Problem Set 1

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Question 1

a. (5 points)

Simulate the price process,

$$P_t = \mu + P_{t-1} + \varepsilon_t,$$

for $t = 1, 2, \ldots, T$, where $T = 100$, $\varepsilon_t = N(0, \sigma^2)$, $\mu = 0.00024$ and $\sigma = 0.013$. You may initiate the simulation with $P_0 = 100$. A correct solution will simply include the simulation code.

Solution:

```
P<-rep(0, times=100)
sigma=0.013
u=0.00024
P[1]<-100
for(i in 2:100){
  P[i] <- u + P[i-1] + rnorm(100, mean = 0, sd = sigma)
}
```

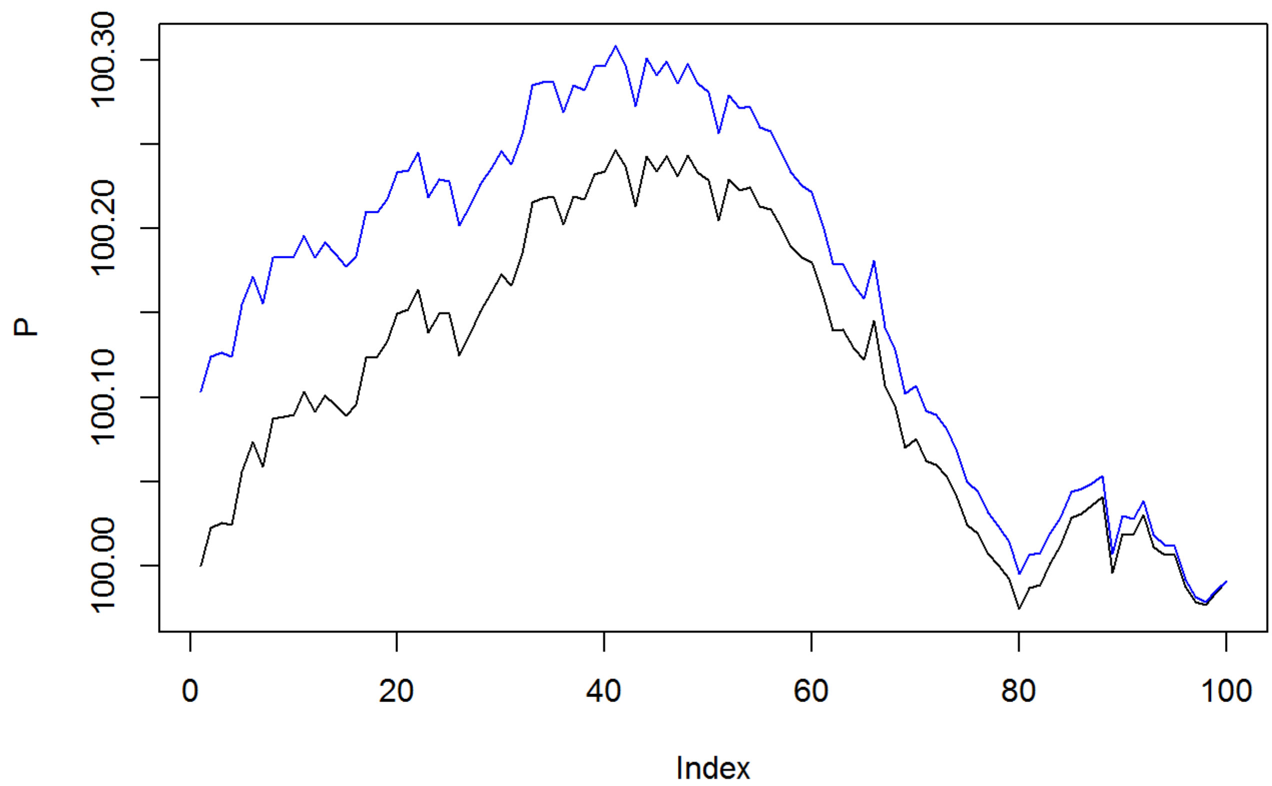
b. (10 points)

Assume that the annualized risk-free rate of interest is $r = 0.0038$. For each $t = 1, 2, \ldots, 100$, compute the value of a futures contract on the asset prices you simulated in part (a), assuming that the contract expires at time $T = 100$, and that the units of time t are days. Plot the trajectories of the futures and asset prices together on a single figure.

Solution:

```
n=100
r=.0038
FP<-rep(0,100)
for(i in n:1){
FP[i]=P[i]*exp(r*(n-i)/365)
}

ymin=min(min(P,na.rm=T),min(FP))
ymax=max(max(P,na.rm=T),max(FP))
plot(P,type="l",ylim=c(ymin,ymax))
lines(FP,col="blue")
```



c. (5 points)

What do you notice about the relationship between the futures price and underlying asset price? How might this relationship change?

Solution:

The futures and the asset price follow roughly the same path but converge at some points. The distance between the two curves is reflected by the rate of interest.As we approach expiration, the two should be identical.

d. (5 points)

Obtain data for the front-month E-mini S&P 500 futures contract and for the S&P 500 index (Yahoo ticker `^GSPC`) for the period Jan 1, 2017 - Mar 9, 2017. Plot both time series in a single figure. What do you notice about the relationship between the futures and spot prices?

Solution:

```
library(Quandl)
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
library(quantmod)
```

```
## Loading required package: TTR
```

```
## Version 0.4-0 included new data defaults. See ?getSymbols.
```

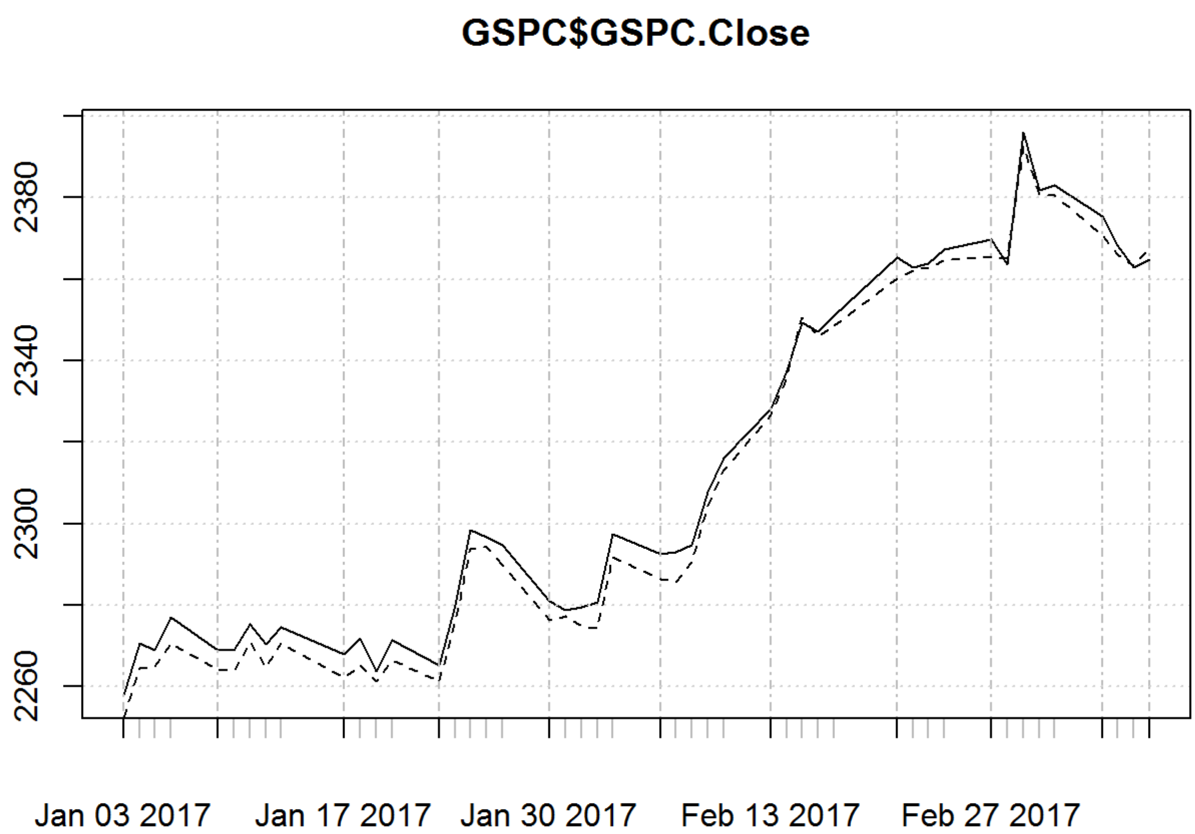
```
getSymbols("^GSPC", from="2017-01-02", to="2017-03-09")
```

```
##     As of 0.4-0, 'getSymbols' uses env=parent.frame() and
##  auto.assign=TRUE by default.
##
##  This  behavior  will be  phased out in 0.5-0  when the call  will
##  default to use auto.assign=FALSE. getOption("getSymbols.env") and
##  getOptions("getSymbols.auto.assign") are now checked for alternate defaults
##
##  This message is shown once per session and may be disabled by setting
##  options("getSymbols.warning4.0"=FALSE). See ?getSymbols for more details.
```

```
## [1] "GSPC"
```

```
ES1 = Quandl("CHRIS/CME_ES1",start_date="2017-01-02",end_date="2017-03-09",type="xts")
datesTilExp = as.numeric(as.Date("2017-03-17")-index(ES1))

plot(GSPC$GSPC.Close,type="l")
lines(ES1$Last,lty=2)
```



The two lines move in tandem.

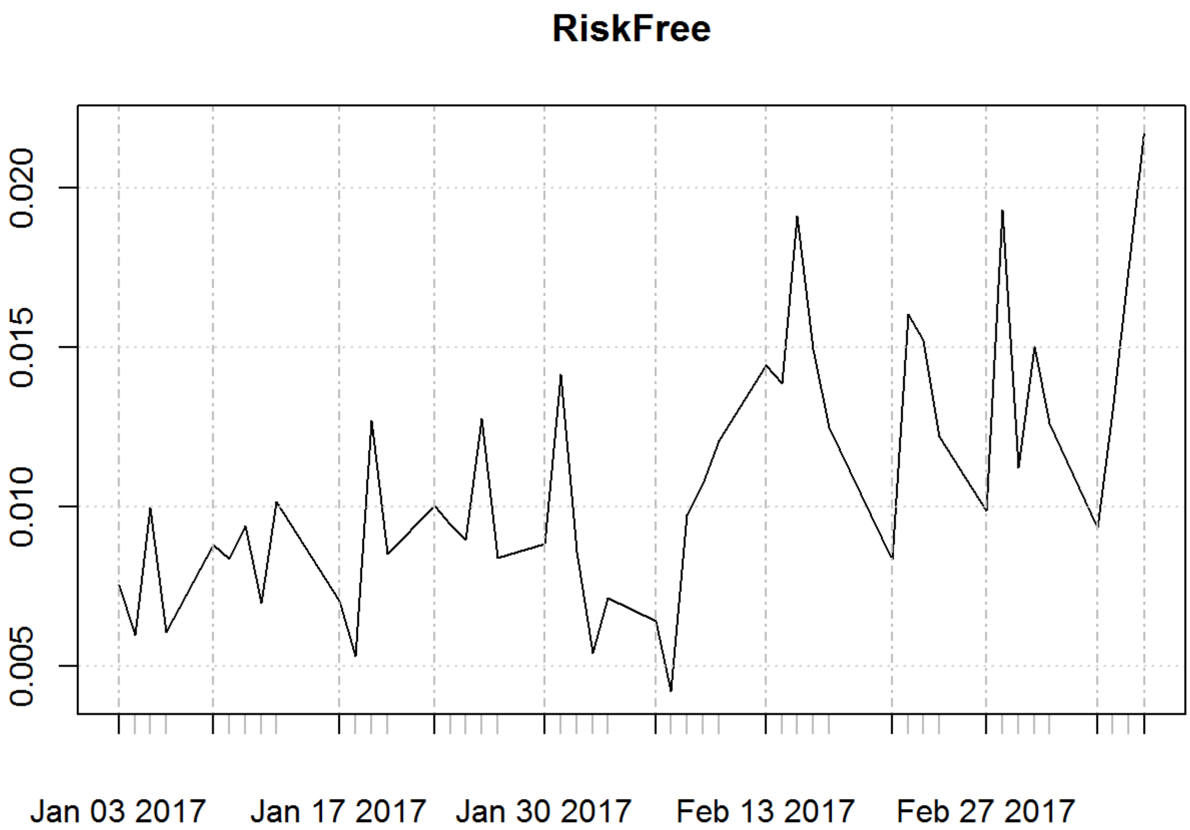
e. (10 points)

Compute and plot the implied risk-free interest rate from the futures and spot prices in part (d), assuming that the S&P 500 index has a dividend yield of 1.7% per year. Assume that the expiry date of the contract is Mar 17, 2017.

Solution:

$$R = \frac{Log(F_t) - Log(S_t)}{T} + Dividend$$

```
RiskFree<-(log(ES1$Last)-log(GSPC$GSPC.Close))/.25 + .017
plot(RiskFree,type="l")
```



Question 2

Suppose you would like to buy cocoa futures on the ICE exchange.

a. (2 points)

What are the price units and size for a cocoa contract?

Solution:

The price units for a cocoa contract are USD per metric ton, with a contract size of 10 metric tons ($10,000)

b. (3 points)

What was the settlement price of ICE cocoa futures on Apr 11, 2017 and what was the resulting value of a single contract on that day?

Solution:

The settlement price is $1,972 of ICE cocoa futures on April 11, 2017. The value of a contract was $1,972.

c. (5 points)

Suppose that the cocoa contract has an initial margin of 1450 USD. If the price of the contract decreases by 1% in the next day, what will be your percentage loss (on the original base of 1450 USD)? If the price increases by 1%, what would be your percentage gain?

Solution:

A 1% decrease in the \$19,720 contract leads to a \$197.20 loss (One percent of the contract price). Therefore, \$197.20 dollar loss on the initial margin you committed of \$1450 falls to \$1350; a 6.89% loss.

A 1% increase in the \$19,720 contract leads to a \$197.20 gain. Therefore, \$1972.20 dollar gain on the initial margin you committed of \$1450 rises to \$1550; a 6.89 gain% .

$$Loss = 1450 - 197.20 = 1252.8 \rightarrow \frac{(1450 - 1252.8)}{1450} = 13.6\%$$
$$Gain = 1450 + 197.20 = 1647.2 \rightarrow \frac{(1450 - 1647.2)}{1450} = -13.6\%$$

d. (5 points)

Calculate the leverage ratio and explain the relationship between the leverage ratio and percentage gain.

Solution:

$$\text{Leverage Ratio} = \frac{\text{Settlement Price} \cdot \text{Contract Size}}{\text{Initial Margin}}$$
$$\text{Leverage Ratio} = \frac{1972 \cdot 10}{1450} = 13.6\%$$

e. (5 points)

Suppose maintenance margin is 1400 USD. What is the lowest settlement price which does not involve a margin call?

Solution:

$$\text{Leverage Ratio} = \frac{\text{Settlement Price} \cdot \text{Contract Size}}{\text{Initial Margin}}$$
$$\frac{X}{1400} = 13.6\%$$
$$X = \$19,040$$

Question 3

a. (3 points)

Suppose $APR = 5\%$ and it is semi-annually compounded. What is EAR?

Solution:

```
#i=stated interest
#n=number of compounding periods
EAR<-function(i,n){
  y<-(1+(i/n))^n-1
  return(y)
}
```

```
EAR(.05,2)
```

```
## [1] 0.050625
```

b. (3 points)

Suppose $APR = 5\%$ and it is continuously compounded. What is EAR?

Solution:

```
EAR_Continuous<-function(i){
  y<-exp(i)-1
  return(y)
}
```

```
EAR_Continuous(.05)
```

```
## [1] 0.0512711
```

c. (5 points)

Suppose $APR = 5\%$ and it is semi-annually compounded. What is the equivalent APR with conutinuous compounding?

Solution:

```
#i=EAR
APR<-function(i){
  y<-log(1+i)
  return(y)
}
```

```
x<-EAR(.05,2)
APR(x)
```

```
## [1] 0.04938523
```

d. (6 points)

Suppose $APR = 5\%$ and it is continuously compounded, using an actual/actual day count convention. What is the equivalent APR with semi-annually compounding, using an actual/360 day count convention?

Solution:

$$.05127 + 1 = ((1 + \frac{r}{2})^2)^{\frac{365}{360}}$$
$$(1.05127)^{.4932} = 1 + \frac{r}{2}$$
$$1.02496 - 1 = \frac{r}{2}$$
$$.02496 \cdot 2 = r$$
$$\boxed{r = .04992}$$

Question 4

a. (5 points)

Consider holding a long position with a forward contract in gold. Express the realized payoff of the forward contract in period T as a function of the forward price and the future spot price.

Solution:

$Payoff_T = S_t - F_0$

b. (5 points)

Suppose the current forward price is \$100 and you expect that the future spot price will be \$99 with probability 0.9 and \$111 with probability 0.1. Calculate the ex-ante payoff of the forward contract when you hold a long position.

Solution:

$ExpectedPayoff = \$99 * 0.9 + \$111 * 0.1 - \$100$ $ExpectedPayoff = \$.20$

Question 5

a. (6 points)

Consider forward contracts on an asset. Suppose the foward price is strictly higher than the opportunity cost of holding the asset for T periods. Explain how you can exploit the opportunity for arbitrage.

Solution:

$F_t > S_0 * e^r t$ Short position on the contract: Price you get is F_t Buy at S_t, borrow and store the good, repay $S_t * e^r t$ Compare which one is larger; by definition F_t is. The difference between the two is your arbitrage profit.You are selling the commodity at a higher price so your profit is the difference between the cost of borrowing (and storage) and the price*quantity you get from the future contract.Price adjust in the direction so futures equation holds in equality. Once people start doing this more and more the market will equilibrate.

b. (6 points)

Suppose the foward price is strictly lower than the opportunity cost of holding the asset for T periods. Explain how you can exploit the opportunity for arbitrage.

Solution:

$F_t < S_0 * e^r t$ Price is low so we take a long position on the contract, paying F_t. This is the price you are paying for the cheaper asset.Borrow asset (cash) today at current rate and pay back $S_T * e^r t$ in the future. When you have to pay back the money, you anticipate the value of the future will rise. The difference in the purchase price of the future and market price will be the profit (minus the cost of borrowing)

c. (6 points)

Consider a future contracts on GBP. The spot price is the value of 1 GBP in USD today. Suppose future price is lower than the opportunity cost of holding 1 GBP for T periods. Explain how you can exploit the opportunity for arbitrage.

Solution:

$e_f^r * t < \frac{S_t * e^r t}{F_t}$

In these circumstances, an investor could take a long position. Borrow the currency and sell it in the future at a higher price. Holding GBP for T periods, the return on this, is what you are forgoing. The procedure is essentially the same as part b.