IDENTIFICATION OF CNTRL, AVG DIRECT/INDIRECT EFFECTS

Let Ω_M denote the range of M. In a randomized study or an observational study with measured confounders $\mathbf{X}_i$, for every $\mathbf{x}$,

$$M_i(0), M_i(1), \{Y_i(0,m), Y_i(1,m) : m \in \Omega_M\} \perp\!\!\!\perp Z_i \mid \mathbf{X}_i = \mathbf{x}.$$

Then

$$\begin{aligned} E(Y_i \mid Z_i = z, M_i = m, \mathbf{X}_i = \mathbf{x}) &= E(Y_i(z,m) \mid Z_i = z, M_i = m, \mathbf{X}_i = \mathbf{x}) \\ &= E(Y_i(z,m) \mid Z_i = z, M_i(z) = m, \mathbf{X}_i = \mathbf{x}) \\ &= E(Y_i(z,m) \mid M_i(z) = m, \mathbf{X}_i = \mathbf{x}). \end{aligned}$$

Thus

$$\begin{aligned} &E(Y_i \mid Z_i = 1, M_i = m, \mathbf{X}_i = \mathbf{x}) - E(Y_i \mid Z_i = 0, M_i = m, \mathbf{X}_i = \mathbf{x}) \\ &\quad = E(Y_i(1,m) \mid M_i(1) = m, \mathbf{X}_i = \mathbf{x}) - E(Y_i(0,m) \mid M_i(0) = m, \mathbf{X}_i = \mathbf{x}), \end{aligned}$$

which is not a causal comparison in general.