Problem Background

Write an R program to find the efficient frontier, the tangency portfolio, and the minimum variance portfolio, and plot on "risk-reward space" the location of each of the six stocks, the efficent frontier, the tangency portfolio, and the line of efficient portfolios.

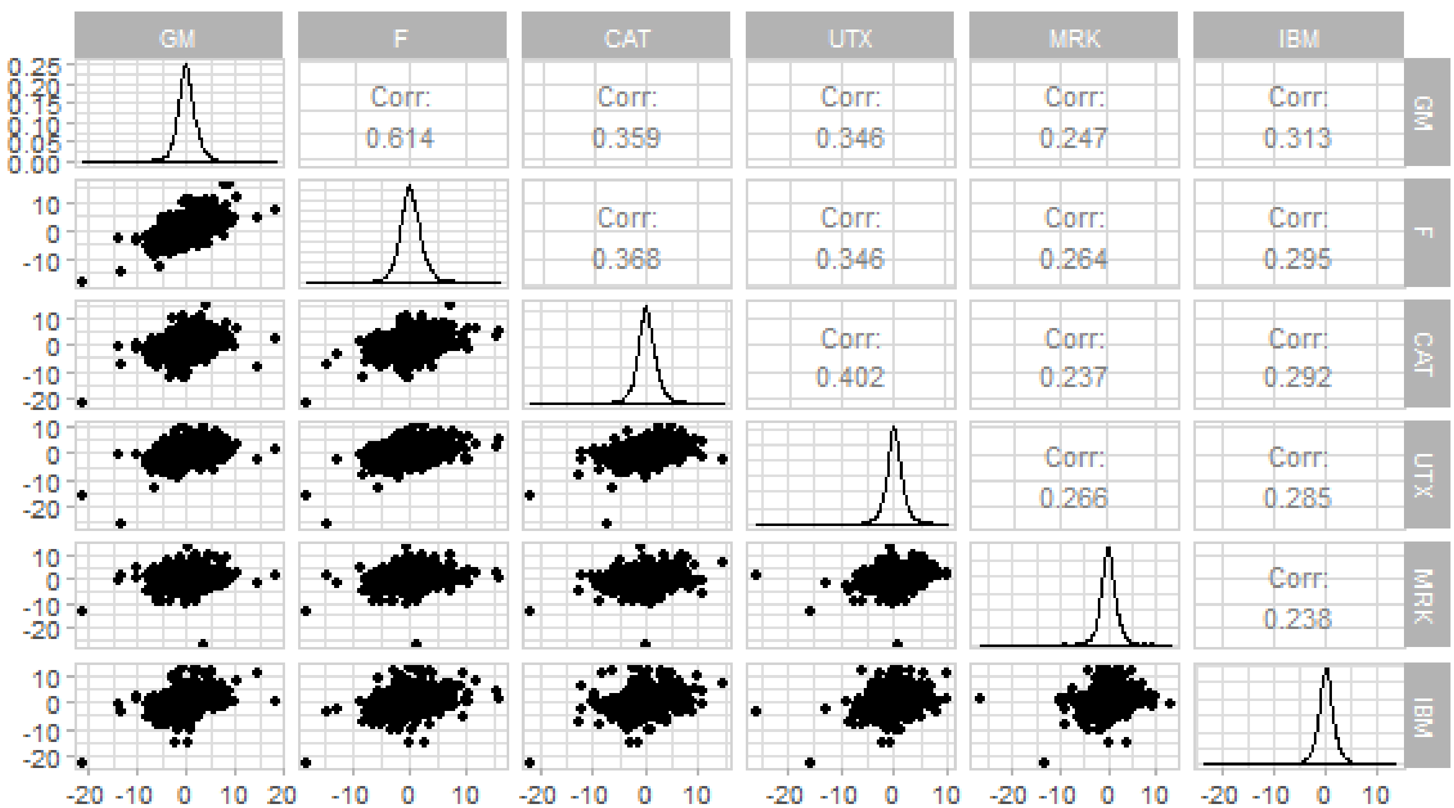
Use the constraints that, $-0.1 \leq w_j \leq 0.5$ for each stock.

The first constraint limits the short sales but does not rule them out completely.

The second constraint prohibits more than 50% of the investment in any single stock.

Assume that the annual risk-free rate is 3%.

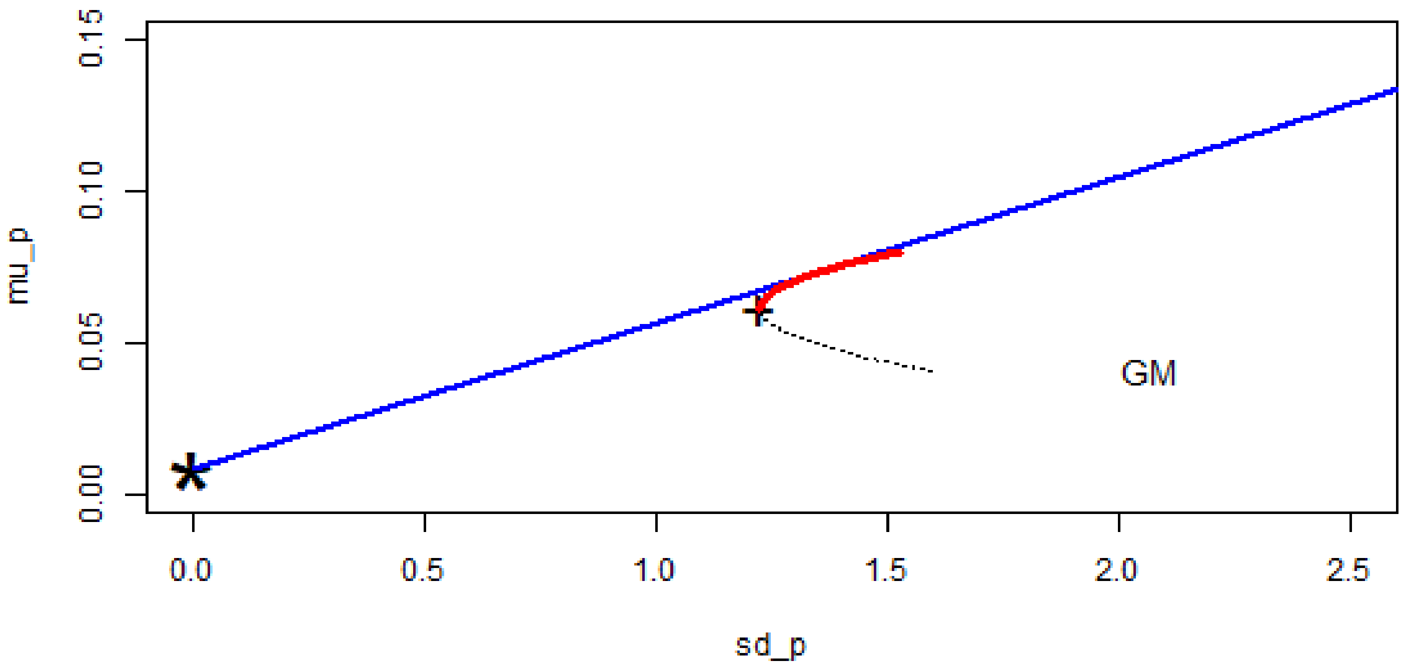
```
data <- data.table(read.csv(paste0(data.dir, "Stock_Bond.csv"),
                   header = T))

prices <- cbind(GM = data$GM_AC, F = data$F_AC, CAT = data$CAT_AC, UTX = data$UTX_AC,
                MRK = data$MRK_AC, IBM = data$IBM_AC)

n <- nrow(prices)

returns <- data.table(100 * (prices[2:n,] / prices[1:(n-1),] - 1))
```



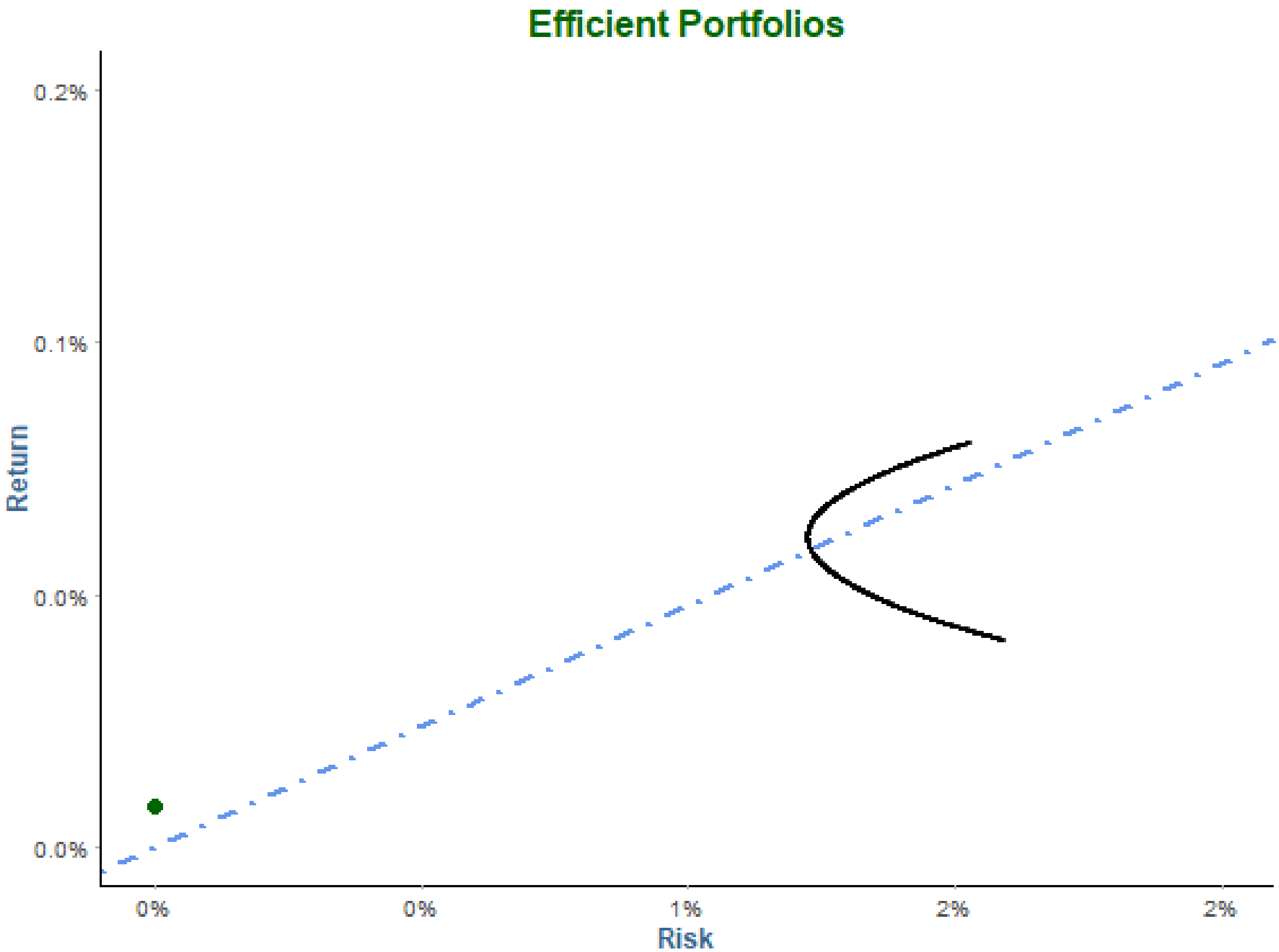
```
cov.mat <- cov(returns)
mean.vec <- colMeans(returns)
sd.vec <- sqrt(diag(cov.mat))

rfr <- 3.0 / 365 # Daily risk-free rate

n.sims <- 500 # Simulations to find optimal allocation.
n.stocks <- ncol(prices) # Stocks to allocate.

# Storage.

mu_p <- seq(min(mean.vec), max(mean.vec), length = n.sims)
sd_p <- mu_p

out.weights <- matrix(0, nrow = n.sims, ncol = n.stocks)

# ?solve.QP

c.vec <- c(-.10, .50) # Allocations between -10% and 50%.

A.mat <- cbind(rep(1, n.stocks), mean.vec)
b.vec <- c(1, NaN)

# Lower-bound
A.mat <- cbind(A.mat, diag(1, n.stocks))
b.vec <- c(b.vec, c.vec[1]*rep(1, n.stocks))

# Upper-bound
A.mat <- cbind(A.mat, -diag(1, n.stocks))
b.vec <- c(b.vec, -c.vec[2]*rep(1, n.stocks))

for ( i in 1:n.sims )  # find the optimal portfolios for each target expected return
{
  b.vec[2] = +mu_p[i]  # constraint vector

  result =
    solve.QP( Dmat = 2*cov.mat,
              dvec = rep(0, n.stocks),
              Amat = A.mat,
              bvec = b.vec,
              meq = 2)

  sd_p[i] = sqrt(result$value)
  out.weights[i, ] = result$solution
```

```
}

# Find maximum Sharpe's ratio
sharpe.ratio <- ( mu_p - rfr) / sd_p
tangent.index <- which.max(sharpe.ratio)

# Get the weights of the tangency portfolio.
tangent.weights <- out.weights[tangent.index,]

stopifnot(sum(tangent.weights) == 1)

# Find & show the minimum variance portfolio
minvar.index <- (sd_p == min(sd_p))

# Find & show the efficent frontier
efficent.frontier = (mu_p > mu_p[minvar.index])

# Find Sharpe Ratio of tangent portfolio.
sharpe <- (mu_p[tangent.index]-rfr)/sd_p[tangent.index]
```

```
plot(sd_p, mu_p, type="l", xlim=c(0,2.5), ylim=c(0,.15),lty=3)  #  plot
points(0, rfr, cex=4,pch="*") # rfr

lines( c(0, 3), rfr + c(0, 3)*(mu_p[tangent.index]-rfr)/sd_p[tangent.index],
       lwd=2, lty=1, col = "blue")

points( sd_p[minvar.index], mu_p[minvar.index], cex=2, pch="+")

lines( sd_p[efficent.frontier], mu_p[efficent.frontier], type="l",xlim=c(0, .25),
      ylim=c(0,.3),lwd=3, col = "red")

text(sd.vec[1], mean.vec[1], "GM", cex = 1.15)
```




```r
suppressWarnings(print({
   ggplot(data.table(cbind(sd_p, mu_p))) +
      geom_abline(aes(intercept = 0, slope = sharpe), col = "cornflowerblue", lty = 4, lwd = 1)
      geom_point(aes(sd_p, mu_p), col = "black", lwd = .15, alpha = .7) +
      geom_point(aes(0, rfr), col = "darkgreen", size = 3) +
      scale_y_continuous(limits = c(0, .15), labels = scales::percent_format(scale = 1)) +
      scale_x_continuous(limits = c(0, 2), labels = scales::percent_format(scale = 1)) +
      labs(title = "Efficient Portfolios", x = "Risk", y = "Return") +
      theme(axis.line = element_line(colour = "black"),
         panel.grid.major = element_blank(),
         panel.grid.minor = element_blank(),
         panel.border = element_blank(),
         panel.background = element_blank())
}))
```


Efficient Portfolios
0.2%
0.1%
0.0%
0.0%
Return
0%
0%
1%
2%
2%
Risk