PLSC 502: “Statistical Methods for Political Research”

Exercise Four
September 29, 2016

Introduction

The purpose of this exercise is twofold: to familiarize you with the various distributions we discussed in class, and to develop your skills at generating, manipulating, and describing (graphically and in words) random variables using statistical software. There are no data for this exercise. In writing up your homework, be sure to include all the code necessary to replicate your work *exactly*.

Exercise

1. **Binomial**
   (a) Generate 100 random draws from a binomial(10,0.8) distribution. Briefly describe (graphically and in words) the resulting “data,” and compare them to the theoretical distribution of values from that distribution.
   (b) Generate another 100 random draws, this time from a binomial(10,0.2) density. Describe these “data,” and compare them to those in part (a).
   (c) Generate 50,000 draws from a binomial(10,0.8) density, and compare the results both to those above and to the theoretical distribution of values. What’s different, and why?

2. **Poisson**
   (a) Generate 1,000 random draws from a Poisson distribution with $\lambda = 0.5$. Compare the proportions observed to the theoretical values.
   (b) Do the same 1,000 draws from Poisson densities with $\lambda = 1.0$, $\lambda = 5.0$, and $\lambda = 10.0$. Discuss how the observed distributions of values change as λ increases.

3. **Normal / Chi-Square/ *t* / *F***
   (a) Draw 5,000 random $N(-2, 2)$ values, 5,000 $N(5, 2)$ values, and 5,000 $N(-2, 9)$ values. Plot and discuss the differences between the three sets of values.
   (b) Using random draws (however many you choose) of a standard normal variate Z, illustrate graphically that $Z^2 \sim \chi^2_1$.
   (c) Similarly, illustrate that $W_3 = Z_1^2 + Z_2^2 + Z_3^2 \sim \chi^2_3$.
   (d) Build on (b) and (c) to show that $\frac{Z}{W_3/3} \sim t_3$.

4. A *Gumbel* distribution is a two-parameter ($\alpha \in \mathcal{R}, \beta > 0$) distribution with $f(x) = \frac{1}{\beta} \exp\left[\frac{-(x-\alpha)}{\beta}\right] \times \exp\left\{-\exp\left[\frac{-(x-\alpha)}{\beta}\right]\right\}$ and $F(x) = 1 - \exp\left\{-\exp\left[\frac{-(x-\alpha)}{\beta}\right]\right\}$. It is related to the standard uniform distribution by $X = \alpha - \beta \log[-\log(U_{0,1})]$.
   (a) Generate 5,000 draws from a Gumbel(1,2) distribution via transformation of a standard uniform variate.
   (b) Plot those values, and discuss/describe the resulting empirical distribution as compared to the theoretical density.

This exercise is due by 5:00 p.m. EST on Thursday, October 6, 2016, and is worth the usual 50 possible points.