Problem Background

Fitting Time Series Models

In this lab we are going to fit time series models to data sets consisting of daily returns on various instruments.

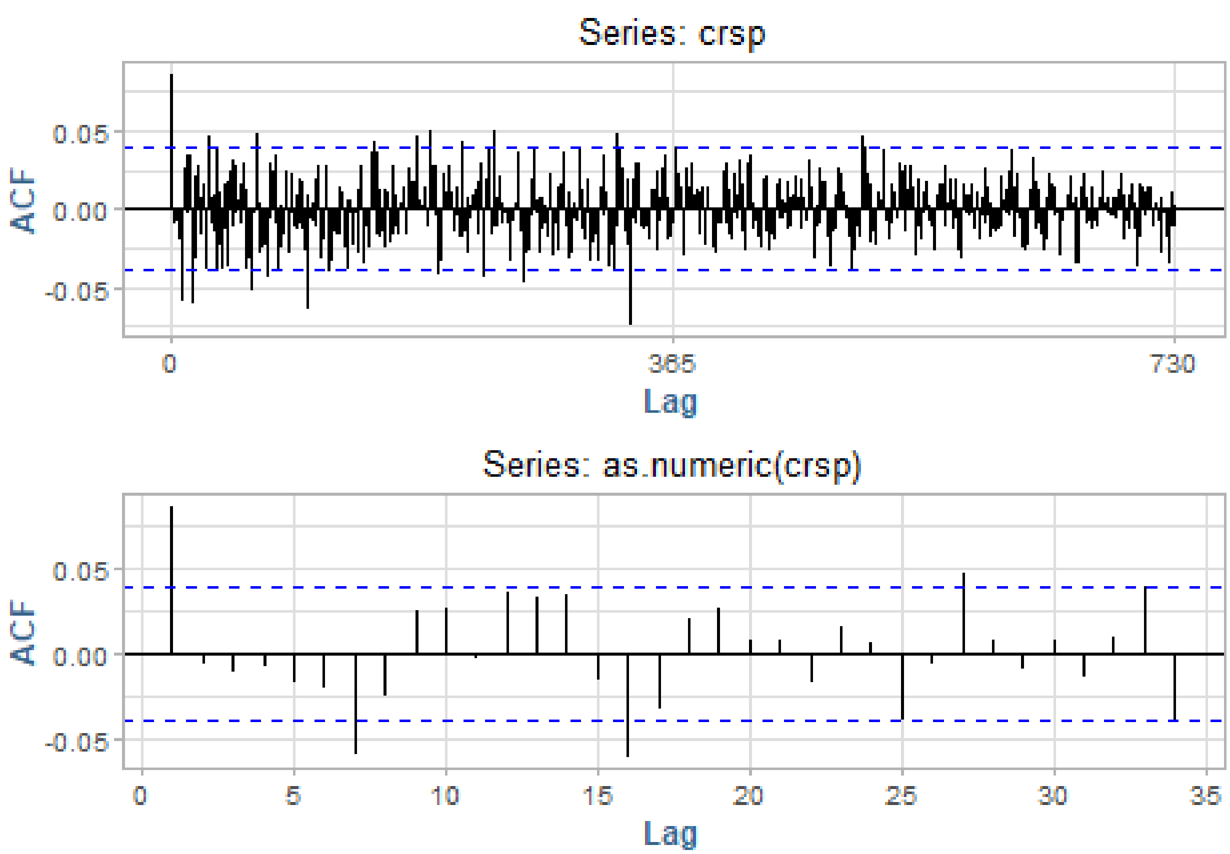
First, we will look a set of CRSP daily returns.

```
data("CRSPday")

crsp <- CRSPday[, 7]
```

Problem 1

Explain what "lag" means in the two ACF plots. Why does lag differ between the plots?

```
p1 <- ggAcf(crsp)
p2 <- ggAcf(as.numeric(crsp))

grid.arrange(p1, p2, nrow =2)
```



```r
head(crsp) # peek the ts object
```

```
Time Series:
Start = c(1969, 1)
End = c(1969, 6)
Frequency = 365
[1] -0.007619  0.013016  0.002815  0.003064  0.001633 -0.001991
```

Lag is a function of the frequency of the time series object *crsp*, which set the unit of time inverval represented by each data point.

From the quick summary of the data, we see that the frequency is 365 (days/year), so the first plot represents and interval of 1/365, or 0.00274. When we cast the data to a pure numeric representation (*as.vector*), this truncates the frequency property from the time series object, and the default reverts to 7/365, or 0.01918. The charts have the same data, just displayed on different time scales.

At what values of lag are there significant autocorrelations in the CRSP returns?

Let's grab the data from the plot for analysis.

```r
vals <- as.data.table(p2$data)[, .(Acf = Freq, Lag = lag)]

sig.vals <- vals[vals$Acf > 0.05 | vals$Acf < -0.05]

pretty_kable(sig.vals, "Significant Autocorrelations", dig = 2)
```

Table 1: Significant Autocorrelations

Acf	Lag
0.09	1
-0.06	7
-0.06	16

We can see the lags with the most significant values are at: 1, 7 and 16.

For which of these values do you think the statistical significance might be due to chance?

We can run a Ljung-Box test on these lags to further test for significance which test successive lags for stronger confidence.

```r
Box.test(crsp, lag = 1, type = "Ljung-Box")
```

```

    Box-Ljung test

data:  crsp
X-squared = 18.41, df = 1, p-value = 1.781e-05
```

At lag 1, we strongly reject the null hypothesis and conclude serial correlation.

```
Box.test(crsp, lag = 7, type = "Ljung-Box")
```

```

	Box-Ljung test

data:  crsp
X-squared = 29.509, df = 7, p-value = 0.0001168
```

At lag 7, we still reject the null and conclude there is serial correlation, but with less confidence than at 1 lag.

```
Box.test(crsp, lag = 16, type = "Ljung-Box")
```

```

	Box-Ljung test

data:  crsp
X-squared = 53.068, df = 16, p-value = 7.355e-06
```

At lag 16, we accept the null hypothesis and conclude this is i.i.d, and the correlation is from randomness.

Problem 2

Next, we will fit AR(1) and AR(2) models to the CRSP returns:

```r
(fit1 <- arima(crsp, order = c(1, 0, 0)))
```

```
Call:
arima(x = crsp, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.0853      7e-04
s.e.  0.0198      2e-04

sigma^2 estimated as 5.973e-05:  log likelihood = 8706.18,  aic = -17406.37
```

```r
(fit2 <- arima(crsp, order = c(2, 0, 0)))
```

```
Call:
arima(x = crsp, order = c(2, 0, 0))

Coefficients:
         ar1      ar2  intercept
      0.0865  -0.0141      7e-04
s.e.  0.0199   0.0199      2e-04

sigma^2 estimated as 5.972e-05:  log likelihood = 8706.43,  aic = -17404.87
```

In comparing these two models we would take the one with lower Akaike information criterion (AIC), or Bayesian information criterion (BIC).

```r
fit.sum <- data.table(Model = c("AR(1)", "AR(2)"),
                      AIC = c(AIC(fit1), AIC(fit2)),
                      BIC = c(BIC(fit1), BIC(fit2)))

pretty_kable(fit.sum, "Model Fit Comparison")
```

Here, we would take AR(1) over AR(2), irrespective of the preferred metric.

Table 2: Model Fit Comparison

Model	AIC	BIC
AR(1)	-17406.37	-17388.86
AR(2)	-17404.87	-17381.53

Find a 95% confidence interval for ϕ for the AR(1) model:

```
alpha <- 0.05

ci <- fit1$model$phi + 0.019 * qnorm(1 - (alpha/2)) * c(-1, 1)

pretty_kable(data.table(Lower = ci[1], Upper = ci[2]), "95 Confidence Interval")
```

Table 3: 95 Confidence Interval

Lower	Upper
0.05	0.12