INTRODUCTION TO INTERFERENCE

So far we have made the stable unit treatment value assumption (SUTVA) (Rubin, 1980):

1. there are not multiple versions of the treatment
2. the values of a unit's potential outcomes depend only on the the treatment assignment administered to that unit, not the assignments of other units.

The first part could be violated if treatment, e.g. a surgical procedure, is performed by different surgeons and a unit's outcome depends on the surgeon.

The second part is called "no interference". The no interference assumption is often reasonable. e.g. a unit's outcome under surgery does not depend on whether another unit is assigned surgery. In social contexts it is sometimes violated:

INTRODUCTION TO INTERFERENCE

e.g. a student's reading scores are affected by the assignment of the other students in his/her class, but not by the assignment of students in other classes (Hong and Raudenbush, 2006).

e.g. Sobel (2006) considers the Moving to Opportunity (MTO) study. Families in housing projects were randomly assigned to three groups:

1. a housing voucher for low-poverty areas plus assistance
2. a housing voucher and no assistance
3. no voucher (control)

Consider two families. Family A may move if assigned a voucher plus assistance and family B, which is friendly with family A, also receives the voucher plus assistance. But if family B is assigned to the control group, family A may not move.

e.g. consider an unvaccinated individual. Contrast the two cases in which all the individuals with whom he is in contact are vaccinated or unvaccinated. In the first, he does not contract the disease. In the second, he does (Hudgens and Halloran, 2008).

INTRODUCTION TO INTERFERENCE

In all examples, an individual's assignment stays the same, but the assignments of other units varies, and the individual gets a different result. Often called a spillover effect or indirect effect.

1. direct effect: the effect on a unit when that unit is exposed to the treatment or not. This effect may depend on the assignments of other units. e.g. in the MTO example, A moves to a low poverty area if B is assigned a voucher plus assistance and does not move if not assigned a voucher plus assistance

2. spillover effect: a unit's assignment is the same, but the assignments of other units vary. e.g. in the MTO example, A receives a voucher plus assistance, and A moves if B receives a voucher plus assistance, and does not move if B does not receive a voucher plus assistance.

Consider a population of two person dyads e.g. a husband and a wife. Each person is exposed or not to an exercise program, and then each member's cardiovascular fitness is measured.

The husband's (wife's outcome) may depend not only on whether or not he (she) was assigned to treatment, but the partner's assignment through spillover. Assume no interference across dyads.

Suppose a simple random sample of size n has been taken from a population $\mathcal{P}$ of dyads. The dyads are labeled $j = 1, \dots, n$, and the units $i = 1$ for husband, $i = 2$ for wife. Let $Z_{1j} = 1$ if the husband in dyad j receives treatment, 0 otherwise. Define Z_{2j} analogously for the wife.

For all pairs (j, j^*) we assume that assignments in dyad j^* do not affect outcomes in dyad j. The potential outcomes for dyad j is $Y_{ij}(z_1, z_2)$, where $i \in \{1, 2\}$, $z_i \in \{0, 1\}$. The observed outcome Y_{ij}. The treatment assignment probabilities are $\Pr(Z_{1j} = z_1, Z_{2j} = z_2)$ for $z_1 = 0, 1$, $z_2 = 0, 1$.

Define the total effect for unit i in dyad j as $Y_{ij}(1, 1) - Y_{ij}(0, 0)$. The total effect may be decomposed as the sum of a direct effect ands a spillover effect, respectively:

$$\begin{aligned} Y_{ij}(1,1) - Y_{ij}(0,0) &= [Y_{ij}(1,1) - Y_{ij}(0,1)] + [Y_{ij}(0,1) - Y_{ij}(0,0)] \\ &= [Y_{ij}(1,0) - Y_{ij}(0,0)] + [Y_{ij}(1,1) - Y_{ij}(1,0)]. \end{aligned}$$

The two decompositions give different values for the constituent effects.

e.g. let $i = 1$.

In decomposition $[Y_j(1,1) - Y_j(0,1)] + [Y_j(0,1) - Y_j(0,0)]$, the total effect of treating both husband and wife vs. treating neither is the sum of the direct effect of treatment for the husband when the wife is also treated and the spillover effect (from treating the wife) onto the husband when he is not treated.

In decomposition $[Y_j(1,0) - Y_j(0,0)] + [Y_j(1,1) - Y_j(1,0)]$, the total effect is the the direct effect of treatment for the husband when the wife is untreated plus the spillover onto the husband from treating the wife when he is treated. Interpretation for $i = 2$ follows upon interchanging the words "husband" and "wife" above.

Averaging over the husbands (wives) in the population gives average potential outcomes $E(Y_{i.}(z_1, z_2))$.

Since this is a randomized study, $\bar{Y}_{i.} = \frac{\sum_{j=1}^{n} Z_{1j} Z_{2j} Y_{ij}}{\sum_{j=1}^{n} Z_{1j} Z_{2j}}$ is a consistent estimate of $E(Y_{i.}(1, 1))$.

Thus, the average direct and spillover effects can be estimated consistently using the sample averages.