

# Mathematics for Political Science

## Lecture 4: Calculus II

### Exercises

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1. For each of the functions:

$$f(x) = 3x^2 - 7x + 2$$

$$g(x) = 8x^3 - 46x^2 + 73x - 35$$

1. Sketch a plot the function on the interval  $[0, 5]$  (calculate  $f(x)$  for integer values of  $x$  to get a general idea of the shape of the function).
  2. Identify the values of  $x$  that generate local maxima or minima (ignoring endpoints).
  3. Show mathematically whether these are maxima or minima.
2. Find the value of  $x$  that maximizes the function  $\ell(x) = 2\ln(x) - x - \ln(2x + 1)$  using the following approach.
1. Take the derivative of  $\ell(x)$  and set it equal to 0.
  2. Manipulate the expression to remove fractions and express it as a quadratic.
  3. Solve for  $x$ .
3. Find the partial derivatives of the function  $(eR(\frac{f}{f+g}))^h$  with respect to  $e$  and  $f$ .
4. (Gill 5.13 [adapted]) Calculate the following indefinite integrals:

a.  $\int 4y^3 dy$

b.  $\int (x^2 - x^{-\frac{1}{2}}) dx$

c.  $\int 360t^6 dt$

5. (Gill 5.10 [adapted]) Solve the following definite integrals using the antiderivative method:

a.  $\int_6^8 x^3 dx$

b.  $\int_1^9 2y^5 dy$

c.  $\int_{-1}^0 (3x^2 - 1) dx$

d.  $\int_{-1}^1 (14 + x^2) dx$

e.  $\int_2^4 e^y dy$

f.  $\int_2^4 \sqrt{t} dt$

6. (Gill 5.11) Calculate the area of the following function that lies above the  $x$ -axis and over the domain  $[-10, 10]$ :

$$f(x) = 4x^2 + 12x - 18$$