

# Math 20: Probability

## Homework 4

Due Date: July 24, 2020

Please specify whether you complete the homework independently or cooperate with (get help from) the TA, your classmates, friends, online resources, etc.

For every problem, show the process and necessary details.

A pdf file is required for submission.

### Problem 1

**4 pts**

Chapter 4.1 Exercise 3

A dice is rolled twice. What is the probability that the sum of the faces is greater than 7, given that

- (a) the first outcome was a 4?
- (b) the first outcome was greater than 3?
- (c) the first outcome was a 1?
- (d) the first outcome was less than 5?

### Problem 2

**4 pts**

Chapter 4.1 Exercise 14

If  $P(\tilde{B}) = \frac{1}{4}$  and  $P(A|B) = \frac{1}{2}$ , what is  $P(A \cap B)$ ?

### Problem 3

4 pts

Chapter 4.1 Exercise 17

Prove that if  $A$  and  $B$  are independent so are

- (a)  $A$  and  $\tilde{B}$ .
- (b)  $\tilde{A}$  and  $\tilde{B}$ .

### Problem 4

4 pts

Chapter 4.1 Exercise 18

A doctor assumes that a patient has one of three diseases  $d_1$ ,  $d_2$  or  $d_3$ . Before any test, he assumes that an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has  $d_1$ , 0.6 if he has disease  $d_2$  and 0.4 if he has disease  $d_3$ . Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?

### Problem 5

4 pts

Chapter 4.1 Exercise 29

A student is applying to Harvard and Dartmouth. He estimates that he has a probability of 0.5 of being accepted at Dartmouth and 0.3 of being accepted at Harvard. He further estimates the probability that he will be accepted by both is 0.2. What is the probability that he is accepted by Dartmouth if he is accepted by Harvard? Is the event “accepted at Harvard” independent of the event “accepted at Dartmouth”?

### Problem 6

4 pts

Chapter 4.1 Exercise 36

A dice is thrown twice. Let  $X_1$  and  $X_2$  denote the outcomes. Define  $X = \min(X_1, X_2)$ . Find the distribution of  $X$ .

### Problem 7

4 pts

Chapter 4.2 Exercise 1

Pick a point  $x$  at random (with uniform density) in the interval  $[0, 1]$ . Find the probability that  $x > \frac{1}{2}$ , given that

- (a)  $x > \frac{1}{4}$ .
- (b)  $x < \frac{3}{4}$ .
- (c)  $|x - \frac{1}{2}| < \frac{1}{4}$ .
- (d)  $x^2 - x + \frac{2}{9} < 0$ .

### Problem 8

4 pts

Chapter 4.2 Exercise 2

A radioactive material emits  $\alpha$ -particles at a rate described by the density function

$$f(t) = 0.1e^{-0.1t}.$$

Find the probability that a particle is emitted in the first 10 seconds, given that

- (a) no particle is emitted in the first second.
- (b) no particle is emitted in the first 5 seconds.
- (c) a particle is emitted in the first 3 seconds.
- (d) a particle is emitted in the first 20 seconds.

### Problem 9

4 pts

Chapter 4.2 Exercise 5 (a) and (c)

Suppose you choose two numbers  $x$  and  $y$ , independently at random from the interval  $[0, 1]$ . Given that their sum lies in the interval  $[0, 1]$ , find the probability that

(a)  $|x - y| < 1$ .

(b)

(c)  $\max\{x, y\} < \frac{1}{2}$ .

(d)

(e)