Mathematics for Political Science

Lecture 2: Algebra

Exercises

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1. Solve the following equations for x:

   1. $12x + 2 = 18x$
   2. $-6 - 4x = -3 - 8x$

2. Express α in terms of the other unknown variables:

   1. $3\alpha - 8\theta = \alpha + 2\beta$
   2. $\alpha x + \alpha y = \alpha x^2 + \alpha y^2 + 4$

3. (Gill 1.6) Solve the following inequalities so that the variable is the only term on the left-hand side:

   1. $x - 3 < 2x + 15$
   2. $11 - \frac{4}{3}t > 3$
   3. $\frac{5}{6}y + 3(y - 1) \leqslant \frac{11}{6}(1 - y) + 2y$

4. Find the values of x where $f(x) = 0$ using factorization:

   1. $x^2 + 5x - 14$
   2. $x^2 - 8x + 16$
   3. $3x^2 + 9x - 30$

5. Solve the following equations for x using the quadratic formula:

   1. $18x^2 + 10x = 3 - 15x$
   2. $20x^2 + 2x - 3 = 5 + 20x - 15x^2$

6. Solve the following systems of equations for a and b using the "direct substitution" approach:

   1. $b + 5a = 2$
      $7b - 6a = 14$
   2. $3(a + b) + 7a = 8(b - 1) + 33$
      $-3a + 4(1 - b) = 4(1 - a) - 15$

7. Solve the following systems of equations for c and d using the "elimination" approach:

1. $3c + 4d = 13$
   $2c + 5d = 4$

2. $c + 4d + 36 = 10d - 3c$
   $2(c + 1) + 2(d + 1) = 6$

8. Solve this system of equations for x and y in terms of α:
   $2x + y = 10\alpha + 5$
   $3x + 3y = 18\alpha + 9$

9. Solve this system of equations for q, r, and s:
   $2q + 4r + s = 1$
   $4(q + 1) + 7(1 - r) = 2s + 16$
   $8q + 4r - 2s = 5q + 19r + 4s$

10. Calculate the dot product of the vectors below.

1. $[3, 4, 1, 7, 0] \cdot [5, 2, 2, 0, 3]$

2. $[4, 1, 3] \cdot [0, 7, 5]$

11. Find the vector norm of the following vectors to determine which is the longest in 3-dimensional space.

1. $\mathbf{a} = [4, 5, 3]$

2. $\mathbf{b} = [4, 4, 4]$

3. $\mathbf{c} = [1, 1, 7]$

12. (Gill 3.9) For the following matrix, calculate $\mathbf{X}^n$ for $n = 2, 3, 4, 5$. Write a rule for calculating higher values of n.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

13. Using the matrix below, show the identities of multiplication and addition for matrices:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

14. Perform the following matrix multiplications, or explain why they are not possible:

1. $\begin{bmatrix} 4 & 5 & 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 8 & 1 \\ 0 & 9 \\ 6 & 4 \end{bmatrix}$
2. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$
3. $\begin{bmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \end{bmatrix} \begin{bmatrix} \lambda & \sigma \end{bmatrix}$

15. Multiply the matrices below to show that order matters for matrix multiplication:

a. $\begin{bmatrix} 4 & 7 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \begin{bmatrix} 4 & 7 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 4 & 8 \\ 1 & 6 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 9 & 6 & 3 \\ 1 & 5 & 3 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} 9 & 6 & 3 \\ 1 & 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 1 & 6 \\ 2 & 2 \end{bmatrix}$

16. Express the system of equations from lecture (and below) in matrix notation, and perform the matrix multiplication to show that the solutions found through elimination are correct.

$$\begin{aligned} p + 2q + 4r &= -7 \\ 3p - 7q + r &= 12 \\ 2p + q + 2r &= 4 \end{aligned}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$