Applied Regression and Time Series Analysis
Live Session 2

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December 28, 2015

1 Agenda

1. Group exercise on Marginal Distributions, Joint Distributions, Conditional Distributions (20 minutes - breakout 10 min; Instructor-led discussion 10 min)

2. Group exercise on linear function on independent random variables (20 minutes - breakout 10 min; Instructor-led discussion 10 min)

3. Group exercise on hypothesis testing and discussion on Type I error , Type II error, power of a test (25 minutes - breakout 10 min; Instructor-led discussion 15 min)

4. Group exercise on the Lama Example (20 minutes)

2 Marginal Distributions, Joint Distributions, Conditional Distributions

1. Draw a graph of the region for which X and Y have positive probability density.

2. Derive the marginal probability density function of X, $f_X(x)$.

3. Derive the marginal probability density function of Y, $f_Y(y)$.

4. Derive the conditional probability function of Y, conditional on X, $f_{Y|X}(Y|x)$.

5. Derive the conditional expectation function of Y, conditional on X, $E(y|x)$.

3 Linear Function of Independent Random Variables

Suppose X and Y are independent continuous random variables, where both of which are uniformly distributed between 0 and 1. Let random variable $Z = X + Y$.

1. Choose a value of z between 0 and 2, and draw a graph depicting the region of the $X - Y$ plane for which Z is less than z.

2. Write down an expression for the cumulative density function $F(Z)$, the probability that Z is less than a value z.

3. Based on the cdf, derive the probability density function, $f(z)$, the probability density at a value, z.

4 W203 Re-visit: Hypothesis Testing and Type I error, Type II error, Power of a test

Students break out to work on the following problem and discuss definitions of Type I error, Type II error, and the power of a test.

An Example
Consider an example where we want to test the null hypothesis that the population mean μ weight of a consumer product is 5 ounces versus an alternative hypothesis that the population mean μ is greater than 5 ounces.

To conduct the test, we collect a random sample of 16 observations, and we choose a significance level of 0.05. Assume the population distribution of the weight of this consumer product follows a normal distribution with standard deviation of 0.1 ounces.

1. Write down the Null Hypothesis

2. Write down the Alternative Hypothesis

3. Write down the decision rule (to reject the null hypothesis)

4. Determine the probability that H_0 will not be rejected if the true mean weight is 5.05 ounces. That is, find the Type II error of the test, if $\mu = 5.05$.

5. Find the power of the test, given that $\mu = 5.05$.