Module 1

Monte Carlo Simulations

Problem Background:

- Suppose a hedge fund owns **$1,000,000** of stock and used *$50,000* of its own capital and *$950,000* in borrowed money for the purchase.
- Suppose that if the value of the stock falls below *$950,000* at the end of any trading day, then the hedge fund will sell all the stock and repay the loan.
- This will wipe out its $50,000 investment.

The hedge fund is said to be leveraged **20:1** since its position is *220* times the amount of its own capital invested.

- Suppose that the daily log returns on the stock have a mean of **0.05**/year and a standard deviation of **0.23**/year.

These can be converted to rates per trading day by dividing by 253 and sqrt(253), respectively.

Initialization

```
niter <- 1e5 # number of iterations

seed.capital <- 5e4
initial.investment <- log(1e6)

profit.threshold <- log(1.1e6)
loss.threshold <- log(9.5e5)
target.profit <- 1e5

simulate_market <- function(days) {
  # generate random returns for N days
  r <- rnorm(days, mean = 0.05 / 253,
             sd = 0.23 / sqrt(253))

  cumsum(r) # return the final log price after N days.
}
```

Problem 4

What is the probability that the value of the stock will be below $950,000 at the close of at least one of the next 45 trading days?

```
set.seed(2009) # reproducible

# setup storage
outcomes <- list(below = rep(0, niter))

# Simulation: Probability dips below $950,000.
for (i in 1:niter) {

  logPrice = initial.investment + simulate_market(45) # simulate 45 trading days.

  minlogP = min(logPrice) # miniumum price over next 45 days

  outcomes$below[i] = as.numeric(minlogP < loss.threshold)
}
```

Probability the value of the stock is below $950,000 at least one of next 45 sessions: **50.99%**

Suppose the hedge fund will:

- sell the stock for a profit of at least **$100,000** if the value of the stock rises to at least **$1,100,000** at the end of one of the first *100* trading days,
- sell it for a loss if the value falls below **$950,000** at the end of one of the first *100* trading days,
- or sell it (for "FMV") after *100* trading days if the closing price has stayed between *$950,000* and *$1,100,000*.

```
set.seed(2009) # reproducible

outcomes <- list(above = rep(0, niter),
                 below = rep(0, niter),
                 middle = rep(0, niter),
                 pnl = rep(0, niter),
                 open = rep(0, niter))

for (i in 1:niter) {

  logPrice = initial.investment + simulate_market(100) # simulate 100 trading days.

  suppressWarnings({
    # ignore Inf returned if condition not meet.
    profit.day <- min(which(logPrice >= profit.threshold))
    loss.day <- min(which(logPrice <= loss.threshold))
  })

  # What was the exit condition of the position, hince the final price of the stock?
  daysOpen <- ifelse(profit.day == Inf && loss.day == Inf, length(logPrice),
                     min(profit.day, loss.day))

  outcomes$above[i] <- min(profit.day) < min(loss.day)
  outcomes$middle[i] <- profit.day == Inf && loss.day == Inf
  outcomes$below[i] <- min(loss.day) < min(profit.day)

  # p&l = ending value - initial investment
  outcomes$pnl[i] <- exp(logPrice[daysOpen]) - exp(initial.investment)
  outcomes$open[i] <- daysOpen
}

stopifnot(sum(outcomes$above) + sum(outcomes$below) + sum(outcomes$middle) == niter)
```

Problem 5

What is the probability that the hedge fund (strategy) will make a profit of at least *$100,000*?

```
# Probability of profit over $100,000.
prob.profit.target <- sum(outcomes$pnl >= target.profit) / length(outcomes$pnl)

p5 <- round(mean(prob.profit.target), 3) * 100
```

Probability the hedge fund (strategy) returns over $100,000 in profit: **38.8%**

Problem 6

What is the probability that the hedge fund (strategy) will suffer a loss?

```
# Probability of loss
prob.loss <- sum(outcomes$below) / length(outcomes$below)

p6 <- round(mean(prob.loss), 3) * 100
```

Probability the hedge fund (strategy) returns a loss: **58.8%**

Problem 7

What is the expected profit from this trading strategy?

```
# floating pnl
floating.pnl <- sum(outcomes$pnl[outcomes$middle == 1])

# expected pnl if we assume:
# above close = $100,000,
# below close = -50,000,
# and middle = sell at market price.

avg.pnl <- (sum(outcomes$above) * 100000 +
   sum(outcomes$below) * -50000 +
   floating.pnl) / length(outcomes$pnl)

p7a <- round(mean(avg.pnl), 3)
```

Expected profit/loss assuming stop limits of $100,000 and -50,000: **$9922.63**

```
# Using only market prices and actual pnl.
ev.pnl <- mean(outcomes$pnl) # Expected P&L
p7b <- round(ev.pnl, 2)
```

Expected profit/loss with market orders: **$8910.54**

Problem 8

What is the expected return?

```
# Expected Return (TW)
strat.returns <- (ifelse(outcomes$pnl < 0,
    -seed.capital, outcomes$pnl) / seed.capital) / outcomes$open
p8 <- round(mean(strat.returns), 5) * 100
```

Expected (time-weighted) return of the hedge fund (strategy): **-1.47%**