# Mathematics for Political Science

Lecture 5: Probability & Looking Ahead

Exercises

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1. Suppose the department is choosing its leadership for next year: a chair, and associate chair, and four field chairs (IR, Comparative, American, Theory). Individuals cannot be both chair and associate chair, but they may hold either of those jobs as well as a field job. If there are 5 IR faculty, 12 Comparative faculty, 13 Americanist faculty, and 6 Theory faculty, how many different leadership groups could be formed? (assuming faculty members are counted according to their primary subfield, so these are non-overlapping groups).

2. For the sets defined in the lecture slides (considering each set of sets separately):

   1. What is the complement of set D? (presidential set)
   2. Draw graphically the complement of set L. (spatial set)
   3. Which other sets are subsets of set T? (number line set)
   4. What is the intersection of sets S and R? (number line set)
   5. What is the union of sets F and G? (dice set)
   6. What is the complement of the union of sets S and V? (number line set)
   7. What is the intersection of set M with the union of sets L and N? (spatial set)
   8. What is the intersection of sets P and I? (presidential set)

3. (Gill 7.6) For some set A, explain $A \cup A$ and $A \cap A$.

4. (Gill 7.5 [adapted]) Suppose you had a pair of four-sided dice, so the set of possible outcomes from summing the results from a single toss is {2,3,4,5,6,7,8}. Determine the probability of each of these outcomes.

5. Calculate the following probabilities:

   1. Flipping a fair coin 7 times and seeing an alternating pattern with no result identical to the previous result.
   2. Drawing 10 cards from a deck (with replacement) and getting exactly three face cards and seven non face cards, in any order.
   3. Rolling a fair, six-sided die 9 times and *not* getting exactly three 3s, three 4s, and three 5s (in any order).

6. (Gill 7.15) Use this joint probability distribution

| | | Y | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| X | 0 | 0.10 | 0.10 | 0.01 |
| | 1 | 0.02 | 0.10 | 0.20 |
| | 2 | 0.30 | 0.10 | 0.07 |

to compute the following:

1. $p(X < 2)$
2. $p(X < 2|Y < 2)$
3. $p(Y = 2|X \leqslant 1)$
4. $p(X = 1|Y = 1)$
5. $p(Y > 0|X > 0)$

7. (Gill 7.10) In rolling two dice labeled X and Y, what is the probability that the sum of the up faces is four, given that either X or Y shows a three?

8. Consider some variations of the Bayes Rule problem from slides 52-53:

1. Suppose the ball drawn had been red instead of blue. What would be the updated conditional probabilities of each urn?
2. Suppose a blue ball were drawn, replaced, and then a second blue ball were drawn. What would be the updated conditional probabilities of each urn? What about after that had been replaced and a third blue ball had been drawn?
3. Suppose instead of beginning with a coin flip to pick which urn, it began with a roll of a fair, 6-sided die, where {1} selects urn A and {2,3,4,5,6} selects urn B. If a blue ball were chosen, what would be the updated conditional probabilities of each urn?

9. Suppose that China has recently completed a large-scale military exercise simulating an amphibious assault on Taiwan, raising levels of tension in the Taiwan Strait. The Chinese leadership is considering whether to launch such an assault, which it would like to do if the United States would stay out but would prefer not to if the United States is strongly committed and thus would intervene. It has a prior belief that there is a 70% chance the United States is strongly committed. Suppose also it believes that a strongly committed United States will respond to the military exercise by sending an aircraft carrier fleet with 90% probability, while a not-strongly committed United States would only send a carrier fleet with 20% probability. If no aircraft carriers are sent, what is China's best estimate of the likelihood the U.S. is strongly committed?