Homework 1
APPM 7400 Spr 2020 Theoretical ML

Due date: Friday, Jan 24, before 1 PM **Instructor**: Prof. Becker
Theme: Introduction, PAC learning, specialized PAC analysis (finite classes, axis-aligned rectangles)

Instructions Collaboration with your fellow students is OK and in fact recommended, although direct copying is not allowed. The internet is allowed for basic tasks (e.g., looking up definitions on wikipedia) but it is not permissible to search for proofs or to *post* requests for help on forums such as `http://math.stackexchange.com/` or to look at solution manuals. Please write down the names of the students that you worked with.

An arbitrary subset of these questions will be graded.

Reading You are responsible for chapters 1, 2, 3 and up to 4.1 in "Understanding Machine Learning" by Shai Shalev-Shwartz and Shai Ben-David (2014, Cambridge University Press). Note: you can buy the book for about $45 on Amazon (or less for an e-book), and the authors host a free PDF copy on their website (but note that this PDF has different page numbers).

Problem 1: Problem 2.1 in Shalev-Shwartz and Ben-David on how thresholded polynomials can memorize a dataset. Given a training set $S = \{(\mathbf{x}_i, f(\mathbf{x}_i))\}_{i=1}^m$ with $\mathcal{X} = \mathbb{R}^d$ and $\mathcal{Y} = \{0, 1\}$, show there exists a polynomial p_S such that $h_S(\mathbf{x}) = 1$ iff $p_S(\mathbf{x}) \geq 0$ where

$$h_s(\mathbf{x}) = \begin{cases} f(\mathbf{x}_i) & \text{if } \exists i \in [m] \, \text{s.t. } \mathbf{x}_i = \mathbf{x} \\ 0 & \text{otherwise} \end{cases}.$$

Problem 2: Problem 2.3 in Shalev-Shwartz and Ben-David on axis aligned rectangles. This has 4 parts. Parts 3 and 4 do not have to be done rigorously. Throughout, assume the distribution $\mathcal{D}$ is a continuous probability distribution. For Part 1, show that A is an ERM *with probability 1*.

Optional Problem 2.2 in Shalev-Shwartz and Ben-David. This is easy yet could be confusing (e.g., the proof is almost trivial).