PLSC 502: "Statistical Methods for Political Research"

Exercise Eight
November 10, 2016

Part I

In Part I, we'll explore the relative sensitivity of (Pearson's) r and (Spearman's) ρ to the "shape" of the underlying distribution of data. Specifically, we want to learn if and when r and ρ are different from one another, as a function of (a) the marginal distributions of the two variables of interest X and Y, and (b) the degree of association between X and Y.

To do this, we'll rely on simulations using the beta distribution, a two-parameter distribution for a random variable with values over the $[0, 1]$ interval. The setup looks like this:

$$\begin{aligned} X &\sim \mathcal{B}(\alpha_X, \beta_X) \\ Y &\sim \lambda(X) + (1 - \lambda)\mathcal{B}(\alpha_Y, \beta_Y) \end{aligned}$$

where $\mathcal{B}(\alpha_{(\cdot)}, \beta_{(\cdot)})$ is the beta distribution with parameters $\alpha_{(\cdot)}$ and $\beta_{(\cdot)}$ and $0 \leq \lambda \leq 1$. The idea is that the parameters α and β allow you to vary the shape of the distributions from which the values of X and Y are drawn, and λ controls the extent of the association between X and Y.

For Part I of the exercise, do the following:

1. At the outset, assume that in all simulations your sample size is $N = 20$, that $\lambda = 0.5$, and that $\alpha_X = \alpha_Y$ and $\beta_X = \beta_Y$ (that is, that the "shape" of the underlying distributions from which X and Y are drawn are the same). Then, consider the cases of $\mathcal{B}(2, 2)$ (unimodal), $\mathcal{B}(0.5, 0.5)$ (bimodal), and $\mathcal{B}(1, 3)$ (right-skewed) distributions for X and Y. Via simulation, examine the differences (if any) in the direction, magnitude, and distribution of estimates of r and ρ.

2. Next, relax the assumption that $\lambda = 0.5$. Specifically, consider a range of values from (say) $\lambda = 0.1$ to $\lambda = 0.9$. Via simulation, present and discuss results that show what differences (if any) exist when the degree of association between X and Y varies.

3. Thirdly: Fix $\lambda = 0.5$ again, and vary the underlying distributions for X and Y. That is, examine what happens when (say) $\alpha_X = 2$ and $\beta_X = 2$ (unimodal), but $\alpha_Y = 0.5$ and $\beta_Y = 0.5$ (bimodal). That means examining unimodal X and bimodal Y, unimodal X and skewed Y, and bimodal X and skewed Y.[1] Again, describe via words and simulation results any significant differences between r and ρ that you find under these varying circumstances.

4. Finally, briefly discuss the possible reasons for your findings in the preceding sections, and the implications for applied researchers.

[1] Because X and Y are effectively interchangeable, and because you've already considered the situation where the two distributions have the same shape, only three additional comparisons are necessary.

Part II

In this part of the exercise, you'll be asked to explore some facets of state political institutions and their consequences. The data are measured at the state level ($N = 50$) during the year 2004. The variables are described and labeled pretty well; more detail (if you need it) is here. *In each case, you'll be left to your own resources in deciding which measure(s) of association to use.*

Answer each of the following questions, using the 2004 state-level data. Use what you've learned in each case to discuss the existence, strength, and (where applicable) direction of any associations described. Discuss each answer in 1-2 short sentences, backed up with whatever statistics, tables, graphs, etc. you see fit to use.

1. Are there significant differences in urbanization across the four regions of the country? How about in ideology (either citizen or elite)? (That is, do more urbanized states tend to be more liberal or conservative?)

2. Are states in particular regions more or less likely to have the death penalty? How about larger or smaller states? States with particular levels of liberalism or conservatism?

3. How strong is the relationship between party control of the upper and lower houses of the state legislatures? Between control of those houses and the governorship?

4. Is there a relationship between citizen or elite ideology and the incidence of divided government (that is, differential party control of the legislature and governorship) at the state level? Do states with divided government tend to have more or less professionalized legislatures? More or less powerful governors? Higher or lower levels of political competition?

As usual, this exercise is worth fifty possible points; it is due by 5:00 p.m. EST on Monday, November 21, 2016.