

PLSC 504 - Fall 2017

Event Counts, II

September 14, 2017

- Truncated Count Models
- Censored Count Models
- “Zero-Inflated” / “Hurdle” Count Models

Running Example: International Conflict(s)

- `conflicts` = N of violent conflicts/year
- `polity` = Rescaled POLITY IV democracy score
- `logPopulation` = $\ln(\text{population})$
- `logGDP` = $\ln(\text{GDP per capita})$
- `GDPGrowth` = growth in GDP
- `logOpenness` = $\ln\left(\frac{\text{Imports} + \text{Exports}}{\text{GDP}}\right)$
- `govshareGDP` = government's % of GDP

Conflict Data

```
> summary(wars)
```

ccode	year	conflicts	conflicts_no_zeros
Min. : 2	Min. :1951	Min. :0.000	Min. :1
1st Qu.:211	1st Qu.:1970	1st Qu.:0.000	1st Qu.:1
Median :439	Median :1981	Median :0.000	Median :1
Mean :439	Mean :1980	Mean :0.304	Mean :1
3rd Qu.:640	3rd Qu.:1991	3rd Qu.:0.000	3rd Qu.:1
Max. :950	Max. :2000	Max. :8.000	Max. :8
			NA's :4075

polity	politysq	population	GDP
Min. :0.000	Min. :0.0000	Min. : 122	Min. : 171
1st Qu.:0.150	1st Qu.:0.0225	1st Qu.: 3054	1st Qu.: 1401
Median :0.450	Median :0.2025	Median : 7725	Median : 3777
Mean :0.527	Mean :0.4278	Mean : 33615	Mean : 6641
3rd Qu.:0.950	3rd Qu.:0.9025	3rd Qu.: 21979	3rd Qu.: 9032
Max. :1.000	Max. :1.0000	Max. :1262474	Max. :84408

openness	govshareGDP	GDPGrowth	logPopulation
Min. : 3.7	Min. : 2.97	Min. : -63.32	Min. : 4.80
1st Qu.: 30.9	1st Qu.:14.64	1st Qu.: -0.90	1st Qu.: 8.02
Median : 50.0	Median :18.94	Median : 2.08	Median : 8.95
Mean : 62.2	Mean :20.95	Mean : 1.92	Mean : 8.99
3rd Qu.: 81.1	3rd Qu.:24.85	3rd Qu.: 4.84	3rd Qu.:10.00
Max. :986.5	Max. :83.68	Max. :125.96	Max. :14.05

logGDP	logOpenness	conflicts_censored	censored
Min. : 5.14	Min. :1.31	Min. :0.000	Min. : -1.00
1st Qu.: 7.25	1st Qu.:3.43	1st Qu.:0.000	1st Qu.: 1.00
Median : 8.24	Median :3.91	Median :0.000	Median : 1.00
Mean : 8.23	Mean :3.87	Mean :0.299	Mean : 0.99
3rd Qu.: 9.11	3rd Qu.:4.40	3rd Qu.:0.000	3rd Qu.: 1.00
Max. :11.34	Max. :6.89	Max. :4.000	Max. : 1.00

Basic Model: Poisson

```
> wars.poisson<-glm(conflicts~polity+politysq+logPopulation+logGDP+
  GDPGrowth+logOpenness+govshareGDP,family="poisson",data=wars)
> summary.glm(wars.poisson)
```

Call:

```
glm(formula = conflicts ~ polity + politysq + logPopulation +
  logGDP + GDPGrowth + logOpenness + govshareGDP, family = "poisson",
  data = wars)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-4.88565	0.36284	-13.47	< 2e-16	***
polity	1.05866	0.39129	2.71	0.0068	**
politysq	-0.95432	0.37292	-2.56	0.0105	*
logPopulation	0.39809	0.01626	24.48	< 2e-16	***
logGDP	-0.05919	0.02919	-2.03	0.0426	*
GDPGrowth	-0.01579	0.00345	-4.58	4.6e-06	***
logOpenness	-0.15187	0.03691	-4.11	3.9e-05	***
govshareGDP	0.03632	0.00235	15.48	< 2e-16	***

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Basic Model: Negative Binomial

```
> wars.nb<-glm.nb(conflicts~polity+politysq+logPopulation+logGDP+GDPGrowth+  
  logOpenness+govshareGDP,data=wars)  
> summary(wars.nb)
```

Call:

```
glm.nb(formula = conflicts ~ polity + politysq + logPopulation +  
  logGDP + GDPGrowth + logOpenness + govshareGDP, data = wars,  
  init.theta = 2.10281397427423, link = log)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-4.987258	0.403221	-12.369	< 2e-16 ***
polity	1.031445	0.429147	2.403	0.016240 *
politysq	-1.006861	0.409911	-2.456	0.014038 *
logPopulation	0.419436	0.019065	22.000	< 2e-16 ***
logGDP	-0.062318	0.032646	-1.909	0.056276 .
GDPGrowth	-0.014965	0.003964	-3.775	0.000160 ***
logOpenness	-0.164250	0.041114	-3.995	6.47e-05 ***
govshareGDP	0.036494	0.002672	13.657	< 2e-16 ***

Theta: 2.103
Std. Err.: 0.322

Zero Truncation

$$\begin{aligned}\Pr(Y_i = 0) &= \frac{\exp(-\lambda_i)\lambda_i^0}{0!} \\ &= \exp(-\lambda_i)\end{aligned}$$

$$\Pr(Y_i > 0) = 1 - \exp(-\lambda_i).$$

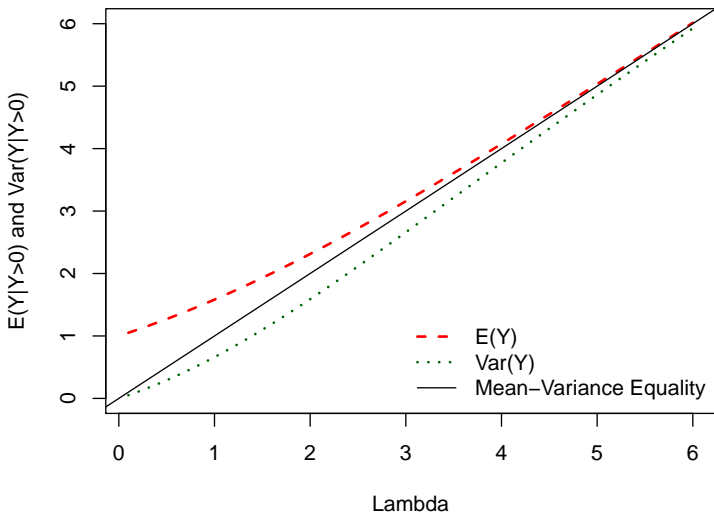
$$\begin{aligned}\Pr(Y_i = y | Y_i > 0) &= \frac{\Pr(Y_i = y)}{\Pr(Y_i > 0)} \\ &= \frac{\exp(-\lambda_i)\lambda_i^y}{y![1 - \exp(-\lambda_i)]}\end{aligned}$$

Zero Truncation (continued)

$$E(Y|Y > 0) = \frac{\lambda}{1 - \exp(-\lambda)}$$

$$\begin{aligned} \text{Var}(Y|Y > 0) &= E(Y|Y > 0) \times \{[1 - \Pr(Y = 0)] E(Y|Y > 0)\} \\ &= \frac{\lambda}{1 - \exp(-\lambda)} \left[1 - \frac{\lambda}{\exp(\lambda) - 1} \right] \end{aligned}$$

Zero Truncation Illustrated



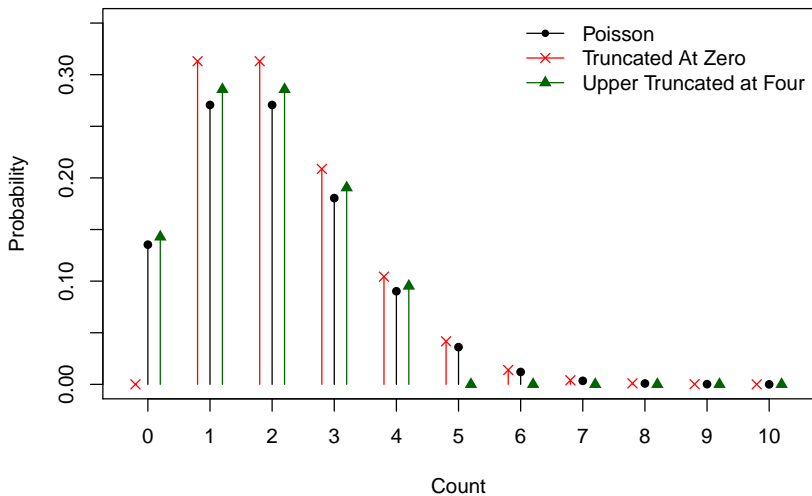
Upper Truncation

$$Y_i = \begin{cases} Y_i^* & \text{if } Y_i^* \leq \tau \\ \text{unobserved} & \text{if } Y_i^* > \tau \end{cases}$$

$$\Pr(Y_i^* \leq \tau) = \sum_{y=0}^{\tau} \frac{\exp(-\lambda_i) \lambda_i^y}{y!}$$

$$\Pr(Y_i = y | Y_i \leq \tau) = \frac{\exp(-\lambda_i) \lambda_i^y}{y! \sum_{y=0}^{\tau} \frac{\exp(-\lambda_i) \lambda_i^y}{y!}}$$

Truncation Illustrated



Truncated Models: Estimation and Interpretation

$$\lambda_i = \exp(\mathbf{X}_i\boldsymbol{\beta})$$

- IRRs, predicted probabilities, etc. as usual
- Using formulae above

Zero-Truncated Models: (Incorrect/Poisson)

Example

```
> wars.poisNo0s<-glm(conflicts_no_zeros~polity+politysq+logPopulation+
  logGDP+GDPGrowth+logOpenness+govshareGDP,family="poisson",data=wars)
> summary(wars.poisNo0s)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-1.4940523	0.3799176	-3.933	8.40e-05	***
polity	-0.3508331	0.3858776	-0.909	0.363	
politysq	0.4273003	0.3734399	1.144	0.253	
logPopulation	0.1317254	0.0181912	7.241	4.45e-13	***
logGDP	0.0389308	0.0320827	1.213	0.225	
GDPGrowth	-0.0005765	0.0031788	-0.181	0.856	
logOpenness	-0.0387960	0.0401573	-0.966	0.334	
govshareGDP	0.0135720	0.0023168	5.858	4.68e-09	***

Null deviance: 396.43 on 1179 degrees of freedom
Residual deviance: 300.56 on 1172 degrees of freedom
(11870 observations deleted due to missingness)
AIC: 2891.2

Number of Fisher Scoring iterations: 4

Zero-Truncated Models: Example

```
> library(VGAM)
> wars.0tpois<-vglm(conflicts_no_zeros~polity+politysq+logPopulation+
  logGDP+GDPGrowth+logOpenness+govshareGDP,pospoisson,data=wars)
> summary(wars.0tpois)
```

Coefficients:

	Value	Std. Error	t value
(Intercept)	-6.9985662	0.7802697	-8.96942
polity	-1.3061668	0.8185705	-1.59567
politysq	1.3202509	0.7876759	1.67613
logPopulation	0.3997250	0.0331791	12.04749
logGDP	0.2326896	0.0608548	3.82369
GDPGrowth	-0.0018478	0.0064828	-0.28503
logOpenness	-0.1045685	0.0779237	-1.34193
govshareGDP	0.0409683	0.0038646	10.60102

Number of linear predictors: 1

Name of linear predictor: log(lambda)

Dispersion Parameter for pospoisson family: 1

Log-likelihood: -806.6696 on 1172 degrees of freedom

Number of Iterations: 5

$$Y_i = \begin{cases} Y_i^* & \text{if } Y_i^* < k \\ k & \text{if } Y_i^* \geq k \end{cases}$$

$$\Pr(Y = y | Y^* < k) = \frac{\exp(-\lambda_i) \lambda_i^y}{y!},$$

$$\Pr(Y = k) = 1 - \sum_{y=0}^{k-1} \frac{\exp(-\lambda_i) \lambda_i^y}{y!}$$

Censored Models

Right Censoring

$$c_i = \begin{cases} 1 & \text{if } Y_i = k \\ 0 & \text{if } Y_i < k \end{cases}$$

$$\ln L = \sum_{i=1}^N (1 - c_i) \ln \left[\frac{\exp(-\lambda_i) \lambda_i^y}{y!} \right] + c_i \ln \left[1 - \sum_{y=0}^{k-1} \frac{\exp(-\lambda_i) \lambda_i^y}{y!} \right]$$

Left Censoring

$$Y_i = \begin{cases} \ell & \text{if } Y_i^* \leq \ell \\ Y_i^* & \text{if } Y_i^* > \ell \end{cases}$$

Double Censoring

$$Y_i = \begin{cases} \ell & \text{if } Y_i^* \leq \ell \\ Y_i^* & \text{if } \ell < Y_i^* < k \\ k & \text{if } Y_i^* \geq k \end{cases}$$

- R :
 - `vglm`, `pospoisson` (in VGAM) (zero truncation)
 - `vglm`, `cens.poisson` (in VGAM) (censored Poisson)
- Stata :
 - `ztp` / `ztnb` (zero truncation)
 - `trpoisson` (general truncation)
 - `cenpois` (censored Poisson)

Example, Again

$$c_i = \begin{cases} 1 & \text{if the observation's count is } \textit{uncensored}, \\ 0 & \text{if the observation's count is } \textit{left-censored}, \text{ and} \\ -1 & \text{if the observation's count is } \textit{right-censored}. \end{cases}$$

```
wars$censoredconflicts<-wars$conflicts  
wars$censoredconflicts<-ifelse(wars$conflicts>3,4,wars$censoredconflicts)  
wars$censindicator<-ifelse(wars$censoredconflicts==4,1,0)
```

Censored Example: (Incorrect) Poisson

```
> wars.poisCensored<-glm(censoredconflicts~polity+politysq+logPopulation+
  logGDP+GDPGrowth+logOpenness+govshareGDP,family="poisson",data=wars)
> summary(wars.poisCensored)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-4.743385	0.364843	-13.001	< 2e-16 ***
polity	1.070801	0.392156	2.731	0.00632 **
politysq	-1.025014	0.374139	-2.740	0.00615 **
logPopulation	0.381121	0.016491	23.111	< 2e-16 ***
logGDP	-0.049747	0.029444	-1.690	0.09111 .
GDPGrowth	-0.015869	0.003469	-4.574	4.78e-06 ***
logOpenness	-0.150489	0.037176	-4.048	5.16e-05 ***
govshareGDP	0.034396	0.002373	14.495	< 2e-16 ***

Null deviance: 5007.5 on 5254 degrees of freedom
Residual deviance: 4059.2 on 5247 degrees of freedom
(7795 observations deleted due to missingness)
AIC: 6644.6
Number of Fisher Scoring iterations: 6

Censored Example: Poisson

```
> wars.censpois<-vglm(SurvS4(censoredconflicts,censindicator)~polity+politysq+logPopulation
+logGDP+GDPGrowth+logOpenness+govshareGDP, cens.poisson,data=wars)
> summary(wars.censpois)
```

Call:

```
vglm(formula = SurvS4(censoredconflicts, censindicator) ~ polity +
      politysq + logPopulation + logGDP + GDPGrowth + logOpenness +
      govshareGDP, family = cens.poisson, data = wars)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.90350	1.22197	1.56	0.119
polity	1.65054	1.75924	0.94	0.348
politysq	-1.96215	1.67550	-1.17	0.242
logPopulation	-0.05184	0.05040	-1.03	0.304
logGDP	0.07472	0.09577	0.78	0.435
GDPGrowth	0.00248	0.00809	0.31	0.759
logOpenness	0.09201	0.14403	0.64	0.523
govshareGDP	-0.01267	0.00741	-1.71	0.088

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Number of linear predictors: 1

Name of linear predictor: loge(mu)

Dispersion Parameter for cens.poisson family: 1

Log-likelihood: -51.72 on 5247 degrees of freedom

Number of iterations: 12

“Zero-Modified” Count Models

- “Zero-Inflated” Models
- “Hurdle” Models

“Zero-Inflated” Count Models

$$Y_i = p_i \times Y_i^*$$

$$\begin{aligned}\Pr(Y_i = 0) &= \Pr(p_i = 0) + [\Pr(p_i = 1) \times \Pr(Y_i^* = 0)] \\ &= (1 - p_i^*) + p_i^*[\exp(-\lambda_i)]\end{aligned}$$

$$\begin{aligned}\Pr(Y_i = y) &= \Pr(p_i = 1) \times \Pr(Y_i^* = y) \\ &= p_i^* \times \frac{\exp(-\lambda_i) \lambda_i^y}{y!}\end{aligned}$$

More on “Zero-Inflated” Models

$$E(Y_i^*) \equiv \lambda_i = \exp(\mathbf{X}_i\boldsymbol{\beta})$$

$$\Pr(p_i = 1) \equiv p_i^* = \frac{1}{1 + \exp(-\mathbf{Z}_i\boldsymbol{\gamma})} \text{ or } \Phi(\mathbf{Z}_i\boldsymbol{\gamma})$$

“Hurdle” Count Models

- $\lambda_0 = \Pr(\text{No War}) = \exp(-\lambda)$
- $\lambda_+ = \Pr(Y \in \{1, 2, 3, \dots\})$

$$\lambda_{0i} = \exp(\mathbf{X}_{0i}\beta_0)$$

$$\lambda_{+i} = \exp(\mathbf{X}_{+i}\beta_+)$$

“Hurdle” Count Models

Define:

$$\delta_i = \begin{cases} 0 & \text{if } Y_i = 0 \\ 1 & \text{if } Y_i > 0 \end{cases}$$

$$\begin{aligned} \ln L = & - \sum_{i=1}^N \delta_i \exp(\mathbf{X}_{0i}\beta_0) + \sum_{i=1}^N (1 - \delta_i) \{ \ln[1 - \exp(-\exp(\mathbf{X}_{0i}\beta_0))] + \\ & Y_i(\mathbf{X}_{+i}\beta_+) - \ln[\exp(\exp(\mathbf{X}_{+i}\beta_+)) - 1] \} \end{aligned}$$

“Hurdle” Models: Details

$$\Pr(Y_i = 0) = 1 - \exp[-\exp(\mathbf{X}_{0i}\beta_0)]$$

- λ_+ defines a *truncated* Poisson process
- Y may be overdispersed, Poisson, or underdispersed

ZIP/ZINB and Hurdle Models: R

Command	Package	Count Distribution(s)	Transition Link(s)
Zero-Inflated Models			
zeroinfl	pscl	Poisson, NB, geometric	probit, logit, cloglog, log, Cauchy
vglm,zipoisson	VGAM	Poisson	logit, probit, cloglog, Cauchy, others
vglm,zinegbinomial	VGAM	Negative Binomial	logit, probit, cloglog, Cauchy, others
cozigam	COZIGAM	various	Various (see documentation)
Hurdle Models			
hurdle	pscl	Poisson, NB, geometric	binomial, Poisson, NB, geometric
vglm,zapoisson	VGAM	Poisson	logit, probit, cloglog, Cauchy, others

ZIP/ZINB and Hurdle Models: Stata

Command	Count Distribution	Transition Link(s)
Zero-Inflated Models		
zip	Poisson	probit or logit
zinb	Negative Binomial	probit or logit
Hurdle Models		
hprobit	Poisson	logit
hpclog	Poisson	complementary log-log
hnbprobit	Negative Binomial	logit
hnbpclog	Negative Binomial	complementary log-log

ZIP Example

```
wars.ZIP<-zeroinfl(conflicts~polity+politysq+logPopulation+logGDP+GDPGrowth+logOpenness+govshareGDP,  
  data=wars,dist="poisson",link="logit")  
summary(wars.ZIP)
```

Call:

```
zeroinfl(formula = conflicts ~ polity + politysq + logPopulation +  
  logGDP + GDPGrowth + logOpenness + govshareGDP, data = wars,  
  dist = "poisson", link = "logit")
```

Pearson residuals:

	Min	1Q	Median	3Q	Max
	-1.4061	-0.5113	-0.3391	-0.0859	31.5392

Count model coefficients (poisson with log link):

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-3.79438	0.50297	-7.54	4.6e-14 ***
polity	-0.34467	0.52364	-0.66	0.51040
politysq	0.85585	0.50196	1.71	0.08819 .
logPopulation	0.27385	0.02336	11.72	< 2e-16 ***
logGDP	-0.14271	0.03993	-3.57	0.00035 ***
GDPGrowth	-0.00931	0.00405	-2.30	0.02138 *
logOpenness	0.20226	0.04963	4.08	4.6e-05 ***
govshareGDP	0.03138	0.00288	10.88	< 2e-16 ***
.				
.				
.				

ZIP Example (continued)

```
.  
. .  
Zero-inflation model coefficients (binomial with logit link):  
      Estimate Std. Error z value Pr(>|z|)  
(Intercept)   0.47668    1.80620   0.26  0.79185  
polity        -3.97446    1.56375  -2.54  0.01103 *  
politysq       5.34458    1.49540   3.57  0.00035 ***  
logPopulation -0.62737    0.09152  -6.86  7.1e-12 ***  
logGDP        -0.20497    0.13193  -1.55  0.12026  
GDPGrowth      0.01898    0.01117   1.70  0.08933 .  
logOpenness    1.73322    0.21124   8.21  2.3e-16 ***  
govshareGDP   -0.03454    0.00889  -3.88  0.00010 ***  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
  
Number of iterations in BFGS optimization: 31  
Log-likelihood: -3.26e+03 on 16 Df
```

Example: Prose

- polity's effect on the probability of being in the “zeros-only” state is curvilinear: it first decreases (as a country goes from being strongly autocratic to transitional) then increases (as it becomes more democratic).
- Growth and openness increase the probability of being in the zeros-only state, while government spending decreases it. e.g.:
 - A one-unit increase in `logOpenness` increases $\Pr(p_i = 0)$ by $(\exp(1.733)) \times 100 = 566$ percent.
 - Similarly, a one-unit (in this case, one-percent) increase in `govshareGDP` decreases $\Pr(p_i = 0)$ by $[1 - (\exp(-0.0345)) \times 100] = 3.4$ percent.
- A one-unit increase in `logOpenness` increases the incidence of armed conflicts by $(\exp(0.202)) \times 100 = 122$ percent.
- A one-unit increase in `logGDP`, by contrast, decreases the incidence of armed conflicts by $(1 - \exp(-0.1427)) \times 100 = 13.3$ percent.

Example: ZINB

```
wars.ZINB<-zeroinfl(conflicts~polity+politysq+logPopulation+logGDP+GDPGrowth+logOpenness+govshareGDP,  
                    data=wars,dist="negbin",link="logit")  
summary(wars.ZINB)
```

Call:

```
zeroinfl(formula = conflicts ~ polity + politysq + logPopulation +  
          logGDP + GDPGrowth + logOpenness + govshareGDP, data = wars,  
          dist = "negbin", link = "logit")
```

Pearson residuals:

	Min	1Q	Median	3Q	Max
	-1.2273	-0.5056	-0.3405	-0.0845	34.0184

Count model coefficients (negbin with log link):

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-3.86793	0.51463	-7.52	5.7e-14 ***
polity	-0.14355	0.54078	-0.27	0.79067
politysq	0.59083	0.52442	1.13	0.25990
logPopulation	0.28574	0.02491	11.47	< 2e-16 ***
logGDP	-0.14299	0.04100	-3.49	0.00049 ***
GDPGrowth	-0.00957	0.00416	-2.30	0.02151 *
logOpenness	0.17351	0.05298	3.28	0.00106 **
govshareGDP	0.03204	0.00313	10.24	< 2e-16 ***
Log(theta)	1.89010	0.37315	5.07	4.1e-07 ***

.
.
.

Example: ZINB (continued)

```
.  
. .  
. .  
Zero-inflation model coefficients (binomial with logit link):  
      Estimate Std. Error z value Pr(>|z|)  
(Intercept)    0.38441    1.98351    0.19  0.84633  
polity         -3.66869    1.73171   -2.12  0.03413 *  
politysq        5.05129    1.64892    3.06  0.00219 **  
logPopulation  -0.65973    0.09735   -6.78  1.2e-11 ***  
logGDP         -0.23621    0.14249   -1.66  0.09737 .  
GDPGrowth      0.01975    0.01181    1.67  0.09457 .  
logOpenness    1.82769    0.22375    8.17  3.1e-16 ***  
govshareGDP   -0.03515    0.00987   -3.56  0.00037 ***  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
  
Theta = 6.62  
Number of iterations in BFGS optimization: 31  
Log-likelihood: -3.25e+03 on 17 Df.
```

Example: Hurdle Poisson

```
> wars.hurdle<-hurdle(conflicts~polity+politysq+logPopulation+logGDP+GDPGrowth+
  logOpenness+govshareGDP,data=wars,dist=c("poisson"),zero.dist=c("poisson"),
  link=c("log"))
> summary(wars.hurdle)
```

Call:

```
hurdle(formula = conflicts ~ polity + politysq + logPopulation + logGDP + GDPGrowth +
  logOpenness + govshareGDP, data = wars, dist = c("poisson"), zero.dist = c("poisson"),
  link = c("log"))
```

Count model coefficients (truncated poisson with log link):

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-6.99856	0.78028	-8.97	< 2e-16 ***
polity	-1.30617	0.81861	-1.60	0.11058
politysq	1.32026	0.78772	1.68	0.09373 .
logPopulation	0.39973	0.03318	12.05	< 2e-16 ***
logGDP	0.23269	0.06085	3.82	0.00013 ***
GDPGrowth	-0.00185	0.00648	-0.29	0.77559
logOpenness	-0.10457	0.07793	-1.34	0.17963
govshareGDP	0.04097	0.00386	10.61	< 2e-16 ***

.
.
.

Example: Hurdle Poisson (continued)

```
.  
.br/>.br/>Zero hurdle model coefficients (censored poisson with log link):  
      Estimate Std. Error z value Pr(>|z|)  
(Intercept)  -3.92477    0.41815   -9.39 < 2e-16 ***  
polity        1.68586    0.44729    3.77 0.00016 ***  
politysq      -1.61143    0.42744   -3.77 0.00016 ***  
logPopulation  0.36963    0.01983   18.64 < 2e-16 ***  
logGDP        -0.14103    0.03409   -4.14 3.5e-05 ***  
GDPGrowth     -0.02152    0.00422   -5.09 3.5e-07 ***  
logOpenness   -0.14945    0.04261   -3.51 0.00045 ***  
govshareGDP    0.02909    0.00293    9.91 < 2e-16 ***  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
  
Number of iterations in BFGS optimization: 27  
Log-likelihood: -3.27e+03 on 16 Df
```