PLSC 502 – Autumn 2016
Bayesian Approaches

December 1, 2016

“Frequentist” Approach

- Probability $=$ *Long-run relative frequency*
- $\Pr(X)$ is a *fixed* but *unknown* quantity

"Bayesian" Probability

Setup:

- Quantity of interest (θ)
- Data (Y)
- *sampling density* [$\Pr(Y|\theta)$]
- We want to know $\Pr(\theta|Y)$
- Likelihood $L(\theta|Y) \propto \Pr(Y|\theta)$

Bayes' Rule

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

and

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}.$$

So:

$$\Pr(A \cap B) = \Pr(B|A)\Pr(A).$$

Substituting, we get

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}.$$

Bayes' Rule Applied

$$\begin{aligned}
\Pr(\theta|Y) &= \frac{\Pr(\theta \cap Y)}{\Pr(Y)} \\
&= \frac{\Pr(Y|\theta)\Pr(\theta)}{\Pr(Y)}.
\end{aligned}$$

- $\Pr(Y|\theta)$ is the *sampling density*
- $\Pr(\theta)$ is the *prior density* of θ
- $\Pr(\theta|Y)$ is the *posterior density* of θ
- $\Pr(Y)$ is the marginal probability of Y

Since Y is fixed in a single sample, we can write:

$$\Pr(\theta|Y) \propto \Pr(Y|\theta)\Pr(\theta).$$

Bayes and Subjective Probability

- Probability is a *belief about the world*

- $\Pr(\theta)$ as our prior / "pre-data" estimate of the value/distribution of θ
- $\Pr(\theta|Y)$ as our posterior / "post-data" estimate

Bayesian Data Analysis

- **Set up a probability model for the data.**
- **Posit one's prior beliefs.**
- **Calculate the posterior distribution using Bayes' Theorem.**
- **Summarize the posterior density.**
- **Conduct post-estimation model checking.**

Bayes: Pros

- **Directly quantifies uncertainty**
- **Provides direct quantities of interest to researchers.**
- **Logically consistent and intuitive**
- **Allow the incorporation of prior information**
- **Allow the fitting complex models**
- **Flexibility**

Bayes: Cons

- **Inherent subjectivity of choosing priors**
- **Computational complexity**
- **Difficulty in knowing when estimates have converged**
- **Lack of software**