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Econ 236 Final Exam

8 June 2015

Write your name below. By doing this you pledge to obey and follow the UCSC Academic Integrity policy.

Name: ______________________________

Question 1 . . . . *10 points*

Explain the relationship between the futures price and the spot price of an asset.

Solution: The relationship between spot and futures prices is given by $F = S_0 e^{r_f T}$, where r_f is the risk free rate of return and T is time to maturity. Assuming the risk-free rate is constant, then the futures price is a deterministic function of the spot price - i.e. it is simply a reflection of the current spot price and varies identically with movements in the spot price. The intuition is that a futures contract is simply an agreement in which I ask you to buy an asset now (for S_0) and hold it for me until I am willing to purchase it from you at a later date. Fair compensation dictates that I should pay you the risk-free rate of interest for the time that you're holding the asset (T), because you otherwise could have earned that rate of interest if you had not purchased that asset for me.

Question 2 . . . . *10 points*

Suppose that a stock price S follows geometric Brownian motion with expected return μ and volatility σ. What is the process followed by the variable e^S?

Solution: Let $G = e^S$. By Ito's Lemma,

$$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right) dt + \frac{\partial G}{\partial S}\sigma S dz$$

Since $\frac{\partial G}{\partial S} = e^S = G$, $\frac{\partial G}{\partial t} = 0$, and $\frac{\partial^2 G}{\partial S^2} = e^S = G$,

$$dG = (\mu SG + \frac{1}{2}\sigma^2 S^2 G)dt + \sigma SGdz$$

Question 3 . . . . *25 points*

On June 8th, 2015, the mid-day price of Netflix (NFLX) stock was \$627.90. The table below is a volatility surface for calls and puts at that time.

Expiry	\$620.00	\$625.00	\$630.00	\$635.00
Calls (%)				
June	22.53	22.57	22.40	22.38
July	38.68	41.05	37.98	37.89
August	40.30	41.42	38.61	40.56
Puts (%)				
June	21.84	21.64	21.38	21.33
July	37.70	37.36	37.10	36.14
August	37.79	37.37	37.67	37.34

(a) (5 points) What is the implied volatility for a June call option that is exactly at the money?

Solution: To obtain the implied volatility for the June call option with $K = \$627.90$, we must linearly interpolate between the implied vols for $K = \$625.00$ and $K = \$630.00$:

$$\sigma^{c}_{bs,627.90} = 22.57 + \frac{627.90 - 625.00}{630.00 - 625.00} \times (22.40 - 22.57) \tag{1}$$

$$= 22.47. \tag{2}$$

(b) (5 points) Assuming 9.5 days until maturity of the June option, and a continuously compounded risk-free rate of 0.7603%, what is the Black-Scholes price of the call option in part (a)?

Solution: We begin by computing the values of d_1 and d_2 for the Black-Scholes pricing formula:

$$\begin{aligned}
d_1 &= \frac{\log(S_0/K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} && (3)\\
&= \frac{\log(1) + (0.007603 + 0.2247^2/2) \times (9.5/252)}{0.2247\sqrt{9.5/252}} && (4)\\
&= 0.02838 && (5)\\
d_2 &= d_1 - \sigma\sqrt{T} && (6)\\
&= 0.02838 - 0.2247\sqrt{9.5/252} && (7)\\
&- 0.01524. && (8)
\end{aligned}$$

The resulting Black-Scholes call option price is:

$$\begin{aligned}
c &= S_0 N(d_1) - Ke^{-r_f T} N(d_2) && (9)\\
&= 627.90 \times 0.5113 - 627.90e^{-0.007603\times(9.5/252)}0.4939 && (10)\\
&= \$11.02. && (11)
\end{aligned}$$

(c) (5 points) What is the implied volatility for a June put option that is exactly at the money?

Solution: Once again, interpolating:

$$\begin{aligned}
\sigma^p_{bs,627.90} &= 21.64 + \frac{627.90 - 625.00}{630.00 - 625.00} \times (21.38 - 21.64) && (12)\\
&= 21.49. && (13)
\end{aligned}$$

(d) (5 points) Assuming 9.5 days until maturity of the June option, and a continuously compounded risk-free rate of 0.7603%, what is the Black-Scholes price of the put option in part (c)?

Solution: Using the same d_1 and d_2 as computed in part (b),

$$\begin{aligned} c &= Ke^{-r_f T}N(-d_2) - S_0 N(-d_1) && (14)\\ &= 627.90e^{-0.007603\times(9.5/252)}0.5061 - 627.90 \times 0.4887 && (15)\\ &= \$10.84. && (16)\end{aligned}$$

(e) (5 points) Does put-call parity hold? If not, what arbitrage opportunity would you exploit?

Solution: Put-call parity cannot hold because the implied volatilities for the call and put (with same strike and maturity) are not the same.

Question 4 . . . . *15 points*

Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.

Solution:

Question 5 .... *15 points*

Companies A and B have been offered the following rates per annum on a \$20 million 5-year loan:

	Fixed Rate (%)	Floating Rate (%)
A	5.0	LIBOR + 0.1
B	6.4	LIBOR + 0.6

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

Solution: The swap will allow A to take out a fixed-rate loan and make floating payments *as if* it took out a floating-rate loan. The swap will also allow B to take out a floating-rate loan and make floating payments *as if* it took out a fixed-rate loan. Under the swap, A receives fixed-rate payments to pay off its fixed-rate loan and B receives floating-rate payments to pay off its floating-rate loan.

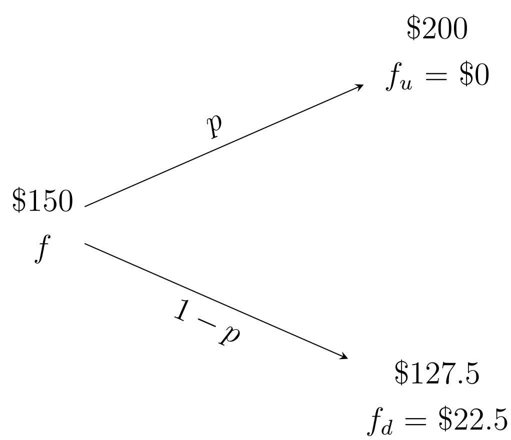
More specifically, A will make payments to the financial intermediary (FI) at $\text{LIBOR} + 0.1$ and receive payments from the FI at a fixed rate of 5.4%. B will make payments to the FI at fixed rate of 6.0% and receive payments from the FI at a floating rate of $\text{LIBOR} + 0.6$.

Under this arrangement, both companies benefit by 0.4% since A receives 5.4% but its loan repayment is only 5.0% while B's loan repayment is only 6.0% instead of the market rate of 6.4%. The FI nets $[(\text{LIBOR} + 0.1) - 5.4] + [6.0 - (\text{LIBOR} + 0.6)] = 0.1\%$.

Question 6 .... *25 points*

Suppose a stock currently trades at a price of \$150. The stock price can go up 33 percent or down 15 percent. The risk-free rate is 4.5 percent.

(a) (15 points) Use a one-period binomial model to calculate the price of a put option with exercise price of \$150.



Solution:

$$200p + 127.5(1-p) = 150e^{0.045(1)} \Rightarrow p = 0.4056$$

$$f = [0p + 22.5(1-p)]e^{-0.045(1)} = \$12.79$$

(b) (10 points) How many units of the underlying stock would be needed at time $t = 0$ to construct a risk-free hedge? Use 10,000 puts.

Solution:

$$200\Delta = 127.5\Delta - 22.5 \Rightarrow \Delta = -0.3103$$

To construct a risk-free hedge, we would need to go long 3103 shares of the stock along with 10000 puts.

Question	Points	Score
1	10	
2	10	
3	25	
4	15	
5	15	
6	25	
Total:	100	

Econ 236 Final Exam

6 June 2016

Write your name below. By doing this you pledge to obey and follow the UCSC Academic Integrity policy.

Name: ______________________________

Question 1 .... *15 points*

Suppose that the 1 year LIBOR interest rate is 10% and the 9-month LIBOR interest rate is 9.4% per annum, both with continuous compounding. Compute the 3-month Eurodollar futures price quote for a contract maturing in 9 months.

Solution: We need to compute the forward interest rate between months 9 and 12. To do so, we use the equation

$$R_f = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where

$$T_1 = \frac{9}{12}, \quad T_2 = \frac{12}{12}, \quad R_1 = 0.094, \quad R_2 = 0.1.$$

Solving, we have

$$\begin{aligned} R_f &= \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} \\ &= \frac{(0.1)(12/12) - (0.094)(9/12)}{\frac{12}{12} - \frac{9}{12}} \\ &= 0.118. \end{aligned}$$

The price quote of the Eurodollar futures contract is given by

$$Q = 100 - R$$

where R is three-month LIBOR interest rate, expressed with quarterly compounding and an actual/360 day count convention. Hence, we need to solve for the quarterly-compounded LIBOR rate, given the continuously-compounded LIBOR interest rate solved above:

$$\begin{aligned} mT_D(e^{\frac{r}{m}} - 1) &= APR * T_C \\ \Rightarrow APR &= m\frac{T_D}{T_C}(e^{\frac{r}{m}} - 1) \\ &= 4 * \frac{90/365}{90/360}(e^{\frac{0.118}{4}} - 1) \\ &= 0.1214 \end{aligned}$$

where we have substituted $m = 4$, $r = 0.118$, $T_C = 90/365$ and $T_D = 90/360$. Hence, the price quote of the 3-month Eurodollar futures contract maturing in 9 months is

$$100 - (100 * APR) = 100 - 12.14 = 87.86.$$

Question 2 . . . . *15 points*

A \$200 million interest rate swap has a remaining life of 10 months. Under the terms of the swap, 6-month LIBOR (compounded semiannually) is exchanged for a fixed rate of 14% per annum (compounded semiannually). The current risk-free interest rate is 10% per annum with continuous compounding. The 6-month LIBOR rate was 9.2% per annum (compounded semiannually) 2 months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed? Ignore day issues for this problem.

Solution: We'll value the swap using the fixed for floating bond method. The floating bond has its next payment in 4 months time. The payment amount is $200 \times 0.092 \times 0.5 = \$9.2$ million. Immediately after this payment, the floating bond's value is \$200 million. The value of the floating bond is therefore

$$V_{float} = (200 + 9.2)e^{-0.1\times 4/12} = \$202.3416\,\text{million}.$$

For the fixed bond, each coupon payment is in the amount of $200 \times 0.14 \times 0.5 = \$14$ million. The value of the fixed bond is given by

$$V_{fix} = 14e^{-0.1\times 4/12} + 214e^{-0.1\times 10/12} = \$210.4305\,\text{million}.$$

The value of the swap to the party paying the floating rate is $V_{fix} - V_{float} = 210.4305 - 202.3416 = \$8.0889$ million and the value to the party paying the fixed rate is $V_{float} - V_{fix} = -\$8.0889$ million.

Question 3 . . . . *10 points*

Consider a put option with price P_t and strike price X. Denote by S the price of the underlying stock.

(a) (5 points) Write the **NET** payoffs (including the cost of the call) to the seller.

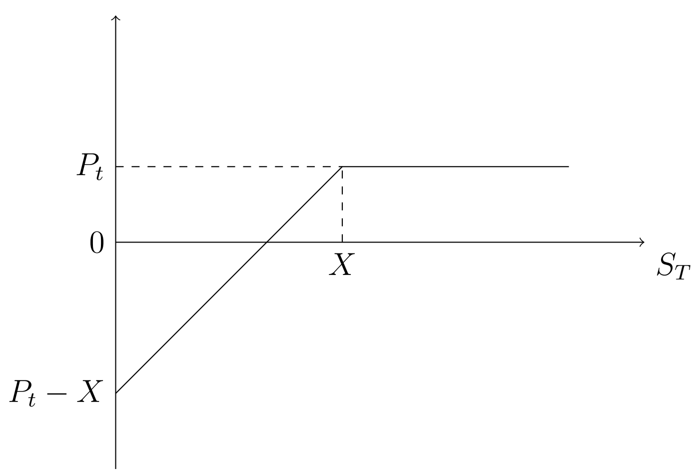
Solution: A put option is only valuable if the price of the underlying asset closes below the strike price: $S_T < X$. In this case, the seller of the put will lose $X - S_T$, which is the payout to the holder of the put option (i.e. the seller must buy the asset from the holder of the put for X, despite the fact that it is only worth S_T on the market). If $S_T > X$ the option goes unexercised and neither the seller nor buyer of the put gain or lose. The resulting gross payoffs are expressed as

$$P_T = \begin{cases} 0 & \text{if } S_T > X \\ -(X - S_T) & \text{if } S_T \leq X. \end{cases}$$

Taking into account the premium, P_t, paid to the seller up front (regardless of the final asset price, S_T), the net payoffs are

$$P_T = \begin{cases} P_t & \text{if } S_T > X \\ -(X - S_T) + P_t & \text{if } S_T \leq X. \end{cases}$$

(b) (5 points) Draw the NET payoff to the seller in a diagram with net payoff on the vertical axis and the stock price on the horizontal axis



Solution:

Question 4 . . . . *15 points*

On March 8th 2013, the closing price of Target stock was \$66.35. A call option with strike price X = \$55.00 and maturity date April 19th 2013 cost \$11.30. Assume a continuously-compounded annual risk-free rate of 0.00096 (0.096%). To compute the proper rate for discounting, assume exactly one month until maturity ($T = 1/12$).

(a) (6 points) If put-call parity holds, what should the price of a put option with the same strike and maturity be, assuming the asset does not pay dividends?

Solution: Put-call parity says that:

$$C + Xe^{-r/12} = S + P.$$

Therefore

$$\begin{aligned} P &= C + Xe^{-r/12} - S \\ &= \$11.30 + \$55.00e^{-0.00096/12} - \$66.35 \\ &= -0.0544. \end{aligned}$$

In reality, P cannot be less than zero, however $P = -0.0544$ is the numerical value that would need to be in force for put-call parity to hold.

(b) (9 points) The actual price of a put option with the same strike and maturity is \$0.08. Does the put-call parity hold? If put-call parity does not hold, describe the arbitrage opportunity that you could exploit.

Solution: Put-call parity does not hold because the actual price of the put is different from the implied price of the put: $0.08 \neq -0.0544$.

In other words the two equivalent strategies – buying a call and a safe asset vs. buying a put and the stock – do not cost the same: the former costs

$$C + Xe^{-r/12} = \$11.30 + \$55.00e^{-0.00096/12} = \$66.30,$$

while the latter costs

$$S + P = \$66.35 + \$0.08 = \$66.43.$$

To exploit the arbitrage opportunity, you can buy the cheap strategy and sell the expensive one: purchase the call, lend the present value of the strike, $PV(X) = Xe^{-r/12}$, short the stock and write a put. The payoffs of the strategy are reported below.

		Future Cash Flow	
Position	Immediate Cash Flow	$S_T < \$55$	$S_T > \$55$
Buy Call	\$-11.3	\$0	S_T-\$55
Lend $PV(X)$	\$-54.996	\$55	\$55
Sell Stock	\$66.35	$-S_T$	$-S_T$
Sell Put	\$0.08	S_T-\$55	\$0
Total	\$0.1344	\$0	\$0

Hence, you can earn \$0.1344 per share, risklessly.

Question 5 .... *10 points*

Suppose that a stock price S follows geometric Brownian motion with expected return μ and volatility σ. What is the process followed by the variable t^2S^3?

Solution: Let $G = t^2S^3$. By Ito's Lemma,

$$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right) dt + \frac{\partial G}{\partial S}\sigma S dZ.$$

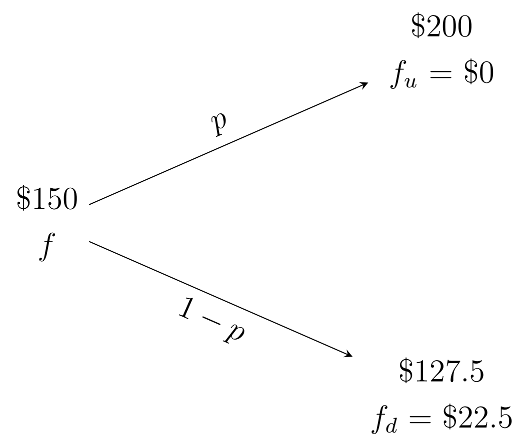
Since $\frac{\partial G}{\partial S} = 3t^2S^2 = 3\frac{G}{S}$, $\frac{\partial G}{\partial t} = 2tS^3 = 2\frac{G}{t}$, and $\frac{\partial^2 G}{\partial S^2} = 6t^2S = 6\frac{G}{S^2}$,

$$dG = \left(3\mu G + 2\frac{G}{t} + 3\sigma^2 G\right) dt + 3\sigma G dZ.$$

Question 6 .... *15 points*

Suppose a stock currently trades at a price of \$150. The stock price can go up 33 percent or down 15 percent in the next year. The risk-free rate is 4.5 percent per annum.

(a) (8 points) Use a one-period binomial model to calculate the price of a put option with exercise price of \$150.



Solution:

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(b) (7 points) How many units of the underlying stock would be needed at time $t = 0$ to construct a risk-free hedge? Use 10,000 puts.

Solution:

$$200\Delta = 127.5\Delta - 22.5 \Rightarrow \Delta = -0.3103$$

To construct a risk-free hedge, we would need to go long 3103 shares of the stock along with 10000 puts.

Question 7 . . . . *20 points*

Suppose the current value of an asset is S_0 and that the continuously-compounded annual risk-free interest rate is r. You are interested in valuing a put option on the asset with strike price X. Recall the Black-Scholes-Merton option pricing formula for a put:

$$P = Xe^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \tag{1}$$

$$d_1 = \frac{\log(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = \frac{\log(S_0/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}, \tag{3}$$

where σ is the volatility of price increments and T is the time to expiry.

(a) (10 points) Consider the case where $\sigma \to 0$. What is the value of the put when $S_0 > Xe^{-rT}$. Prove your answer mathematically.

Solution: Begin with the definition of d_1:

$$\begin{aligned} d_1 &= \frac{\log(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ &= \frac{\log(S_0/X) - \log\left(e^{-rT}\right)}{\sigma\sqrt{T}} + \frac{\sigma^2 T/2}{\sigma\sqrt{T}} \\ &= \frac{\log\left(\frac{S_0}{Xe^{-rT}}\right)}{\sigma\sqrt{T}} + \sigma\sqrt{T}/2. \end{aligned} \tag{4}$$

Equation (4) shows that if $S_0 > Xe^{-rT}$, $d_1 \to \infty$ as $\sigma \to 0$ since the numerator of the first term of Equation (4) is positive and the denominator goes to zero and since the second term goes to zero. Since $d_2 = d_1 - \sigma\sqrt{T}$, $d_2 \to \infty$ as $\sigma \to 0$ as well. The result is that $\Phi(-d_1) \to 0$ and $\Phi(-d_2) \to 0$ as $\sigma \to 0$, which causes $P = Xe^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \to 0$ as $\sigma \to 0$.

(b) (5 points) Consider the case where $\sigma \to 0$. What is the value of the put when $S_0 < Xe^{-rT}$. Prove your answer mathematically.

Solution: Referring to Equation (4), we see that if $S_0 < Xe^{-rT}$, the numerator of the first term is negative, causing the entire first term to go to negative infinity when $\sigma \to 0$. As before, the second term goes to zero. The result is that now $d_1 \to -\infty$ as $\sigma \to 0$. Likewise, since $d_2 = d_1 - \sigma\sqrt{T}$, $d_2 \to -\infty$ as $\sigma \to 0$. As a consequence, $\Phi(-d_1) \to 1$ and $\Phi(-d_2) \to 1$ as $\sigma \to 0$, which causes $P = Xe^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \to Xe^{-rT} - S_0$ as $\sigma \to 0$.

(c) (5 points) Interpret your solutions to parts (a) and (b).

Solution: In each case ($S_0 > Xe^{-rT}$ and $S_0 < Xe^{-rT}$), the value of the put is $\max\left\{Xe^{-rT} - S_0, 0\right\}$. This is equivalent to the present value of a forward contract under each of the conditions, which is what must maintain in a world with no risk ($\sigma \to 0$).

Question	Points	Score
1	15	
2	15	
3	10	
4	15	
5	10	
6	15	
7	20	
Total:	100	

Problem Set 1

Econ 236

Question 1 .... *35 points*

Consider an asset currently worth \$100. An investor plans to sell it in one year and is concerned that the price may have fallen significantly by then. To hedge this risk, the investor enters into a forward contract to sell the asset in one year. Assume that the risk-free rate is 5 percent.

(a) (7 points) Calculate the appropriate price at which this investor can contract to sell the asset in one year.

> **Solution:** The compounding convention (EAR vs. continuous APR) for the stated risk-free rate in this problem is ambiguous. The solutions will provide both sets of results.
>
> **EAR**:
>
> $$F_0 = S_0(1 + r) = \$100 \times 1.05 = \$105.$$
>
> **APR**:
>
> $$F_0 = S_0 e^r = \$100 e^{0.05} = \$105.13.$$

(b) (7 points) Three months into the contract, the price of the asset is \$90. Calculate the gain or loss that has accrued to the forward contract.

> **Solution:** The solutions for EAR and APR separately follow.
>
> **EAR**: The value of the forward contract is now $\$90 - \$105 \times 1.05^{-(1-0.25)} = -\$11.23$. The value has decreased by $\$0 - (-\$11.23) = \$11.23$ which is a *gain* to the investor since he/she is short.
>
> **APR**: The value of the forward contract is now $\$90 - \$105.13 \times e^{-0.75 \times 0.05} = -\$11.26$. The value has decreased by $\$0 - (-\$11.26) = \$11.26$ which is a *gain* to the investor since he/she is short.

(c) (7 points) Assume that five months into the contract, the price of the asset is \$107. Calculate the gain or loss on the forward contract.

Solution: The solutions for EAR and APR separately follow.

EAR: The value of the contract is now $\$107 - \$105 \times 1.05^{-(1-\frac{5}{12})} = \$4.95$. Therefore, compared to part (a) the contract has gained \$4.95 in value which is a loss to the investor.

APR: The value of the contract is now $\$107 - \$105.13 \times e^{-(1-\frac{5}{12})\times 0.05} = \$4.89$. Therefore, compared to part (a) the contract has gained \$4.89 in value which is a loss to the investor.

(d) (7 points) Suppose that at expiration, the price of the asset is \$98. Calculate the value of the forward contract at expiration. Also indicate the overall gain or loss to the investor on the whole transaction.

Solution: The solutions for EAR and APR separately follow.

EAR: At expiration, the contract is worth $\$98 - \$105 \times 1.05^{-(1-\frac{12}{12})} = -\$7$. This is a gain to the investor of \$7. However, the asset has gone down by $\$100 - \$98 = \$2$ so the net gain is \$5. This is unsurprising since we contracted to sell our \$100 asset at a price of \$105 so our net gain must be \$5 no matter what happens to the underlying asset.

APR: At expiration, the contract is worth $\$98 - \$105.13 \times e^{-(1-\frac{12}{12})\times 0.05} = -\$7.13$. This is a gain to the investor of \$7.13. However, the asset has gone down by $\$100 - \$98 = \$2$ so the net gain is \$5.13. This is unsurprising since we contracted to sell our \$100 asset at a price of \$105.13 so our net gain must be \$5.13 no matter what happens to the underlying asset.

(e) (7 points) Now calculate the value of the forward contract at expiration assuming that at expiration, the price of the asset is \$110. Indicate the overall gain or loss to the investor on the whole transaction. Is this amount more or less than the overall gain or loss from the previous part?

Solution: The solutions for EAR and APR separately follow.

EAR: At expiration, the contract is worth $\$110 - \$105 \times 1.05^{-(1-\frac{12}{12})} = \$5$. This is a loss of \$5 for the investor but the asset has gone up by $\$110 - \$100 = \$10$ so the net gain is still \$5.

APR: At expiration, the contract is worth $\$110 - \$105.13 \times e^{-(1-\frac{12}{12})\times 0.05} = \$4.87$. This is a loss of \$4.87 for the investor but the asset has gone up by $\$110 - \$100 = \$10$ so the net gain is still \$5.13.

Question 2 . . . . *14 points*

Assume that a security is currently priced at \$200. The risk-free rate is 5 percent.

(a) (7 points) A dealer offers you a contract in which the forward price of the security with delivery in three months is \$205. Explain the transactions you would undertake to take advantage of the situation.

Solution: We know that the typical forward price relationship *should* hold:

$$F = e^{rt}S \tag{1}$$

However, in this case

$$F = \$205 > \$202.51 = 200e^{0.05\times\frac{3}{12}} = e^{rT}S \tag{2}$$

This means that an arbitrage opportunity is available. When taking advantage of an arbitrage, one should always buy the cheap asset and sell the expensive. In this case, the cheap asset is the security itself and the expensive asset is the forward contract on the security. To accomplish the strategy, you could borrow \$200 at the risk-free rate, purchase the security, and simultaneously enter into a short forward contract. At maturity you deliver the security, receive \$205 and repay your loan, now valued at \$202.51. The total profit is \$205 - \$202.51 = \$2.49.

(b) (7 points) Suppose the dealer were to offer you a contract in which the forward price of the security with delivery in three months is \$198. How would you take advantage of the situation?

Solution: We can short sell the security, invest the proceeds at the risk-free rate, and enter into a forward contract to buy the security at \$198. After selling the security and investing the proceeds, we would have \$202.51 after 3 months. We would use \$198 of that to repurchase the security and close out the short sale. This leaves us with gains of $\$202.51 - \$198 = \$4.51$ which again is in excess of the risk-free gains.

Question 3 . . . . *16 points*

Suppose that zero interest rates with continuous compounding are as follows:

Maturity (Months)	Rate (% per annum)
3	8.0
6	8.2
9	8.4
12	8.5
15	8.6
18	8.7

Calculate forward interest rates for the second, third, fourth, fifth and sixth quarters.

Solution: The forward rate between times T_1 and T_2 are defined as

$$R_F = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$

Quarter	Forward Rate (% per annum)
2	$\frac{8.2(0.5)-8(0.25)}{0.5-0.25} = 8.4$
3	$\frac{8.4(0.75)-8.2(0.5)}{0.75-0.5} = 8.8$
4	$\frac{8.5(1)-8.4(0.75)}{1-0.75} = 8.8$
5	$\frac{8.6(1.25)-8.5(1)}{1.25-1} = 9.0$
6	$\frac{8.7(1.5)-8.6(1.25)}{1.5-1.25} = 9.2$

Question 4 .... *35 points*

Suppose you are holding a portfolio comprised of 7244 shares of QQQ (Nasdaq-100 ETF) and 5468 shares of GLD (gold ETF) and that you would like to hedge using XIV (VIX inverse ETF). Use one year of daily historical data to compute the appropriate hedge ratio for 8 April 2015. How many shares of XIV should you buy or sell? Include `R` or Python code with you answer.

Solution: XIV is an inverse ETF on the VIX. Since the VIX covaries negatively with most assets (like QQQ and GLD), XIV should covary positively. Thus, we want to short XIV. The following code computes the appropriate hedge ratio for the portfolio of QQQ and GLD to be 0.0911. Using valuations on 4/8/2015, the corresponding number of XIV shares to short is 3447, after rounding.

```
library(quantmod)

# Get the data and compute returns
getSymbols(c("QQQ","GLD","XIV"),from="2014-04-09",to="2015-04-08")
qqqRets = dailyReturn(QQQ$QQQ.Adj)
gldRets = dailyReturn(GLD$GLD.Adj)
xivRets = dailyReturn(XIV$XIV.Adj)

# Compute the portfolio returns
qqqShares = 7244
gldShares = 5468
qqqWeight = qqqShares/(qqqShares+gldShares)
portRets = qqqWeight*qqqRets + (1-qqqWeight)*gldRets

# Regress portfolio returns on XIV returns
hRatio = lm(portRets ~ xivRets)$coef[2]
portVal = QQQ$QQQ.Adj["2015-04-08"]*qqqShares + GLD$GLD.Adj["2015-04-08"]*gldShares
xivVal = XIV$XIV.Adj["2015-04-08"]
hShares = hRatio*portVal/xivVal
```

Problem Set 1

Econ 236

Question 1 . . . . *35 points*

Consider an asset currently worth \$100. An investor plans to sell it in one year and is concerned that the price may have fallen significantly by then. To hedge this risk, the investor enters into a forward contract to sell the asset in one year. Assume that the risk-free rate is 5 percent.

(a) (7 points) Calculate the appropriate price at which this investor can contract to sell the asset in one year.

(b) (7 points) Three months into the contract, the price of the asset is \$90. Calculate the gain or loss that has accrued to the forward contract.

(c) (7 points) Assume that five months into the contract, the price of the asset is \$107. Calculate the gain or loss on the forward contract.

(d) (7 points) Suppose that at expiration, the price of the asset is \$98. Calculate the value of the forward contract at expiration. Also indicate the overall gain or loss to the investor on the whole transaction.

(e) (7 points) Now calculate the value of the forward contract at expiration assuming that at expiration, the price of the asset is \$110. Indicate the overall gain or loss to the investor on the whole transaction. Is this amount more or less than the overall gain or loss from the previous part?

Question 2 . . . . *14 points*

Assume that a security is currently priced at \$200. The risk-free rate is 5 percent.

(a) (7 points) A dealer offers you a contract in which the forward price of the security with delivery in three months is \$205. Explain the transactions you would undertake to take advantage of the situation.

(b) (7 points) Suppose the dealer were to offer you a contract in which the forward price of the security with delivery in three months is \$198. How would you take advantage of the situation?

Question 3 . . . . *16 points*

Suppose that zero interest rates with continuous compounding are as follows:

Maturity (Months)	Rate (% per annum)
3	8.0
6	8.2
9	8.4
12	8.5
15	8.6
18	8.7

Calculate forward interest rates for the second, third, fourth, fifth and sixth quarters.

Question 4 .... *35 points*

Suppose you are holding a portfolio comprised of 7244 shares of QQQ (Nasdaq-100 ETF) and 5468 shares of GLD (gold ETF) and that you would like to hedge using XIV (VIX inverse ETF). Use one year of daily historical data to compute the appropriate hedge ratio for 8 April 2015. How many shares of XIV should you buy or sell? Include `R` or Python code with you answer.

Problem Set 2

Econ 236

Question 1 . . . . *10 points*

On 22 April 2016, the settlement prices for the Jun 2016, Sep 2016 and Dec 2016 British Pounds futures contracts at the CME were 1.5218, 1.5209, and 1.5203, respectively. Compute the implied difference between short-term interest rates in each country for each horizon.

Solution: Looking at historical exchange rate data (for example, xe.com), we find that the spot exchange rate, S_0, for GBP/USD on 22 April 2016 is 1.4388. Using the CME website for the GBP/USD futures contract specs (http://bit.ly/1E9tBZx), we see that the futures contracts stop trading on the second business day before the third Wednesday of the expiry months, which correspond to 13 June, 19 September and 19 December. This means that the number of days for each contract is 52, 150 and 241, respectively. In general, we know that the following relationship holds:

$$F_0 = S_0 e^{(r-r_f)T}$$
$$\implies r - r_f = \frac{1}{T}\log(F_0/S_0),$$

where T is the number of days to expiry divided by 365 (we'll assume an actual/actual day count convention). Thus, the implied difference at each horizon is:

$$\frac{365}{52}\log\left(\frac{1.5208}{1.4388}\right) = 0.3937$$
$$\frac{365}{150}\log\left(\frac{1.5209}{1.4388}\right) = 0.1350$$
$$\frac{365}{241}\log\left(\frac{1.5203}{1.4388}\right) = 0.08668.$$

Question 2 . . . . *10 points*

On 22 April 2016, the 3-month interest rates in Switzerland and the United States were, respectively, -0.8% and 0.23% per annum with continuous compounding. The spot price of the Swiss Franc was \$1.0232 and the settlement price of the CME Jun 2016 futures contract was \$1.0238. Is there an arbitrage opportunity? If so, what is it?

Solution: According to Equation 5.9, the futures price of the Swiss Franc should be

$$F_0 = 1.0232e^{(0.0023+0.0080)(52/365)} = \$1.0247$$

where I have used an actual/actual day count. The futures price of the Swiss Franc is actually \$1.0238, suggesting that the Swiss Franc is cheap relative to the US dollar. To take advantage of this arbitrate opportunity, we will do the following:

1. Borrow 1 million Swiss Francs at -0.80% per annum, exchange it to $1,000,000(1.0232) = \$1,023,200$ USD, and invest the dollars at 0.23%.

2. \$1,023,200 grows into $1,023,200e^{0.0023(52/365)} = 1,023,535$, so enter into a Jun 2016 futures contract to sell \$1,023,535 for $1,023,535/1.0238 = 999,741.5$ Francs.

Of the 999,741.5 Francs, we use $1,000,000e^{-0.0080(52/365)} = 998,860.9$ to repay the initial loan which leaves us with an arbitrage profit of $999,741.5 - 998,860.9 = 880.6$ Francs.

Question 3 . . . . *15 points*

Suppose the 300-day LIBOR zero rate is 4% and Eurodollar quotes for contracts maturing in 300, 398, and 489 days are 95.83, 95.62, and 95.48. Calculate the 398-day and 489-day LIBOR zero rates. Assume no difference between forward and future rates for the purposes of your calculations.

Solution: To solve this, we will calculate the Eurodollar forward rates, assuming that the Eurodollar and LIBOR forward rates are the same. Recall that Eurodollar forward rates are expressed as the difference between 100 and the quoted Eurodollar price, and are compounded in quarterly frequency, using an actual/360 day count convention (see Hull(2014), section 6.3, p. 140). This means that the quarterly compounded rates are:

$$f_{q,300} = 100 - 95.83 = 4.17\%$$
$$f_{q,398} = 100 - 95.62 = 4.38\%$$
$$f_{q,489} = 100 - 95.48 = 4.52\%.$$

Using these quarterly rates, and the fact that each futures contract is for 3 months (90/360 days) we can solve for the corresponding continuous rates using the following relationship:

$$\begin{aligned} e^{\left(f_{c,d}\times\frac{90}{365}\right)} &= \left(1+\frac{f_{q,d}}{4}\right)^{4\times\frac{90}{360}} \\ &= \left(1+\frac{f_{q,d}}{4}\right) \\ \implies f_{c,d} &= \frac{365}{90}\log\left(1+\frac{f_{q,d}}{4}\right), \end{aligned}$$

where $d = \{300, 398, 489\}$. Thus,

$$\begin{aligned} f_{c,300} &= (365/90)\ln(1+0.0417/4) = 0.0421 \implies 4.21\% \\ f_{c,398} &= (365/90)\ln(1+0.0438/4) = 0.0442 \implies 4.42\% \\ f_{c,489} &= (365/90)\ln(1+0.0452/4) = 0.0456 \implies 4.56\%. \end{aligned}$$

Noting equation (6.4) from Hull p. 144,

$$R_{i+1} = \frac{F_i(T_{i+1}-T_i)+R_iT_i}{T_{i+1}}$$

it follows that the LIBOR zero rates are

$$\begin{aligned} R_{398}^{\text{LIBOR}} &= \frac{f_{c,300}\times(398-300)+(4\times 300)}{398} = 0.0405 \implies 4.05\% \\ R_{489}^{\text{LIBOR}} &= \frac{f_{c,398}\times(489-398)+(R_{398}^{\text{LIBOR}}\times 398)}{489} = 0.0412 \implies 4.12\% \end{aligned}$$

Question 4 .... *15 points*

It is July 30, 2015. The cheapest-to-deliver bond in a September 2015 Treasury bond futures contract is a 13% coupon bond, and delivery is expected to be made on September 30, 2015. Coupon payments on the bond are made on February 4 and August 4 each year. The term structure is flat, and the rate of interest with semiannual compounding is 12% per annum. The conversion factor for the bond is 1.5. The current quoted bond price is \$110. Calculate the quoted futures price for the contract.

Solution: Recall that the cash price of a US Treasury bond is the quoted price plus the accrued interest since the last coupon date. There are 176 days between February 4 and July 30 and 181 days between February 4 and August 4. The cash price of the bond is

$$110 + \frac{176}{181}\left(100 \times 0.13 \times \frac{1}{2}\right) = \$116.32$$

The next step is to find the futures price of the cheapest-to-deliver bond. Since the bond pays a coupon on August 4, we use the formula

$$F_0 = (S_0 - I)e^{rT}$$

The general formula that relates a continuously compounded APR, r_c with a semi-annually compounded APR, r_s is:

$$e^{r_c} = \left(1 + \frac{r_s}{2}\right)^2$$
$$\Rightarrow r_c = 2\log\left(1 + \frac{r_s}{2}\right).$$

Since the semi-annually compounded spot rate is 0.12, this means that the continuously compounded spot rate is $2\log(1.06) = 0.1165$. Hence, the present value of the August 4 coupon (which is paid in 5 days) is

$$I = 6.5e^{-(5/365)0.1165} = \$6.49$$

There are 62 days until expiry. Therefore, the futures price of the cheapest-to-deliver bond is

$$F_0 = (116.32 - 6.49)e^{0.1165(62/365)} = \$112.03$$

We then need to find the futures price of the Treasury bond futures contract. The following formula allows us to relate the two:

$$F_0 = (\text{Most recent settlement price} \times \text{Conversion factor}) + \text{Accrued interest}$$

At delivery, there are 57 days of accrued interest (since the last coupon payment day). Denoting the futures price of the Treasury bond futures contract as P, we have

$$\$112.03 = P \times 1.5 + 6.5\left(\frac{57}{181}\right)$$
$$\Rightarrow P = \$73.32.$$

Question 5 . . . . *15 points*

A Eurodollar futures quote for the period between 5.1 and 5.35 years in the future is 97.1. The standard deviation of the change in the short-term interest rate in one year is 1.4%. Estimate the forward interest rate in an FRA.

Solution: The futures rate is $100 - 97.1 = 2.9\%$. Using Equation 6.3,

$$\text{Forward rate} = 0.029 - \frac{1}{2}(0.014^2)(5.1)(5.35) = 0.0263 \implies 2.63\%$$

Question 6 . . . . *15 points*

The 1-year LIBOR rate is 10% with annual compounding. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. The 2- and 3-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the 2- and 3-year LIBOR zero rates.

Solution: The two-year swap rate implies that a two-year LIBOR bond with a coupon of 11% sells for par:

$$\frac{11}{1.10} + \frac{111}{(1+R_2)^2} = 100 \Rightarrow R_2 = 0.1106 \text{ or } 11.06\%$$

Similarly, the three-year swap rate implies that a three-year LIBOR bond with a coupon of 12% sells for par:

$$\frac{12}{1.10} + \frac{12}{1.1106^2} + \frac{112}{(1+R_3)^3} = 100 \Rightarrow R_3 = 0.1217 \text{ or } 12.17\%$$

Question 7 . . . . *20 points*

Under the terms of an interest rate swap, a financial institution has agreed to pay 10% per annum and to receive 3-month LIBOR in return on a notional principal of $100 million with payments being exchanged every 3 months. The swap has a remaining life of 14 months. The average of the bid and offer fixed rates currently being swapped for 3-month LIBOR is 12% per annum for all maturities. The 3-month LIBOR rate 1 month ago was 11.8% per annum. All rates are compounded quarterly. What is the value of the swap?

Solution: We'll first value the swap using the bond approach. For the discount rate, we convert the 12% rate with quarterly compounding to 11.82% with continuous compounding using the formula

$$APR = [(1 + EAR)^{\frac{1}{n}} - 1] \times n$$
$$r_c = \left[\left(\frac{r}{2} + 1\right)^{\frac{1}{2}} - 1\right] \times 4$$

The floating bond has its next payment in two months time. The payment is in the amount of $100(0.118)\frac{1}{4} = \$2.95$ million since its interest rate is determined by last month's 3-month LIBOR rate. Immediately after this payment, the floating bond's value is \$100 million. The value of the floating bond is therefore

$$(100 + 2.95)e^{-0.1182(2/12)} = \$100.94 \text{ million}$$

For the fixed bond, each coupon payment is in the amount of $100(0.1)\frac{1}{4} = \$2.5$ million. The value of the fixed bond is given by

$$2.5e^{-0.1182(2/12)} + 2.5e^{-0.1182(5/12)} + 2.5e^{-0.1182(8/12)} + 2.5e^{-0.1182(11/12)} + 102.5e^{-0.1182(14/12)} = \$98.68 \text{ million}$$

The value of the swap is then $100.94 - 98.68 = \$2.26$ million.

The second method to value the swap is to treat is as a portfolio of FRA's. Using this approach, the swap is valued at

$$(2.95 - 2.5)e^{-0.1182(2/12)} + (3 - 2.5)e^{-0.1182(5/12)} + (3 - 2.5)e^{-0.1182(8/12)}$$
$$+ (3 - 2.5)e^{-0.1182(11/12)} + (3 - 2.5)e^{-0.1182(14/12)} = \$2.26 \text{ million}$$

where the floating payment in 2 months uses last month's 3-month LIBOR but all later floating payments the current 3-month LIBOR (since the term structure is flat, all forward 3-month LIBOR rates must be equal to the current 3-month LIBOR). As expected, we get the same answer using both methods.

Problem Set 2

Econ 236

Question 1 .... *10 points*

On 22 April 2016, the settlement prices for the Jun 2016, Sep 2016 and Dec 2016 British Pounds futures contracts at the CME were 1.5218, 1.5209, and 1.5203, respectively. Compute the implied difference between short-term interest rates in each country for each horizon.

Question 2 .... *10 points*

On 22 April 2016, the 3-month interest rates in Switzerland and the United States were, respectively, -0.81% and 0.01% per annum with continuous compounding. The spot price of the Swiss Franc was \$1.0468 and the settlement price of the CME Jun 2016 futures contract was \$1.0478. Is there an arbitrage opportunity? If so, what is it?

Question 3 .... *15 points*

Suppose the 300-day LIBOR zero rate is 4% and Eurodollar quotes for contracts maturing in 300, 398, and 489 days are 95.83, 95.62, and 95.48. Calculate the 398-day and 489-day LIBOR zero rates. Assume no difference between forward and future rates for the purposes of your calculations.

Question 4 .... *15 points*

It is July 30, 2015. The cheapest-to-deliver bond in a September 2015 Treasury bond futures contract is a 13% coupon bond, and delivery is expected to be made on September 30, 2015. Coupon payments on the bond are made on February 4 and August 4 each year. The term structure is flat, and the rate of interest with semiannual compounding is 12% per annum. The conversion factor for the bond is 1.5. The current quoted bond price is \$110. Calculate the quoted futures price for the contract.

Question 5 .... *15 points*

A Eurodollar futures quote for the period between 5.1 and 5.35 years in the future is 97.1. The standard deviation of the change in the short-term interest rate in one year is 1.4%. Estimate the forward interest rate in an FRA.

Question 6 .... *15 points*

The 1-year LIBOR rate is 10% with annual compounding. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. The 2- and 3-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the 2- and 3-year LIBOR zero rates.

Question 7 . . . . *20 points*

Under the terms of an interest rate swap, a financial institution has agreed to pay 10% per annum and to receive 3-month LIBOR in return on a notional principal of $100 million with payments being exchanged every 3 months. The swap has a remaining life of 14 months. The average of the bid and offer fixed rates currently being swapped for 3-month LIBOR is 12% per annum for all maturities. The 3-month LIBOR rate 1 month ago was 11.8% per annum. All rates are compounded quarterly. What is the value of the swap?

Problem Set 3

Econ 236

Question 1 . . . . *10 points*

On May 15th, 2015, the closing price of Target stock was \$78.53. A call option with strike price X = \$70.00 and maturity date June 19th, 2015 costs \$8.30. A put option with the same strike and maturity costs \$0.31. Assume a continuously-compounded risk-free rate of 0.00096 (0.096%) per annum. Further, assume that the options are European and that the stock does not pay dividends.

(a) (5 points) If put-call parity holds, what should the price of the put option be?

> **Solution:** According to put-call parity
>
> $$c + Ke^{-rT} = p + S_0.$$
>
> Since there are exactly 35 days until expiration, $T = 35/365 = 0.09589$. Thus,
>
> $$\begin{aligned} p &= c + Ke^{-rT} - S_0 \\ &= 8.3 + 70e^{-0.00096\times 0.09589} - 78.53 \\ &= -0.2364, \end{aligned}$$
>
> or -\$0.23. Since a put price can never be negative, the price should be zero.

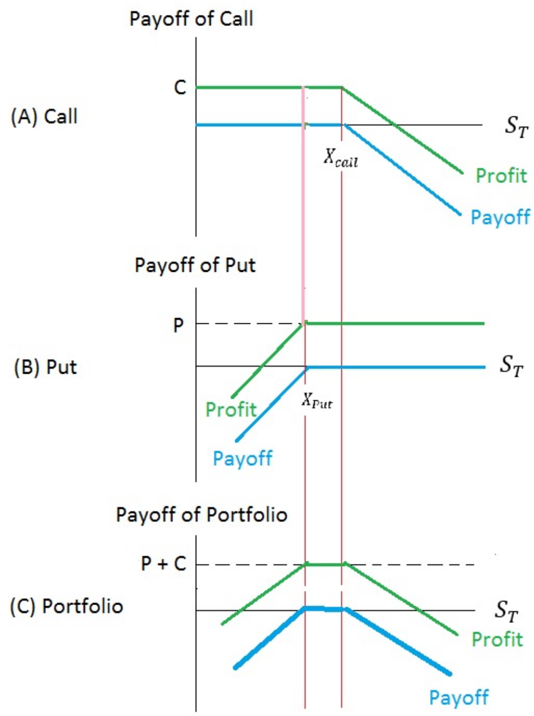
(b) (5 points) If put-call parity holds, what should the price of the call option be?

> **Solution:** Again, by the put-call parity relationship,
>
> $$\begin{aligned} c &= p + S_0 - Ke^{-rT} \\ & 0.31 + 78.53 - 70e^{-0.00096\times 0.09589} \\ &= 8.846, \end{aligned}$$
>
> or \$8.85.

Question 2 . . . . *20 points*

Consider the following options portfolio. You write an April call option on IBM with exercise price \$85. You write an April IBM put option with exercise price \$80. The price of the call option is \$0.95 and the price of the put is \$2.45.

(a) (5 points) Graph the payoff of this portfolio at option expiration as a function of IBM's stock price at that time.



(b) (5 points) What will be the profit/loss on this position if IBM is selling at \$83 on the option maturity date. What if IBM is selling at \$90?

> **Solution:** If IBM is selling at \$83 on the maturity date, then neither the call option nor the put option are in the money so the profit is $\$0.95 + \$2.45 = \$3.40$. If IBM is selling for \$90 instead, then the call option is in the money. In this case, profit is $\$3.40 + (\$85 - \$90) = -\$1.60$.

(c) (5 points) At what two stock prices will you just break even on your investment?

> **Solution:** We will break even when either option is in the money by an amount equal to \$3.40. For the call option, this price is $\$85 + \$3.40 = \$88.40$. For the put option, this price is $\$80 - \$3.40 = \$76.60$.

(d) (5 points) What kind of bet is this investor making; that is, what must this investor believe about IBM's stock price to justify this position?

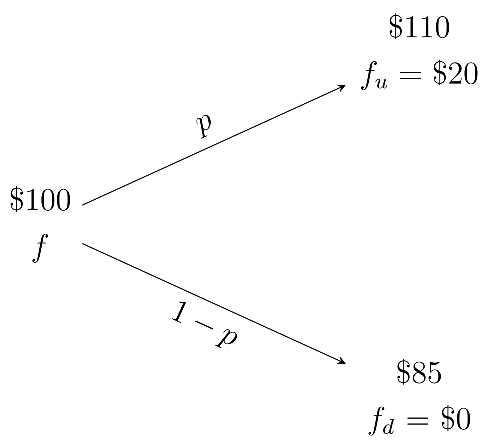
> **Solution:** The investor believes that IBM's stock price will have low volatility (in both directions) from now until option maturity.

Question 3 .... *15 points*

A stock currently trades at a price of \$100. The stock price can go up 10 percent or down 15 percent. The risk-free rate is 6.5 percent.

(a) (7 points) Use a one-period binomial model to calculate the price of a call option with an exercise price of \$90.

> **Solution:** Assume that the maturity date is one year from now. The value of the stock can either go up to $100(1.10) = \$110$ (in which case the option is worth $110 - 90 = \$20$) or go down to $100(0.85) = \$85$ (in which case the option is worthless).



> There are two ways to find the option price. For the first approach, we'll build a riskless portfolio by going long on shares and by shorting 1 option. Let Δ be the number of shares we want to hold in the portfolio to totally hedge against risk. By definition, a riskless portfolio will have the same value whether the stock goes up or down. Therefore,
>
> $$110\Delta - 20 = 85\Delta \Rightarrow \Delta = 0.8$$
>
> and the value of the portfolio at maturity is $110(0.8) - 20 = \$68$. The present value of the portfolio is thus $68e^{-0.065(1)} = \$63.72$. But we also know that the present value of the portfolio is $100(0.8) - f$ where f is spot price of the option. This gives $f = 100(0.8) - 63.72 = \$16.28$.

The second way to price the option is via risk neutrality. Letting p be the probability of the stock going up, the value of today's portfolio compounded at the risk-free rate must equal the expected future value of the portfolio:

$$110p + 85(1-p) = 100e^{0.065(1)} \Rightarrow p = 0.8686$$

Furthermore, the present value of the option must be equal to the discounted expected future value of the option:

$$f = [20p + 0(1-p)]e^{-0.065(1)} = \$16.28$$

(b) (4 points) Suppose the call price is currently \$17.50. Show how to execute an arbitrage transaction that will earn more than the risk-free rate. Use 100 call options.

Solution: Build the riskless portfolio as in the previous part by going long 80 shares and shorting 100 options. The current value of this portfolio is $100(80) - 17.50(100) =$ \$6250. The future value of this portfolio (no matter whether the stock goes up or down) is \$6800. This implies a 1-year interest rate of $\ln(\frac{6800}{6250}) = 0.08434$ which exceeds the risk-free rate.

(c) (4 points) Suppose the call price is currently \$14. Show how to execute an arbitrage transaction that replicates a loan that will earn less than the risk-free rate. Use 100 call options.

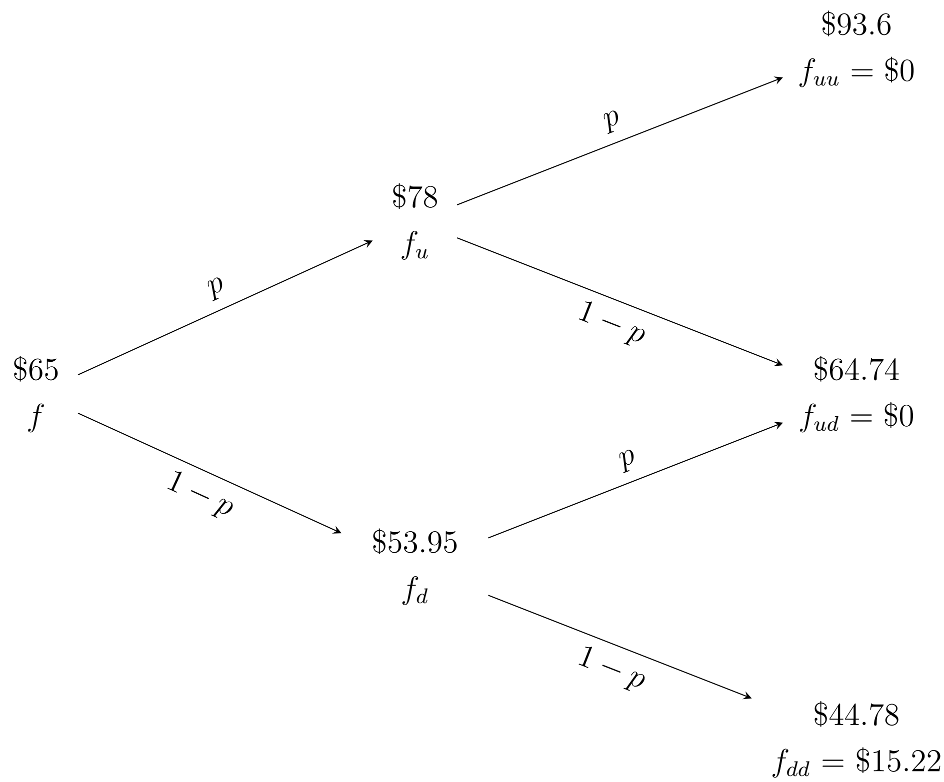
Solution: The riskless portfolio now implies a 1-year interest rate of $\ln(\frac{6800}{100(80)-14(100)}) =$ 0.02985 which is less than the risk-free rate. The arbitrage play here is to short the riskless portfolio (by shorting shares and going long on the options) and investing the proceeds at the risk-free rate.

Question 4 . . . . *40 points*

Consider a two-period binomial model in which a stock currently trades at a price of \$65. The stock price can go up 20 percent or down 17 percent each period. The risk-free rate is 5 percent.

(a) (10 points) Calculate the price of a European put option expiring in two periods with exercise price of \$60.

Solution: Assume that each of the two periods lasts one year ($\Delta t = 1$).



The first step is to calculate the option prices in the second period, f_{uu}, f_{ud}, and f_{dd}. The next step is to calculate the prices of the options in the first period (f_u and f_d) using the prices in the second period. Since $f_{uu} = f_{ud} = 0$, f_u must also have zero value. To find f_d, we must first find p using the fact that the the value of the stock invested at the risk-free rate must be equal to the expected future value of the stock:

$$53.95e^{0.05(1)} = 64.74p + 44.78(1 - p) \Rightarrow p = 0.5980$$

Then, f_d must be equal to the present value of the expected value of the options in the second period:

$$f_d = [0 \cdot p + 15.22(1 - p)]e^{-0.05(1)} = \$5.82$$

This same argument can be used to find the price of the option at time zero, f:

$$f = [0 \cdot p + 5.82(1 - p)]e^{-0.05(1)} = \$2.23$$

(b) (5 points) Based on your answer in part (a), calculate the number of units of the underlying stock that would be needed at each point in the binomial tree in order to construct a risk-free hedge. Use 10,000 puts.

Solution: We need to calculate Δ, Δ_u, and Δ_d:

$$\Delta = \frac{f_u - f_d}{78 - 53.95} = \frac{0 - 5.82}{78 - 53.95} = -0.2420$$
$$\Delta_u = \frac{f_{uu} - f_{ud}}{93.6 - 64.74} = \frac{0 - 0}{93.6 - 64.74} = 0$$
$$\Delta_d = \frac{f_{ud} - f_{dd}}{64.74 - 44.78} = \frac{0 - 15.22}{64.74 - 44.78} = -0.7625$$

Unlike for call options, Δ's for put options are negative. This means we would need to go long on both the shares and the options or short on both the shares and the options in order to construct a riskless portfolio. At time zero, we would need to go long $|10000\Delta| = 2420$ shares and 10000 puts. If the stock goes up after one year, we would not be able to hedge risk without using a different option. If the stock goes down after one year, we would need to go long $|10000\Delta_d| = 7625$ shares and 10000 puts.

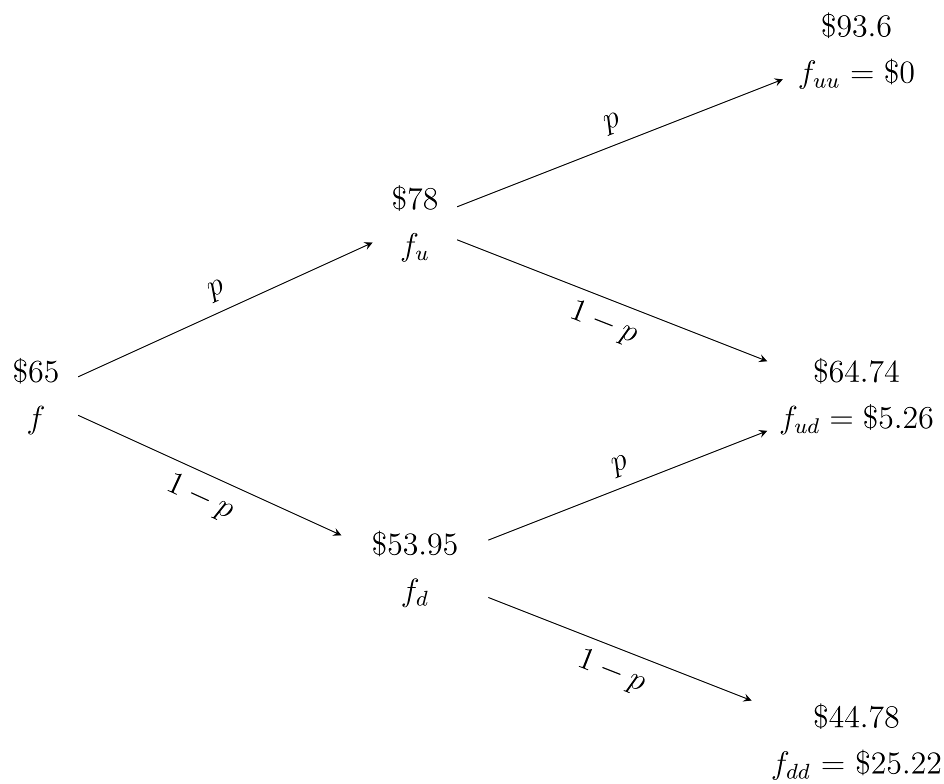
(c) (5 points) Is the price of an American put option different? If so, compute the price.

Solution: If the option were American instead of European, then its price could potentially change if it were optimal to exercise the option early. For this problem, if the stock goes down after one year, then it is optimal to exercise the option early since $60 - 53.95 = 6.05 > f_d$. Therefore, the price of the American option is

$$f = [0 \cdot p + 6.05(1 - p)]e^{-0.05(1)} = \$2.31$$

(d) (10 points) Calculate the price of a European put option expiring in two periods with exercise price of \$70.

Solution: As in part (a), $p = 0.5980$ and $f_{uu} = 0$. However, f_{ud} and f_{dd} are different than in part (a).



$$f_u = [0 \cdot p + 5.26(1-p)]e^{-0.05(1)} = \$2.01$$
$$f_d = [5.26p + 25.22(1-p)]e^{-0.05(1)} = \$12.64$$
$$f = [2.01p + 12.64(1-p)]e^{-0.05(1)} = \$5.98$$

(e) (5 points) Based on your answer in part (d), calculate the number of units of the underlying stock that would be needed at each point in the binomial tree in order to construct a risk-free hedge. Use 10,000 puts.

Solution:

$$\Delta = \frac{f_u - f_d}{78 - 53.95} = \frac{2.01 - 12.64}{78 - 53.95} = -0.4420$$
$$\Delta_u = \frac{f_{uu} - f_{ud}}{93.6 - 64.74} = \frac{0 - 5.26}{93.6 - 64.74} = -0.1823$$
$$\Delta_d = \frac{f_{ud} - f_{dd}}{64.74 - 44.78} = \frac{5.26 - 25.22}{64.74 - 44.78} = -1$$

(f) (5 points) Is the price of an American put option different? If so, compute the price.

Solution: If the option were American, then f_d changes from \$12.64 to $70 - 53.95 =$ \$16.05. Therefore,

$$f = [2.01p + 16.05(1 - p)]e^{-0.05(1)} = \$7.28$$

Question 5 .... *15 points*

Suppose you are a financial adviser and that you have a client who believes the common stock price of TRT materials (currently \$58 per share) could move substantially in either direction in reaction to an expected court decision involving the company. The client currently owns no TRT shares, but asks you for advice about implementing a strangle strategy to capitalize on the possible stock price movement. Suppose that a 90-day call option with strike of \$60 is priced at \$5 and that a 90-day put option with strike of \$55 is priced at \$4.

(a) (3 points) Would you recommend a long strangle or short strangle strategy to achieve the client's objective?

Solution: The client should implement a long strangle to take advantage of high volatility in the stock price.

(b) For the recommended strategy, calculate the:

i. (4 points) Maximum possible loss per share.

Solution: The maximum loss per share is $5 + 4 =$ \$9 and is incurred when the stock price is between \$55 and \$60.

ii. (4 points) Maximum possible gain per share.

Solution: The maximum possible gain per share if the stock moves downward is $55 - 9 = \$46$. The possible gain per share if the stock moves upward is unlimited since in theory the stock price can go towards infinity.

iii. (4 points) Break even stock price(s).

Solution: The break even prices are $55 - 9 = \$46$ and $60 + 9 = \$69$.

Problem Set 3

Econ 236

Question 1 . . . . *10 points*

On May 15th, 2015, the closing price of Target stock was \$78.53. A call option with strike price X = \$70.00 and maturity date June 19th, 2015 costs \$8.30. A put option with the same strike and maturity costs \$0.31. Assume a continuously-compounded risk-free rate of 0.00096 (0.096%) per annum. Further, assume that the options are European and that the stock does not pay dividends.

(a) (5 points) If put-call parity holds, what should the price of the put option be?

(b) (5 points) If put-call parity holds, what should the price of the call option be?

Question 2 . . . . *20 points*

Consider the following options portfolio. You write an April call option on IBM with exercise price \$85. You write an April IBM put option with exercise price \$80. The price of the call option is \$0.95 and the price of the put is \$2.45.

(a) (5 points) Graph the payoff of this portfolio at option expiration as a function of IBM's stock price at that time.

(b) (5 points) What will be the profit/loss on this position if IBM is selling at \$83 on the option maturity date. What if IBM is selling at \$90?

(c) (5 points) At what two stock prices will you just break even on your investment?

(d) (5 points) What kind of bet is this investor making; that is, what must this investor believe about IBM's stock price to justify this position?

Question 3 . . . . *15 points*

A stock currently trades at a price of \$100. The stock price can go up 10 percent or down 15 percent. The risk-free rate is 6.5 percent.

(a) (7 points) Use a one-period binomial model to calculate the price of a call option with an exercise price of \$90.

(b) (4 points) Suppose the call price is currently \$17.50. Show how to execute an arbitrage transaction that will earn more than the risk-free rate. Use 100 call options.

(c) (4 points) Suppose the call price is currently \$14. Show how to execute an arbitrage transaction that replicates a loan that will earn less than the risk-free rate. Use 100 call options.

Question 4 .... *40 points*

Consider a two-period binomial model in which a stock currently trades at a price of \$65. The stock price can go up 20 percent or down 17 percent each period. The risk-free rate is 5 percent.

(a) (10 points) Calculate the price of a European put option expiring in two periods with exercise price of \$60.

(b) (5 points) Based on your answer in part (a), calculate the number of units of the underlying stock that would be needed at each point in the binomial tree in order to construct a risk-free hedge. Use 10,000 puts.

(c) (5 points) Is the price of an American put option different? If so, compute the price.

(d) (10 points) Calculate the price of a European put option expiring in two periods with exercise price of \$70.

(e) (5 points) Based on your answer in part (d), calculate the number of units of the underlying stock that would be needed at each point in the binomial tree in order to construct a risk-free hedge. Use 10,000 puts.

(f) (5 points) Is the price of an American put option different? If so, compute the price.

Question 5 .... *15 points*

Suppose you are a financial adviser and that you have a client who believes the common stock price of TRT materials (currently \$58 per share) could move substantially in either direction in reaction to an expected court decision involving the company. The client currently owns no TRT shares, but asks you for advice about implementing a strangle strategy to capitalize on the possible stock price movement. Suppose that a 90-day call option with strike of \$60 is priced at \$5 and that a 90-day put option with strike of \$55 is priced at \$4.

(a) (3 points) Would you recommend a long strangle or short strangle strategy to achieve the client's objective?

(b) For the recommended strategy, calculate the:

  i. (4 points) Maximum possible loss per share.

  ii. (4 points) Maximum possible gain per share.

  iii. (4 points) Break even stock price(s).

Problem Set 4

Econ 236

Question 1 . . . . *80 points*

On 29 May 2015 the closing price of Goldman Sachs (GS) stock was \$208.48.

(a) (10 points) Use daily adjusted closing prices from 30 May 2014 - 29 May 2015 to estimate the *annual* volatility for Goldman.

Solution: Using the following code, we find that the standard deviation of daily GS returns between 30 May 2014 and 29 May 2015 is 0.01119, or 1.119%. To annualize, we multiply by the square root of 252, resulting in an annual volatility of 0.1776 or 17.76%.

```
library(quantmod)
getSymbols('GS',from='2014-05-30',to='2015-05-29')
rets = dailyReturn(GS)
sd(rets)*sqrt(252)
```

(b) (10 points) The closing price of the June 2015 E-mini futures contract, expiring on 20 June 2015, was 2121.75 while the closing value of the S&P 500 index was 2120.79. Assuming a zero dividend yield for the S&P 500, what is the annual risk-free rate of interest implied by these prices? Use actual/252 to annualize.

Solution: We know that the futures price abides by the relationship $F = S_0 e^{r_f T}$, where S_0 is the current price of the underlying asset, r_f is the annual risk-free rate and T is the remaining life of the futures contract, expressed as a fraction of the year. This means,

$$\begin{aligned} r_f &= \frac{1}{T}\log(F/S_0) \\ &= \frac{252}{15}\log(2121.75/2120.79) \\ &= 0.007603, \end{aligned}$$

where $T = 15/252$ because there are 15 trading days between 29 May 2015 and 20 June 2015. Thus, the annual risk-free rate is 0.7603%.

(c) (10 points) Using the Black-Scholes formula and the values that you computed in parts

(a) and (b) to compute the theoretical call price of a Goldman option that expires on 19 June 2015 with strike $K = 205.00$.

Solution: According to Black-Scholes,

$$c = S_0 N(d_1) - K e^{-r_f T} N(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{\log(S_0/K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} \\ &= \frac{\log(208.48/205.00) + (0.007603 + 0.1776^2/2)(15/252)}{0.1776\sqrt{15/252}} \\ &= 0.4206 \\ d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.4206 - 0.1776\sqrt{15/252} \\ &= 0.3773. \end{aligned}$$

Using R to evaluate $N(d_1)$ and $N(d_2)$,

$$\begin{aligned} c &= S_0 N(d_1) - K e^{-r_f T} N(d_2) && (1) \\ &= 208.48 \times 0.6630 - 205.00 e^{-0.007603 \times 15/252} \times 0.6470 && (2) \\ &= 5.64. && (3) \end{aligned}$$

(d) (10 points) Using the Black-Scholes formula and the values that you computed in parts (a) and (b) to compute the theoretical put price of a Goldman option that expires on 19 June 2015 with strike $K = 205.00$.

Solution: According to Black-Scholes,

$$\begin{aligned} p &= K e^{-r_f T} N(-d_2) - S_0 N(-d_1) \\ &= 205.00 e^{-0.007603 \times 15/252} \times 0.3530 - 208.48 \times 0.3370 \\ &= 2.07, \end{aligned}$$

where d_1 and d_2 are the same values that were computed in the previous part.

(e) (10 points) Does put-call parity hold? If not, what arbitrage opportunity could you employ?

Solution: Yes, put-call parity holds. One way to show this is to simply plug in the numbers and verify it for this problem. Instead, we'll show that it must hold in general for the theoretical option prices. Put-call parity holds if

$$c + Ke^{-rT} = p + S_0$$

which can be re-arranged into the equivalent statement

$$c - p = S_0 - Ke^{-rT}.$$

Plugging in the Black-Scholes equations for c and p,

$$\begin{aligned} c - p &= [S_0 N(d_1) - Ke^{-r_f T} N(d_2)] - [Ke^{-r_f T} N(-d_2) - S_0 N(-d_1)] \\ &= S_0[N(d_1) + N(-d_1)] - Ke^{-r_f T}[N(d_2) + N(-d_2)] \\ &= S_0 - Ke^{-r_f T} \end{aligned}$$

where the third equality is because $N(d_1) + N(-d_1) = N(d_2) + N(-d_2) = 1$ since the standard normal distribution is symmetric around zero. Thus, put-call parity holds.

(f) (15 points) On 29 May 2015, the closing call option price for Goldman with strike $K =$ 205.00 and expiry 19 June 2015 was \$3.90. Given this value, use `R` to compute the Black-Scholes implied volatility.

Solution: This can be solved by guess an check, or you could use the function `optimize` in `R` to find value of volatility that minimizes the pricing error of the Black-Scholes price relative to the observed price. I computed the latter, by minimizing the squared distance between the Black-Scholes price and the observed price. I found the implied vol to be 0.07323, with the code below.

```
bsCallError = function(sigma,callPrice,S0,K,rf,tt){
d1 = (log(S0/K) + (rf+(sigma^2)/2)*tt)/(sigma*sqrt(tt))
d2 = d1 - sigma*sqrt(tt)
```

```
bsCallPrice = S0*pnorm(d1) - K*exp(-rf*tt)*pnorm(d2)
return((callPrice - bsCallPrice)^2)
}
optimize(f=bsCallError, interval=c(0,1),callPrice=3.90,
         S0=208.48,K=205, rf=0.007603, tt=(15/252))
```

(g) (15 points) On 29 May 2015, the closing put option price for Goldman with strike $K = 205.00$ and expiry 19 June 2015 was \$2.38. Given this value, use R to compute the Black-Scholes implied volatility.

Solution: Using the following code, I found the implied vol to be 0.1948.

```
bsPutError = function(sigma,putPrice,S0,K,rf,tt){
d1 = (log(S0/K) + (rf+(sigma^2)/2)*tt)/(sigma*sqrt(tt))
d2 = d1 - sigma*sqrt(tt)
bsPutPrice = K*exp(-rf*tt)*pnorm(-d2) - S0*pnorm(-d1)
return((putPrice - bsPutPrice)^2)
}
optimize(f=bsPutError, interval=c(0,1),putPrice=2.07,
         S0=208.48, K=205, rf=0.007603, tt=(15/252))
```

Question 2 .... *20 points*

Suppose that a stock price S follows geometric Brownian motion with expected return μ and volatility σ. What is the process followed by the variable S^n?

Solution: Let $G = S^n$. By Ito's Lemma,

$$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right) dt + \frac{\partial G}{\partial S}\sigma S dz$$

Since $\frac{\partial G}{\partial S} = nS^{n-1} = \frac{nG}{S}$, $\frac{\partial G}{\partial t} = 0$, and $\frac{\partial^2 G}{\partial S^2} = n(n-1)S^{n-2} = \frac{n(n-1)G}{S^2}$,

$$dG = (\mu + \frac{1}{2}(n-1)\sigma^2)nGdt + n\sigma G dz$$

Problem Set 4

Econ 236

Question 1 .... *80 points*

On 29 May 2015 the closing price of Goldman Sachs (GS) stock was \$208.48.

(a) (10 points) Use daily adjusted closing prices from 30 May 2014 - 29 May 2015 to estimate the *annual* volatility for Goldman.

(b) (10 points) The closing price of the June 2015 E-mini futures contract, expiring on 20 June 2015, was 2121.75 while the closing value of the S&P 500 index was 2120.79. Assuming a zero dividend yield for the S&P 500, what is the annual risk-free rate of interest implied by these prices? Use actual/252 to annualize.

(c) (10 points) Using the Black-Scholes formula and the values that you computed in parts (a) and (b) to compute the theoretical call price of a Goldman option that expires on 19 June 2015 with strike $K = 205.00$.

(d) (10 points) Using the Black-Scholes formula and the values that you computed in parts (a) and (b) to compute the theoretical put price of a Goldman option that expires on 19 June 2015 with strike $K = 205.00$.

(e) (10 points) Does put-call parity hold? If not, what arbitrage opportunity could you employ?

(f) (15 points) On 29 May 2015, the closing call option price for Goldman with strike $K = 205.00$ and expiry 19 June 2015 was \$3.90. Given this value, use `R` to compute the Black-Scholes implied volatility.

(g) (15 points) On 29 May 2015, the closing put option price for Goldman with strike $K = 205.00$ and expiry 19 June 2015 was \$2.38. Given this value, use `R` to compute the Black-Scholes implied volatility.

Question 2 .... *20 points*

Suppose that a stock price S follows geometric Brownian motion with expected return μ and volatility σ. What is the process followed by the variable S^n?