Math 20: Probability

Homework 2

Due Date: July 10, 2020

Please specify whether you complete the homework independently or cooperate with (get help from) the TA, your classmates, friends, online resources, etc.

For every problem, show the process and necessary details.

Problem 1

4 pts

Chapter 1.2 Exercise 6

A dice is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.) What is the probability of getting an even number in one throw?

Problem 2

4 pts

Chapter 1.2 Exercise 14

Let X be a random variable with distribution function $m_X(x)$ defined by

$$m_X(-1) = \frac{1}{5}, \qquad m_X(0) = \frac{1}{5}, \qquad m_X(1) = \frac{2}{5}, \qquad m_X(2) = \frac{1}{5}.$$

1. Let Y be the random variable defined by the equation $Y = X + 3$. Find the distribution function $m_Y(y)$ of Y.
2. Let Z be the random variable defined by the equation $Z = X^2$. Find the distribution function $m_Z(z)$ of Z.

Problem 3

4 pts

Chapter 1.2 Exercise 22

A dice is rolled until the first time that a six turns up. We shall see that the probability that this occurs on the nth roll is $(\frac{5}{6})^{n-1}\frac{1}{6}$. Using this fact, describe the appropriate infinite sample space and distribution function for the experiment of rolling a dice until a six turns up for the first time. Verify that for your distribution function $\sum_{\omega} m(\omega) = 1$.

Problem 4

4 pts

Chapter 1.2 Exercise 26

Two cards are drawn successively from a deck of 52 cards. Find the probability that the second card is higher in rank than the first card. Hint: Show that $1 = P(\text{higher}) + P(\text{lower}) + P(\text{same})$ and use the fact that $P(\text{higher}) = P(\text{lower})$.

Problem 5

5 pts

Chapter 2.2 Exercise 2

Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f(x) = Cx,$$

where C is a constant.

(a) Find C.

(b) Find $P(E)$, where $E = [a, b]$ is a subinterval of $[2, 10]$.

(c) Find $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.

Problem 6

4 pts

Chapter 2.2 Exercise 5

Suppose you are watching a radioactive source that emits particles at a rate described by the exponential density

$$f(t) = \lambda e^{-\lambda t},$$

where $\lambda = 1$, so that the probability $P(0, T)$ that a particle will appear in the next T seconds is $P([0, T]) = \int_0^T \lambda e^{-\lambda t} dt$. Find the probability that a particle (not necessary the first) will appear

(a) within the next second.

(b) within the next 3 seconds.

(c) between 3 and 4 seconds from now.

(d) after 4 seconds from now.

Problem 7

4 pts

Chapter 2.2 Exercise 6

Assume that a new light bulb will burn out after t hours, where t is chosen from $[0, \infty)$ with an exponential density

$$f(t) = \lambda e^{-\lambda t}.$$

In this context, λ is often called the **failure rate** of the bulb.

(a) Assume that $\lambda = 0.01$, and find the probability that the bulb will not burn out before T hours. This probability is often called the **reliability** of the bulb.

(b) For what T is the reliability of the bulb $= \frac{1}{2}$.

Problem 8

4 pts

Chapter 5.2 Exercise 2

Choose a number U from the interval $[0, 1]$ with uniform distribution. Find the cumulative distribution and density for the random variables

(a) $Y = \frac{1}{U+1}$.

(b) $Y = \log(U + 1)$.

Problem 9

4 pts

Chapter 5.2 Exercise 10

Let U, V be random numbers chosen independently from the interval $[0, 1]$. Find the cumulative distribution and density for the random variables

(a) $Y = \max(U, V)$.

(b) $Y = \min(U, V)$.