

PLSC 502 – Autumn 2016

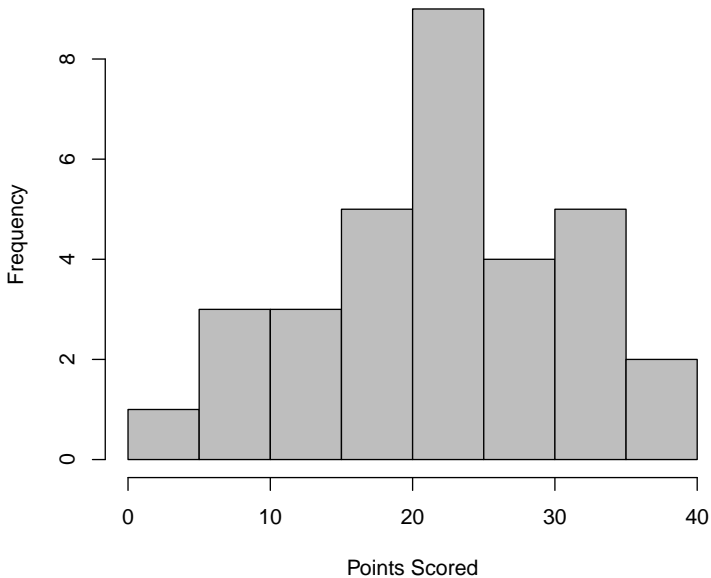
Central Tendency

September 13, 2016

Data: NFL Week One Points

Redskins	16	Giants	20	Bengals	23	Ravens	13
Jets	22	Dolphins	10	Chiefs	33	Patriots	23
Texans	23	Steelers	38	Jaguars	23	Titans	16
Lions	39	Falcons	24	Seahawks	12	Bills	7
Rams	0	Eagles	29	Buccaneers	31	Saints	34
Bears	14	Colts	35	Panthers	20	Chargers	27
Cardinals	21	49ers	28	Cowboys	19	Browns	10
Vikings	25	Packers	27	Broncos	21	Raiders	35

NFL Points: Histogram



The Arithmetic Mean

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N x_i$$

\bar{X} Minimizes Squared Deviations

$$\begin{aligned} f(X) &= \sum_{i=1}^N (X_i - \mu)^2 \\ &= \sum_{i=1}^N (X_i^2 + \mu^2 - 2\mu X_i) \end{aligned}$$

$$\frac{\partial f(X)}{\partial X} = \sum_{i=1}^N (2\mu - 2X_i)$$

$$\sum_{i=1}^N (2\mu - 2X_i) = 0$$

$$2N\mu - 2 \sum_{i=1}^N X_i = 0$$

$$2N\mu = 2 \sum_{i=1}^N X_i$$

$$\mu = \frac{1}{N} \sum_{i=1}^N X_i \equiv \bar{X}$$

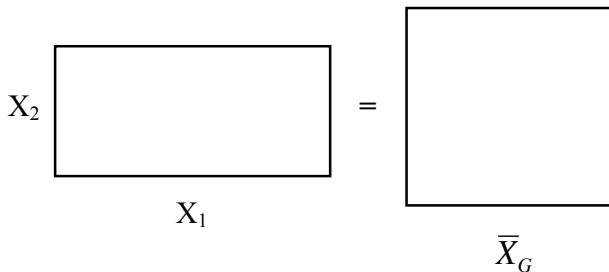
Means from Sums of Frequencies

Points	Frequency
3	1
7	1
10	5
13	2
\vdots	\vdots
41	1

$$\bar{X} = \frac{1}{N} \sum_{j=1}^J f_j X_j$$

Geometric Mean

$$\begin{aligned}\bar{X}_G &= \left(\prod_{i=1}^N X_i \right)^{\frac{1}{N}} \\ &= \sqrt[N]{X_1 \cdot X_2 \cdot \dots \cdot X_N} \\ &= \exp \left[\frac{1}{N} \sum_{i=1}^N \ln X_i \right]\end{aligned}$$



Harmonic Mean

$$\begin{aligned}\bar{X}_H &= \frac{N}{\sum_{i=1}^N \frac{1}{X_i}} \\ &= \frac{1}{\left(\frac{1}{\bar{X}}\right)}\end{aligned}$$

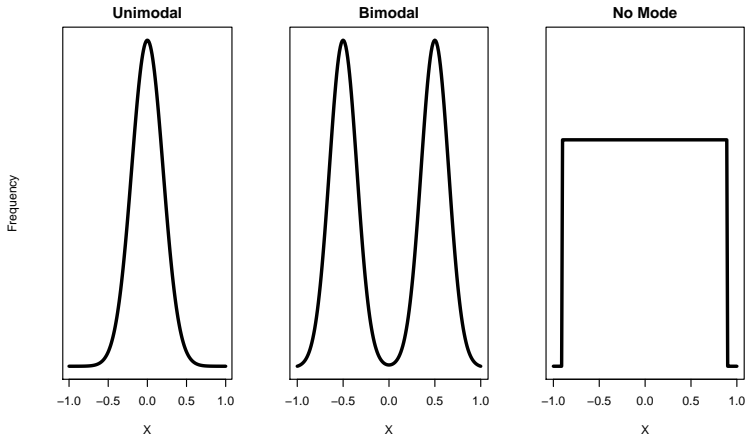
$$\bar{X}_H \leq \bar{X}_G \leq \bar{X}$$

$$\begin{aligned}\check{X} &= \text{“middle observation” of } X \\ &= \text{50th percentile of } X.\end{aligned}$$

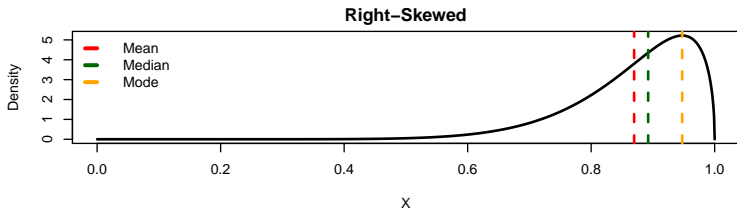
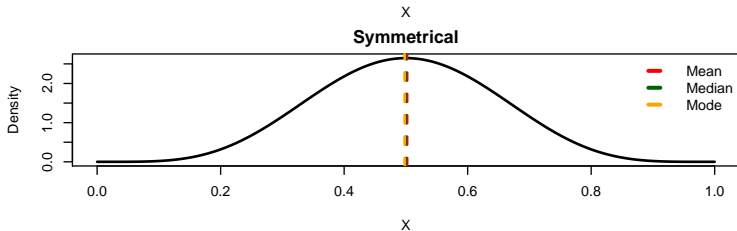
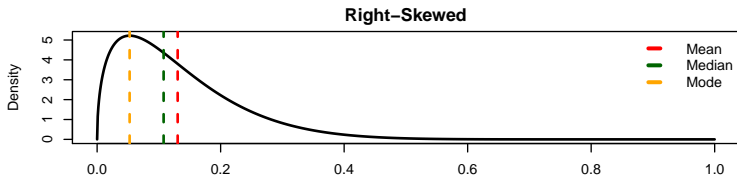
Minimizes *absolute* distance:

$$\check{X} = \min \left(\sum_{i=1}^N |X_i - c| \right).$$

The Mode



Means, Medians, Modes, and Skewness



Central Tendencies: NFL Data

