Problem Background

Fitting Copula Models

In this lab we are going to fit copula models to a bivariate data set of daily returns on IBM and S&P500 Index.

First, we need to fit a model with the univariate marginal t-distributions and a t-copula.

This model will have three degrees of freedom parameters:

- IBM tail index
- S&P 500 Index tail index
- Joint tail index (the copula)

```
net.returns <- read.csv(paste0(data.dir, "IBM_SP500_04_14_daily_netRtns.csv"),
                        header = T)

ibm <- net.returns[, "IBM"]
sp500 <- net.returns[, "SP500"]

suppressWarnings({
  est.ibm = as.numeric( fitdistr( ibm, "t" )$estimate )
  est.sp500 = as.numeric( fitdistr( sp500, "t" )$estimate )
})

est.ibm[2] = est.ibm[2] * sqrt( est.ibm[3] / (est.ibm[3]-2) )
est.sp500[2] = est.sp500[2] * sqrt(est.sp500[3] / (est.sp500[3]-2) )
```

The univariate estimates will be used as starting values when we estimate the *meta-t* distribution is fit by maximum liklihood. Before we do that, we need to compute an estimate of the correlation coefficent in the *t*-copula.

Problem 1

Using Kendall's tau, compute omega, which is the estimate of the Pearson correlation from Kendall's tau.

From **8.27** we have Kendall's tau, ρ_{τ}, =

$\rho_{\tau}(Y_i, Y_j) = \frac{2}{\pi} arcsin(\Omega_{i,j})$.

Inverting, we derive that:

$\Omega_{i,j} = sin[\frac{\pi}{2}\rho_{\tau}(Y_i, Y_j)]$

```
cor_tau = cor(ibm, sp500, method = "kendall")
omega = sin((pi/2) * cor_tau)
```

$\Omega = 0.701835$

The *t*-copula using omega as the correlation parameter and 4 as the degrees of freedom:

```
cop_t_dim2 <- tCopula(omega, dim = 2, dispstr = "un", df = 4)
```

```
t-copula, dim. d = 2
Dimension:  2
Parameters:
  rho.1    = 0.7018346
  df       = 4.0000000
```

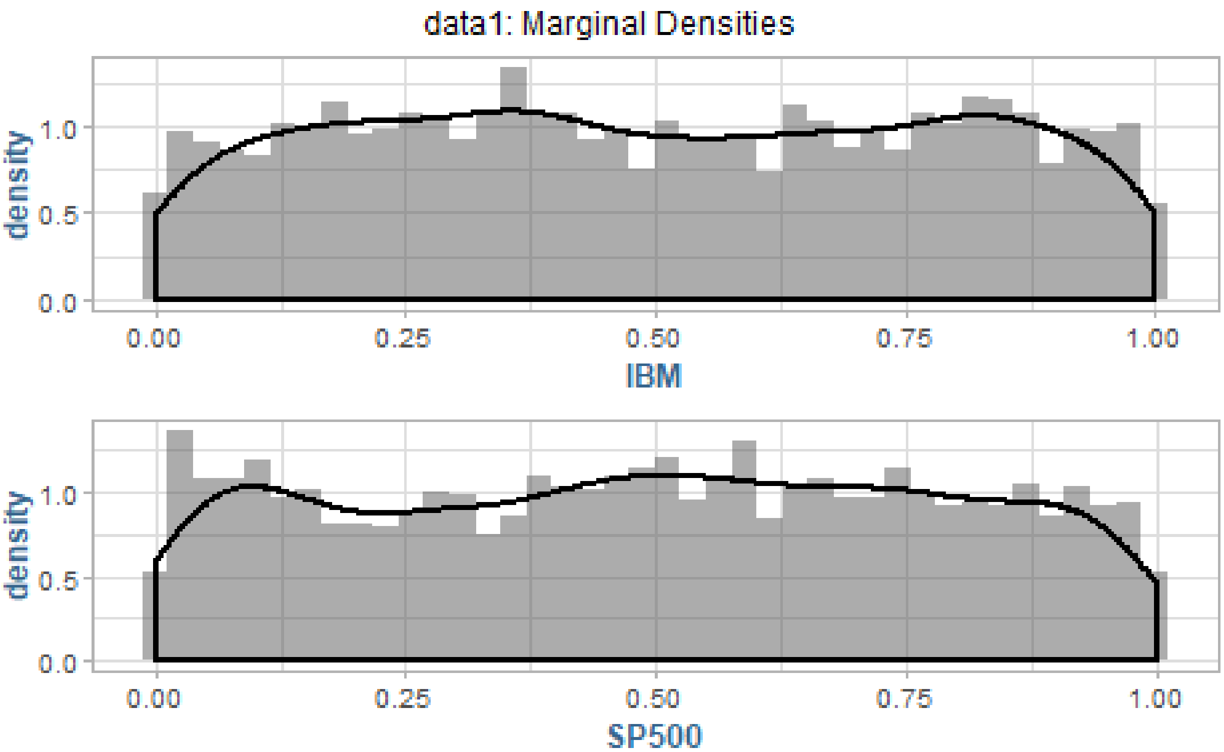
Now fit copulas to the uniformed-transformed data:

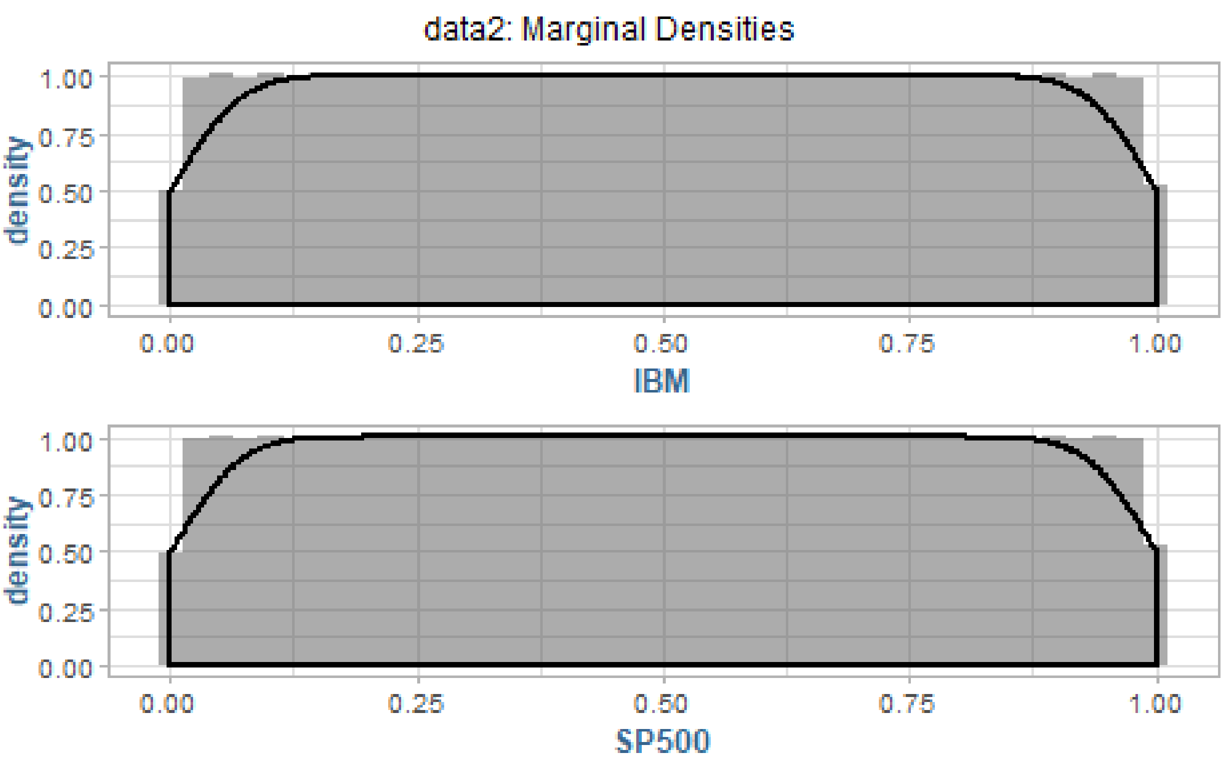
```
n = nrow(net.returns)

data1 = cbind( pstd( ibm, mean=est.ibm[1], sd=est.ibm[2], nu=est.ibm[3] ),
               pstd( sp500, mean=est.sp500[1], sd=est.sp500[2], nu=est.sp500[3] ) )

data2 = cbind( rank(ibm)/(n+1), rank(sp500)/(n+1) )

ft1 = fitCopula(cop_t_dim2, data=data1, method="ml", start=c(omega,4))
ft2 = fitCopula(cop_t_dim2, data=data2, method="ml", start=c(omega,4))
```

Problem 2

Explain the difference between methods used to obtain the two estimates *ft1* and *ft2*.

For *ft1* we pass in the probability


data1: Marginal Densities
density
0.0
0.5
1.0
0.00
0.25
0.50
0.75
1.00
IBM
density
0.0
0.5
1.0
0.00
0.25
0.50
0.75
1.00
SP500


data2: Marginal Densities
density
0.00
0.25
0.50
0.75
1.00
0.00
0.25
0.50
0.75
1.00
IBM
density
0.00
0.25
0.50
0.75
1.00
0.00
0.25
0.50
0.75
1.00
SP500

Do the two estimates seem significantly different (*in a practical sense*)?.

```
summary(ft1)
```

```
Call: fitCopula(copula, data = data, method = "ml", start = ..2)
Fit based on "maximum likelihood" and 2516 2-dimensional observations.
t-copula, dim. d = 2
      Estimate Std. Error
rho.1   0.7022      0.012
df      2.9834      0.269
The maximized loglikelihood is 967.2
Optimization converged
Number of loglikelihood evaluations:
function gradient
      40        9
```

```
summary(ft2)
```

```
Call: fitCopula(copula, data = data, method = "ml", start = ..2)
Fit based on "maximum likelihood" and 2516 2-dimensional observations.
t-copula, dim. d = 2
      Estimate Std. Error
rho.1   0.7031      0.012
df      3.0222      0.278
The maximized loglikelihood is 964.6
Optimization converged
Number of loglikelihood evaluations:
function gradient
      38        9
```

Problem 3

Next, we will define a meta-*t*-distribution by specifying its *t*-copula and its univariate marginal distributions.

```
mvdc_t_t = mvdc( cop_t_dim2, c("std","std"), list(
           list(mean=est.ibm[1],sd=est.ibm[2],nu=est.ibm[3]),
           list(mean=est.sp500[1],sd=est.sp500[2],nu=est.sp500[3])))
```

```
mvdc_t_t
```

```
Multivariate Distribution Copula based ("mvdc")
 @ copula:
t-copula, dim. d = 2
Dimension:  2
Parameters:
  rho.1   = 0.7018346
  df      = 4.0000000
 @ margins:
[1] "std" "std"
   with 2 (not identical)  margins; with parameters (@ paramMargins)
List of 2
 $ :List of 3
  ..$ mean: num 0.05015879
  ..$ sd  : num 1.42823
  ..$ nu  : num 3.254383
 $ :List of 3
  ..$ mean: num 0.07918415
  ..$ sd  : num 1.968172
  ..$ nu  : num 2.249776
```

Problem 4