

PLSC 503 – Spring 2018

Variable Selection and Specification Bias

March 15, 2018

Requires that:

- $\text{Cov}(\mathbf{X}, \mathbf{u}) = 0$, and
- the distribution of **X** does not depend on either β or σ^2 .

“Truth”:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

Fitted model:

$$Y_i = \gamma_0 + \gamma_1 X_{1i} + e_i$$

Then:

$$e_i = \beta_2 X_{2i} + u_i$$

$$\begin{aligned} E(e) &= E(\beta_2 X_2 + u) \\ &= X_2 E(\beta_2) + E(u) \\ &\neq 0 \end{aligned}$$

$$\begin{aligned} E(\gamma_1) &= \beta_1 + \frac{\sum_{i=1}^N (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2)}{\sum_{i=1}^N (X_{1i} - \bar{X}_1)^2} \beta_2 \\ &= \beta_1 + b_{X_2 X_1} \beta_2 \end{aligned}$$

where $b_{X_2 X_1}$ is the “slope” coefficient one obtains from regressing X_2 on X_1 .

Omitted Variable Bias, continued

If $\text{Cov}(X_1, X_2) = 0$ then

- $E(\hat{\gamma}_1) = \beta_1$, but
- $E(\hat{\gamma}_0) \neq \beta_0$.

If $\text{Cov}(X_1, X_2) \neq 0$ then

- $E(\hat{\gamma}_1) \neq \beta_1$ and $E(\hat{\gamma}_0) \neq \beta_0$
- In the simple bivariate case,
 - if $\text{Cov}(X_1, X_2) > 0$ then $E(|\hat{\gamma}_1|) > |\beta_1|$,
 - if $\text{Cov}(X_1, X_2) < 0$ then $E(|\hat{\gamma}_1|) < |\beta_1|$.

Omitted Variables and Inference

Recall that for one X :

$$\widehat{\text{Var}}(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum_{i=1}^N (X_i - \bar{X})^2}.$$

and for two X s:

$$\widehat{\text{Var}}(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum_{i=1}^N (X_i - \bar{X})^2 (1 - R_{X_1 X_2}^2)}$$

Also, because $\hat{e}_i \neq \hat{u}_i$,

$$E(\sigma_e^2) = \sigma_u^2 + f(\beta_2, X_1) \leftarrow \text{Bias}$$

Multivariate Regression

For the “true” DGP

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

and fitted model

$$\mathbf{Y} = \mathbf{Z}\boldsymbol{\Gamma} + \mathbf{e}$$

where $\mathbf{Z} \subset \mathbf{X}$, we have

$$\begin{aligned}\boldsymbol{\Gamma} &= (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{u}) \\ &= (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{X}\boldsymbol{\beta} + (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{u}\end{aligned}$$

and so

$$\begin{aligned}E(\boldsymbol{\Gamma}) &= (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{X}\boldsymbol{\beta} \\ &= \mathbf{P}\boldsymbol{\beta}.\end{aligned}$$

Now assume a “true” model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

and fitted model:

$$\mathbf{Y} = \mathbf{Z}\boldsymbol{\Gamma} + \mathbf{e}$$

where $\mathbf{X} \subset \mathbf{Z}$. This means:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\boldsymbol{\Theta} + \mathbf{u}$$

where $\boldsymbol{\Theta} = \mathbf{0}$.

Results:

- $E(\hat{\beta}) = \beta$ and $E(\hat{\sigma}^2) = \sigma^2$, but
- $\widehat{\text{Var}}(\beta) > \text{Var}(\beta) \leftarrow \text{Inefficiency}$

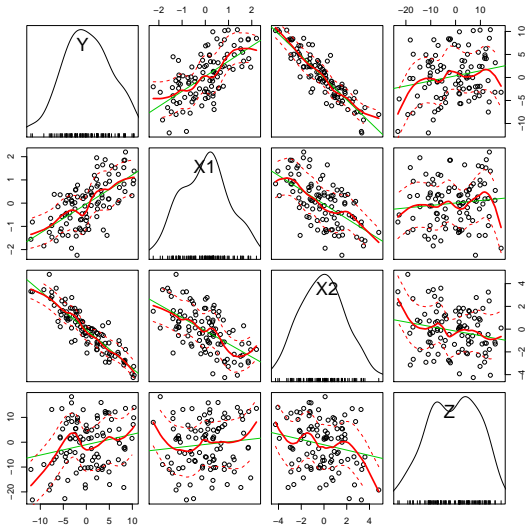
Implication: *Pre-Test Bias*

Omitted Variable Bias: Simulated Example

$$Y_i = 0 + 1.0X_{1i} - 2.0X_{2i} + u_i$$

```
> N <- 100
> X1<-rnorm(N)                # <- X1
> X2<-(-X1)+1.5*(rnorm(N))    # <- correlated w/X1
> Y<-X1-(2*X2)+(2*(rnorm(N))) # <- Y
> Z<-10*rnorm(N)              # <- irrelevant
> data <- data.frame(Y=Y,X1=X1,X2=X2,Z=Z)
```

Scatterplot Matrix



Correctly Specified Model

```
> correct<-lm(Y~X1+X2)
> summary(correct)
```

Residuals:

Min	1Q	Median	3Q	Max
-5.721	-1.209	0.093	1.198	5.915

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.03311	0.21249	-0.156	0.87651
X1	0.81690	0.26718	3.057	0.00288 **
X2	-2.13652	0.13844	-15.433	< 2e-16 ***

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 2.116 on 97 degrees of freedom

Multiple R-squared: 0.8295, Adjusted R-squared: 0.826

F-statistic: 236 on 2 and 97 DF, p-value: < 2.2e-16

Overspecified Model

```
> overspec<-lm(Y~X1+X2+Z)
> summary(overspec)
```

Residuals:

Min	1Q	Median	3Q	Max
-5.9809	-1.0442	-0.0265	1.2609	6.0201

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.01570	0.21420	-0.073	0.94173
X1	0.82148	0.26785	3.067	0.00281 **
X2	-2.11735	0.14105	-15.011	< 2e-16 ***
Z	0.01662	0.02202	0.755	0.45220

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 2.12 on 96 degrees of freedom
Multiple R-squared: 0.8306, Adjusted R-squared: 0.8253
F-statistic: 156.8 on 3 and 96 DF, p-value: < 2.2e-16

Underspecified Model

```
> incorrect<-lm(Y~X1)
> summary(incorrect)
```

Residuals:

Min	1Q	Median	3Q	Max
-9.3297	-2.9762	-0.0672	2.4828	8.7787

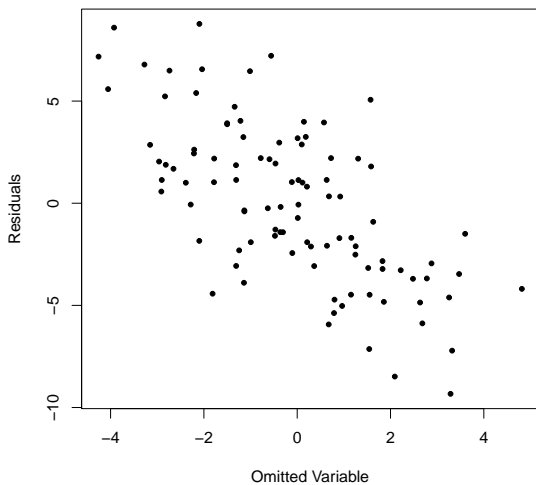
Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.2704	0.3913	0.691	0.491
X1	3.2783	0.3964	8.270	6.71e-13 ***

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 3.913 on 98 degrees of freedom
Multiple R-squared: 0.411, Adjusted R-squared: 0.405
F-statistic: 68.39 on 1 and 98 DF, p-value: 6.714e-13

Omitted Variable Plot



Nothing Beats a Good Theory. Period.

Also:

- “Model specification tests” ← meh
- Examine residuals
- Proxy variables...
- *Resist the urge to overspecify!*