PLSC 308: Introduction to Political Research

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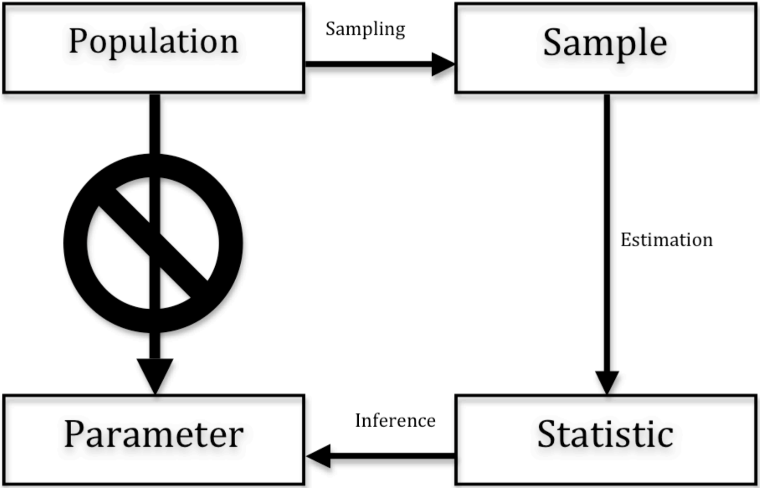

Population
Sampling
Sample
Estimation
Parameter
Inference
Statistic

Inference: Approaches

1. Hypothesis testing

2. Interval estimation

Hypothesis Testing: Moving Parts

- A *null hypothesis*, usually denoted H_0
- an *alternative* (or *research*) *hypothesis* H_a or H_1
- a *test statistic* $\theta =$ f(sample data $\mathbf{X}$)
- a *rejection region* for the null in the space of the sample statistic.

Hypothesis Testing: An Example

October 20, 2008 Zogby poll ($N = 737$ registered PA voters):

- Obama: 51.6 percent
- McCain: 40.2 percent

Q: What is Pr(Obama's Vote Share $> 50\%$), or (equivalently) Pr(Vote For Obama) $\equiv \pi > 0.5$?

Hypothesis Testing: An Example

Null hypothesis:

$$H_0 : \pi = 0.5$$

Alternative hypothesis:

$$H_a : \pi > 0.5$$

Test statistic:

$$\hat{\pi} = 0.516 \; (N = 737)$$

Rejection region: "How confident do we want to be in our decision regarding rejection?"

Types of Errors

- A **Type I error** occurs when we reject the null hypothesis and, in fact, that null hypothesis is true. Think of this as a "false positive."
- A **Type II error** occurs when we fail to reject the null hypothesis when it is not, in fact, true. Think of this as a "false negative."

	Reailty / Population	
Test Statistic / Sample	H_a	H_0
H_a	Correct	Type I error
H_0	Type II Error	Correct

By convention:

$$\Pr(\text{Type I Error}) = \alpha$$

and

$$\Pr(\text{Type II Error}) = \beta.$$

In general, we focus on α...

Hypothesis Testing

In the example, we know that:

$$\hat{\pi} \sim \mathcal{N}(\pi, \sigma^2_{\hat{\pi}}).$$

Because $\sigma^2_{\hat{\pi}} = \frac{\sigma^2}{N}$, and (in the Bernoulli case) $\sigma^2 = \pi(1-\pi)$, then for $H_0 : \pi = 0.5$,

$$\begin{aligned} \hat{\sigma}^2 &= 0.516(1-0.516) \\ &= 0.250 \end{aligned}$$

and

$$\begin{aligned} \hat{\sigma}^2_{\hat{\pi}} &= \frac{0.250}{737} \\ &= 0.00034. \end{aligned}$$

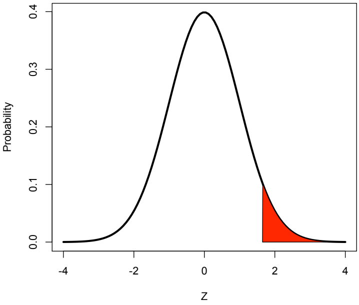
Hypothesis Testing (continued)

Convert our test statistic $\hat{\pi}$ to a z-score:

$$\frac{\hat{\pi} - \pi}{\sigma_{\hat{\pi}}} = Z \sim \mathcal{N}(0, 1).$$

Then create a decision rule:

$$\text{Reject } H_0 \text{ if } Z \geq z_\alpha.$$



Hypothesis Testing (continued)

Suppose we set $\alpha = 0.05$. Then we know that 95 percent of the area of the standard normal density lies below the value $z = 1.65$. Thus, if $Z \geq 1.65$, we would *reject the null hypothesis* that $\pi = 0.5$ at the 95 percent confidence level.

In the example:

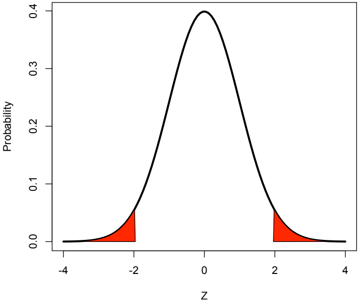
$$Z = \frac{0.516 - 0.50}{0.0184} = 0.87$$

indicating that we *cannot reject the null hypothesis* ($\pi = 0.5$) at the 95 percent level of confidence.

“Tailedness”

“Two-Tailed” Hypothesis tests:

$$H_a : \pi \neq 0.5.$$



P-Values

- AKA "attained significance level"
- *The smallest level of significance α for which the observed data indicate that the null hypothesis should be rejected.*
- Example:
  - $Z = 0.87$ corresponds to a P-value of 0.19.
  - If we'd set α equal to 0.20, we would have rejected the null hypothesis that $\pi = 0.5$.
  - Equivalently: A researcher could reject the null of $\pi = 0.5$ for any significance level up to and including 81 percent.

Confidence Intervals

Goal: Greate a range of possible values for $\hat{\theta}$ that:

1. Contain θ, and
2. Are as narrow as possible.

Define the two-sided interval $[\hat{\theta}_L, \hat{\theta}_U]$; we can also have one-sided intervals like $(-\infty, \hat{\theta}_U]$ and $[\hat{\theta}_L, \infty)$.

Tradeoff:

- Wider intervals are more likely to contain θ, but
- Narrower intervals are more useful.

Confidence Intervals

For a given level of confidence α, we know that

$$\Pr(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 1 - \alpha.$$

We also typically know that $E(\hat{\theta}) = \theta$.

Finally, we usually know that $\hat{\theta} \sim \mathcal{N}(\theta, \sigma^2_{\hat{\theta}})$.

- This allows us to determine the range of values of $\hat{\theta}$ associated with a given level of probability α...

Creating Confidence Intervals

- Estimate $\hat{\theta}$,
- Calculate $\hat{\theta}$'s standard error,
- Choose α, which implies the critical value Z,
- Multiply the standard error by the critical value,
- Add and subtract this from the estimate to get the interval.

Confidence Interval: Example

In the 2008 PA survey example:

- $\hat{\pi} = 0.519$
- Standard error $= \hat{\sigma}_{\hat{\pi}} = \sqrt{\frac{(0.519)(1-0.519)}{737}} = 0.0184$
- If we choose $\alpha = 0.05$, then that gives $Z = 1.96$
- So:
  - $\hat{\theta}_L = 0.519$ - $(1.96 \times 0.0184) = \mathbf{0.483}$
  - $\hat{\theta}_U = 0.519 + (1.96 \times 0.0184) = \mathbf{0.555}$

We say: "In repeated samples, confidence intervals constructed in this fashion will contain the true population value θ $(1 - \alpha) \times 100$ percent of the time."

C.I.s and Hypothesis Testing

One is the "inverse" of the other...

We might say:

> *"Do not reject H_0 at $P = \alpha$ if θ lies within a $(1 - \alpha) \times 100$-percent confidence interval around $\hat{\theta}$, and reject H_0 if it does not."*

Important Things To Remember

- **P-values are not "the probability that the null hypothesis is false."**
- **One does not, in general, "accept" the null hypothesis.**
- **A statistic can never be "significant in the wrong direction."**
- **Identical P-values are not "better" or "more reliable" if they are based on a larger sample.**
- **Statistical significance does not equate to substantive significance.**