Mathematics for Political Science

Lecture 2: Algebra

Exercise Solutions

1.

    1. $x = \frac{1}{3}$

    2. $x = \frac{3}{4}$

2.

    1. $\alpha = \beta + 4\theta$

    2. $\alpha = \frac{4}{(x+y-x^2-y^2)}$

3.

    1. $x > -18$

    2. $t < 6$

    3. $y \leqslant \frac{29}{22}$

4.

    1. $x = 2$ or $x = -7$

    2. $x = 4$

    3. $x = 2$ or $x = -5$

5.

    1. $x = \frac{1}{9}$ or $x = -1.5$

    2. $x = -\frac{2}{7}$ or $x = \frac{4}{5}$

6.

    1. $a = 0, b = 2$

    2. $a = 5, b = 5$

7.

    1. $c = 7, d = -2$

    2. $c = -3, d = 4$

8. $x = 4\alpha + 2$, $y = 2\alpha + 1$

9. $q = 1$, $r = -1$, $s = 3$

10.

   1. 25

   2. 22

11. **c** is the longest

    1. $\|\mathbf{a}\| = \sqrt{50}$

    2. $\|\mathbf{b}\| = \sqrt{48}$

    3. $\|\mathbf{c}\| = \sqrt{51}$

12. Odd powers are identical to the matrix given; even powers are the identity matrix.

13.
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a+0 & b+0 \\ c+0 & d+0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a*1+b*0 & a*0+b*1 \\ c*1+d*0 & c*0+d*1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

14.

   1. $\begin{bmatrix} 56 & 70 \end{bmatrix}$

   2. $\begin{bmatrix} ap+bq+cr \\ dp+eq+fr \\ gp+hq+ir \end{bmatrix}$

   3. The inner dimensions do not conform, so these matrices cannot be multiplied in this order.

15. Multiply the matrices below to show that order matters for matrix multiplication:

   1. $\begin{bmatrix} 17 \end{bmatrix}$ or $\begin{bmatrix} 12 & 21 & 3 \\ 0 & 0 & 0 \\ 20 & 35 & 5 \end{bmatrix}$

   2. $\begin{bmatrix} 44 & 64 & 36 \\ 15 & 36 & 21 \\ 20 & 22 & 12 \end{bmatrix}$ or $\begin{bmatrix} 48 & 114 \\ 15 & 44 \end{bmatrix}$

16.

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & -7 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -7 \\ 12 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & -7 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 1*5+2*0+4*-3 \\ 3*5+-7*0+1*-3 \\ 2*5+1*0+2*-3 \end{bmatrix} = \begin{bmatrix} -7 \\ 12 \\ 4 \end{bmatrix}$$