Problem Set 2

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Question 1

Suppose that you own 2500 shares of Apple stock.

a. (15 points)

Obtain data for the period 01 Jan 2017 - 31 Mar 2017 and estimate the hedge ratio for your portfolio. Use the 3-month U.S. T-bill for the risk-free interest rate. [Notes: (1) T-bill rates are expressed in annual terms - you will need to deannulaize to daily frequency; (2) Do not average the T-bill values - use them day-by-day.]

Solution:

$$\text{Calculate the returns}\frac{\delta P}{P_0}$$

$$r_f = \frac{APR}{(365 * 100)}$$

```
rf = Quandl("FRED/DTB3", start_date="2017-01-01", end_date="2017-03-31",type="xts",order=c("asc"))

getSymbols(c("AAPL","^GSPC"), from="2017-01-01", to="2017-03-31")
```

```
##     As of 0.4-0, 'getSymbols' uses env=parent.frame() and
##  auto.assign=TRUE by default.
##
##  This  behavior  will be  phased out in 0.5-0  when the call  will
##  default to use auto.assign=FALSE. getOption("getSymbols.env") and
##  getOptions("getSymbols.auto.assign") are now checked for alternate defaults
##
##  This message is shown once per session and may be disabled by setting
##  options("getSymbols.warning4.0"=FALSE). See ?getSymbols for more details.
```

```
## [1] "AAPL" "GSPC"
```

```
AAPLreturn<-dailyReturn(AAPL)
GSPCreturn<-dailyReturn((GSPC))

rf<-rf/(365*100)

# Excess Return

ExcessAppPort<-AAPLreturn-rf
ExcessGSPCMarket<-GSPCreturn-rf

data<-data.frame(ExcessAppPort,ExcessGSPCMarket,rf)


reg<-lm(ExcessAppPort~ExcessGSPCMarket,data)
summary(reg)
```

```
##
## Call:
## lm(formula = ExcessAppPort ~ ExcessGSPCMarket, data = data)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.009610 -0.004263 -0.001713  0.001124  0.057889
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.002823   0.001160   2.433   0.0180 *
## ExcessGSPCMarket 0.894054   0.279419   3.200   0.0022 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.008978 on 60 degrees of freedom
## Multiple R-squared:  0.1458, Adjusted R-squared:  0.1315
## F-statistic: 10.24 on 1 and 60 DF,  p-value: 0.002199
```

```
hedge<-reg$coefficients[2]
print(hedge)
```

```
## ExcessGSPCMarket
##        0.8940535
```

b. (10 points)

How many E-mini S&P 500 futures contracts would you purchase or sell in order to hedge your portfolio on 31 Mar 2017?

Solution:

$$N* = \beta * \frac{V_p}{V_{e-mini}}$$

```
ES1<-Quandl("CHRIS/CME_ES1",start_date="2017-03-31", end_date="2017-03-31",type="xts")
APP<-(2500*AAPL[62,4])
EMIN<-50*ES1$Last
N=hedge*(APP/EMIN)
print(N)
```

```
##            AAPL.Close
## 2017-03-31   2.724066
```

c. (10 points)

Suppose you would like to synthesize a portfolio beta of 1. What strategy would you employ?

Solution:

$$N* = (\beta - \beta*) * \frac{V_p}{V_{e-mini}}$$

$$\beta * \text{ will be set equal to } 1$$

```
contract<-(hedge-1)*(APP/EMIN)
print(contract)
```

```
##            AAPL.Close
## 2017-03-31 -0.3228053
```

Question 2 (15 points)

Sixty futures contracts are used to hedge an exposure to the price of silver. Each futures contract is on $5,000$ ounces of silver. At the time the hedge is closed out, the basis is $\$0.20$ per ounce. What is the effect of the basis on the hedgerÃ¢Â€Â™s financial position if (a) the trader is hedging the purchase of silver and (b) the trader is hedging the sale of silver?

Solution:

$$Basis = S_T - F_T$$

$$60 * 5000 * .2 = 60000$$

a

The effect of the basis in this example is bad. The hedger is going to purchase silver in the future at a higher price than he would have. The basis increases the cost of the purchase

$$\$60000 - \$50000$$

b

When you hedge the sale of silver the basis is a positive effect on you. You are selling in the future at a higher price so the net gain will be

$$\$60,000 - \$50,000$$

Question 3

a. (5 points)

Suppose that LIBOR rates for maturities of $1, 2, 3, 4, 5$, and 6 months are 2.6%, 2.9%, 3.1%, 3.2%, 3.25%, and 3.3% with continuous compounding. What are the forward rates for future 1-month periods?

Solution:

$$r_f = \frac{R_t * T_t - R_{t-1} * T_{t-1}}{T_t - T_{t-1}}$$

```
forwardrate<-function(R2,R1,T2,T1){
  forward<-((R2*T2)-(R1*T1))/(T2-T1)
  print(forward)
}

forwardrate(2.9,2.6,2,1)
```

```
## [1] 3.2
```

```
forwardrate(3.1,2.9,3,2)
```

```
## [1] 3.5
```

```
forwardrate(3.2,3.1,4,3)
```

```
## [1] 3.5
```

```
forwardrate(3.25,3.2,5,4)
```

```
## [1] 3.45
```

```
forwardrate(3.3,3.25,6,5)
```

```
## [1] 3.55
```

b. (5 points)

Prove that a forward rate is strictly larger (smaller) than the two zero rates used to calculate the forward rate when the slope of zero curve at the interval is strictly positive (negative).

Solution:

$$r_f = r_2 + (r_2 - r_1)\frac{t_1}{t_2 - t_1}$$

$$r_2 > r_1$$

In this case, the slope of the curve will be positive. This shows that r_f will be strictly greater than the zero rates used in the equation because r_2 + some positive number will always be bigger.

$$r_2 < r_1$$

In this case, the slope of the curve will be negative. This shows that r_f will be strictly less than the zero rates used in the equation becuase r_2 + some negative number will always be smaller.

If the zero curve is upward sloping between t_1 and t_2, $r_f > r_2 > r_1$

If the zero curve is downward sloping between t_1 and $t_2, r_f < r_2 < r_1$

c. (5 points)

Suppose you can borrow or lend at LIBOR rates. Explain how you can lock in the forward rate for future 1-month period starting in a month.

Solution:

$$e^{0.026(\frac{1}{12})} = 1.0021699$$
$$e^{0.029(\frac{2}{12})} = 1.004845$$
$$1.0021699e^{r(\frac{1}{12})} = 1.004845$$
$$ln(e^{r(\frac{1}{12})}) = ln(1.002669)$$
$$r(\frac{1}{12}) = .002666$$
$$r = (.002666) \cdot 12 = .03199$$

Question 4 (15 points)

It is March 10, 2017 today. The cheapest-to-deliver bond in a December 2017 Treasury bond futures contract is an 8% coupon bond, and delivery is expected to be made on December 31, 2017. Coupon payments on the bond are made on March 1 and September 1 each year. The rate of interest with continuous compounding is 5% per annum for all maturities. The conversion factor for the bond is 1.2191. The current quoted bond price is $137. Calculate the quoted futures price for the contract.

Solution:

Spot Price $= 137 + [\frac{9}{9+173}] * (\frac{8}{2}) = 137.198$

PV Coupon

$$= \frac{173}{365} = .47397$$

$$4e^{-0.05(.47397)} = 3.906$$

$$F_0 : \frac{296}{365} = .81096 \rightarrow (137.198 - 3.906)e^{.05 \cdot .81096} = 138.808$$

At expiration, there are 122 days of accrued interest

Quoted Futures Price

$$= 138.808 - 4(\frac{122}{122+61}) = 136.141$$

Conversion Factor, Quoted Futures Price for the Contract =

$$\frac{136.141}{1.2191} = 111.67$$

Question 5

Suppose that it is April 27, 2017 and the quoted price of a December, 2017 expiry Eurodollar futures contract is 98.52.

a. 5 (points)

What is the expiry date of the futures contract?

Solution:

The CME Group specifies that expiry is the 2nd business day prior to the third Wednesday of December. Therefore, we are using expiry of Monday, December 18, 2017

b. (5 points)

What is the implied forward rate and for what period does it apply? Express the rate per year and in terms of the actual period.

Solution:

The rate per year is the APR :

$$P = 100 - 98.52 = 1.48\%$$

The actual period, or the 3-month rate following expiry of said contract, is the quarterly compounded APR:

$$\frac{1.48}{4} = 0.37\%$$

c. (5 points)

What is the cash price of the contract?

Solution:

$$\text{Contract Price:} P = (100 - .25R) * 10,000$$
$$P = (100 - .25(100 - Q)) * 10,000$$
$$P = (100 - .25(100 - 98.52)) * 10,000$$
$$P = (100 - .37) * 10,000 = 996,300$$

d. (5 points)

If the price of the futures contract falls, who benefits and why?

Solution:

The borrower benefits. If the price of a future contract falls that means the interest rate has risen. This is good for the borrower because he/she locked in a lower rate on the contract. If the borrower didnt borrow, and decided to in the future, he would have to take a higher rate. The lender could have lent at a higher rate but instead lent at the lower rate like a chump.