PLSC 503 – Spring 2018
Simultaneity and Endogeneity

March 20, 2018

Endogeneity

Consider:

$$Y_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

$$Y_2 = \mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2$$

Rewrite:

$$\begin{aligned}
Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1[\mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2] + \mathbf{u}_1 \\
&= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\gamma_2 Y_1 + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 - \gamma_1\gamma_2 Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
(1 - \gamma_1\gamma_2) Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 &= \mathbf{X}_1\left(\frac{1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_1\right) + \mathbf{X}_2\left(\frac{\gamma_1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_2\right) + \left(\frac{\gamma_1\mathbf{u}_2 + \mathbf{u}_1}{1-\gamma_1\gamma_2}\right) \\
&= \Delta_1\mathbf{X}_1 + \Delta_2\mathbf{X}_2 + \mathbf{e}
\end{aligned}$$

"Reduced Form"

$$Y_1 = \mathbf{X}_1 \left(\frac{1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_1 \right) + \mathbf{X}_2 \left(\frac{\gamma_1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_2 \right) + \left(\frac{\gamma_1\mathbf{u}_2 + \mathbf{u}_1}{1-\gamma_1\gamma_2} \right)$$

means

$$\frac{\partial Y_1}{\partial X_\ell} = \frac{\beta_\ell}{1-\gamma_1\gamma_2}.$$

But

$$\hat{\Delta}_1 \neq \hat{\boldsymbol{\beta}}_1.$$

Simultaneity Bias

For (e.g.)

$$Y_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

we have:

$$\mathrm{E}(Y_2, \mathbf{u}_1) = \frac{\gamma_2}{1 - \gamma_1\gamma_2}\sigma^2_{\mathbf{u}}$$

What To Do

- OLS
- Lagged Variables
- Two-Stage Least Squares (2SLS)
- Systems of Equations / 3SLS / etc.

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

has

$$\hat{\boldsymbol{\beta}}_{OLS} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}.$$

Suppose $\text{Cov}(\mathbf{X}, \mathbf{u}) \neq \mathbf{0}$, but we have $\mathbf{Z}$ with

- $\text{Cov}(\mathbf{Z}, \mathbf{X}) \neq \mathbf{0}$ and
- $\text{Cov}(\mathbf{Z}, \mathbf{u}) = \mathbf{0}$.

Then

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{IV} &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Y} \\ &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{u}) \\ &= \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{u} \end{aligned}$$

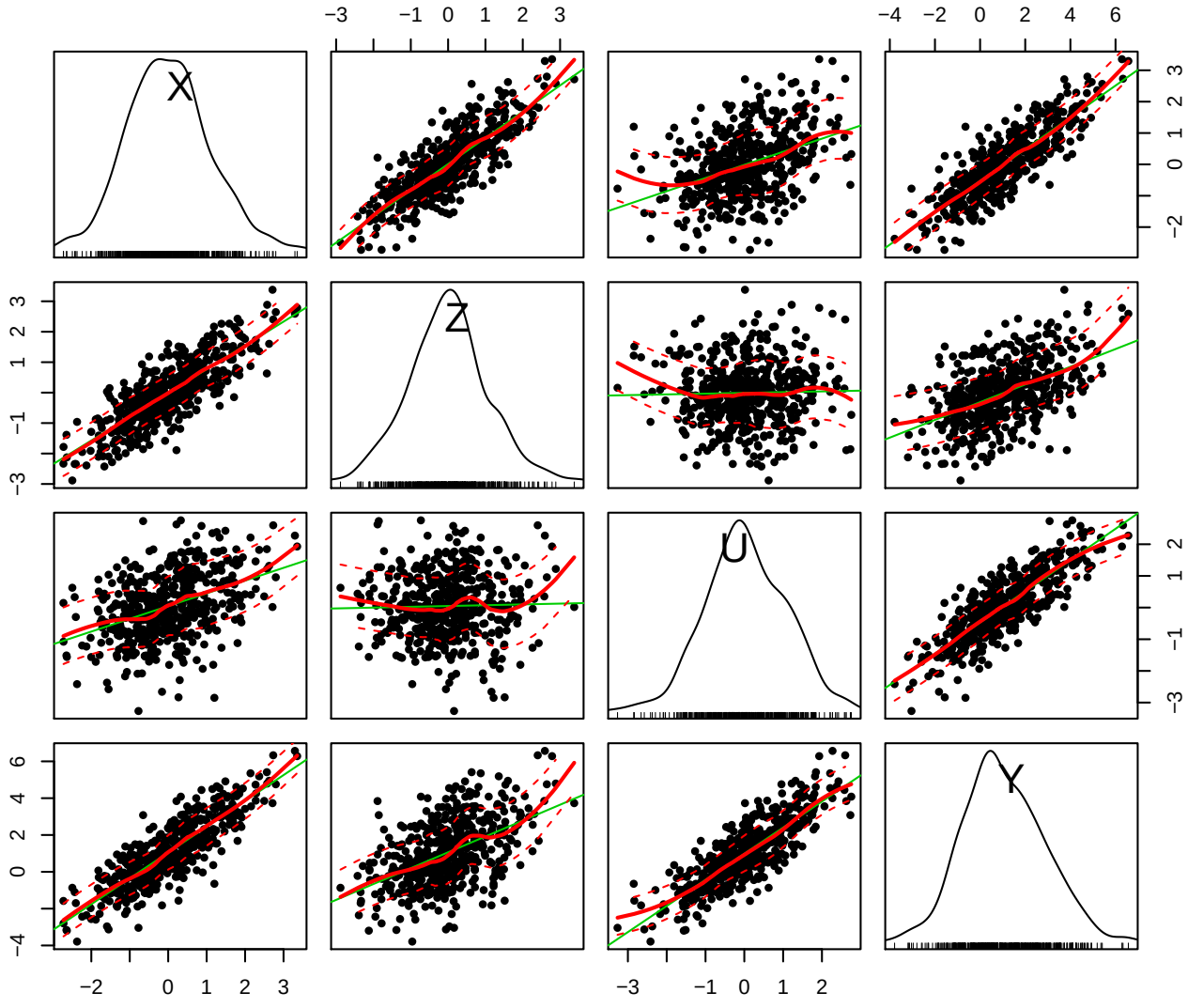
is consistent.

2SLS: How-To

- Regress endogenous $\mathbf{X}$s variables on $\{\mathbf{Z}, \mathbf{X}\}$
- Generate $\hat{\mathbf{X}}$s
- Regress Y on $\hat{\mathbf{X}}$ to get β_{2SLS}.
- Adjust standard error estimates

IV Estimation

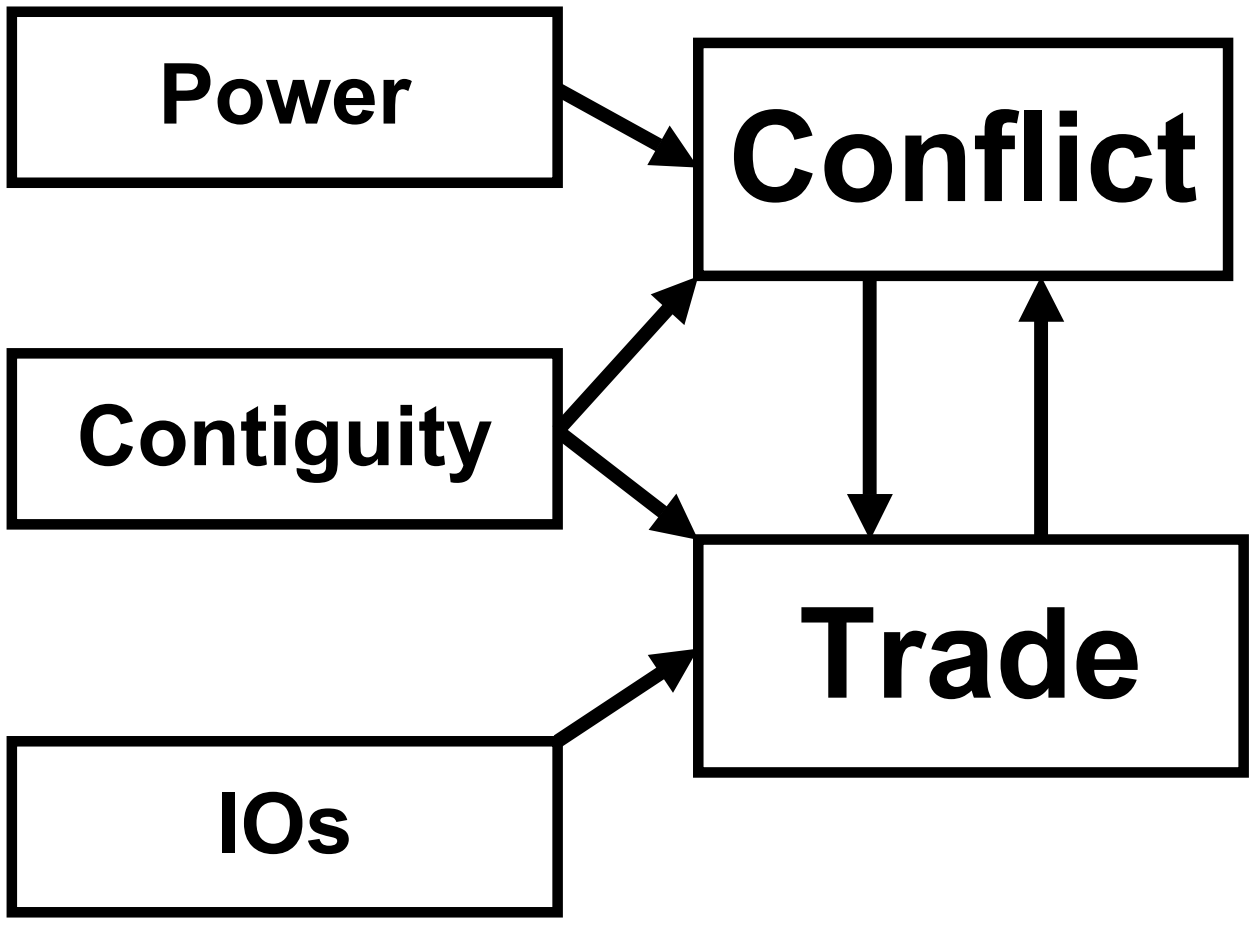
```
library(MASS)
library(sem)
library(car)

seed<-1337
set.seed(seed)

mu<-c(0,0,0) # <== X, Z, U
Sigma<-matrix(c(1,0.8,0.4,0.8,1,0,0.4,0,1),nrow=3,byrow=TRUE) #
Vars<- mvrnorm(500,mu,Sigma)
colnames(Vars)<-c("X","Z","U")
Vars<-data.frame(Vars)

Vars$Y<- 1 + Vars$X + Vars$U
```


X
Z
U
Y

```
> OLS<- lm(Y~X,data=Vars)
> summary(OLS)

Call:
lm(formula = Y ~ X, data = Vars)

Residuals:
    Min      1Q  Median      3Q     Max
-3.3809 -0.6058 -0.0102  0.6320  2.9470

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.04770    0.04209   24.89   <2e-16 ***
X            1.40254    0.04005   35.02   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.9413 on 498 degrees of freedom
Multiple R-squared:  0.7112,Adjusted R-squared:  0.7106
F-statistic:  1226 on 1 and 498 DF,  p-value: < 2.2e-16
```

```
> TSLS<-tsls(Y~I(X),data=Vars,instruments=~Z)
> summary(TSLS)

2SLS Estimates

Model Formula: Y ~ I(X)

Instruments: ~Z

Residuals:
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-3.29300 -0.68210 -0.06139  0.00000  0.76270  2.70300

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) 1.0491828  0.0456017 23.00754 < 2.22e-16 ***
I(X)        1.0302012  0.0536909 19.18763 < 2.22e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.0196738 on 498 degrees of freedom
```

IV: A (Toy) Example



```
> summary(IRData)
     dyadid         logdisputes         logtrade            IOs        
 Min.   :  2020   Min.   :-0.6931   Min.   :-0.6931   Min.   : 4.579  
 1st Qu.:135155   1st Qu.:-0.6931   1st Qu.: 2.4079   1st Qu.:19.500  
 Median :220484   Median :-0.6931   Median : 5.5786   Median :27.704  
 Mean   :275526   Mean   :-0.2627   Mean   : 4.6518   Mean   :30.891  
 3rd Qu.:385710   3rd Qu.: 0.0000   3rd Qu.: 7.1248   3rd Qu.:39.289  
 Max.   :900920   Max.   : 3.4965   Max.   :11.5037   Max.   :93.700  
   contiguity        capratio         GDPgrowth       
 Min.   :0.0000   Min.   :   1.081   Min.   :-9.0800  
 1st Qu.:0.0000   1st Qu.:   4.849   1st Qu.:-0.2923  
 Median :0.0000   Median :  26.577   Median : 0.8363  
 Mean   :0.3207   Mean   : 196.310   Mean   : 0.5097  
 3rd Qu.:1.0000   3rd Qu.: 144.035   3rd Qu.: 1.7106  
 Max.   :1.0000   Max.   :7451.982   Max.   : 7.0460  
```

```
> OLSWar<-lm(logdisputes~logtrade+contiguity+capratio)
> summary(OLSWar)

Residuals:
     Min       1Q   Median       3Q      Max
-0.82840 -0.32644 -0.26860 -0.08972  3.45504

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -4.253e-01  6.020e-02  -7.065 3.46e-12 ***
logtrade     8.558e-03  1.057e-02   0.809   0.4185
contiguity   4.623e-01  7.124e-02   6.489 1.50e-10 ***
capratio    -1.296e-04  6.467e-05  -2.003   0.0455 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.853 on 813 degrees of freedom
Multiple R-squared: 0.08301,Adjusted R-squared: 0.07962
F-statistic: 24.53 on 3 and 813 DF,  p-value: 3.345e-15
```

```
> library(sem)
> TwoSLSWar<-tsls(logdisputes~contiguity+capratio+I(logtrade),
   instruments=~contiguity+capratio+IOs)
> summary(TwoSLSWar)

2SLS Estimates

Model Formula: logdisputes ~ contiguity + capratio + I(logtrade)

Instruments: ~contiguity + capratio + IOs

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-1.21e+00 -5.24e-01 -2.26e-01 -7.44e-17 -2.10e-02  3.65e+00

              Estimate Std. Error t value  Pr(>|t|)
(Intercept) -0.1515180  8.562e-02  -1.770 7.717e-02
contiguity   0.6263774  8.111e-02   7.722 3.353e-14
capratio    -0.0002664  7.252e-05  -3.674 2.543e-04
I(logtrade) -0.0558374  1.769e-02  -3.157 1.652e-03

Residual standard error: 0.8723 on 813 degrees of freedom
```

"By-Hand"

```
> ITrade<-lm(logtrade~contiguity+IOs+capratio)
> summary(ITrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.0385 -1.7666  0.4139  1.6154  7.6029

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.7319793  0.1912570   3.827 0.000140 ***
contiguity   1.3386037  0.1816041   7.371 4.17e-13 ***
IOs          0.1218373  0.0055313  22.027  < 2e-16 ***
capratio    -0.0013913  0.0001626  -8.555  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.239 on 813 degrees of freedom
Multiple R-squared: 0.5535,Adjusted R-squared: 0.5519
F-statistic: 335.9 on 3 and 813 DF,  p-value: < 2.2e-16
```

"By-Hand"

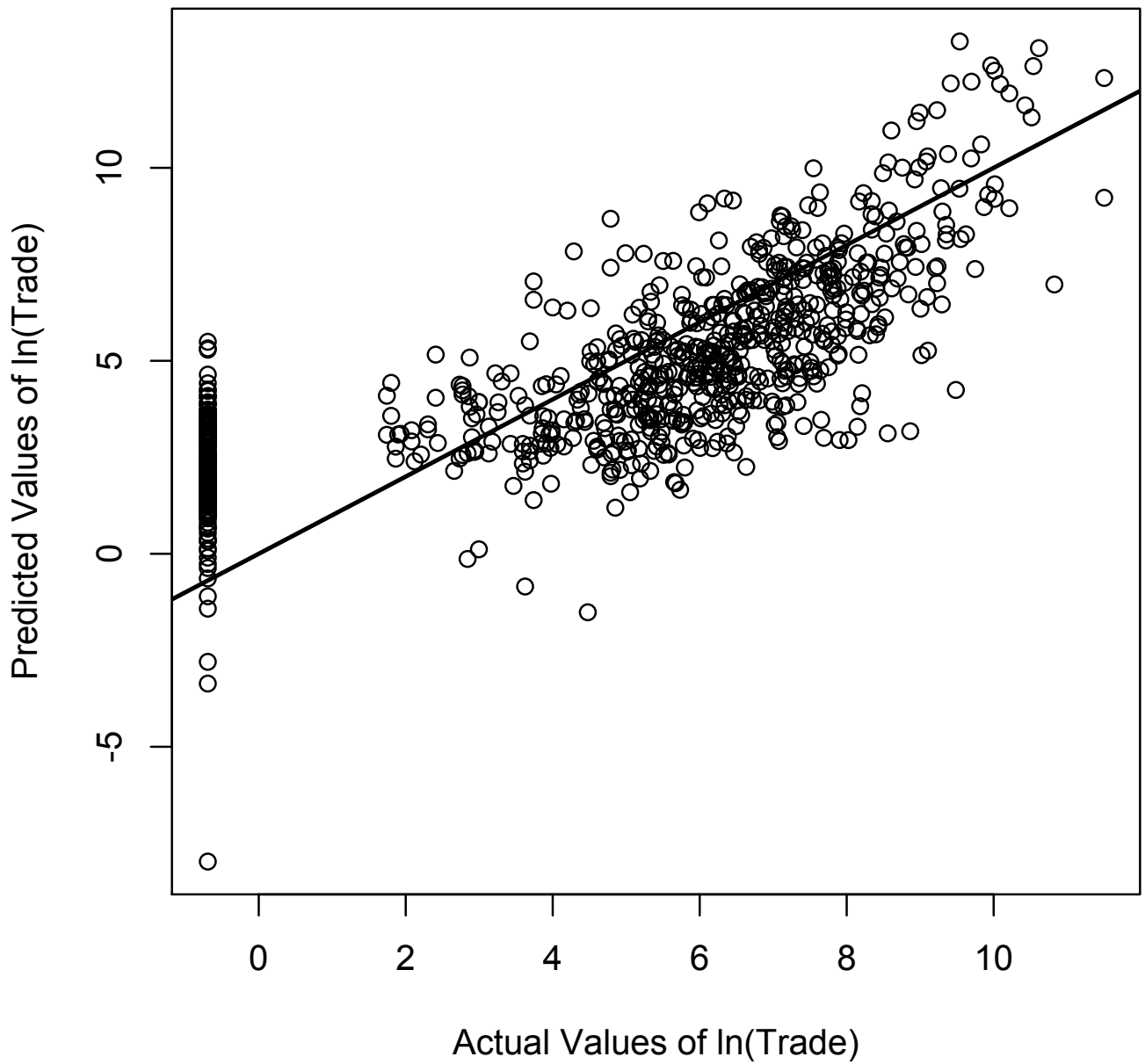
```
> IVWarByHand<-lm(logdisputes~capratio+contiguity
   +(ITrade$fitted.values))
> summary(IVWarByHand)

Residuals:
    Min      1Q  Median      3Q     Max
-1.0055 -0.3618 -0.2782 -0.0492  3.5301

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)          -1.515e-01  8.323e-02  -1.821 0.069050 .
capratio             -2.664e-04  7.049e-05  -3.780 0.000168 ***
contiguity            6.264e-01  7.884e-02   7.944 6.49e-15 ***
ITrade$fitted.values -5.584e-02  1.719e-02  -3.248 0.001210 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.8479 on 813 degrees of freedom
Multiple R-squared: 0.09402,Adjusted R-squared: 0.09068
F-statistic: 28.12 on 3 and 813 DF,  p-value: < 2.2e-16
```

Weak Instruments

```
> OLSTrade<-lm(logtrade~logdisputes+contiguity+IOs)
> summary(OLSTrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.2467 -2.2067  0.4275  1.6659  6.1264

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.191111   0.182875   1.045    0.296
logdisputes 0.408116   0.095067   4.293 1.98e-05 ***
contiguity  1.357557   0.193109   7.030 4.38e-12 ***
IOs         0.133778   0.005614  23.831  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.312 on 813 degrees of freedom
Multiple R-squared: 0.5241,Adjusted R-squared: 0.5223
F-statistic: 298.4 on 3 and 813 DF,  p-value: < 2.2e-16
```

```
> TwoSLSTrade<-tsls(logtrade~contiguity+IOs+I(logdisputes),
    instruments=~contiguity+capratio+IOs)
> summary(TwoSLSTrade)

2SLS Estimates

Model Formula: logtrade ~ contiguity + IOs + I(logdisputes)

Instruments: ~contiguity + capratio + IOs

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-2.57e+01 -1.46e+00  1.36e+00  2.84e-14  4.00e+00  1.09e+01

                Estimate Std. Error t value  Pr(>|t|)
(Intercept)        2.150    0.85122   2.526 1.173e-02
contiguity        -2.728    1.52615  -1.787 7.427e-02
IOs                0.172    0.02045   8.408 2.220e-16
I(logdisputes)     7.371    2.45198   3.006 2.727e-03

Residual standard error: 6.3721 on 813 degrees of freedom
```

Pretty Good Instrument (Trade)



Crappy Instrument (War)