PLSC 502 – Autumn 2016
Measures of Association
Interval/Ratio Data

November 8, 2016

Linearity

$$\frac{\partial Y}{\partial X} = m;$$

$$Y = mX + b$$

Other alternatives:

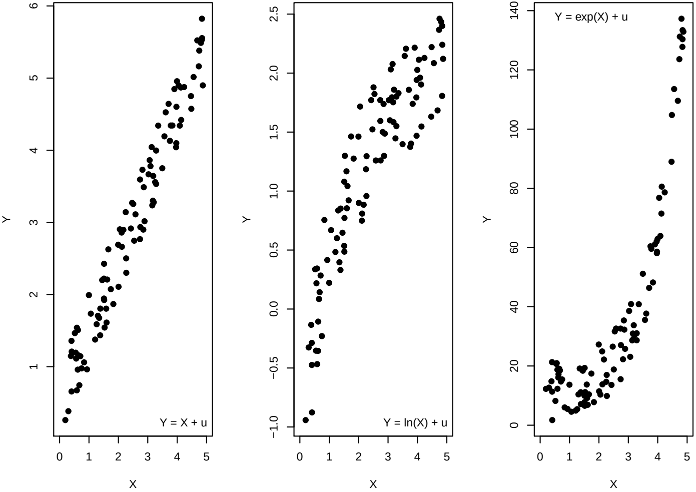
- *Logarithmic*:

$$\frac{\partial^2 Y}{\partial X \partial X} < 0$$

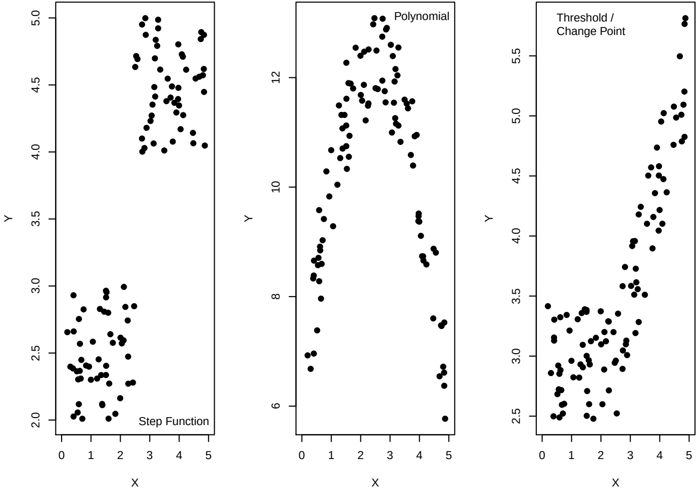
- *Exponential*:

$$\frac{\partial^2 Y}{\partial X \partial X} > 0$$

Linear, Logarithmic, Exponential



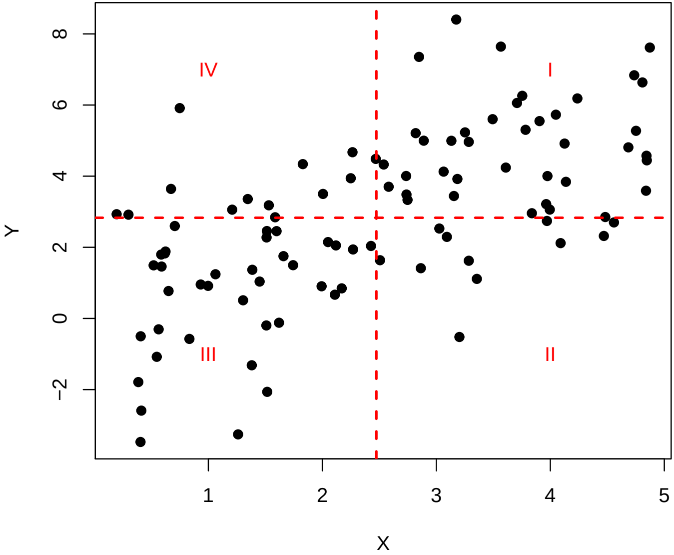
Other Possibilities



Linear Association: Pearson's r

$$
\begin{aligned}
r &= \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y} \\
&= \frac{\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{N}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}} \\
&= \frac{\sum_{i=1}^{N}\left(\frac{X_i - \bar{X}}{s_X}\right)\left(\frac{Y_i - \bar{Y}}{s_Y}\right)}{N - 1}
\end{aligned}
$$

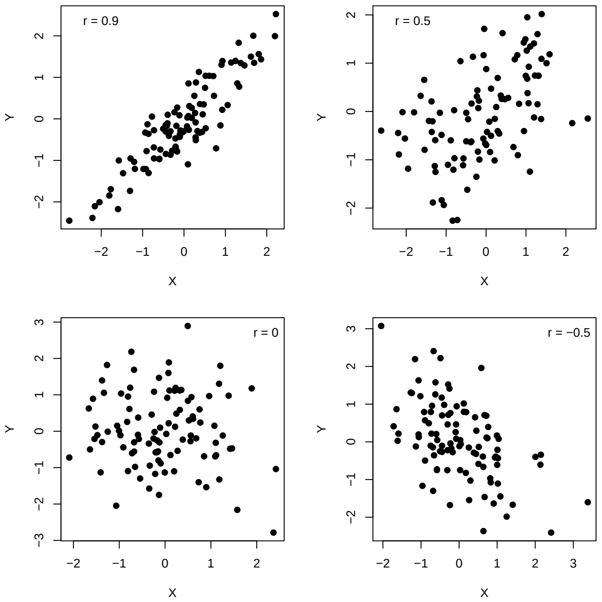
Pearson's r: Intuition



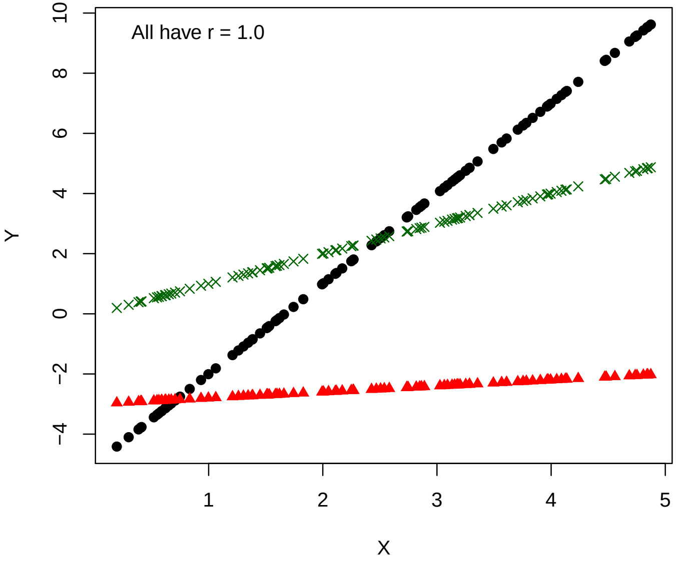
Pearson's r: Characteristics

- $r \in [-1, 1]$
- $r = 0 \leftrightarrow$ no association between Y and X.
- Sign(r) $\rightarrow$ "direction" of the *linear* association
- $|r| \rightarrow$ "strength" of the *linear* association
- In general:
    - $|r| < 0.3 \rightarrow$ "weak" association
    - $0.3 < |r| < 0.7 \rightarrow$ "moderate" association
    - $|r| > 0.7 \rightarrow$ "strong" association

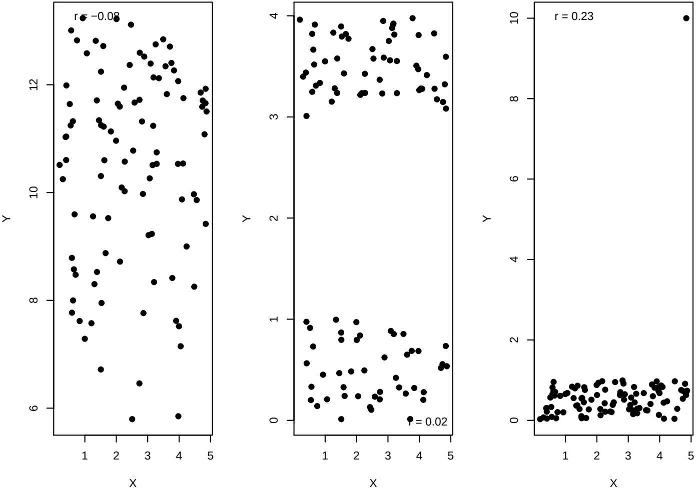
Examples



$r = \pm 1.0 \to ?$



Nonlinearity, etc.



Inference on r

The sampling distribution of r is:

- complex, and
- skewed as $|r| \to 1.0$.

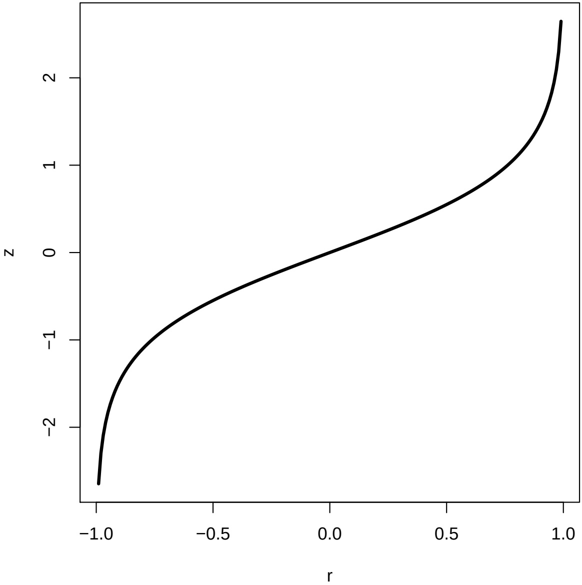
Fisher:

$$\hat{w} \equiv \frac{1}{2}\ln\left(\frac{1+\hat{r}}{1-\hat{r}}\right) \sim \mathcal{N}\left[\frac{1}{2}\ln\left(\frac{1+\hat{r}}{1-\hat{r}}\right), \frac{1}{\sqrt{N-3}}\right]$$

implying:

$$z_r = \frac{\frac{1}{2}\ln\left(\frac{1+\hat{r}}{1-\hat{r}}\right) - \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right)}{\sqrt{\frac{1}{N-3}}} \sim \mathcal{N}(0,1)$$

Fisher's z Transformation of r



Alternative Formula (t)

$$\frac{\hat{r}\sqrt{N-2}}{\sqrt{1-\hat{r}^2}} \sim t_{N-2}.$$

Note that this converges to z as $N \to \infty$.

Alternative Measure: Spearman's ρ

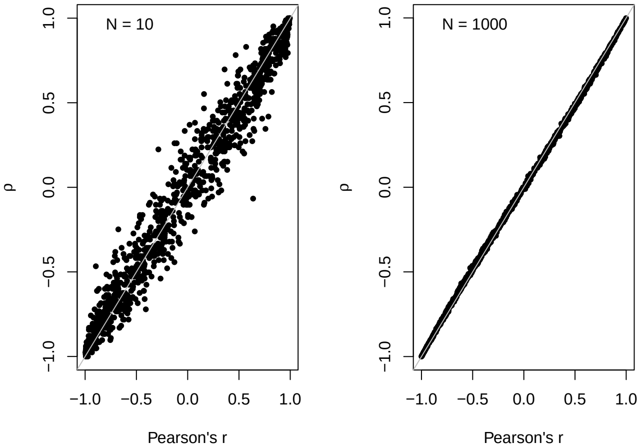
For sorted data on X and Y, where R_{Y_i} and R_{X_i} are the respective ranks,

$$\rho = 1 - \frac{6 \sum_{i=1}^{N} (R_{Y_i} - R_{X_i})^2}{N(N^2 - 1)}$$

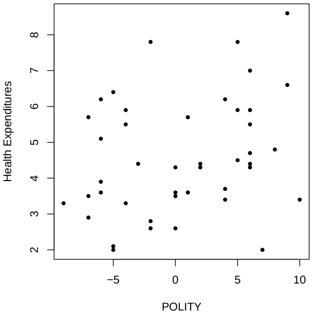
Characteristics:

- $\rho \in [-1, 1]$
- Same interpretation as r.
- Also appropriate for use with ordinal data; but
- When many "ties" occur, calculate Pearson's r on the ranks R_{Y_i} and R_{X_i}, and assign "partial" (or "half") ranks to tied individuals.

r vs. ρ Comparison



Africa Example: POLITY and Healthcare $



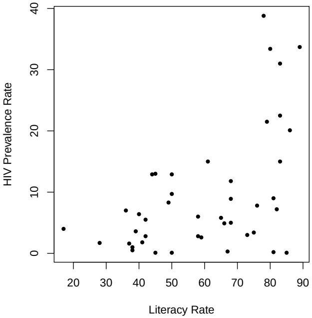

```
> with(Africa, cor.test(polity,healthexp))

Pearson's product-moment correlation

data:  polity and healthexp
t = 1.909, df = 41, p-value = 0.06328
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.01601428  0.53973343
sample estimates:
      cor
0.2857038

> with(Africa, SpearmanRho(polity,healthexp))
[1] 0.2809225
```

Africa Example: Literacy and HIV $



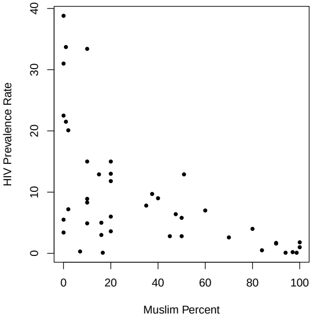

```
> with(Africa, cor.test(literacy,adrate))

Pearson's product-moment correlation

data:  literacy and adrate
t = 3.8459, df = 41, p-value = 0.0004112
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.2537995 0.7060508
sample estimates:
      cor
0.5148944

> with(Africa, SpearmanRho(literacy,adrate))
[1] 0.4570738
```

Africa Example: Muslim Percent and HIV $




```
> with(Africa, cor.test(muslperc,adrate))

Pearson's product-moment correlation

data:  muslperc and adrate
t = -4.4527, df = 41, p-value = 6.391e-05
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.7437363 -0.3265790
sample estimates:
       cor
-0.5709233

> with(Africa, SpearmanRho(muslperc,adrate))
[1] -0.6313474
```

Summary: Measures of Association

		X			
		Nominal	Binary	Ordinal	Interval/Ratio
Y	Nominal	χ^2	χ^2	χ^2	t-test (and η)
	Binary	χ^2	ϕ, Q	γ, τ_c	t-test
	Ordinal	χ^2	γ, τ_c	γ, τ_a, τ_b	Spearman's ρ
	Interval / Ratio	t-test (and η)	t-test	Spearman's ρ	r