The idea is further substantiated by the proposition that the distance to a city is usually not greater than the distance to a highway as cities are generally located on highways.

The following page shows a set of comparison histograms for the *distanceToHighway* variable with the different transformations. A histogram of *distanceToCity* is included as a reference.

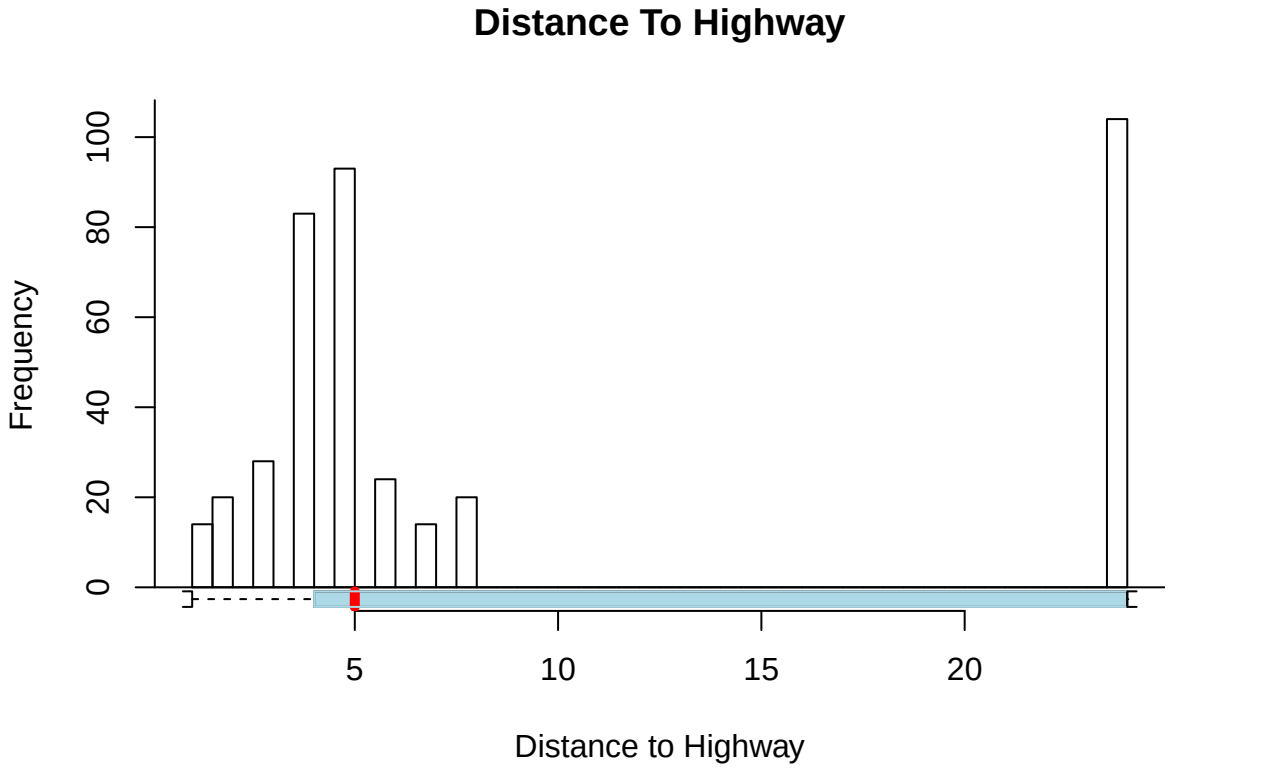
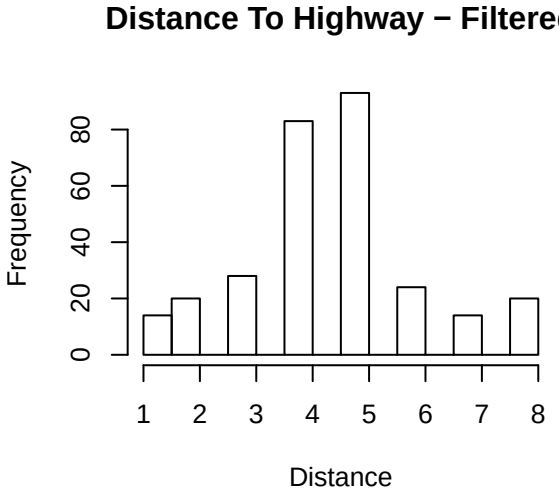


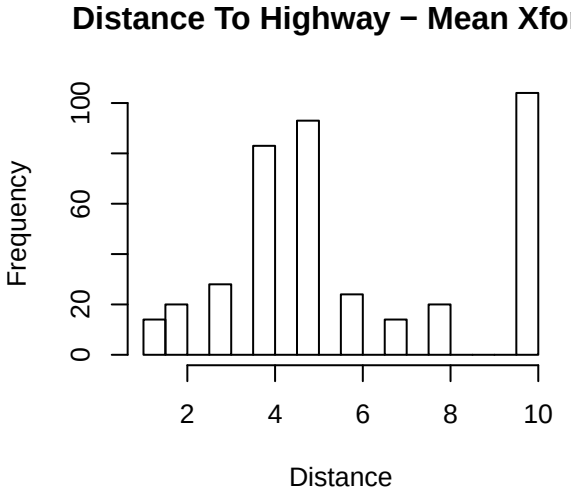
Table 2: Filtered Dataset

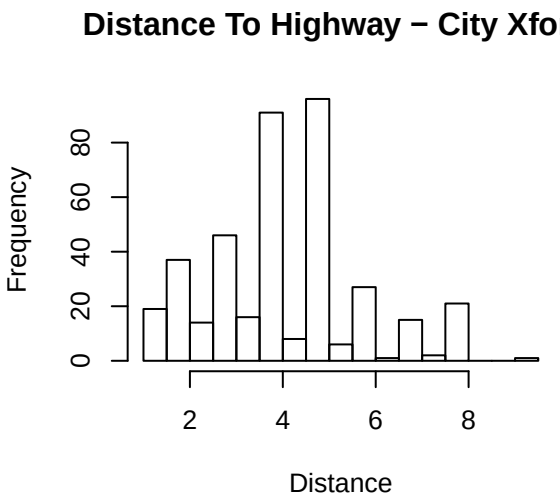
Statistic	N	Mean	St. Dev.	Min	Max
distanceToHighway	296	4.517	1.636	1	8

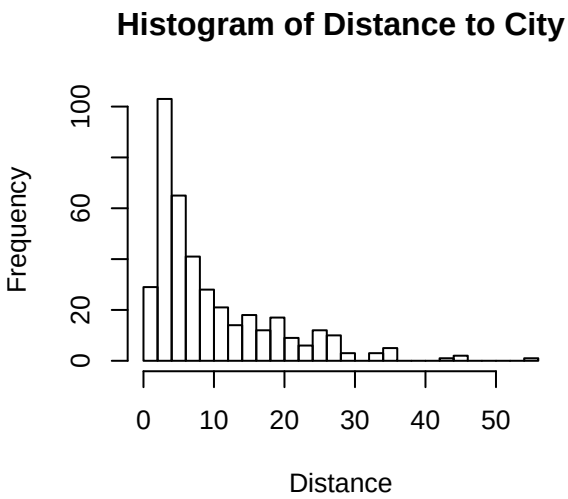
Table 3: Full Dataset

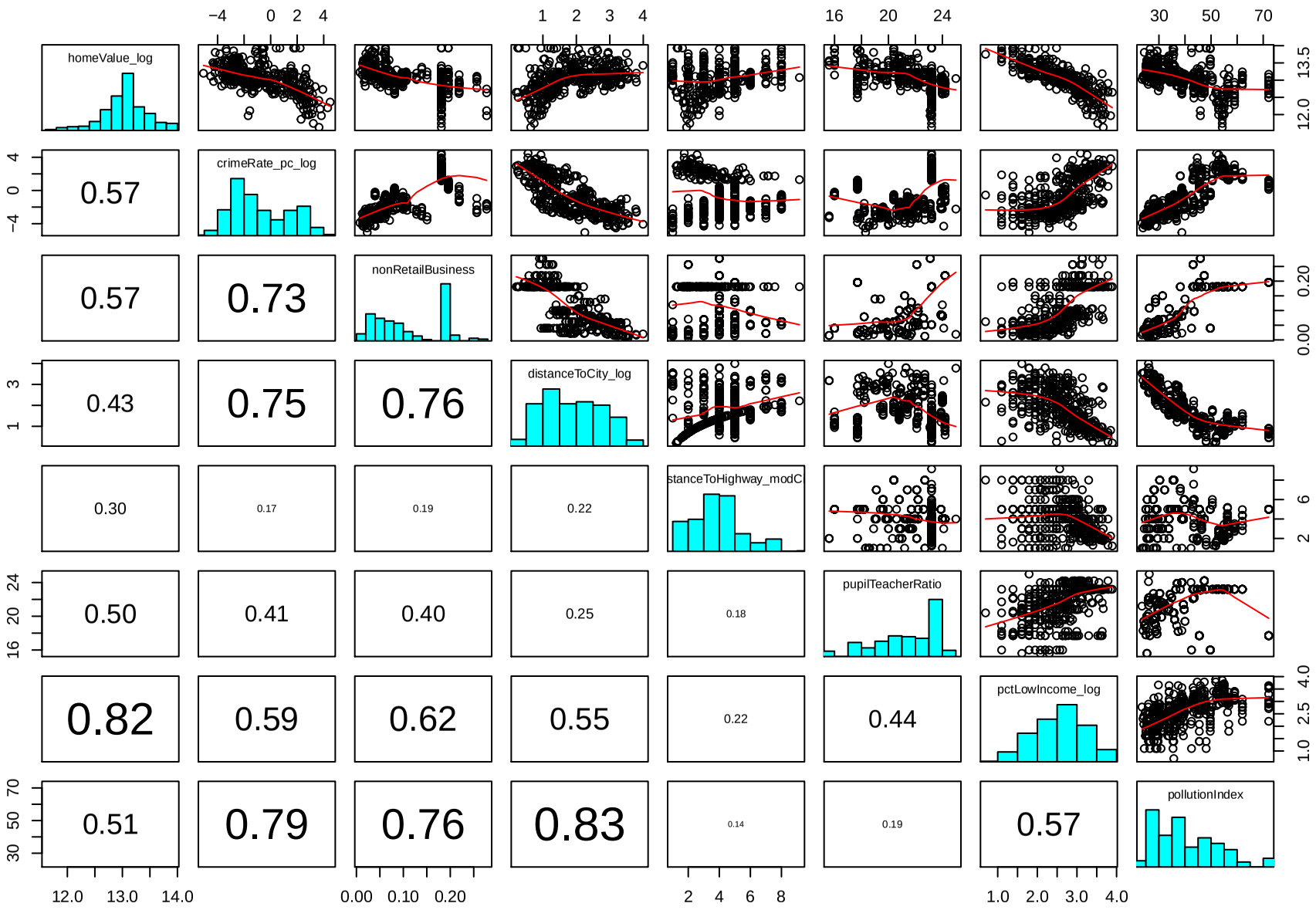
Statistic	N	Mean	St. Dev.	Min	Max
distanceToHighway_modMean	400	5.834	2.632	1.000	9.582
distanceToHighway_modCity	400	4.192	1.698	1.000	9.159





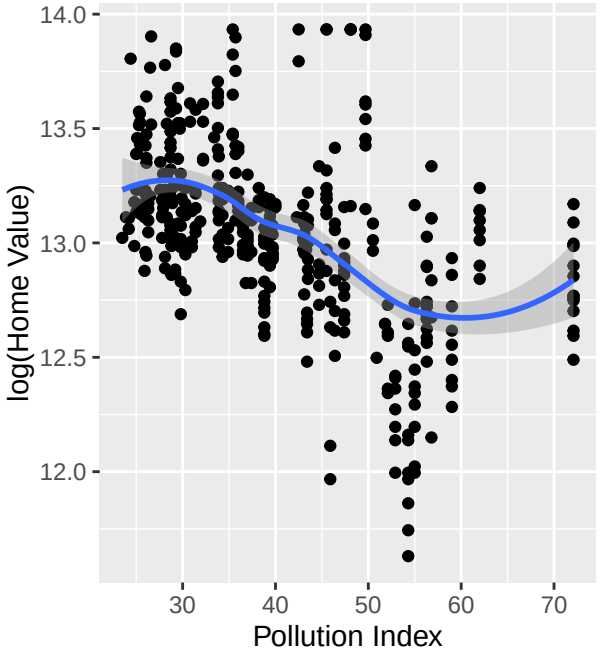


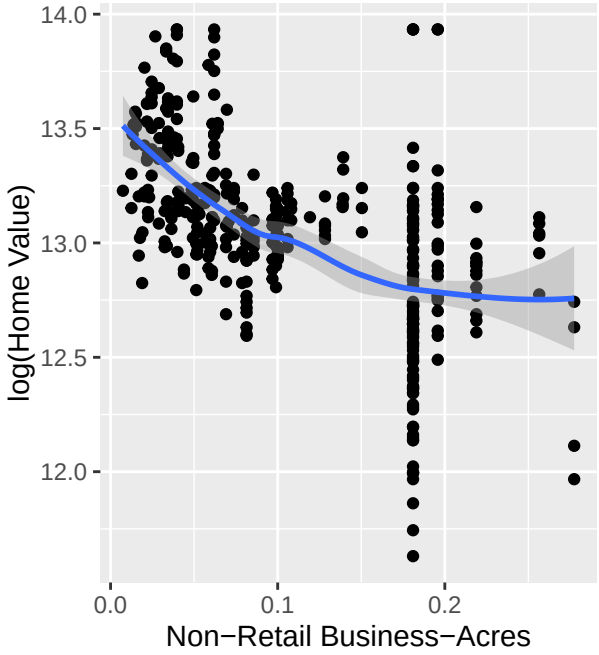


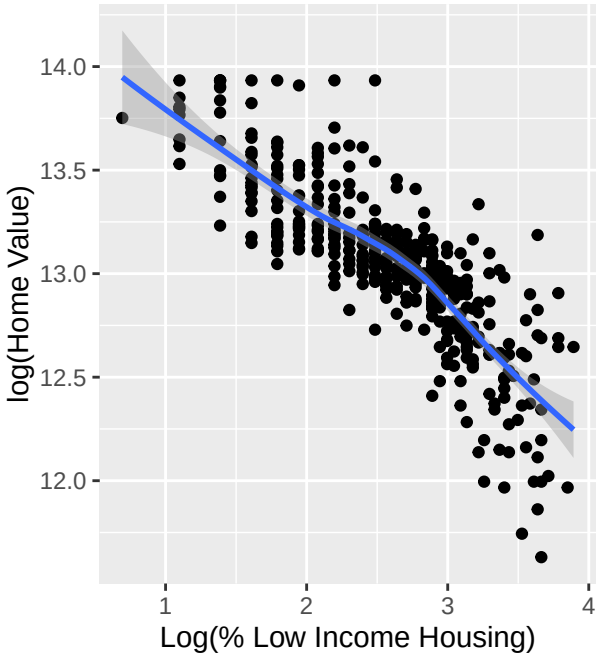

Scatterplot Matrix of Transformed Environmental Variables
homeValue_log
crimeRate_pc_log
nonRetailBusiness
distanceToCity_log
stanceToHighway_modC
pupilTeacherRatio
pctLowIncome_log
pollutionIndex
0.57
0.57
0.73
0.43
0.75
0.76
0.30
0.17
0.19
0.22
0.50
0.41
0.40
0.25
0.18
0.82
0.59
0.62
0.55
0.22
0.44
0.51
0.79
0.76
0.83
0.14
0.19
0.57

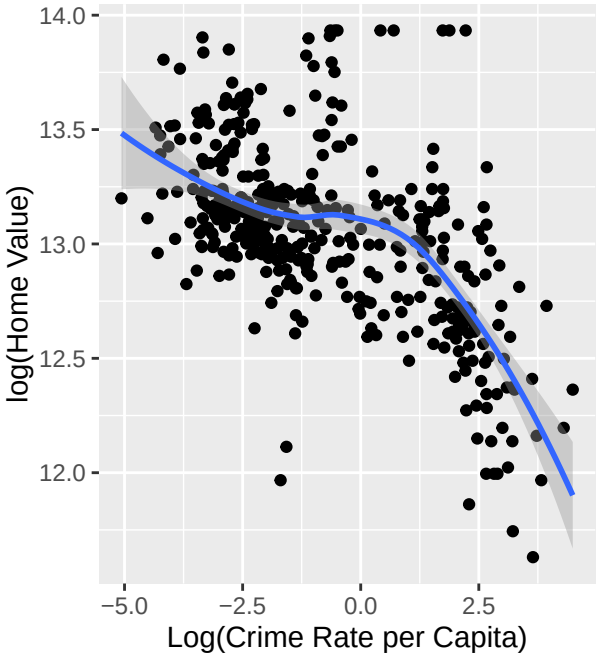
Multivariate Data Analysis

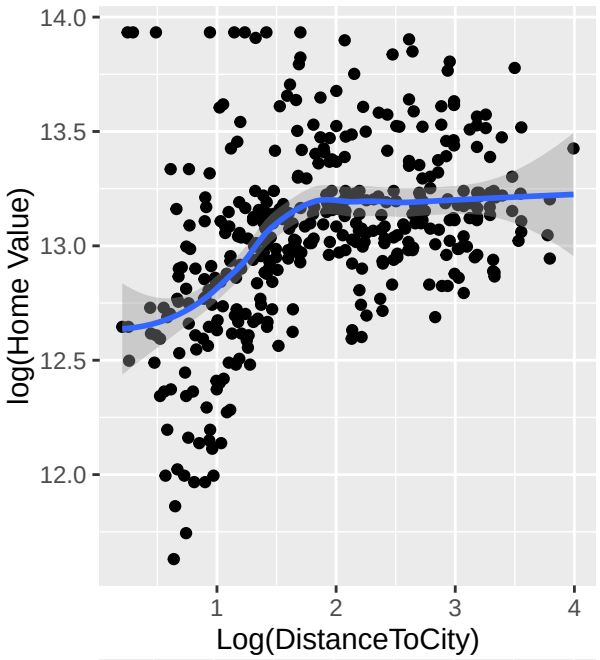
First we will examine the relationships of the environment variables on home values graphically. We can see that there are definitely relationships between most of the environmental variables and home values, as shown on the next two pages of graphs.

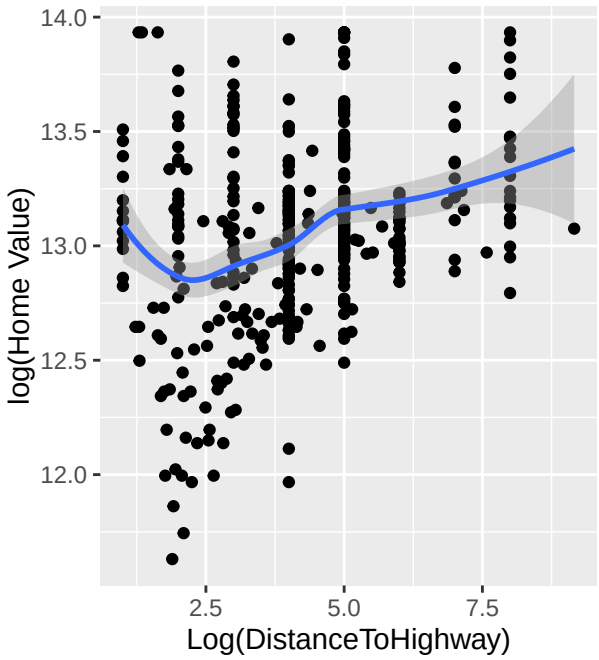


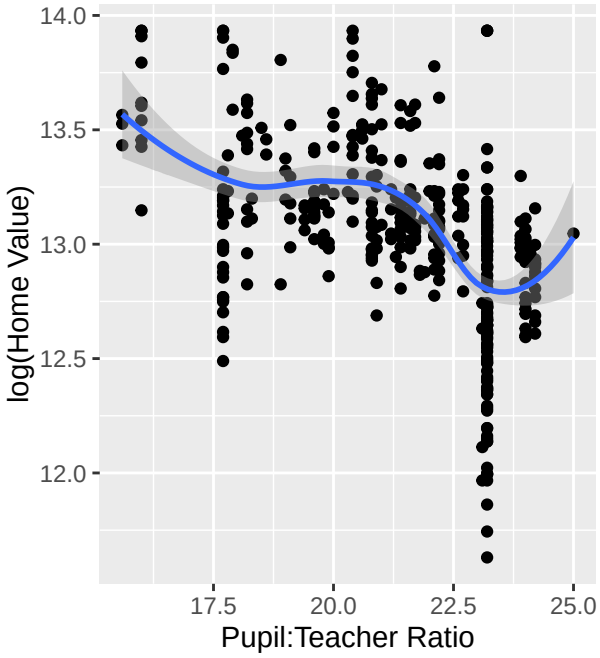




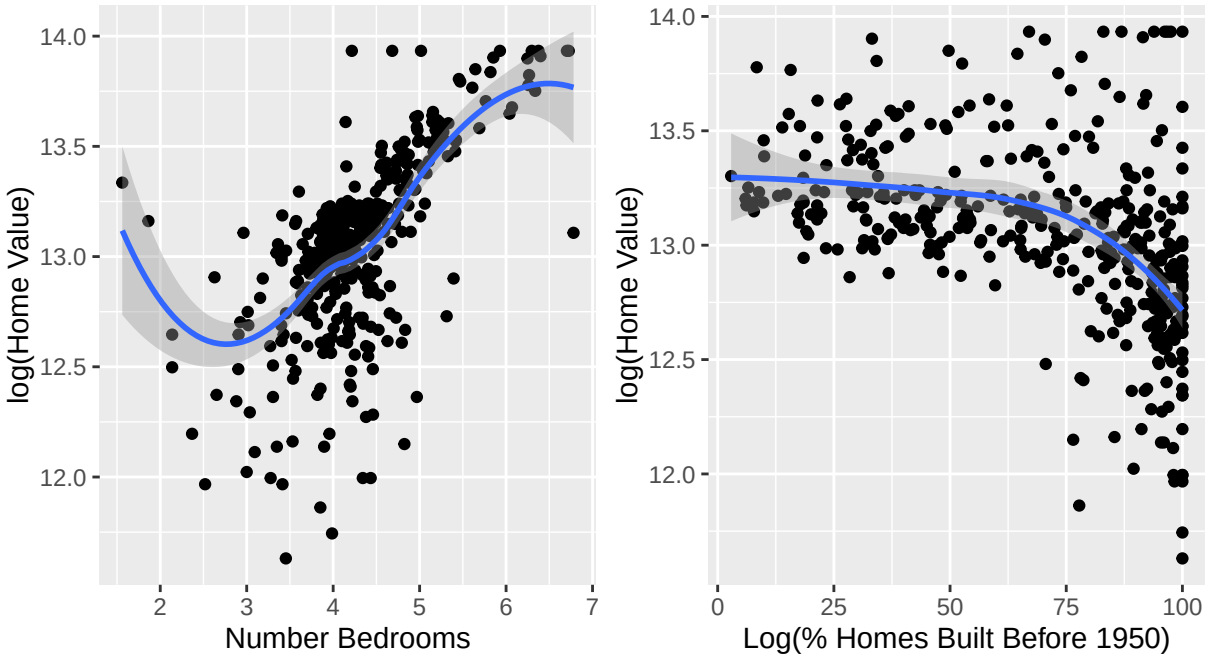



14.0
13.5
13.0
12.5
12.0
log(Home Value)
1
2
3
4
Log(DistanceToCity)


14.0
13.5
13.0
12.5
12.0
log(Home Value)
2.5
5.0
7.5
Log(DistanceToHighway)


14.0
13.5
13.0
12.5
12.0
log(Home Value)
17.5
20.0
22.5
25.0
Pupil:Teacher Ratio

These final two graphs show relationships between home attributes and home values.



Models Incorporating Environment Variables

The linear regression models are built towards increasing saturation where collinearity was not introduced that biases the model. Of all the environment parameters *pollutionIndex* and *nonRetailBusiness* added increased collinearity in the model. In fact, the *pollutionIndex* estimated coefficient was not distinguishable from 0 and was subsequently left out of the more saturated models. A similar situation occurred with the *nonRetailBusiness* variable.

The saturated regression model for environmental variables is expressed as:

$$log(homeValue) = 15.5784 + 0.5128log(TeacherPupilRatio) + 0.1193distanceToHighway$$
$$+0.025withWater - 0.0192log(crimeRate\_pc) - 0.4238log(lowIncomeHousing)$$

Teacher:Pupil Ratio: more teachers per pupil is correlated with increased home values. However it is not possible to tell if an area with higher value homes can afford to hire more teachers or if more teachers makes for better schools which attract more affluent families. A 1% increase in the Teacher to pupil ration will result in a 1/2% increase in the home values, holding the other variables constant.

Distance To Highway: Interestingly, a longer distance to highway is correlated with higher home values. An increase of 1 mile in the distance results in the increase of about 1/10% of the average value of a home, controlling for the other variables in the model.

Close To Water: On average, home values increase by .025% if the home is within 5 miles of water when controlling for other variables in the model. While this result is statistically significant it is not practically significant. One can imagine that homes actually "on the water" have higher values than those that aren't. However the practical significance of being close to a body of water is small.

Crime Rate: Unsurprisingly, crime rate has a negative effect on average home value. An increase of 1% in the crime rate results in a reduction in average home value of about .02%, controlling for the other variables. This may not seem like very much, but crime rates tend to be very low in the areas with the highest home values. A doubling of the crime rate from 1 per 1000 to 2 per 1000 results in a 100% increase in the crime rate and a 2% reduction in price. Because this is a non-linear relationship we can't take extrapolate much further but the general idea is apparent.

Low Income Housing: Low income housing is an obvious correlator for average home values. However we can imagine that while the numeric averages are reduded as lower income housing increases in an area the effect of higher concentrations of low income housing tends to pull down home values within a certain distance. Controlling for the other variables in the model, an increase of 1% in low income housing can decrease average home values in the area by about 1/2%.

Distance To City: It is interesting that *distanceToCity* did not factor into our models. It is highly correlated with the *crimeRate_pc* variable in that the closer to the city the higher the crime rate.

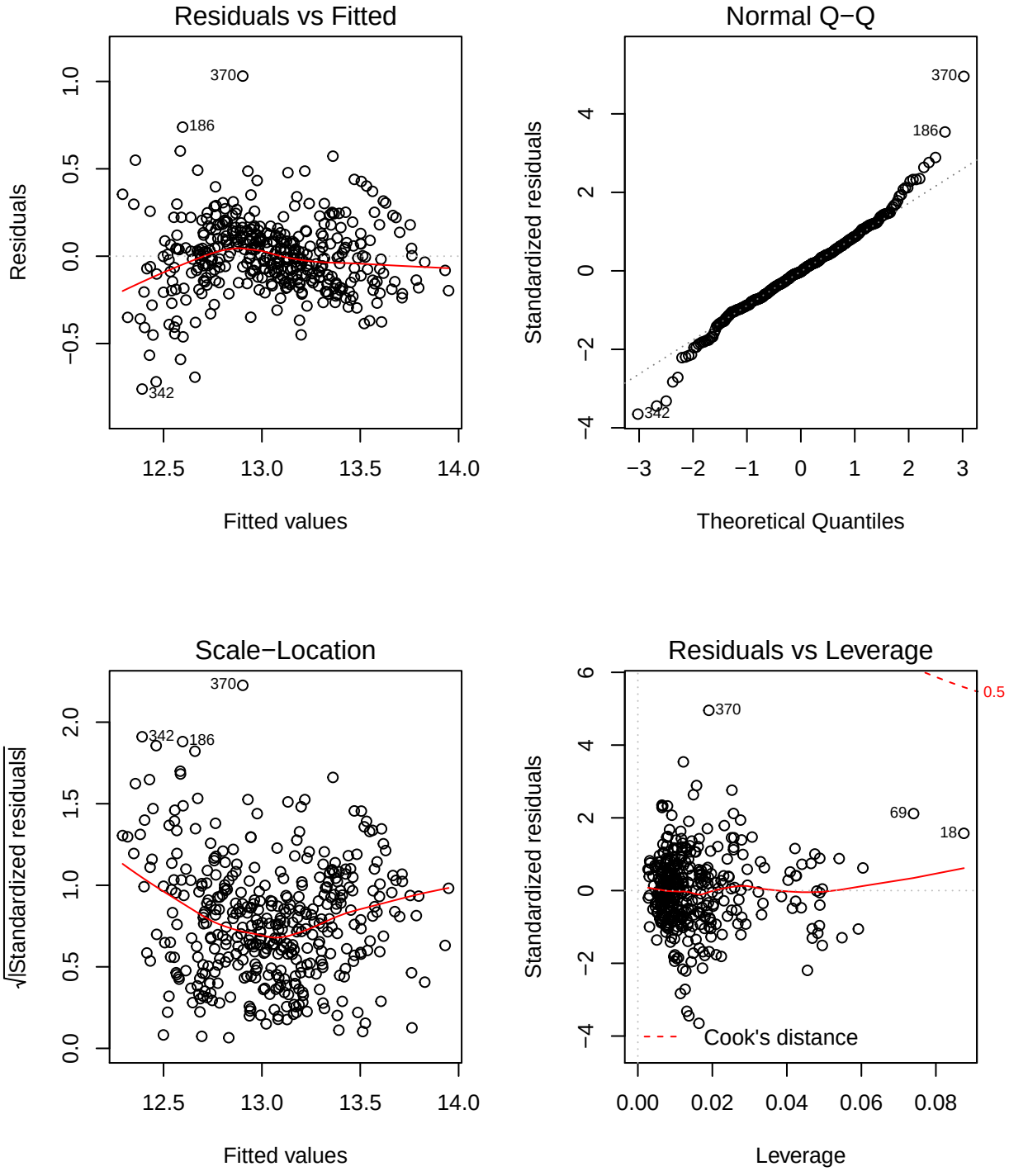
This model only reflects the correlations of a small group of environment variables on the average price of a home. It is not a model to predict the average price of a home given these factors. This model gives some ideas about how some factors could affect the average values of homes in the area, however. Some biases in the model include the correlation between the low income housing and home values. It is difficult to use that observation as an indicator for other things like the type of people in the area, the extent to which social services are offered, or how the type of housing interacts with crime rate.

Table 4: Regression Model Comparison

	Dependent variable:					
	log(Home Values)					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Low Income Housing)	-0.5178^{***}	-0.4694^{***}	-0.4743^{***}	-0.4611^{***}	-0.4484^{***}	-0.4238^{***}
	(0.0179)	(0.0219)	(0.0225)	(0.0218)	(0.0216)	(0.0217)
log(Crime Rate)		-0.0237^{***}	-0.0297^{***}	-0.0260^{***}	-0.0244^{***}	-0.0192^{***}
		(0.0064)	(0.0087)	(0.0064)	(0.0062)	(0.0062)
Pollution Index			0.0016			
			(0.0016)			
Close To Water					0.0271^{***}	0.0250^{***}
					(0.0065)	(0.0064)
Distance To Highway				0.1444^{***}	0.1371^{***}	0.1193^{***}
				(0.0441)	(0.0433)	(0.0424)
log(Teacher:Pupil Ratio)						0.5128^{***}
						(0.1116)
Constant	14.3803^{***}	14.2376^{***}	14.1814^{***}	14.2046^{***}	14.0599^{***}	15.5784^{***}
	(0.0475)	(0.0606)	(0.0821)	(0.0607)	(0.0689)	(0.3374)
Observations	400	400	400	400	400	400
R^2	0.6770	0.6878	0.6887	0.6961	0.7088	0.7236
Adjusted R^2	0.6762	0.6863	0.6863	0.6938	0.7059	0.7201
Residual Std. Error	0.2259	0.2223	0.2223	0.2197	0.2153	0.2100
F Statistic	834.3370^{***}	437.3986^{***}	291.9644^{***}	302.2906^{***}	240.3876^{***}	206.3106^{***}

Note: *p<0.1; **p<0.05; ***p<0.01

Finally we examine the residual plots to diagnose the selected model, we can see that the assumptions of normality, zero-condition mean, and homoskedasticity can be held for the model.



Part 2 - Modeling and Forecasting a Real-World Macroeconomic Financial Time Series

Exploratory Data Analysis

In order to better understand the time series and analyze the possible underlying processes we must first observe and explore the time series. A summary of the time series is:

[1] "function"

Table 5: DXCM Series Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Max
DXCM Series	2,332	23.2	23.4	1.4	101.9

The first set of plots reveals:

- The series is non-stationary; it has a persistent upward trend, interrupted by shocks;
- There are shocks at approximately time periods 500, 1200, 1800 and 2200;
- There appears to be seasonality in the series;
- The autocorrelation shows a very slight decay over the entire correlagram;
- The partial autocorrelation shows barely significant results at lags 14 and 32;
- We do not know the frequency of the time series;
- The series is of closing prices of DXCM

To remove the trend from the series we take the first difference and replot to check the results.

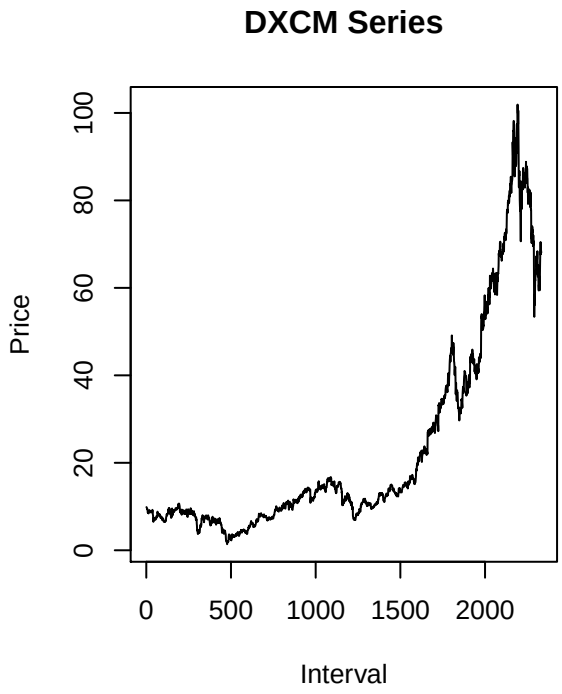
[1] "function"

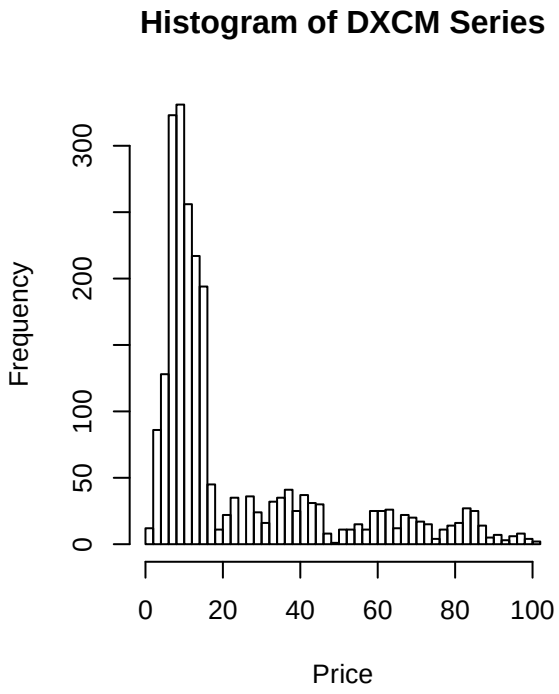
Table 6: Differenced Series Descriptive Statistics

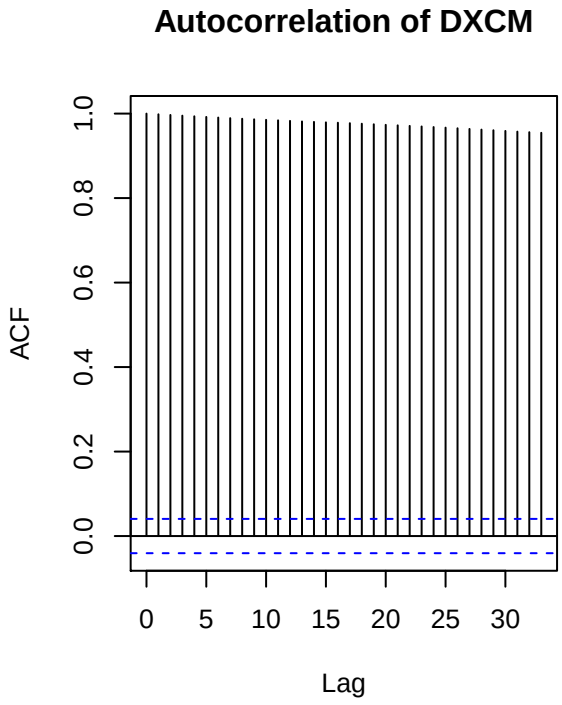
Statistic	N	Mean	St. Dev.	Min	Max
DXCM Differenced Series	2,331	0.02	0.9	-8.6	13.7

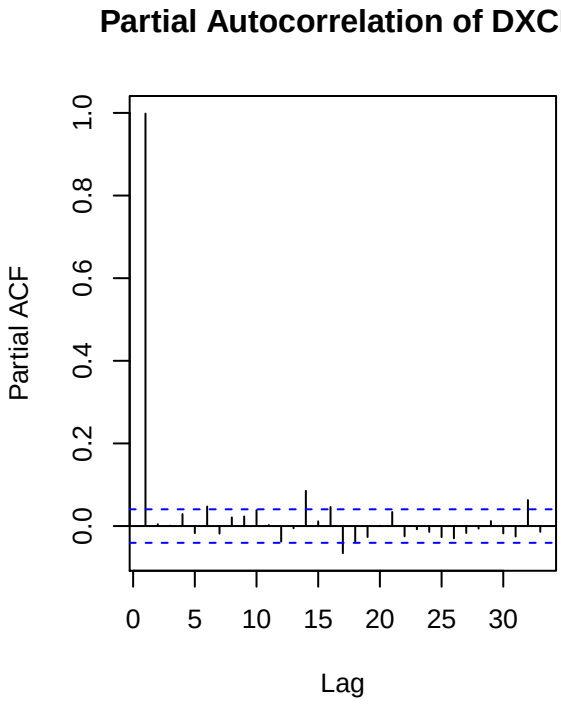
In the differenced series we observe:

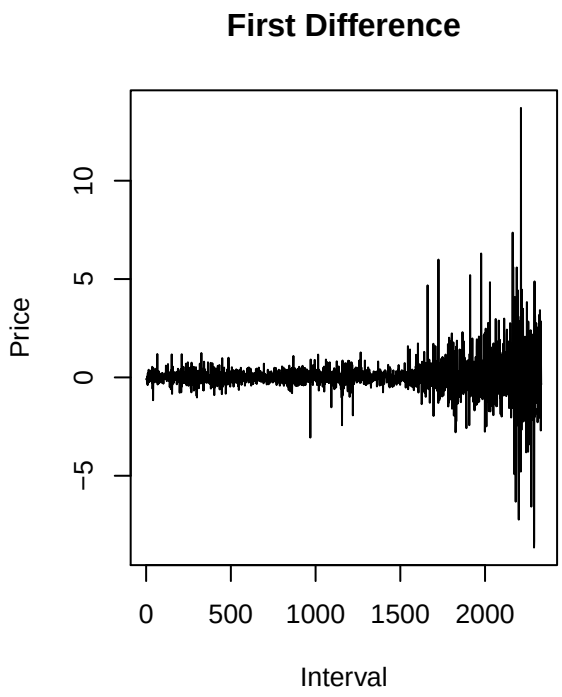
- The first difference series has a more or less white noise appearance until approximately time interval 1600 where the volatility of the series increases dramatically. This corresponds to the sudden, persistent upward trend in the original series.
- The autocorrelation shows marginally significant results at lags 13, 15, 16, 24, 31
- The partial autocorrelation shows a cyclic behavior that doesn't appear to decline, with significant results at lags 11, 13, 14, 15, 16, 24, 25, 31

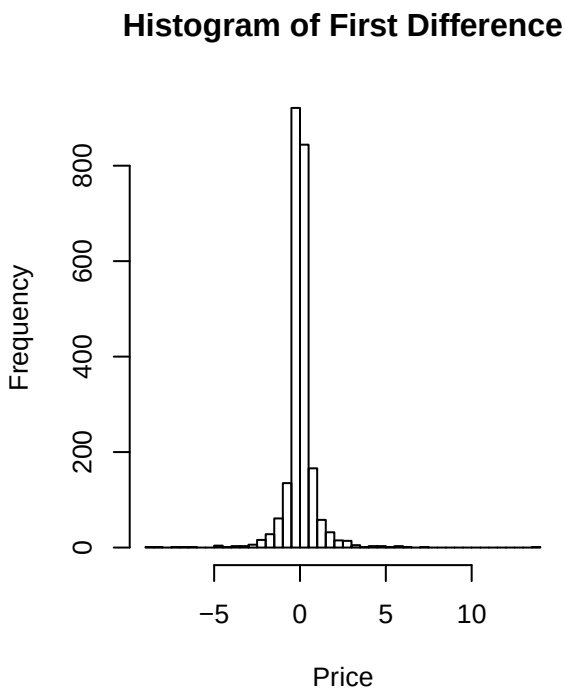

DXCM Series
Price
Interval
0
20
40
60
80
100
0
500
1000
1500
2000

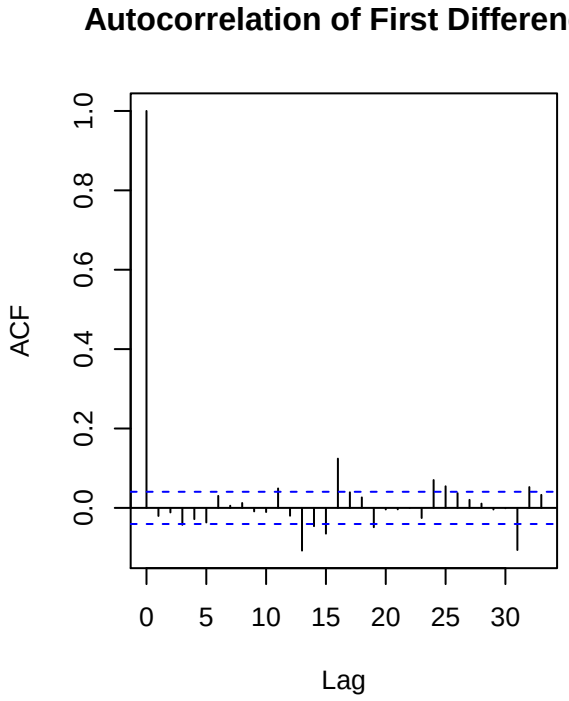

Histogram of DXCM Series
Frequency
Price
0
50
100
200
300
0
20
40
60
80
100

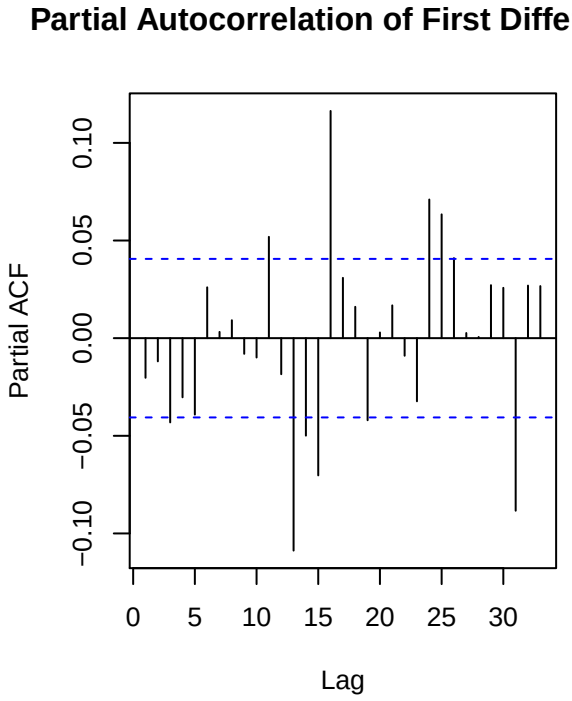

Autocorrelation of DXCM
ACF
Lag
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30


Partial Autocorrelation of DXCM
Partial ACF
Lag
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30


First Difference
Price
Interval
0
500
1000
1500
2000
−5
0
5
10

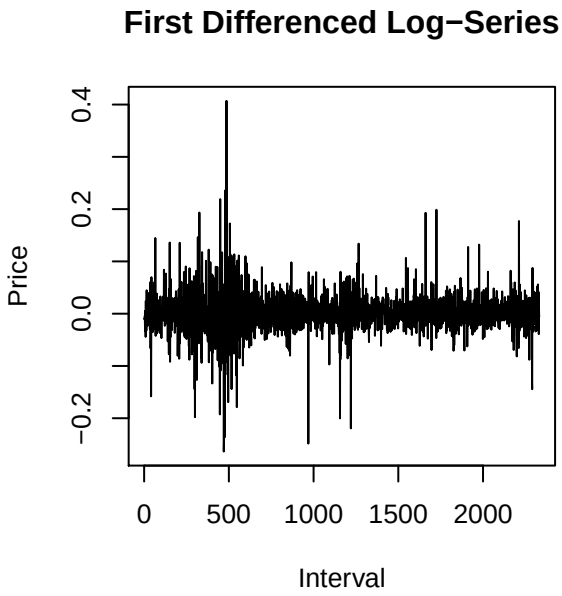

Histogram of First Difference
Frequency
Price
0
200
400
600
800
−5
0
5
10

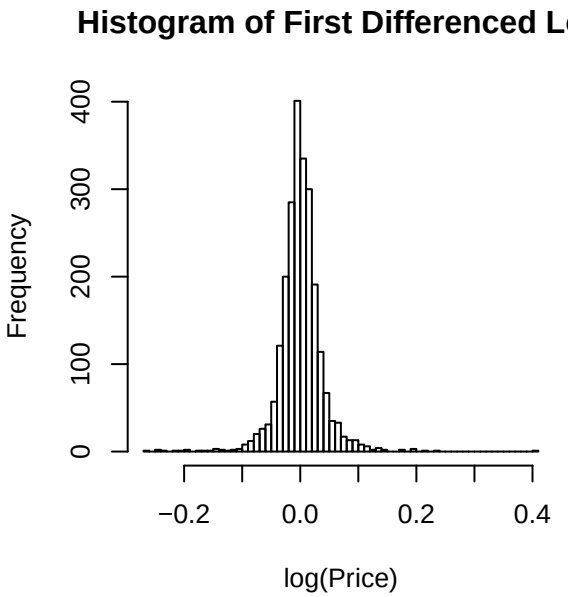

Autocorrelation of First Difference
ACF
Lag
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30

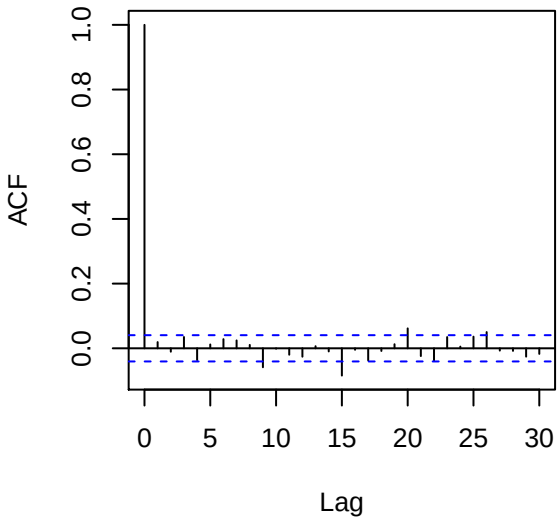

Partial Autocorrelation of First Differen
Partial ACF
Lag
−0.10
−0.05
0.00
0.05
0.10
0
5
10
15
20
25
30

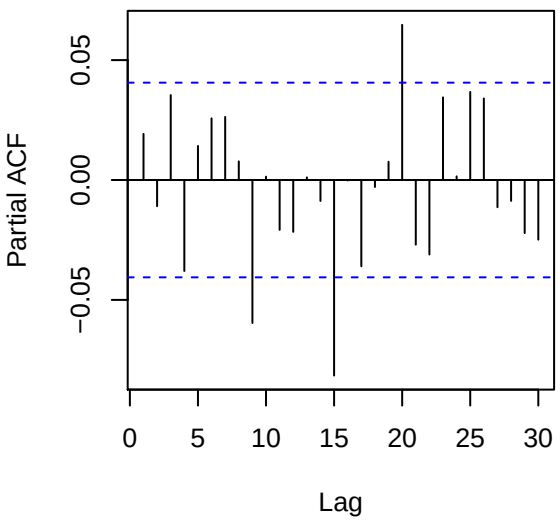
We also examine the first difference of the log of the series and replot to check results. In the differenced log series we observe:

- The volatility appears to be reversed such that it is around interval 300-500, and overall the volatility of the differenced log series is higher.
- The ACF shows only a small results at lag 15 and 20.
- The PACF shows a cyclic behavior with significant results at lags 9, 15, 20



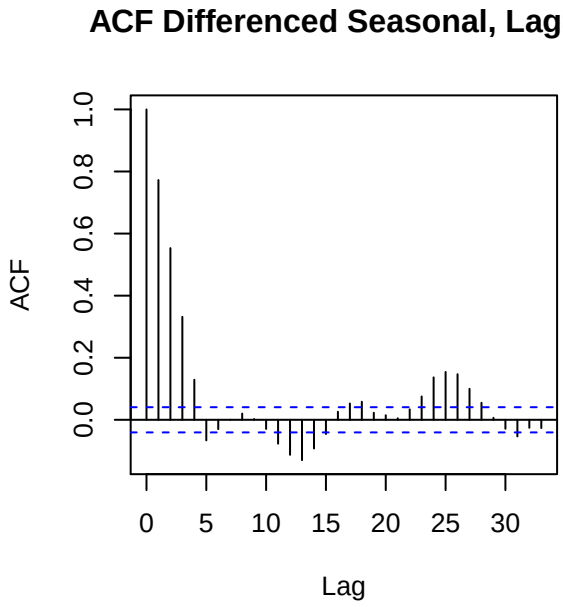


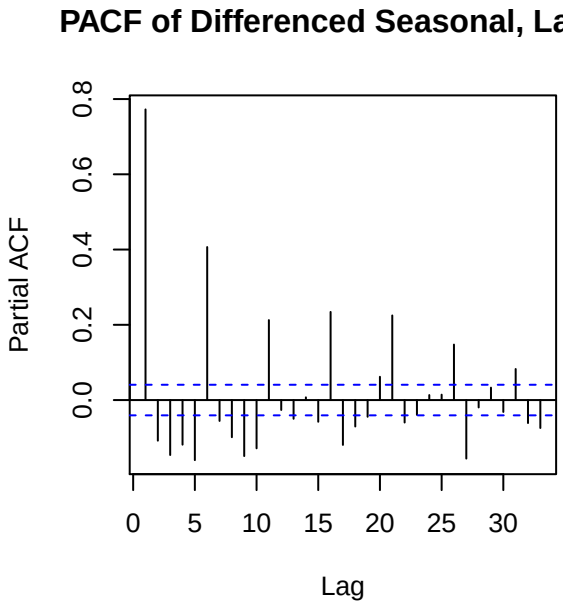


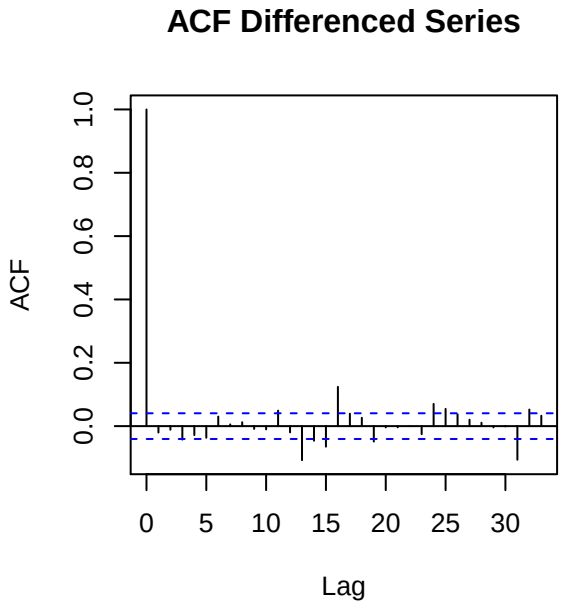


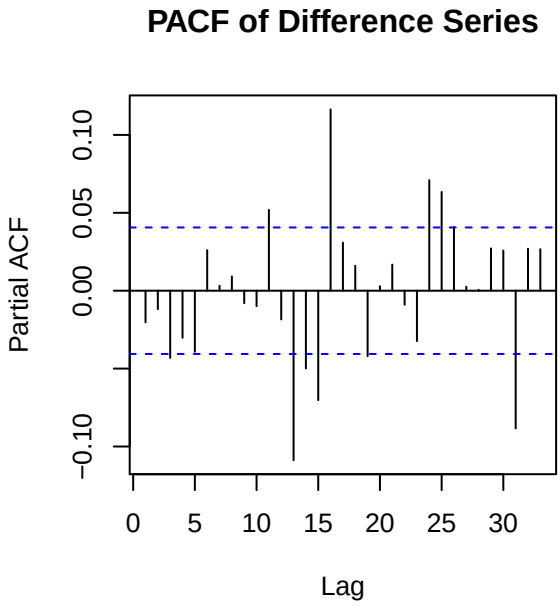
The seasonality of the series is not strong in the ACF of the differenced series. However, there are hints of a 5 day cycle that corresponds to the weekly frequency. There are stronger spikes that appear in lags 15 and 30 that support a multiple or harmonic of 5.

The ACF and PACF of the differenced seasonal series show evidence of an underlying MA(5) process









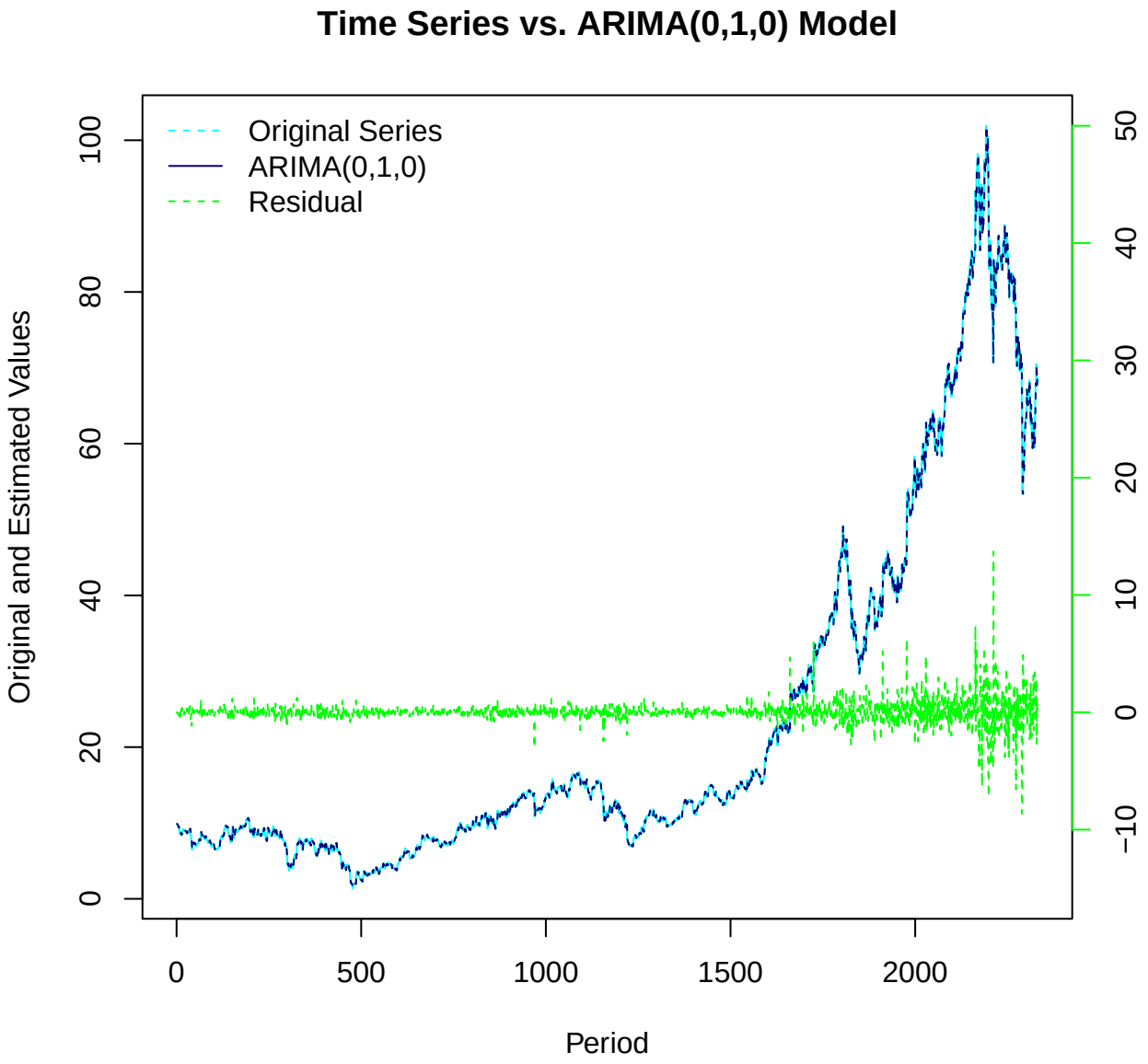
Model Selection

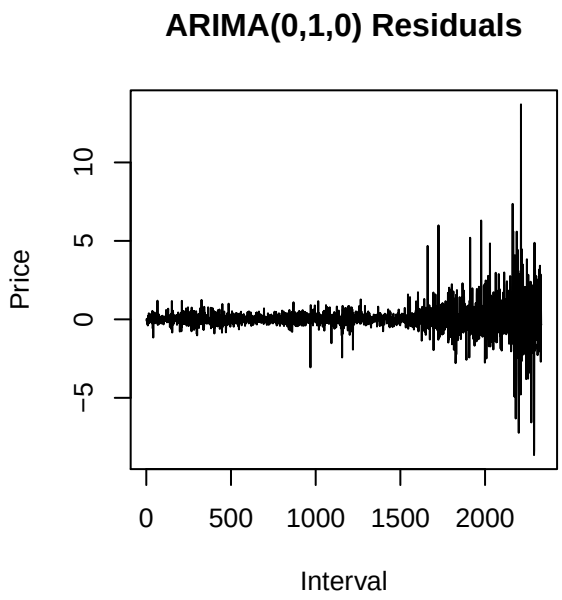
We use the `get.best.arima` function from the async materials and the Introductory Time Series with R book [Cowpertwait, Metcalfe - 2009] to perform a search for the best ARIMA model. The procedure results in an ARIMA(0,1,0) estimated model.

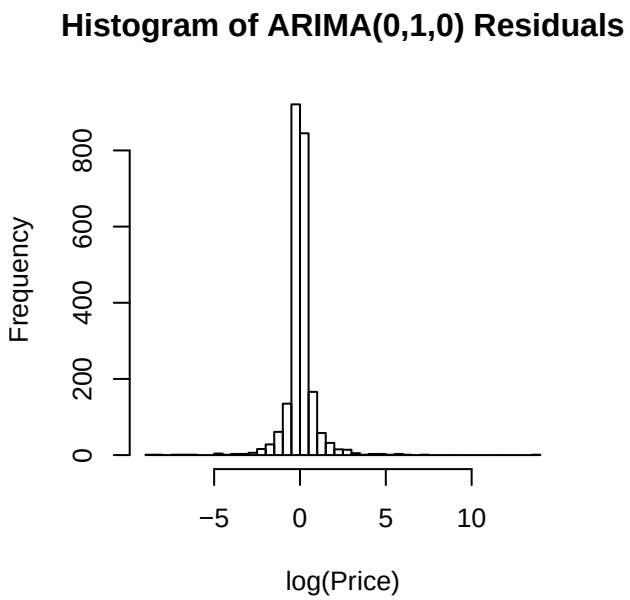
[1] "function"

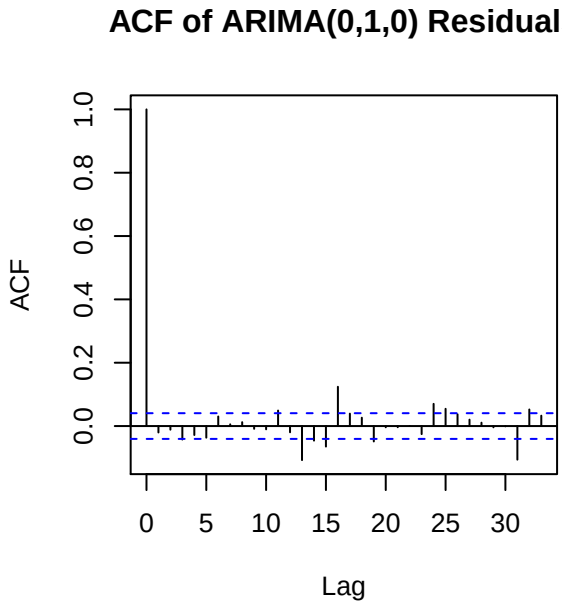
Table 7: Comparative Statistics

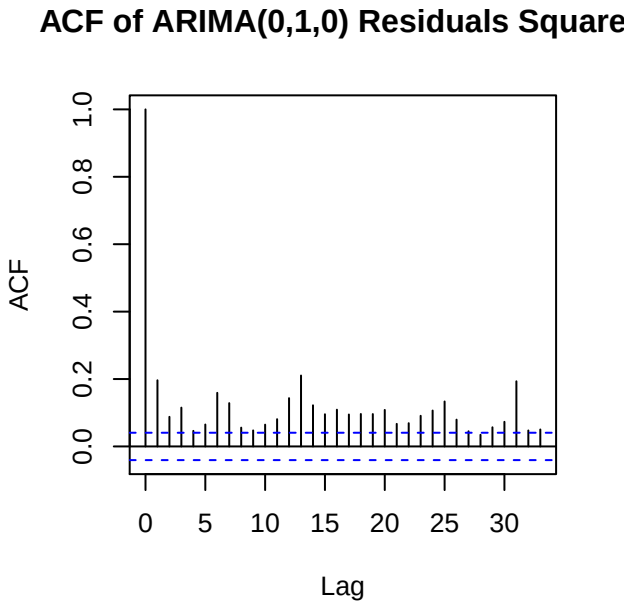
Statistic	N	Mean	St. Dev.	Min	Max
DXCS Series	2,332	23.2	23.4	1.4	101.9
ARIMA(0,1,0) Model	2,332	23.2	23.4	1.4	101.9
Residuals	2,332	0.02	0.9	−8.6	13.7

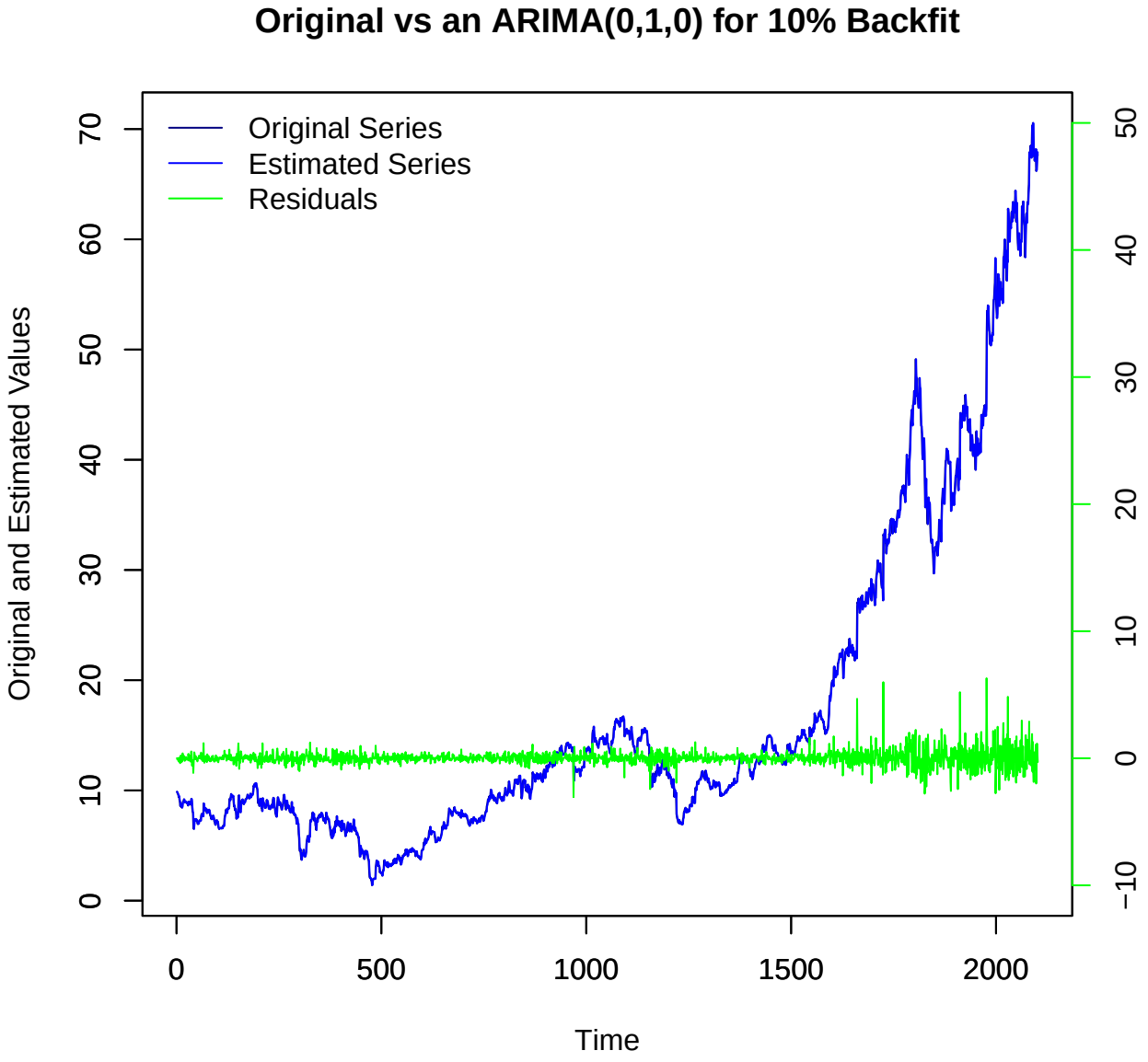



ARIMA(0,1,0) Residuals
Price
10
5
0
−5
0
500
1000
1500
2000
Interval


Histogram of ARIMA(0,1,0) Residuals
Frequency
800
600
400
200
0
−5
0
5
10
log(Price)


ACF of ARIMA(0,1,0) Residuals
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
Lag

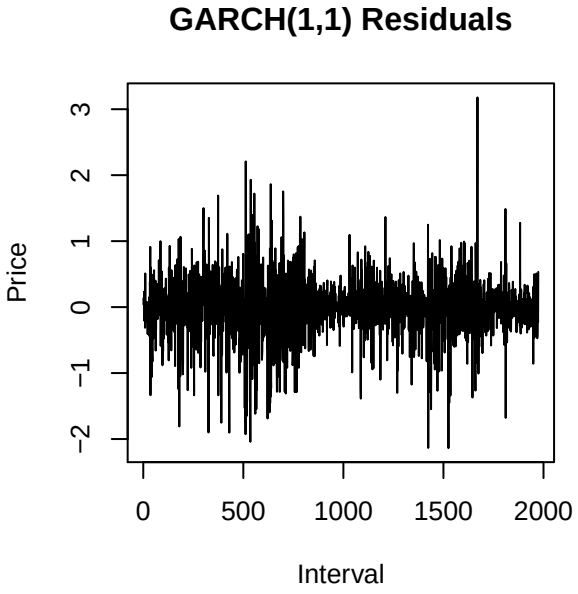

ACF of ARIMA(0,1,0) Residuals Square
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
Lag

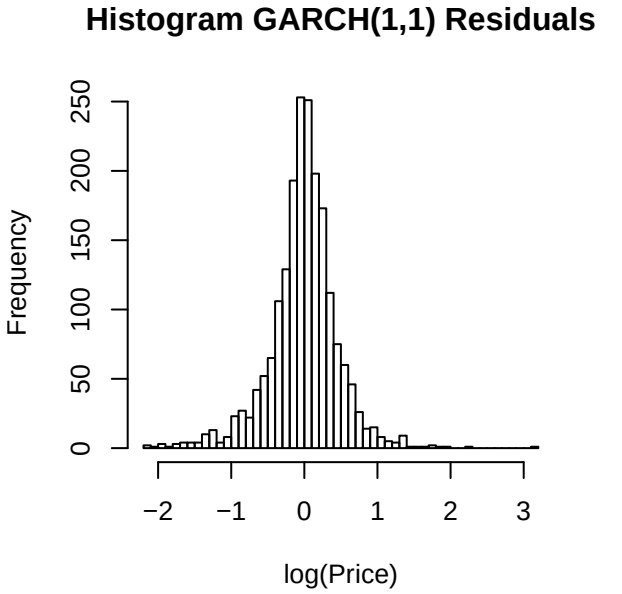

Original vs an ARIMA(0,1,0) for 10% Backfit
Original Series
Estimated Series
Residuals
Original and Estimated Values
Time
0
500
1000
1500
2000
0
10
20
30
40
50
60
70
−10
0
10
20
30
40
50

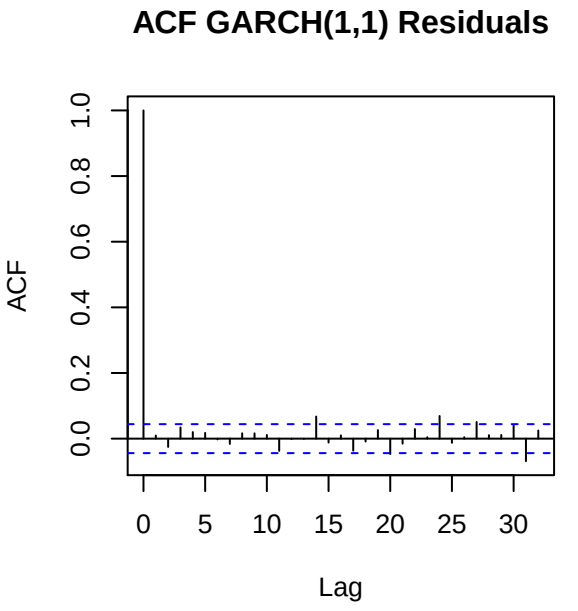
The residuals show an increased volatility at the right end of the residuals graph, and the PACF of the squared residuals shows definite autocorrelation of the residuals.

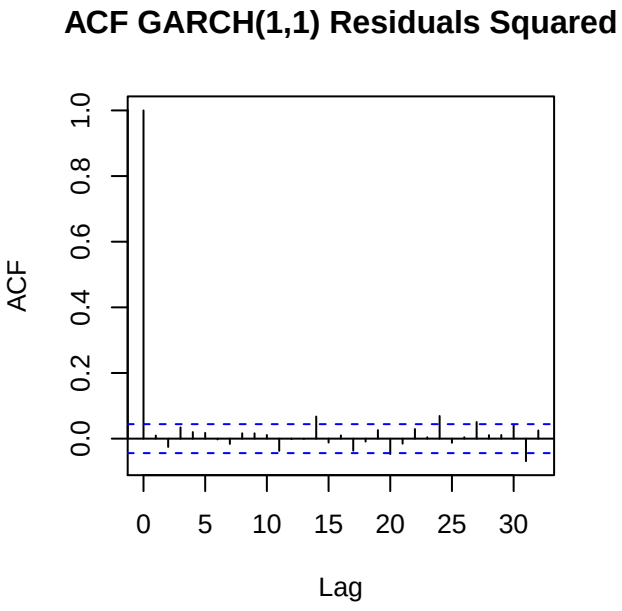
The ARIMA(0,1,0) model of the captures a good deal of the time series' behavior and it is a very parsimonious model.

Since the residuals of the ARIMA(0,1,0) model shows time-dependency we fit a GARCH model to the residual of the ARIMA model. The result is a GARCH(1,1) model where all parameters are significant. We fail to reject the hypothesis that the residuals are IID based on the results of the Ljung-Box test on the squared residuals. The summary of the GARCH model output is shown below.









	GARCH(1,1)
mu	−0.006
	(0.008)
omega	0.011***
	(0.003)
alpha1	0.153***
	(0.026)
beta1	0.806***
	(0.033)
Num. obs.	1974
AIC	1.125
Log Likelihood	1106.608

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

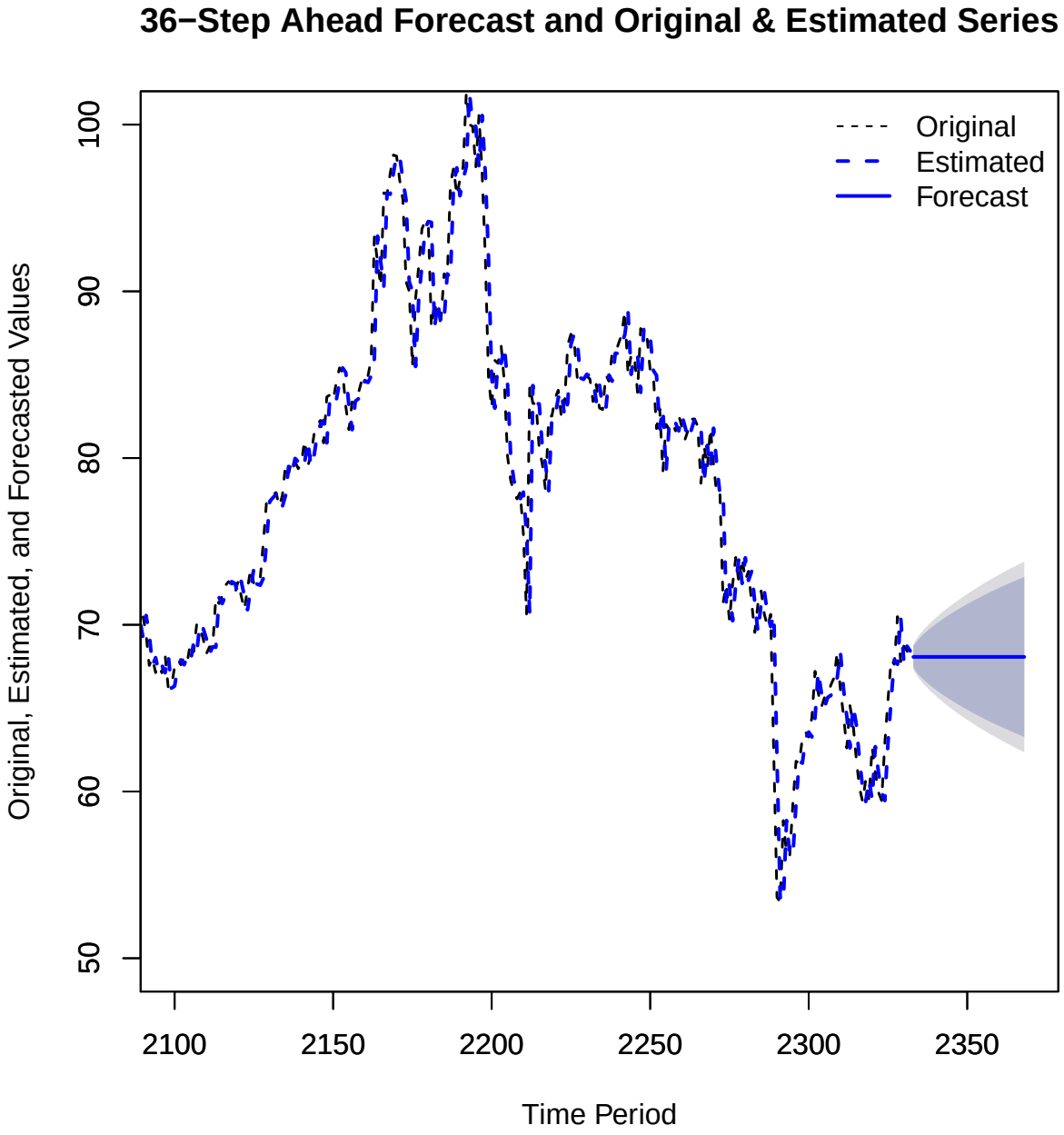
```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ts1.fit1$residuals ~ garch(1, 1), trace = FALSE)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fedfc67c000>
##  [data = dem2gbp]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## -0.0061903   0.0107614   0.1531341   0.8059737
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.006190    0.008462   -0.732 0.464447
## omega   0.010761    0.002838    3.793 0.000149 ***
## alpha1  0.153134    0.026422    5.796  6.8e-09 ***
## beta1   0.805974    0.033381   24.144  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -1106.608    normalized:  -0.5605916
##
## Description:
##  Sun Apr 24 22:54:35 2016 by user:
##
##
```

```
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1059.851  0
##  Shapiro-Wilk Test  R    W      0.9622848 0
##  Ljung-Box Test     R    Q(10)  10.12141  0.4299066
##  Ljung-Box Test     R    Q(15)  17.04349  0.3162711
##  Ljung-Box Test     R    Q(20)  19.29764  0.5025619
##  Ljung-Box Test     R^2  Q(10)  9.062556  0.5261773
##  Ljung-Box Test     R^2  Q(15)  16.07769  0.3769072
##  Ljung-Box Test     R^2  Q(20)  17.50715  0.6198388
##  LM Arch Test       R    TR^2   9.771217  0.6360238
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 1.125236 1.136559 1.125228 1.129396
```

The complete ARIMA(0,1,0)-GARCH(1,1) model looks like:

$$y_t - y_{t-1} = .010761 + .153134\epsilon_{t-1}^2 + .805974\hat{h}_{t-1}$$

All of these models exhibit time-dependent residuals so we fit a GARCH model to the residuals of the resulting model


36−Step Ahead Forecast and Original & Estimated Series
Original
Estimated
Forecast
Original, Estimated, and Forecasted Values
50
60
70
80
90
100
2100
2150
2200
2250
2300
2350
Time Period

Part 3 - Forecast Web Search Activity for "Global Warming"

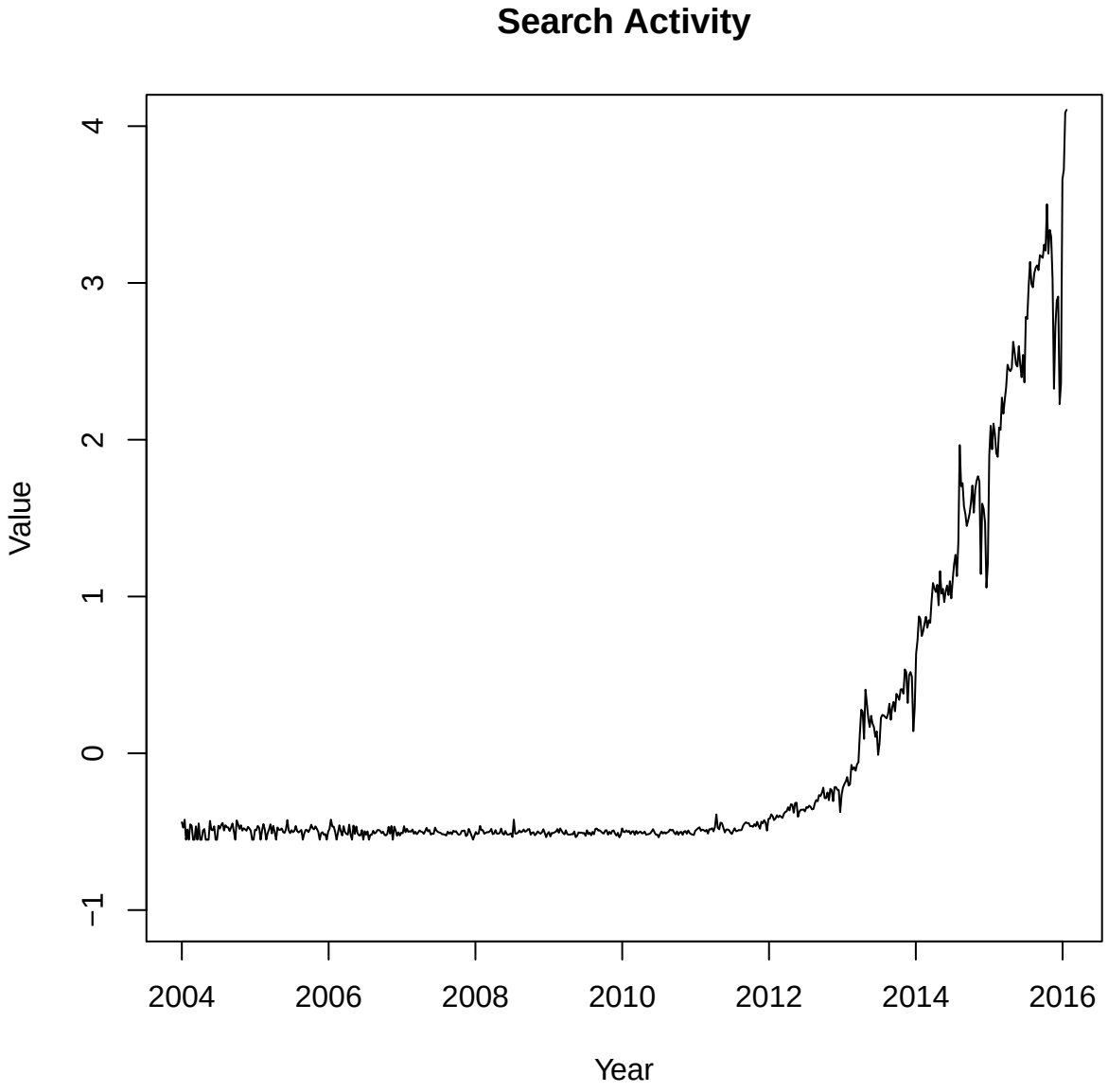
Exploratory Data Analysis

The time series is a weekly measurement of the frequency of a search phrase over a period from 1/4/2004 to 1/24/16 and we have no other information. The plot reveals that from 2004 to around 2012 there is very little activity; after 2012 the activity begins increasing persistently at a steep rate.

[1] "function"

Table 8: Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Max
Search Series	630	0.0000	1.00	−0.55	4.10



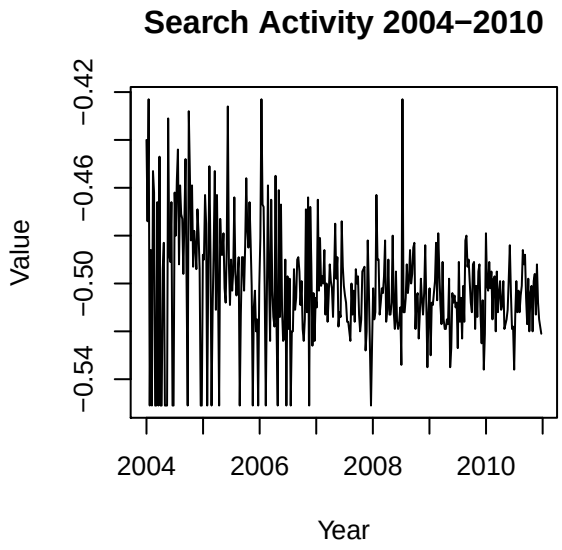
We can fit a model that does a better job of forecasting if we take that part of the series that contains the data since about 2012 on which to estimate a model. An analysis of the series indicates that the split point is around 2011. We split the series into 2004-2010 and 2011-2016 in order to capture the variation in the later part of the model. We measure this by observing the autocorrelation in each portion of the series to minimize the autocorrelation in the early series and maximize it in the later series.

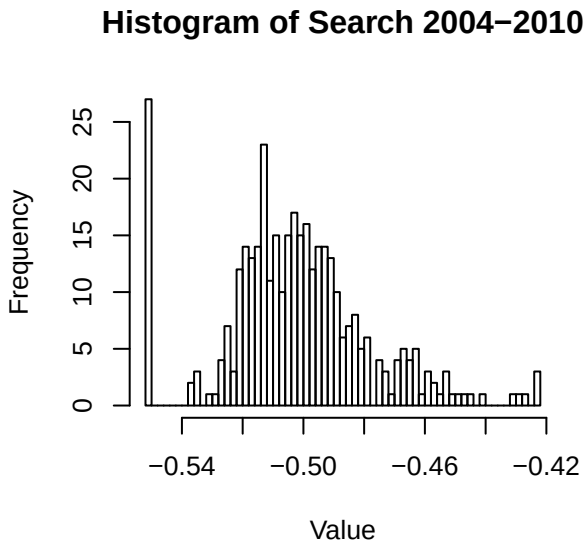
The following two pages show the comparative time series plots, ACF and PACF for the early and later time series.

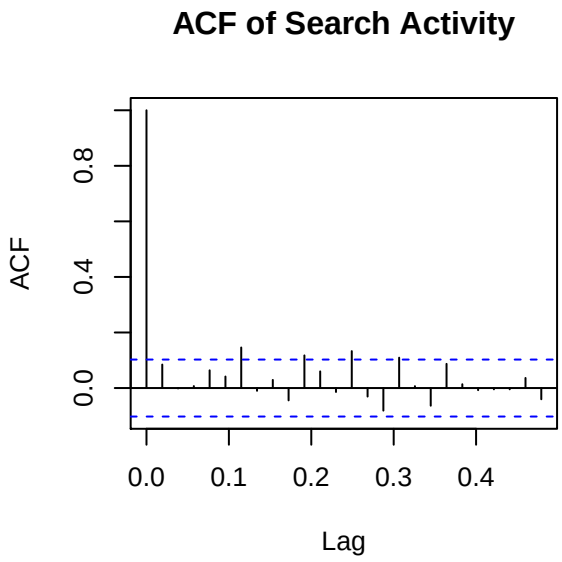
[1] "function"

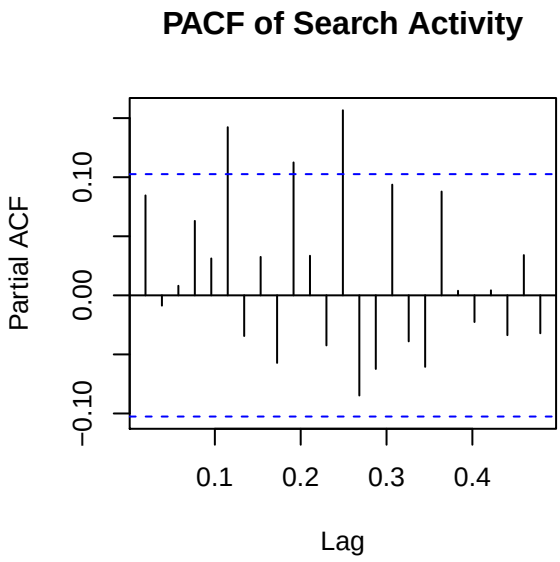
Table 9: Comparative Statistics

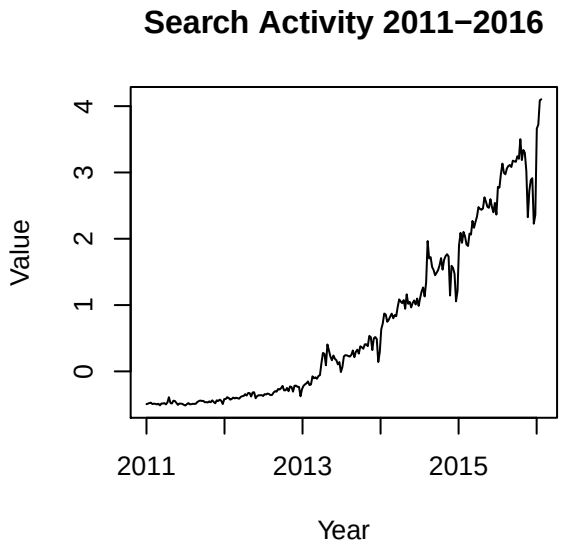
Statistic	N	Mean	St. Dev.	Min	Max
2004-2010	365	−0.50	0.02	−0.55	−0.42
2011-2016	265	0.69	1.25	−0.52	4.10

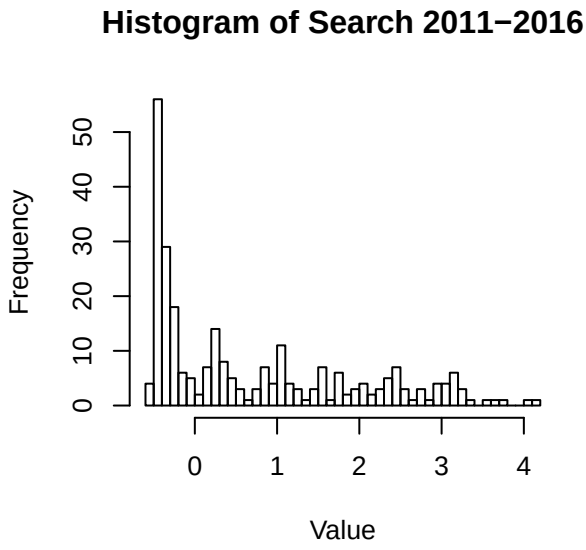

Search Activity 2004−2010
Value
−0.54
−0.50
−0.46
−0.42
2004
2006
2008
2010
Year

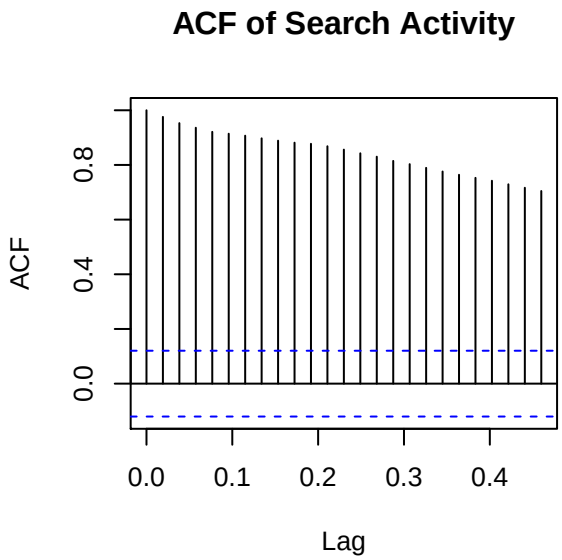

Histogram of Search 2004−2010
Frequency
0
5
10
15
20
25
−0.54
−0.50
−0.46
−0.42
Value

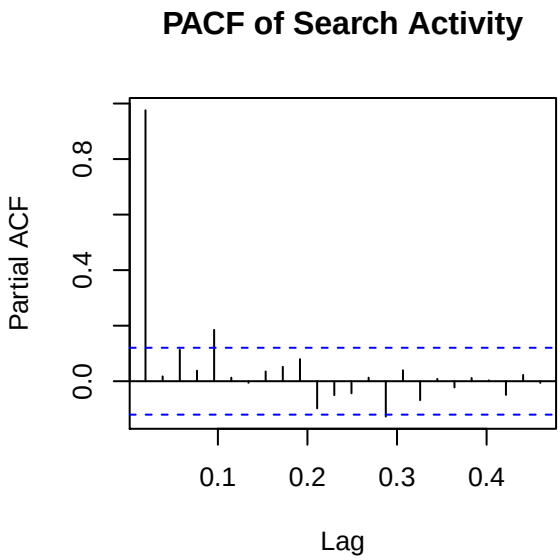

ACF of Search Activity
ACF
0.0
0.4
0.8
0.0
0.1
0.2
0.3
0.4
Lag


PACF of Search Activity
Partial ACF
−0.10
0.00
0.10
0.1
0.2
0.3
0.4
Lag


Search Activity 2011−2016
Value
0
1
2
3
4
2011
2013
2015
Year

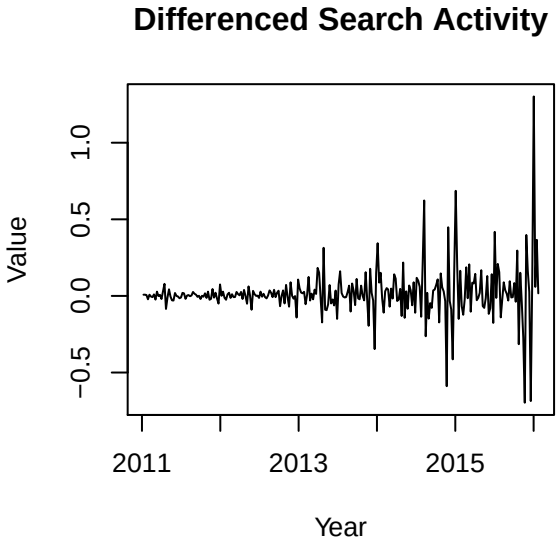

Histogram of Search 2011−2016
Frequency
0
10
20
30
40
50
0
1
2
3
4
Value

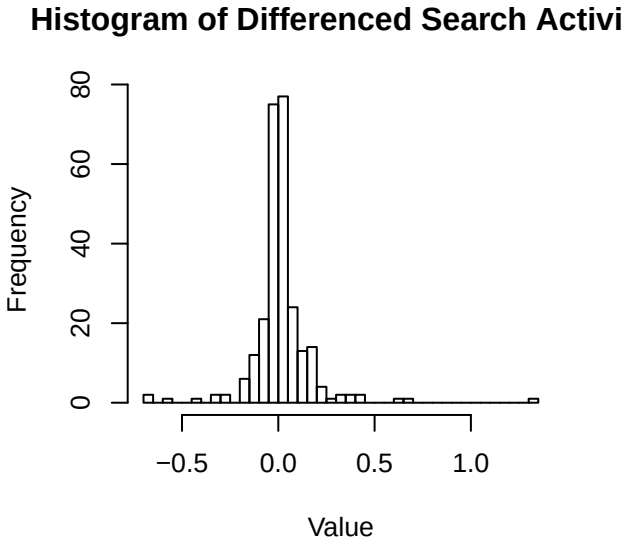

ACF of Search Activity
ACF
0.0
0.4
0.8
0.0
0.1
0.2
0.3
0.4
Lag

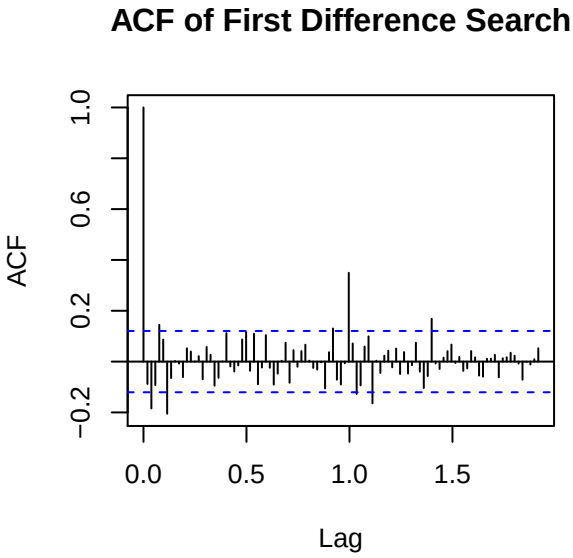

PACF of Search Activity
Partial ACF
0.0
0.4
0.8
0.1
0.2
0.3
0.4
Lag

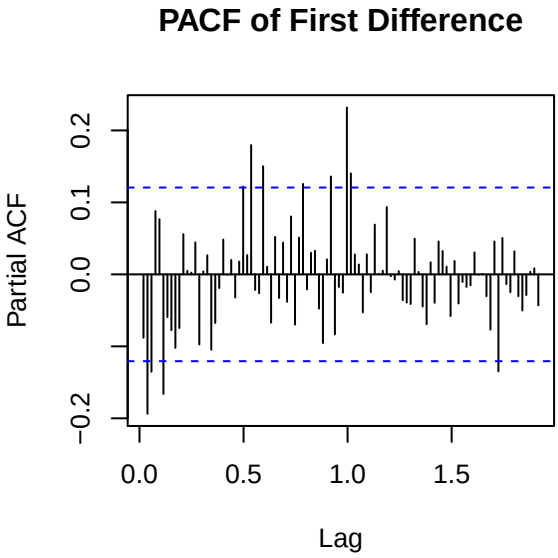
Time Series 2011-2015 Data Exploration

We can see from the plots on the previous page that the time series is not stationary. It has a persistent trend and exhibits seasonality with an increasing variance. We first examine the first difference of the series.



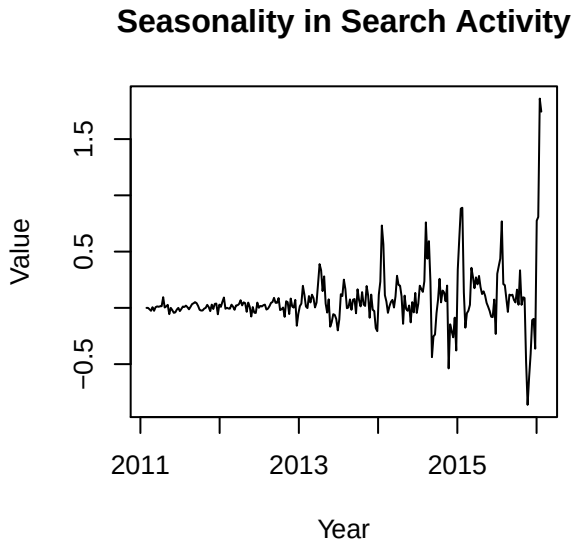


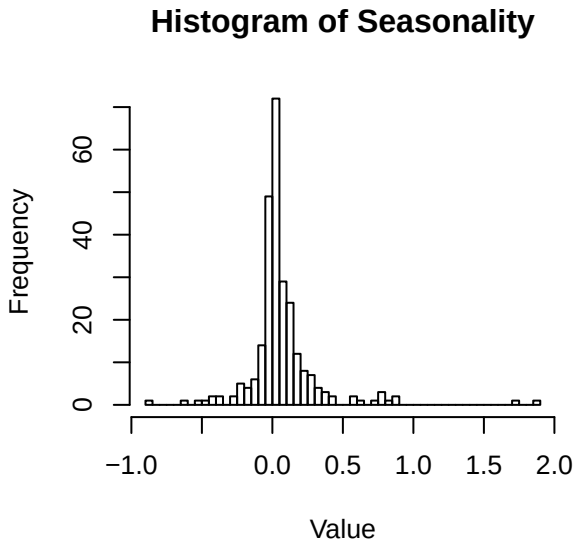


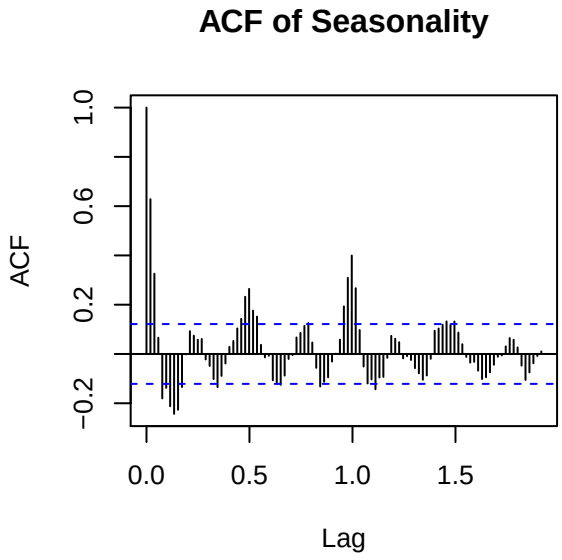


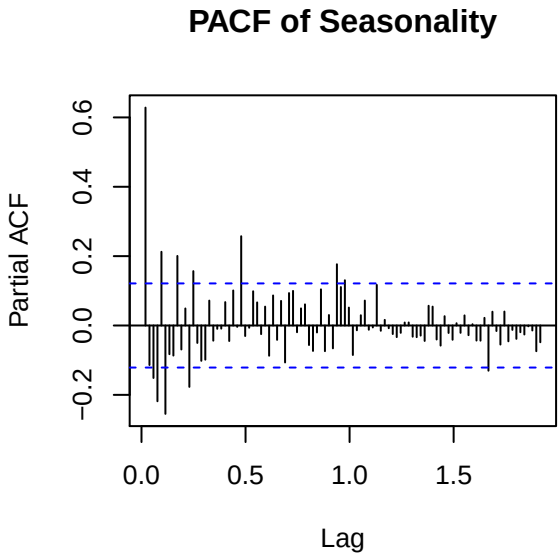
The resulting differenced series has increasing volatility over time. The ACF is indicative of an AR(2) process and the PACF indicates an MA process as well.

The seasonal component is present and seems to be in multiples of 4 and 13.









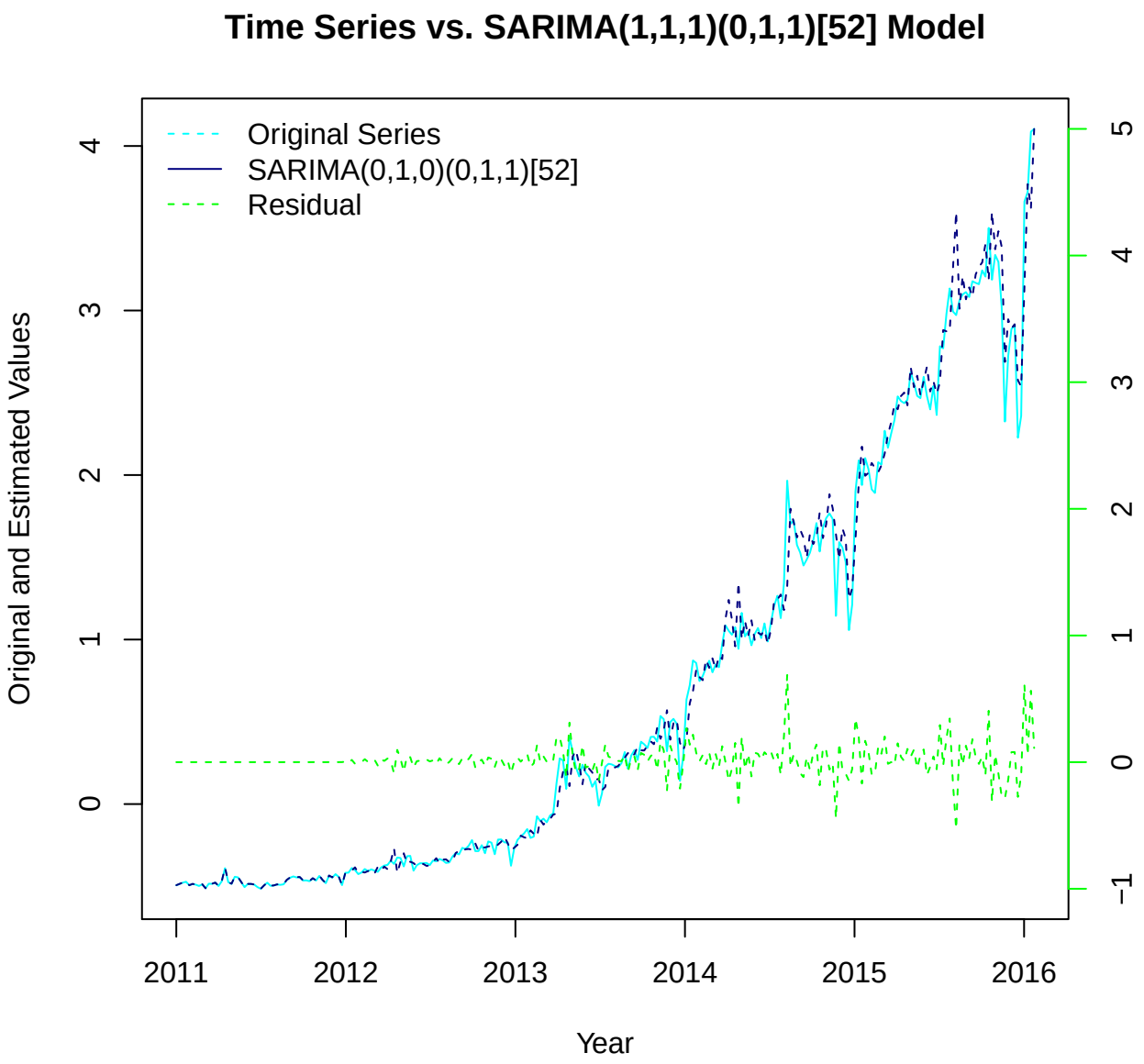
Modeling

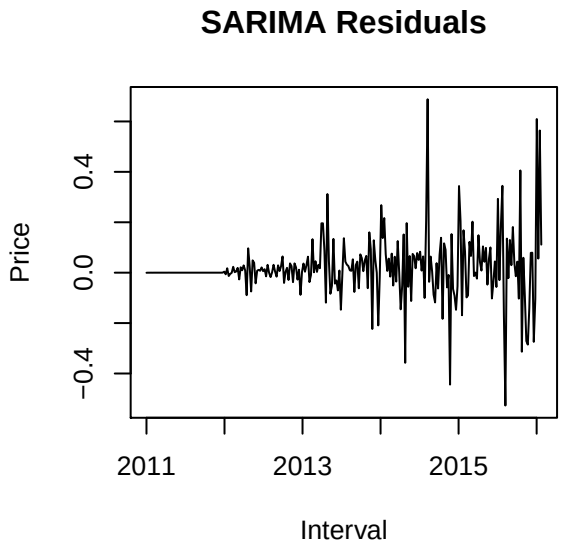
We use the `auto.arima` function to estimate the best SARIMA model and we obtain a SARIMA(1,1,1)(0,1,1)[52]. The in-fit plot, below, shows that the estimated series is a reasonable approximation of the original series and captures most of the dynamics of that series. However, note that the volatility of the residual series increases with time, indicating possible heteroskedasticity of the residuals.

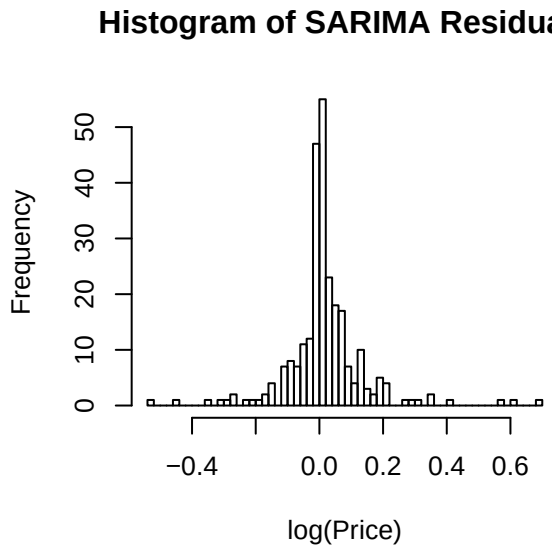
[1] "function"

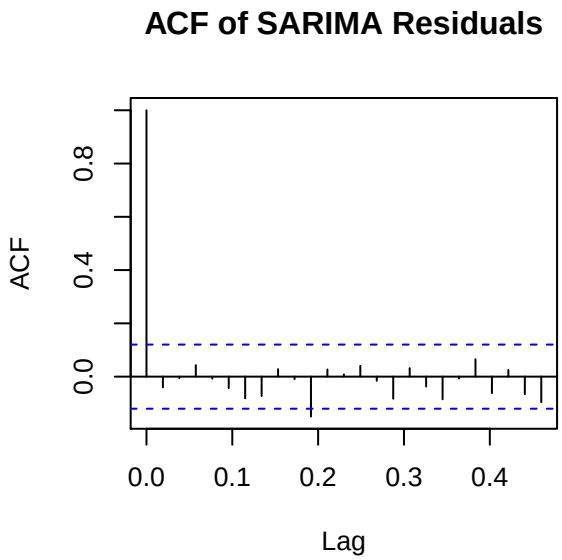
Table 10: Comparative Statistics

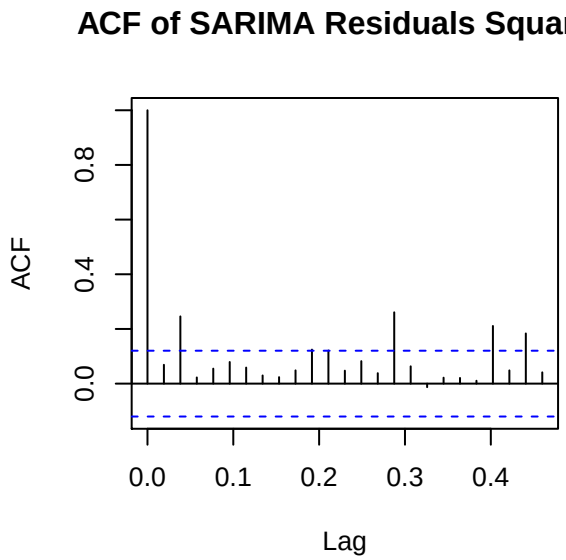
Statistic	N	Mean	St. Dev.	Min	Max
2011-2016 Series	265	0.7	1.2	−0.5	4.1
SARIMA(0,1,0)(0,1,1)[52]	265	0.7	1.2	−0.5	4.0
Residuals	265	0.02	0.1	−0.5	0.7







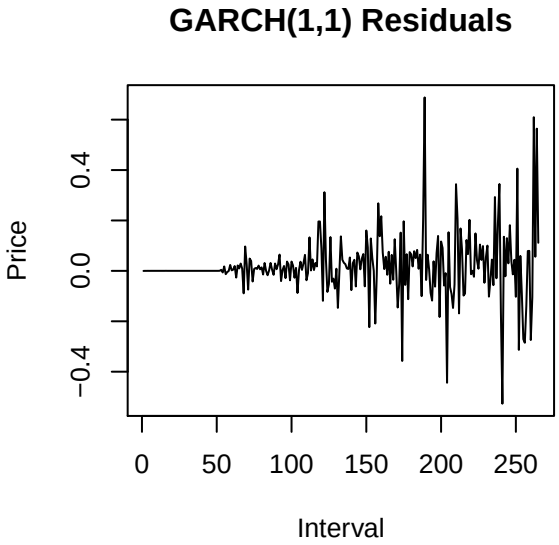


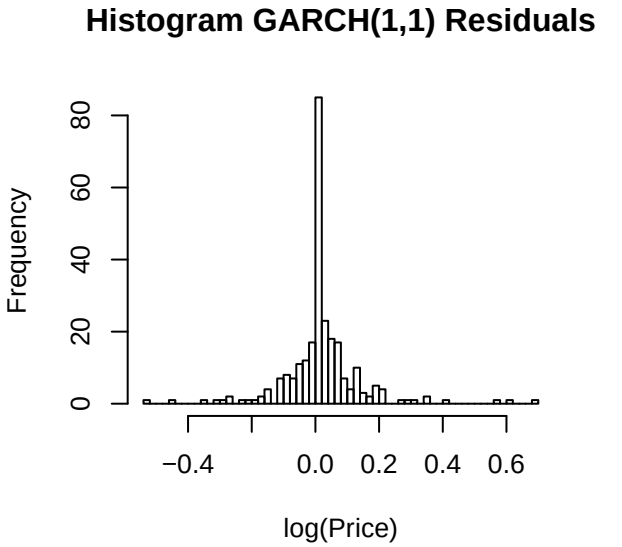


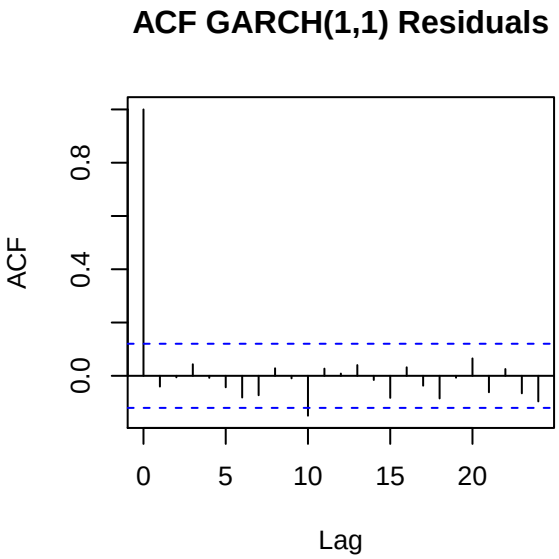
The residual continues to show increasing volatility over time, and the autocorrelation of the squared residuals confirms the heteroskedasticity of the residuals.

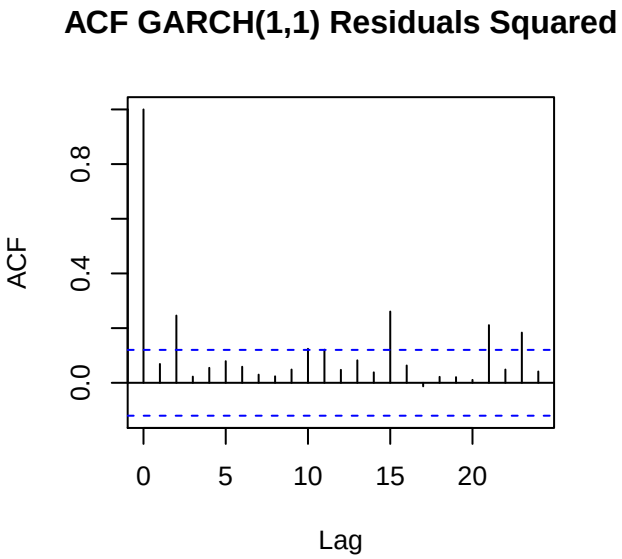
A GARCH(1,1) model is estimated for the SARIMA model residuals and the results are explored in the next few graphs.

Garch model



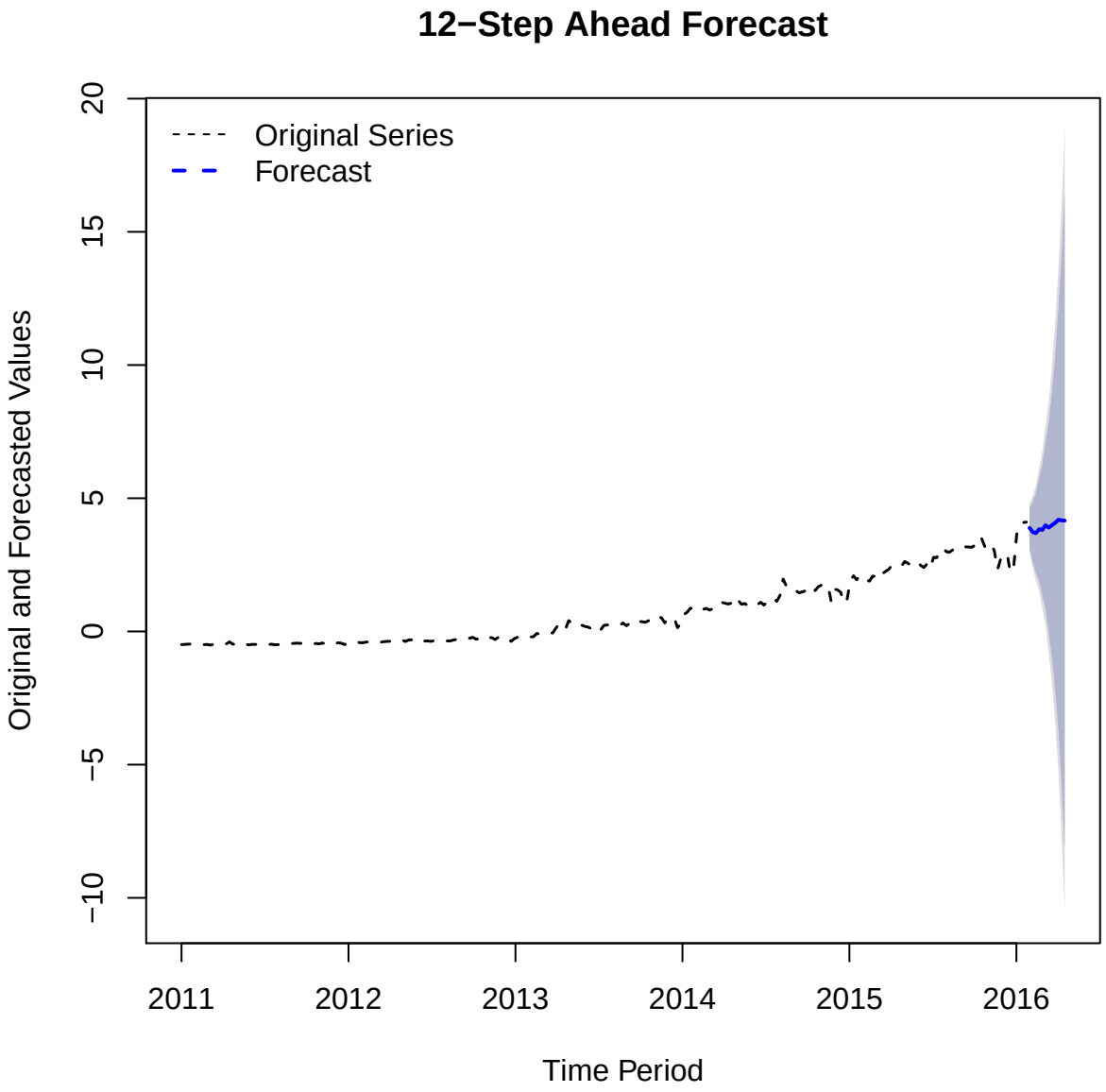






The Box-Ljung tests run as part of the `garchFit()` output show that for lags up to 20 the resulting residuals are independent while the Jarque-Bera test and Shapiro-Wilk test show that the residual distribution is not normal. However the squared residual plot seems to indicate otherwise.

Forecast

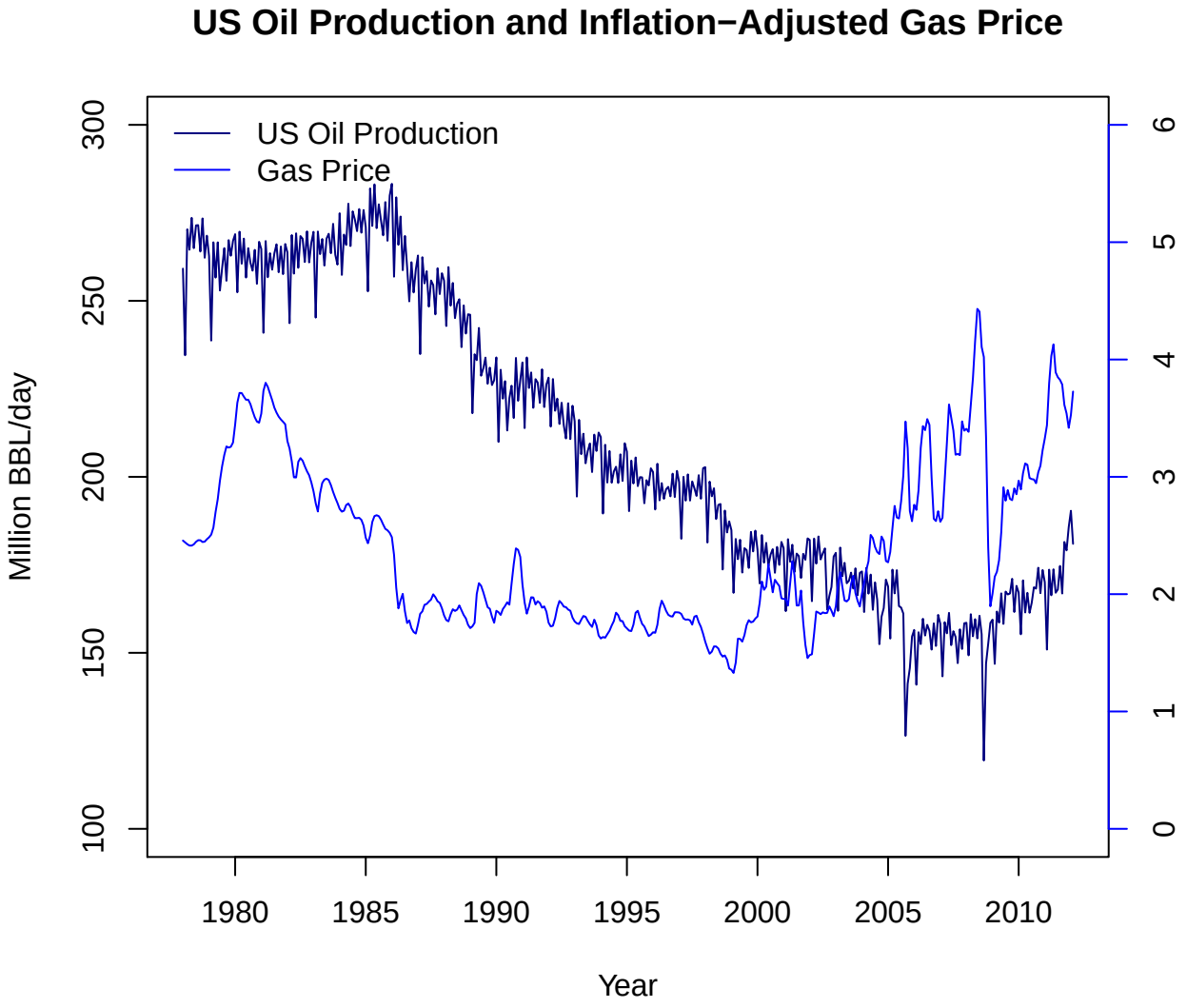


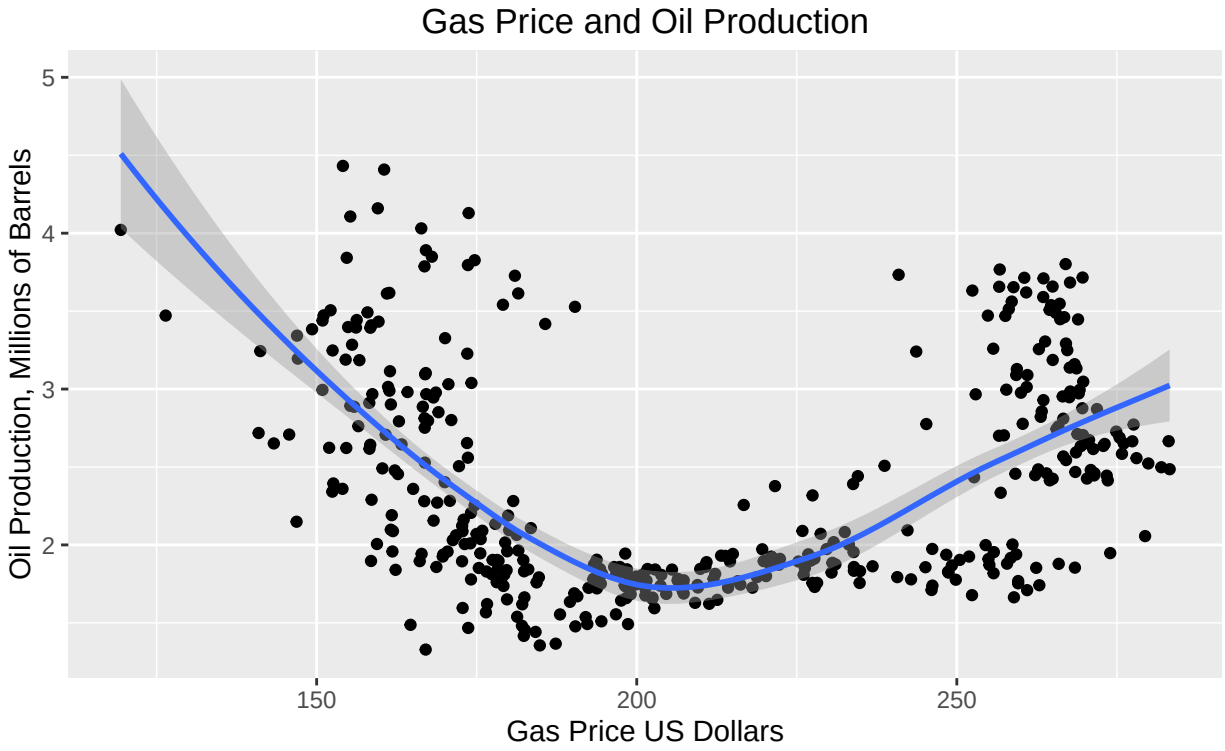
The forecast is projected 12 steps ahead as depicted in the graph, above. The confidence intervals have been adjusted for the GARCH estimated process of the residuals.

Part 4 - Forecast Inflation-Adjusted Gas Price

Exploratory Data Analysis

The *gasOil* series consists of 410 observations of two variables, *Production* and *Price*. As described the *Production* variable is in units of Million Barrels of Oil and *Price* is in inflation adjusted US Dollars. The observations are monthly beginning 1978/1/1 and ending 2012/2/1. The plots of the two variables indicate that both series appear similar to random walks: they have varying trends up and down over time. The *Production* series appears to have a seasonal component. The *Price* series may have a seasonal component as well but it isn't as prominent. A scatterplot of the two variables does not reveal any obvious correlation.



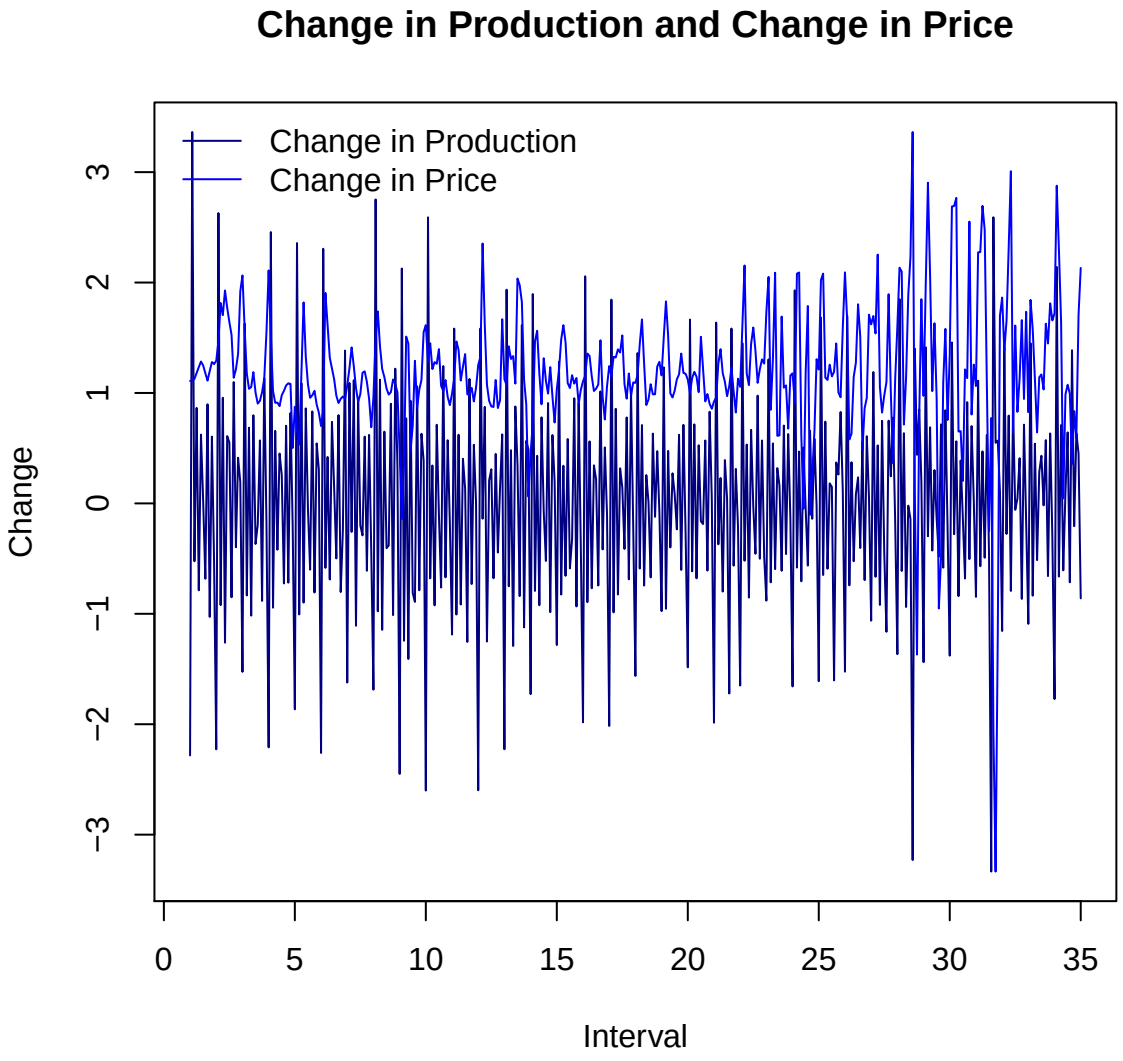


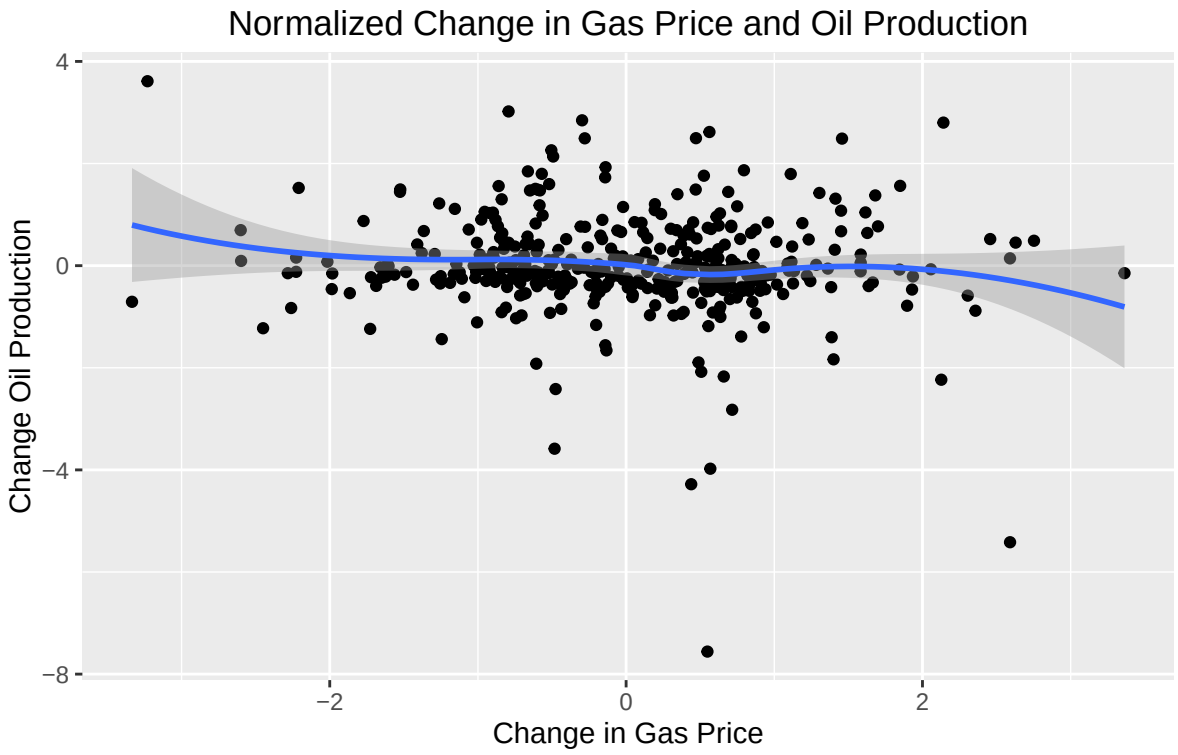
Change in Production and Change in Price

We take the difference of each series in order to compate the change in production to the change in price. We also scale the two variables so as to have zero mean and a standard deviation of 1.0 to make them more comparable. Now the scatterplot appears as a cluster about the origin of the graph.

Table 11: Descriptive Statistics of Differenced Series

Statistic	N	Mean	St. Dev.	Min	Max
Change in Production	409	−0.19	10.70	−35.85	35.78
Change in Price	409	0.003	0.13	−0.95	0.46
Scaled Change in Production	409	0.00	1.00	−3.33	3.36
Scaled Change in Price	409	0.00	1.00	−7.56	3.61


Change in Production and Change in Price
Change in Production
Change in Price
Change
Interval
−3
−2
−1
0
1
2
3
0
5
10
15
20
25
30
35



Model Recreation

The AP analysis states that there is no statistically significant correlation between oil production and gas prices. We can reconstruct a means of measuring a correlation and its significance using linear regression. We perform a regression on both the *Production* and *Price* variables as well as the *Change in Production* and *Change in Price*. The results of the regressions are summarized in the table, below.

The regression on the *change in production* and the *change in price* shows a marginally significant p-value but only at the $\alpha = 0.1$ level. The standard error shows that the estimate range crosses 0 so it is not distinguishable from 0. As expected all regression coefficients are not distinguishable from 0 and this reproduces the AP analysis conclusion that there is no statistically significate correlation between the two variables.

Critique

Comparing *Price* and *Production* in this way does not take into account the dependency on time. Each variable has it's own seasonal effects and its own volatility. A more careful analysis would compare the variables as time series and analyse the lagged relationships and seasonal factors.

Table 12: Oil Production and Gas Price Model Summary

	Dependent variable:			
	Change in Production	Production	Change in Price	Price
	(1)	(2)	(3)	(4)
Change in Price	−0.08615*			
	(0.04938)			
Price		1.66562		
		(2.96964)		
Change In Production			−0.08615*	
			(0.04938)	
Production				0.00046
				(0.00082)
Constant	0.00000	206.02400***	0.00000	2.29431***
	(0.04932)	(7.39724)	(0.04932)	(0.17660)
Observations	409	410	409	410
R^2	0.00742	0.00077	0.00742	0.00077
Adjusted R^2	0.00498	−0.00168	0.00498	−0.00168
Residual Std. Error	0.99751 (df = 407)	41.91082 (df = 408)	0.99751 (df = 407)	0.69843 (df = 408)
F Statistic	3.04323* (df = 1; 407)	0.31459 (df = 1; 408)	3.04323* (df = 1; 407)	0.31459 (df = 1; 408)
Note:				*p<0.1; **p<0.05; ***p<0.01

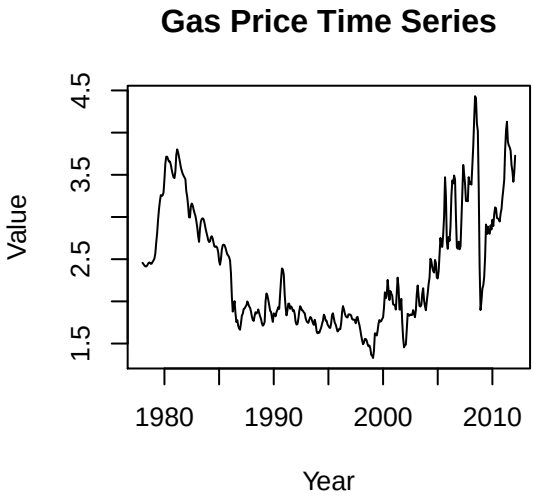
Time Series Analysis

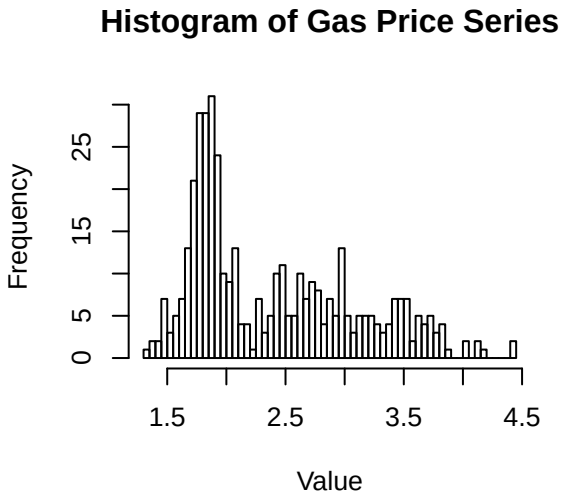
Observations of the *Price* series are:

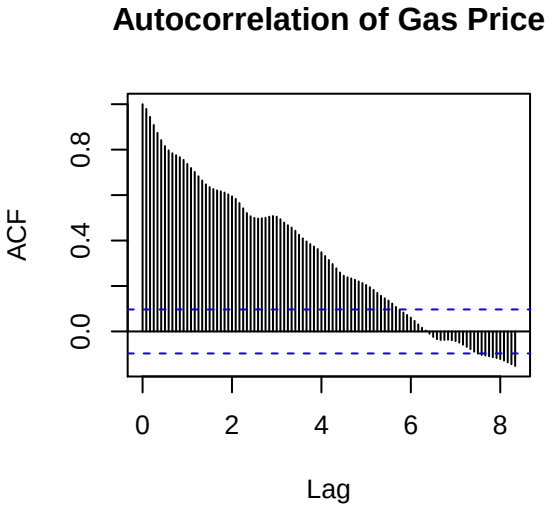
- The series appears like a random walk; it has periods when it is trending down and other periods when it trends up but it is not a constant trend in either direction.
- The autocorrelation plot shows very high autocorrelation and the partial autocorrelation shows very little lagged correlation. It appears that it could be a AR() underlying process but may have a small MA() component.

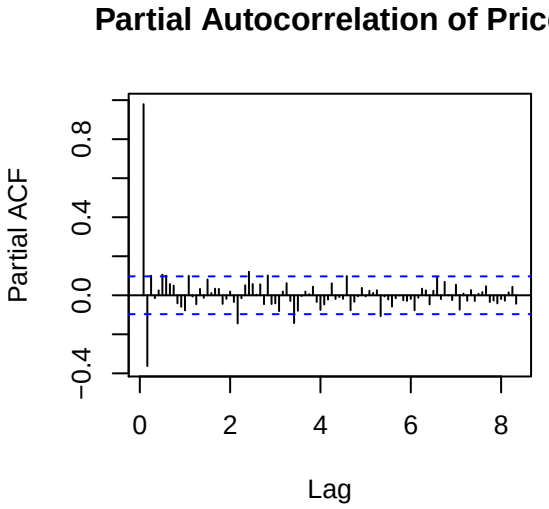
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.329	1.823	2.096	2.391	2.909	4.432

Table 13: Price Series Statistical Summary

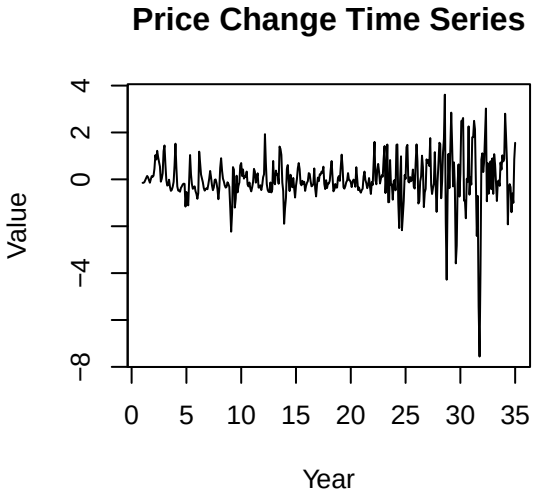


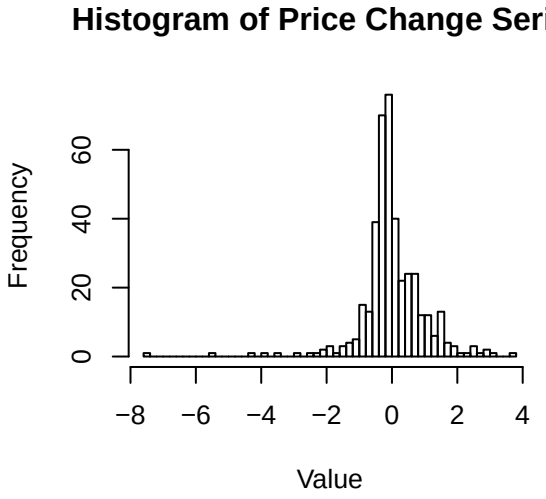


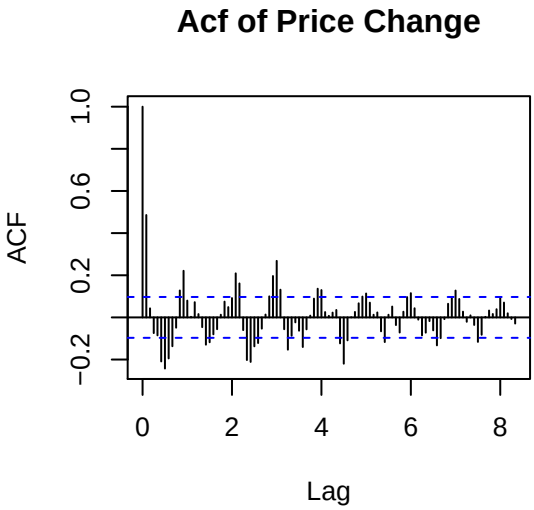


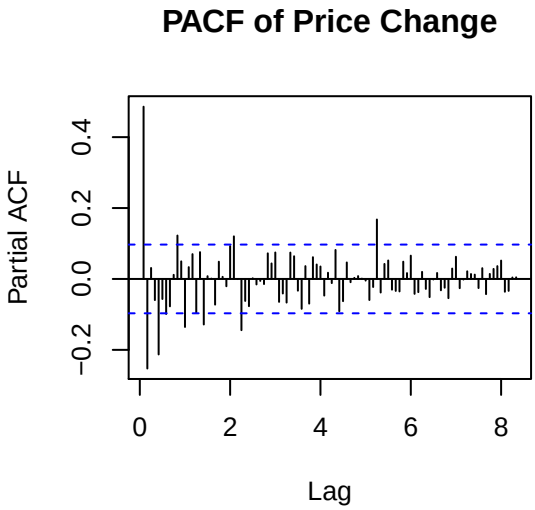


We calculated the differenced time series previously and we show the plots for the *Price* variable here for analysis. There appears to be a seasonal component in the ACF at around 6 lags.









	ARIMA(1,1,3)
ar1	0.76
	(0.09)
ma1	−0.15
	(0.10)
ma2	−0.39
	(0.06)
ma3	−0.24
	(0.05)
AIC	-675.65
AICc	-675.50
BIC	-655.58
Log Likelihood	342.82

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

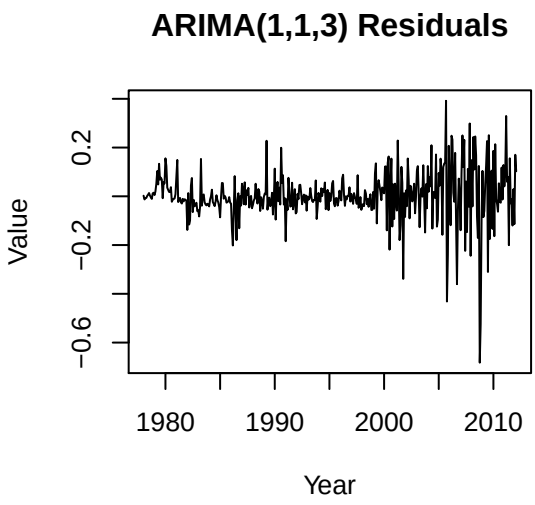
Table 14: ARIMA(1,1,3) Model Parameters

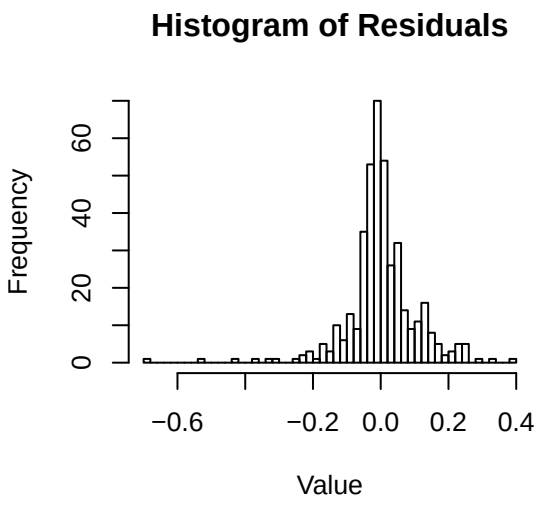
Model Estimation

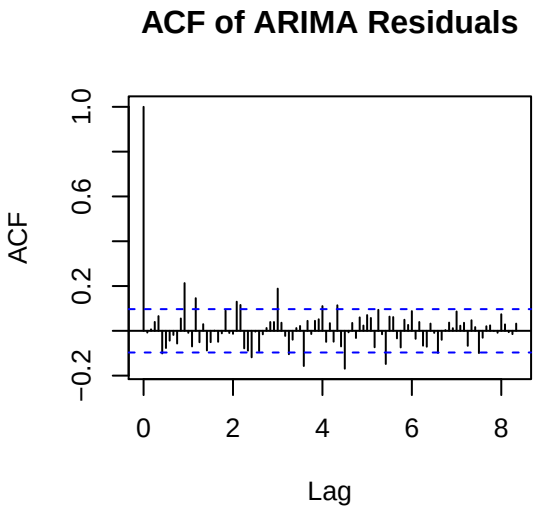
Using the `auto.arima` function in R we obtain an estimated ARIMA(1,1,3) model with no seasonal components. The estimated coefficients of the model are tabulated and plots of the resulting residuals and residual autocorrelation are shown. The residuals from the estimated ARIMA(1,1,3) model show a time dependency for which we can fit a GARCH model.

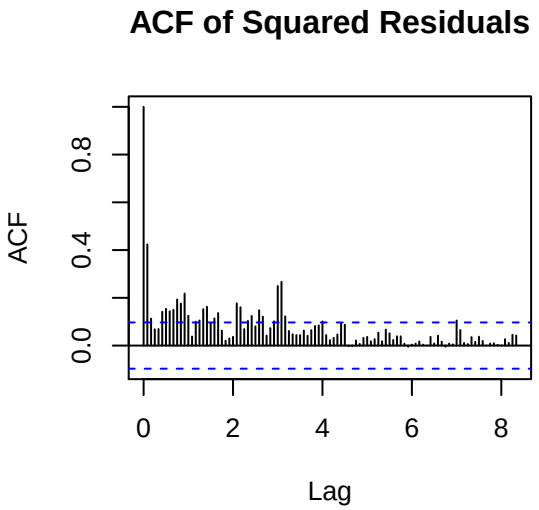
	2.5 %	97.5 %
ar1	0.57	0.94
ma1	-0.35	0.06
ma2	-0.51	-0.28
ma3	-0.34	-0.14

Table 15: ARIMA(1,1,3) Confidence Intervals


ARIMA(1,1,3) Residuals
Value
0.2
−0.2
−0.6
1980
1990
2000
2010
Year


Histogram of Residuals
Frequency
0
20
40
60
−0.6
−0.2
0.0
0.2
0.4
Value


ACF of ARIMA Residuals
ACF
1.0
0.6
0.2
−0.2
0
2
4
6
8
Lag


ACF of Squared Residuals
ACF
0.8
0.4
0.0
0
2
4
6
8
Lag

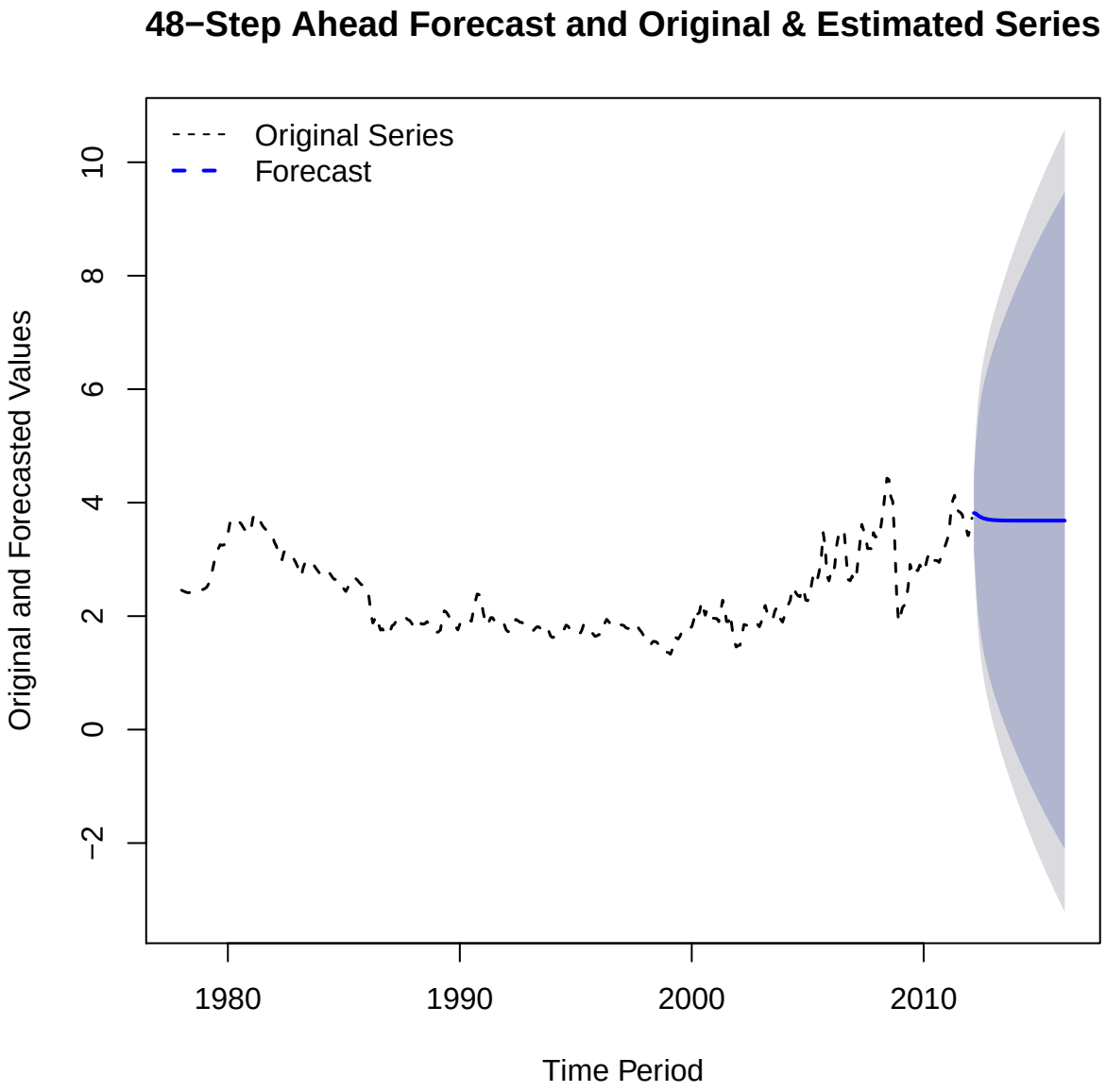
	GARCH(1,1)
mu	−0.01
	(0.01)
omega	0.01***
	(0.00)
alpha1	0.15***
	(0.03)
beta1	0.81***
	(0.03)
Num. obs.	1974
AIC	1.13
Log Likelihood	1106.61

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 16: GARCH(1,1) Model Parameters

Estimating the GARCH model on the residuals of the ARIMA(1,1,3) model we obtain a GARCH(1,1) model. The residuals of the GARCH model are shown and exhibit no correlation or time dependency and the Box-Ljung test indicates we are unable to reject the null hypothesis of an IID residual series.

Forecast


48−Step Ahead Forecast and Original & Estimated Series
Original Series
Forecast
Original and Forecasted Values
10
8
6
4
2
0
−2
1980
1990
2000
2010
Time Period