SOME RANDOMIZED EXPERIMENTS

Last time we introduced:

1. the potential outcomes notation
2. unit and average treatment effects
3. unbiased estimation of the average treatment effects

SOME RANDOMIZED EXPERIMENTS

Randomized experiments are the bridge to observational studies so we need to understand these. This module is focused on randomized experiments:

1. Lesson 1: different types of randomized experiments

2. Lesson 2: randomization based inference: the null hypothesis of 0 effect

3. Lesson 3: estimation of the Sample Average Treatment Effect (SATE)

SOME RANDOMIZED EXPERIMENTS

We will study several types of randomized experiments in this module. The key to understanding how these differ are the rules by which subjects are assigned to either a treatment group or a control group.

First, some notation

NOTATION

n units, $i = 1, ..., n$

row vector $(Z_i, \mathbf{X}_i', Y_i(0), Y_i(1))$

Z_i is the treatment assignment of unit i

$\mathbf{X}_i$ a column vector of covariates (pre-treatment characteristics) associated with unit i

$(Y_i(0), Y_i(1))$–i's potential outcomes.

Let $\mathbf{Z} = (Z_1, ..., Z_n)'$ denote the column vector of assignments, $\mathrm{I}(X, \mathbf{Y}(0), \mathbf{Y}(1))$ denote the corresponding matrix with rows $(\mathbf{X}_i', Y_i(0), .Y_i(1))$.

ASSIGNMENT RULES

As in module 1, we are interested in the relationship between treatment assignment, potential outcomes and covariates:

$$\Pr(\mathbf{Z} = \mathbf{z} \mid X, \mathbf{Y}(0), \mathbf{Y}(1)), \tag{1}$$

where $\mathbf{Z} \in \Omega \subseteq \{(0,1)\}^n$

ASSIGNMENT RULES

We shall be interested primarily in assignment rules of the form:

$$\begin{aligned}\Pr(\mathbf{Z} = \mathbf{z} \mid X, \mathbf{Y}(0), \mathbf{Y}(1)) &= k \prod_{i=1}^{n} \Pr(Z_i = z_i \mid \mathbf{X}_i, Y_i(0), Y_i(1)) \\ &= k \prod_{i=1}^{n} \Pr(Z_i = z_i \mid \mathbf{X}_i) = k \prod_{i=1}^{n} e(X_i)^{z_i}(1 - e(X_i)^{1-z_i}), \quad (2)\end{aligned}$$

where k is chosen so that $\sum_{Z \in \Omega} \Pr(\mathbf{Z} = \mathbf{z} \mid X, \mathbf{Y}(0), \mathbf{Y}(1)) = 1$ and $0 < \Pr(Z_i = 1 \mid X_i)$ and $0 < e(X_i) < 1$ for all i

ASSIGNMENT RULES

1. individualistic assignment: i's assignment depends on own covariates and potential outcomes, but not assignments of other units.

2. unconfounded assignment: i's assignment does not depend on potential outcomes

3. every unit i has positive probability of exposure to treatment and its absence.

When 2. holds, we say, following the terminology introduced by Rosenbaum and Rubin (1983), treatment assignment is strongly ignorable, given covariates.

$e(\mathbf{X}_i)$ is called the propensity score—the propensity score will be very important later on

ASSIGNMENT RULES FOR SOME RANDOMIZED EXPERIMENTS

1. Bernoulli experiment (coin tossing)

For all i, let $0 < \lambda = \Pr(Z_i = 1 \mid X_i) < 1$.

Here $\Omega = \{0, 1\}^n$, $k = 1$, and each unit has the same known probability of assignment λ to treatment

i.e. the propensity score is the same for all units.

ASSIGNMENT RULES FOR SOME RANDOMIZED EXPERIMENTS

2. completely randomized experiment where the n_1 treated units and $n - n_1 = n_0$ untreated units

Let $\Omega = \{\mathbf{Z} : \sum_{i=1}^{n} Z_i = n_1\}$.

All $\frac{n!}{n_1!n_0!}$ assignments in Ω are equally likely and $e(X_i) = n_1/n$ for all i.

ASSIGNMENTS RULES FOR SOME RANDOMIZED EXPERIMENTS

3. randomized block experiment where we group the n units into S strata on the basis of covariates and conduct a completely randomized experiment in each block.

Let $f(X_i) = s$, $s \in \{1, ..., S\}$.

n_s units per stratum, n_{s1} assigned to the treatment group.

Let $Z_{si'} = 1$ is unit i' in stratum s is assigned to treatment, 0 otherwise so that $\mathbf{Z}_s$ is a column vector of assignments in stratum s: $\mathbf{Z}_s = (Z_{s1}, ..., Z_{sn_s})'$.

$\Omega_s = \{\mathbf{Z}_s : \sum_{si'=1}^{n_s} Z_{si'} = n_{s1}\}$ so that Ω is the Cartesian product of the sets Ω_s: $\Omega = \prod_{s=1}^{S} \Omega_s$,
$\Pr(Z_1 = z_1, ..., Z_S = z_s \mid X_1, ..., X_n) = \prod_{s=1}^{S} \frac{n!_s}{n_{s1}!(n_s - n_{s1})!}]^{-1}$

ASSIGNMENT RULES FOR SOME RANDOMIZED EXPERIMENTS

4. paired randomized experiment, a special case of the randomized block experiment where $n_s = 2$, $n_{s1} = 1$, $s = 1, ..., S$.

Investigator groups units into pairs matched on the basis of their "closeness" on covariates, then assigns each member of the pair to receive treatment with probability $1/2$.