Problem Background

Write an R program to find the efficient frontier, the tangency portfolio, and the minimum variance portfolio, and plot on "risk-reward space" the location of each of the six stocks, the efficent frontier, the tangency portfolio, and the line of efficient portfolios.

Use the constraints that, $-0.1 \leq w_j \leq 0.5$ for each stock.

The first constraint limits the short sales but does not rule them out completely.

The second constraint prohibits more than 50% of the investment in any single stock.

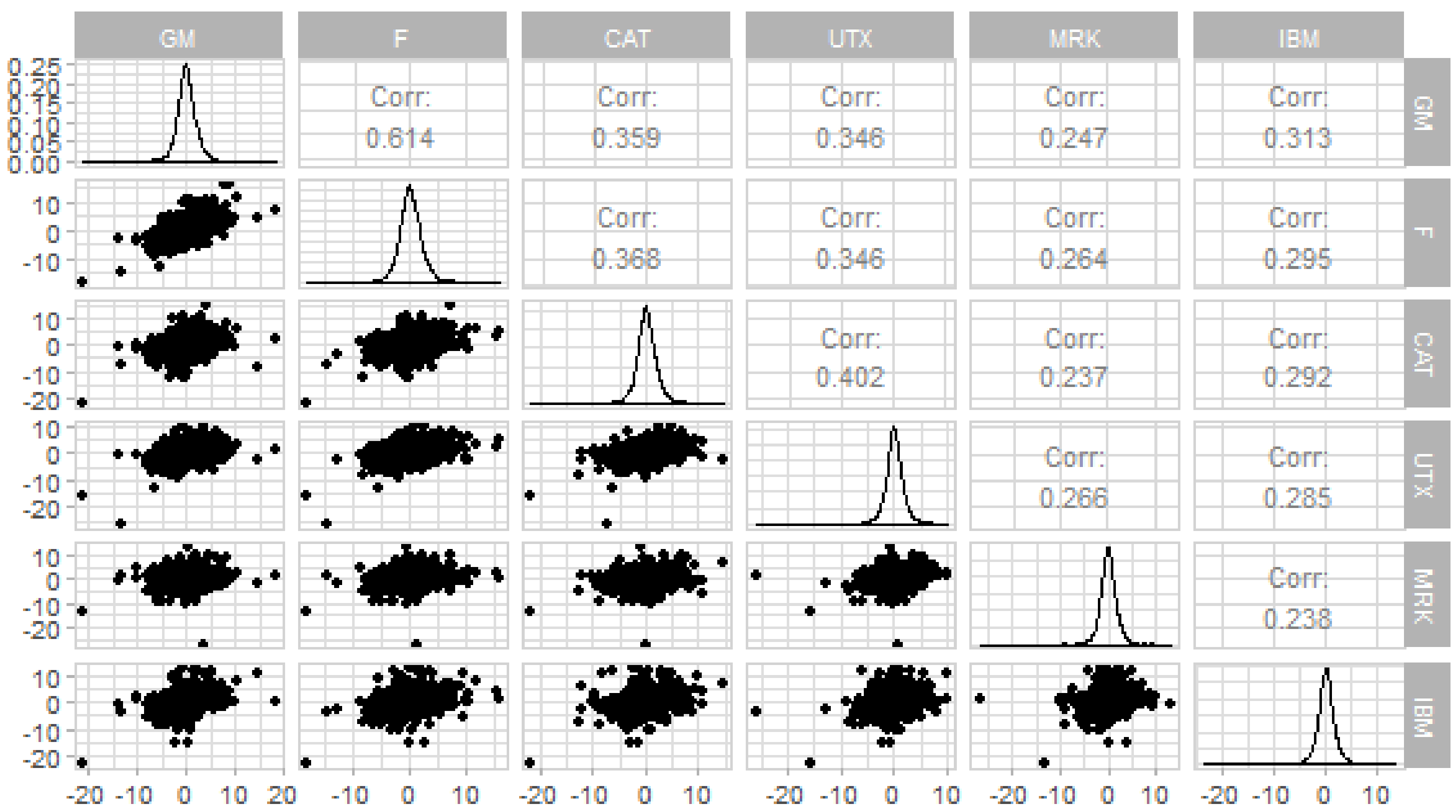
Assume that the annual risk-free rate is 3%.

```
data <- data.table(read.csv(paste0(data.dir, "Stock_Bond.csv"),
                   header = T))

prices <- cbind(GM = data$GM_AC, F = data$F_AC, CAT = data$CAT_AC, UTX = data$UTX_AC,
                MRK = data$MRK_AC, IBM = data$IBM_AC)

n <- nrow(prices)

returns <- data.table(100 * (prices[2:n,] / prices[1:(n-1),] - 1))
```



```r
cov.mat <- cov(returns)
mean.vec <- colMeans(returns)
sd.vec <- sqrt(diag(cov.mat))

rfr <- 3.0 / 365 # Daily risk-free rate

n.sims <- 500 # Simulations to find optimal allocation.
n.stocks <- ncol(prices) # Stocks to allocate.
c.vec <- c(-.10, .50) # Allocations between -10% and 50%.

# Storage.

mu_p <- seq(min(mean.vec), max(mean.vec), length = n.sims)
sd_p <- mu_p

out.weights <- matrix(0, nrow = n.sims, ncol = n.stocks)

# ?solve.QP

A.mat <- cbind(rep(1, n.stocks), mean.vec)
b.vec <- c(1, NaN)

# Lower-bound
A.mat <- cbind(A.mat, diag(1, n.stocks))
b.vec <- c(b.vec, c.vec[1]*rep(1, n.stocks))

# Upper-bound
A.mat <- cbind(A.mat, -diag(1, n.stocks))
b.vec <- c(b.vec, -c.vec[2]*rep(1, n.stocks))

# Find the optimal portfolios for each target expected return.
for ( i in 1:n.sims )
{
   # constraint vector
   b.vec[2] = +mu_p[i]

   result =
      solve.QP( Dmat = 2*cov.mat,
                dvec = rep(0, n.stocks),
                Amat = A.mat,
                bvec = b.vec,
                meq = 2)

   sd_p[i] = sqrt(result$value)
```

```
   out.weights[i, ] = result$solution
}

# Find maximum Sharpe's ratio
sharpe.ratios <- ( mu_p - rfr) / sd_p

# Find & save the tangency portfolio
tangent.index <- which.max(sharpe.ratios)

dt.tan.weights <- data.table(matrix(out.weights[tangent.index,], nrow = 1))
colnames(dt.tan.weights) <- colnames(prices)

# Find & save the minimum variance portfolio
minvar.index <- which.min(sd_p)

dt.minvar.weights <- data.table(matrix(out.weights[minvar.index,], nrow = 1))
colnames(dt.minvar.weights) <- colnames(prices)

# Portfolio weights must sum to 100%.
stopifnot(round(sum(dt.tan.weights), 4) == 1 &
          round(sum(dt.minvar.weights), 4) == 1)

# Find Sharpe Ratio of tangent portfolio.
tangent.sharpe <- sharpe.ratios[tangent.index]
```

```
# Plot Efficent Frontier

ef.data <- data.table(cbind(sd_p, mu_p))[, Index := .I]
ef.data[, ef := ifelse(Index > minvar.index, mu_p, NA)]

stocks.data <- data.table(x = sd.vec, y = mean.vec, symbol = colnames(prices))

suppressWarnings(print({
   ggplot(ef.data) +

      geom_abline(aes(intercept = rfr, slope = tangent.sharpe), col = "cornflowerblue", lty = 4

      geom_point(aes(sd_p, mu_p), col = "darkred", lwd = .15, alpha = .7) +
      geom_point(aes(sd_p, ef), col = "darkgreen", lwd = .15, alpha = .7) +

      geom_point(aes(0, rfr), col = "darkgreen", size = 3) +
      geom_text_repel(data = ef.data[tangent.index],
                aes(0, rfr, label = "Risk-free Rate"),
                size          = 4,
```

```
                 box.padding   = 1.5,
                 force         = 20,
                 segment.size  = 0.2,
                 segment.color = "grey50",
                 direction     = "x") +

      geom_point(data = ef.data[tangent.index], aes(sd_p, mu_p), col = "lightblue", size = 3) +
      geom_text_repel(data = ef.data[tangent.index],
                 aes(sd_p, mu_p, label = "Tangency"),
                 size = 4, box.padding = 1.5,
                 force = 30, segment.size  = 0.2,
                 segment.color = "grey50",
                 direction     = "x") +

      geom_point(data = ef.data[minvar.index], aes(sd_p, mu_p), col = "darkgrey", size = 3) +
      geom_text_repel(data = ef.data[minvar.index],
                 aes(sd_p, mu_p, label = "Minimum Variance"),
                 size = 4, box.padding = 1.5, force = 15,
                 segment.size  = 0.2, segment.color = "grey50",
                 direction = "x") +

      geom_text_repel(data = ef.data[1,],
                 aes(sd_p, mu_p, label = "Efficent Frontier"),
                 size = 4, box.padding = 1.5,
                 force = 10, segment.size  = 0.2,
                 segment.color = "grey50",
                 direction = "y") +

      geom_text(data = stocks.data, aes(x, y, label = symbol, col = symbol)) +

      geom_text(data = stocks.data, aes(mean(x), mean(y), nudge_y = -.5,
                                     label = "Individual Stocks"), color = "black", size = 5

      scale_y_continuous(limits = c(0, .1), labels = scales::percent_format(scale = 100)) +
      scale_x_continuous(limits = c(0, 2.5)) +

      labs(x = "Risk", y = "Return") +
      guides(color = "none") +
      theme(axis.line = element_line(colour = "black"),
         panel.grid.major = element_blank(),
         panel.grid.minor = element_blank(),
         panel.border = element_blank(),
         panel.background = element_blank())
}))
```

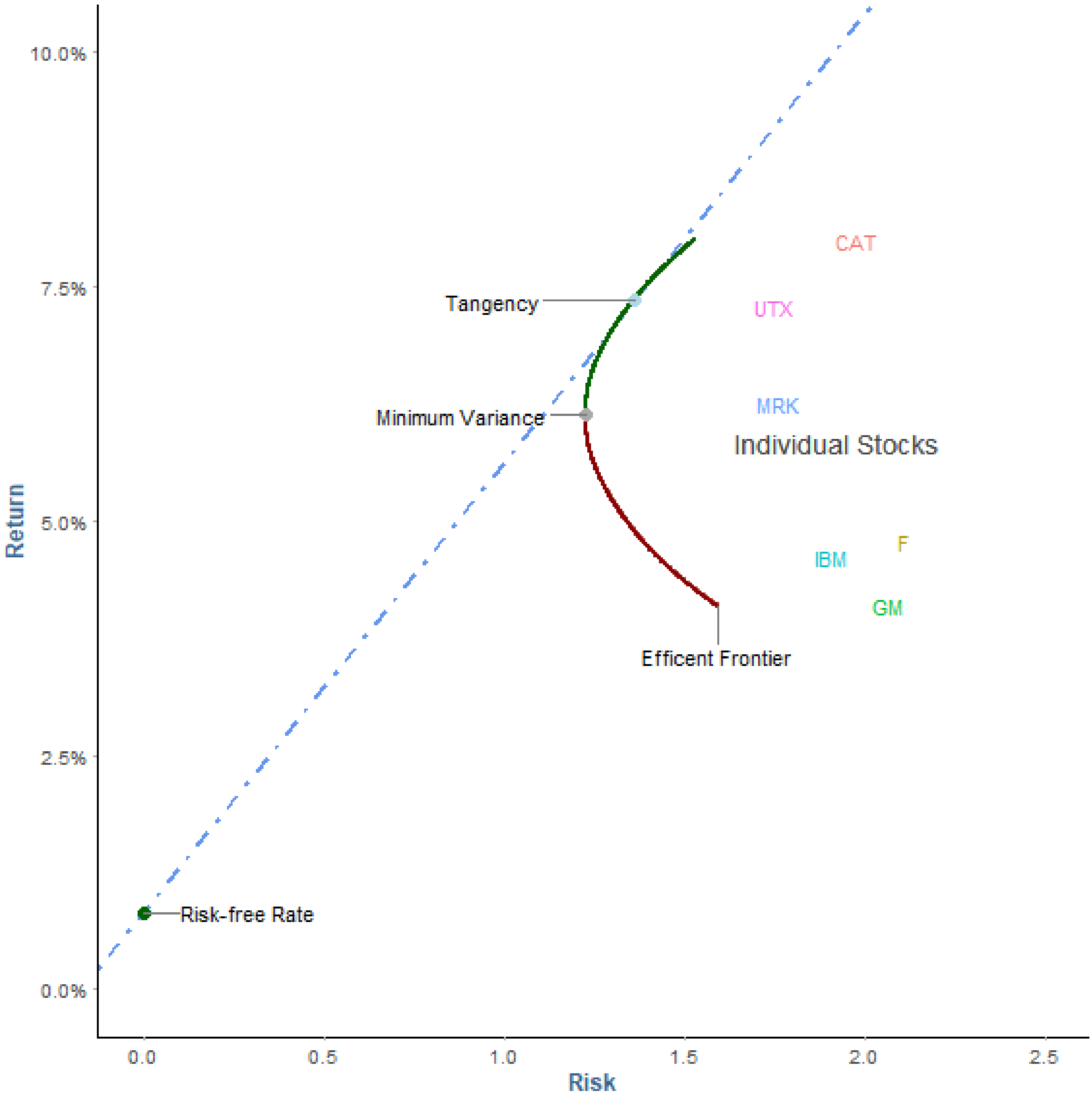


Table 1: Optimal Portfolio Weights

Portfolio	GM	F	CAT	UTX	MRK	IBM
Tangency	-0.09166	-0.00308	0.33589	0.38413	0.31955	0.05518
Minimum Variance	0.08340	0.05791	0.12815	0.23484	0.29593	0.19977