

Mathematics for Political Science

Lecture 3: Calculus I

Exercises

1. (Gill 5.1 [adapted]) Find the following finite limits:

1. $\lim_{x \rightarrow 4} [x^2 - 6x + 4]$

2. $\lim_{x \rightarrow 0} [\frac{x - 25}{x + 5}]$

3. $\lim_{x \rightarrow 4} [\frac{x^2}{3x - 2}]$

4. $\lim_{x \rightarrow 1} [\frac{x^2 - 1}{x - 1}]$

2. (Gill 5.3 [adapted]) Find the following infinite limits and graph:

1. $\lim_{x \rightarrow \infty} [\frac{9x^2}{x^2 + 3}]$

2. $\lim_{x \rightarrow \infty} [\frac{3x - 4}{x + 3}]$

3. $\lim_{x \rightarrow \infty} [\frac{2^x - 3}{2^x + 1}]$

3. (Gill 5.5 [adapted]) Calculate the following derivatives:

a. $\frac{d}{dx} 3x^{\frac{1}{3}}$

b. $\frac{d}{dt} (14t - 7)$

c. $\frac{d}{dy} (y^3 + 3y^2 - 12)$

d. $\frac{d}{dx} (x^2 + 1)(x^3 - 1)$

e. $\frac{d}{dy} (y^3 - 7)(1 + \frac{1}{y^2})$

f. $\frac{d}{dy} (y - y^{-1})(y - y^{-2})$

g. $\frac{d}{dx} \frac{4x - 12x^2}{x^3 - 4x^2}$

h. $\frac{d}{dy} e^{y^2 - 3y + 2}$

i. $\frac{d}{dx} \ln(2\pi x^2)$

4. Consider the function $k(x) = 2(8(x^4 + 2) - 1)^2$. Find the derivative by:

1. Expanding the polynomial and calculating the derivative using the power rule.

2. Expressing $k(x)$ as the result of three nested functions $f(g(h(x)))$ and applying the chain rule.

Show that these approaches yield the same answer.