Problem Background

Write an R program to find the efficient frontier, the tangency portfolio, and the minimum variance portfolio, and plot on "risk-reward space" the location of each of the six stocks, the efficent frontier, the tangency portfolio, and the line of efficient portfolios.

Use the constraints that, $-0.1 \leq w_j \leq 0.5$ for each stock.

The first constraint limits the short sales but does not rule them out completely.

The second constraint prohibits more than 50% of the investment in any single stock.

Assume that the annual risk-free rate is 3%.

```
# Stock/Bond/FX data.
stocks <- as.data.table(read.csv(paste0(data.dir, "Stock_FX_Bond_2004_to_2005.csv"),
                                 header=T))
stocks$Date <- as.Date(stocks$Date, format = "%d-%b-%y")
stocks_subset <- stocks[, .(Date, GM_AC, F_AC, UTX_AC, MRK_AC)]
stocks_diff <- data.table(Date = stocks_subset[-1]$Date,
                          apply(log(stocks_subset[, .(GM_AC, F_AC, UTX_AC, MRK_AC)]), 2, diff))

# French-Farma data.
FF_data <- as.data.table(read.table(paste0(data.dir, "FamaFrenchDaily.txt"),
                                    header=T))
FF_data$Date <- as.Date(as.character(FF_data$date), format = "%Y%m%d")
FF_data$date <- NULL

# Combine into one data.table.
consolidated.data <- merge(stocks_diff, FF_data, on = c("Date"))
capm.data <- consolidated.data[, .(Date,
                                   GM = GM_AC - RF,
                                   Ford = F_AC - RF,
                                   UTX = UTX_AC - RF,
                                   Merck = MRK_AC - RF,
                                   Mkt.RF, SMB, HML, RF)]


summary(fit <- lm(as.matrix(cbind(GM, Ford, UTX, Merck))
                  ~ Mkt.RF + SMB + HML, data = capm.data))
```

```
Response GM :

Call:
lm(formula = GM ~ Mkt.RF + SMB + HML, data = capm.data)

Residuals:
      Min        1Q    Median        3Q       Max
```

```
-0.139498 -0.007662  0.000968  0.010022  0.147197

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0106077  0.0008924 -11.887   <2e-16 ***
Mkt.RF       0.0138621  0.0015652   8.856   <2e-16 ***
SMB         -0.0024251  0.0023081  -1.051   0.2939
HML          0.0063736  0.0027274   2.337   0.0198 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01989 on 499 degrees of freedom
Multiple R-squared:  0.1647,    Adjusted R-squared:  0.1597
F-statistic:  32.8 on 3 and 499 DF,  p-value: < 2.2e-16


Response Ford :

Call:
lm(formula = Ford ~ Mkt.RF + SMB + HML, data = capm.data)

Residuals:
      Min        1Q    Median        3Q       Max
-0.058213 -0.009663  0.000516  0.009240  0.096103

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.005e-02  7.083e-04 -14.185   <2e-16 ***
Mkt.RF       1.348e-02  1.242e-03  10.854   <2e-16 ***
SMB         -7.779e-05  1.832e-03  -0.042   0.9661
HML          3.780e-03  2.165e-03   1.746   0.0814 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01578 on 499 degrees of freedom
Multiple R-squared:  0.2477,    Adjusted R-squared:  0.2431
F-statistic: 54.76 on 3 and 499 DF,  p-value: < 2.2e-16


Response UTX :

Call:
lm(formula = UTX ~ Mkt.RF + SMB + HML, data = capm.data)

Residuals:
```

```
      Min        1Q    Median        3Q       Max
-0.028567 -0.006099 -0.000599  0.005712  0.037245

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0080963  0.0004199 -19.282  < 2e-16 ***
Mkt.RF       0.0102592  0.0007365  13.929  < 2e-16 ***
SMB         -0.0028475  0.0010861  -2.622  0.00901 **
HML          0.0003584  0.0012834   0.279  0.78013
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.009357 on 499 degrees of freedom
Multiple R-squared:  0.315, Adjusted R-squared:  0.3109
F-statistic: 76.48 on 3 and 499 DF,  p-value: < 2.2e-16


Response Merck :

Call:
lm(formula = Merck ~ Mkt.RF + SMB + HML, data = capm.data)

Residuals:
      Min        1Q    Median        3Q       Max
-0.303331 -0.005931  0.001621  0.007577  0.123256

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.008695   0.000926  -9.389  < 2e-16 ***
Mkt.RF       0.007066   0.001624   4.350 1.65e-05 ***
SMB         -0.004095   0.002395  -1.710  0.08795 .
HML         -0.009191   0.002830  -3.247  0.00124 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02064 on 499 degrees of freedom
Multiple R-squared:  0.06266,   Adjusted R-squared:  0.05702
F-statistic: 11.12 on 3 and 499 DF,  p-value: 4.479e-07
```