PLSC 504 - Fall 2017
Competing Risks and Repeated Events

October 5, 2017

Competing Risks

Competing Risks

R multiple kinds of events:

$$T_i \in T_{i1}, ..., T_{iR}$$

Observed duration:

$$T_i = \min(T_{i1}, ... T_{iR})$$

Event indicator:

$$D_i = r \text{ iff } T_i = T_{ri}$$

R censoring indicators:

$$C_{ir} = \begin{cases} 1 \text{ if observation } i \text{ experienced event } r \\ 0 \text{ otherwise} \end{cases}$$

Likelihoods

$$L_i = f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r)$$

$$\begin{aligned} L &= \prod_{i=1}^{N} \left\{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r) \right\} \\ &= \prod_{r=1}^{R} \prod_{i=1}^{N_r} \{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) \, S_r(T_i|\mathbf{X}_{ir}, \beta_r) \} \\ &= \prod_{r=1}^{R} \prod_{i=1}^{N} [f_r(T_i|\mathbf{X}_{ir}, \beta_r)]^{C_{ir}} \, [S_r(T_i|\mathbf{X}_{ri}, \beta_r)]^{1-C_{ir}} \end{aligned}$$

Practical Estimation

- Independent risks = separate models
- Otherwise identical estimation, interpretation, etc.
- No identification problem
- Discrete-Time $\rightarrow$ MNL
- See (e.g.) Diermeier and Stevenson 1999; Zorn and Van Winkle 2000; Goemans 2008

Independent Risks

- Key: Conditional independence
- $\rightsquigarrow$ Model specification
- Dependent risks:
  - Using frailties (Gordon 2002)
  - Discrete-time: strategic (Fukumoto 2009)
  - Discrete-time: bivariate probit (Quiros Flores 2012)
  - SUR?

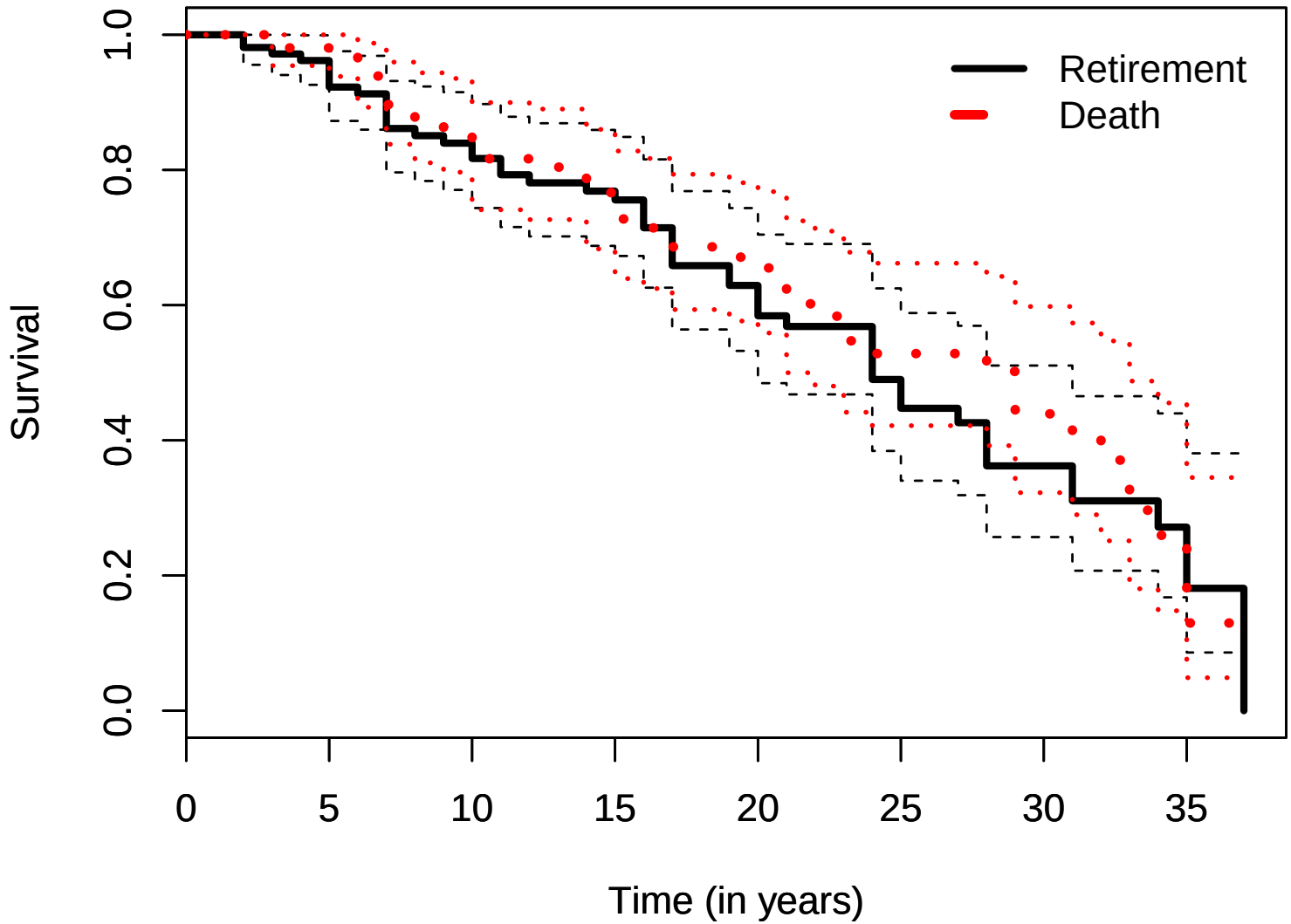
Example: SCOTUS Vacancies

- Supreme Court Vacancies, 1789-1992 ($NT = 1783$)
- Departures $\in \{$Retirement, Mortality$\}$
- Independent competing risks models: Cox + MNL

SCOTUS Data

```
> summary(scotus)
    justice       svcstart     service       retire
 Min.   :  1   Min.   : 0   Min.   : 1   Min.   :0.00
 1st Qu.: 26   1st Qu.: 4   1st Qu.: 5   1st Qu.:0.00
 Median : 51   Median : 9   Median :10   Median :0.00
 Mean   : 53   Mean   :11   Mean   :12   Mean   :0.03
 3rd Qu.: 79   3rd Qu.:16   3rd Qu.:17   3rd Qu.:0.00
 Max.   :109   Max.   :36   Max.   :37   Max.   :1.00
     death          chief          south           age
 Min.   :0.00   Min.   :0.00   Min.   :0.00   Min.   :32
 1st Qu.:0.00   1st Qu.:0.00   1st Qu.:0.00   1st Qu.:56
 Median :0.00   Median :0.00   Median :0.00   Median :62
 Mean   :0.03   Mean   :0.12   Mean   :0.31   Mean   :62
 3rd Qu.:0.00   3rd Qu.:0.00   3rd Qu.:1.00   3rd Qu.:69
 Max.   :1.00   Max.   :1.00   Max.   :1.00   Max.   :91
    pension        pagree        threecat
 Min.   :0.0   Min.   :0.00   Min.   :0.00
 1st Qu.:0.0   1st Qu.:0.00   1st Qu.:0.00
 Median :0.0   Median :1.00   Median :0.00
 Mean   :0.2   Mean   :0.61   Mean   :0.08
 3rd Qu.:0.0   3rd Qu.:1.00   3rd Qu.:0.00
 Max.   :1.0   Max.   :1.00   Max.   :2.00
```

SCOTUS: Death and Retirement



Independent Risks (Cox) Models

	Combined	Retirement	Death
Age	0.06	0.07	0.04
	(0.02)	(0.03)	(0.02)
Chief	−0.03	−0.23	0.09
	(0.30)	(0.44)	(0.40)
South	0.29	0.06	0.45
	(0.23)	(0.34)	(0.33)
Pension Eligibility	0.59	2.04	−0.48
	(0.28)	(0.55)	(0.41)
Party Agreement	−0.01	0.10	−0.10
	(0.21)	(0.29)	(0.31)
AIC	713.26	356.70	348.83
Num. events	99	52	47

Multinomial Logit

	Retirement	Death
Intercept	−7.77	−8.28
	(1.45)	(1.28)
Age	−0.29	0.00
	(0.45)	(0.42)
Chief	0.06	0.48
	(0.34)	(0.32)
South	0.07	0.06
	(0.03)	(0.02)
Pension Eligibility	1.40	−0.56
	(0.42)	(0.41)
Party Agreement	0.03	−0.26
	(0.30)	(0.31)
log(Time)	−0.30	0.51
	(0.27)	(0.29)
AIC	847.51	847.51
BIC	924.31	924.31
Log Likelihood	-409.75	-409.75

Repeated Events

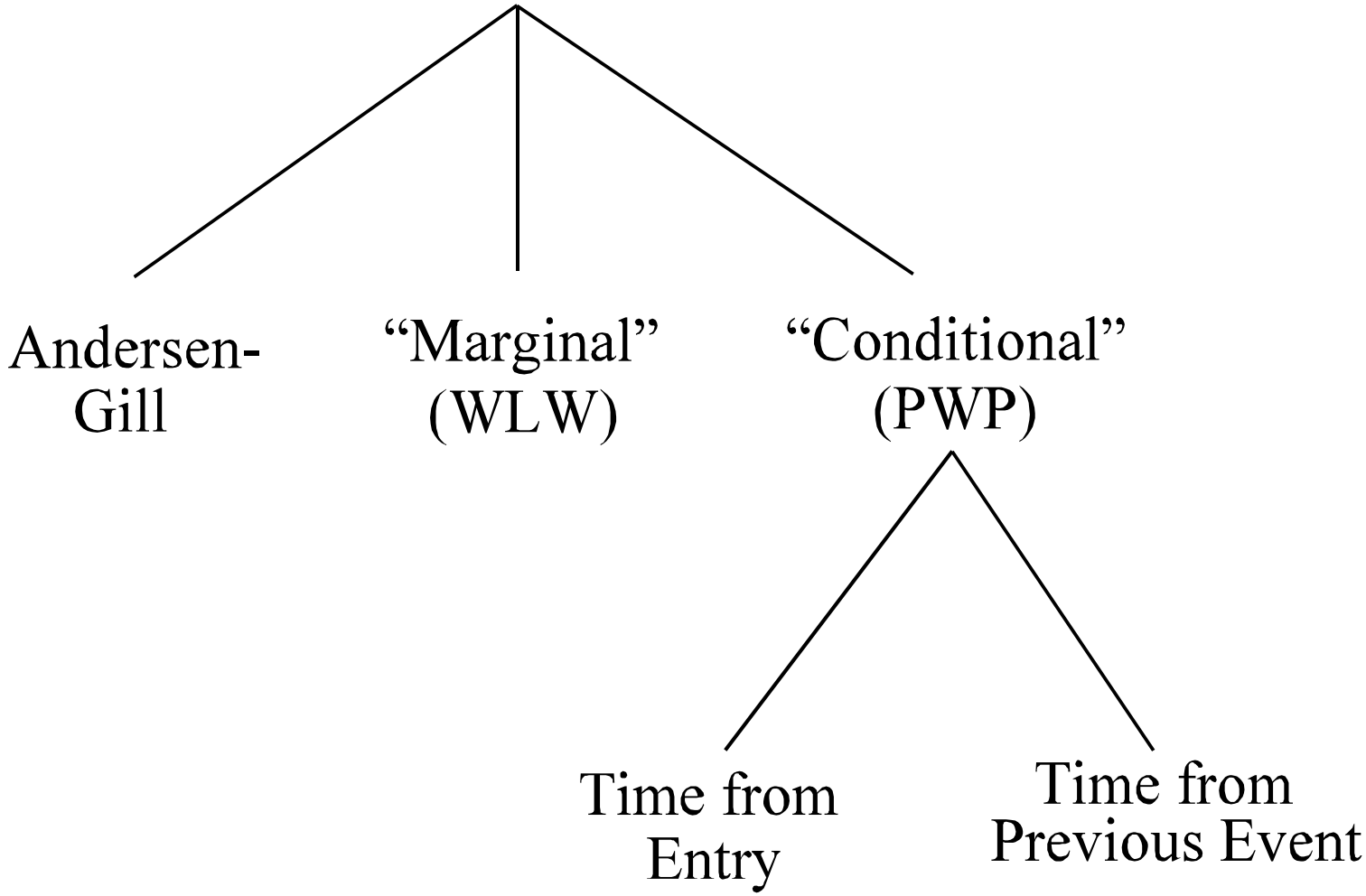
Multiple / Repeated Events

Events are not "absorbing" $\rightarrow$ capable of repetition

Raises (at least) two issues:

- Dependence across events
- Parameter variability

Variance-Correction Models



Variance Correction Model Properties

Model Property	Andersen-Gill (AG)	Marginal (WLW)	Conditional (PWP), Elapsed Time	Conditional (PWP), Gap Time
Risk Set for Event k at Time t	Independent Events	All Subjects that Haven't Experienced Event k at Time t	All Subjects that Have Experienced Event k - 1, and Haven't Experienced Event k, at Time t	
Time Scale	Duration Since Starting Observation	Duration Since Starting Observation	Duration Since Starting Observation	Duration Since Previous Event
Robust standard errors?	Yes	Yes	Yes	
Stratification by Event?	No	Yes	Yes	

Data Organization

```
> OR<-OR[order(OR$dyadid,OR$year),]
> OR$one<-rep(1,times=nrow(OR))
> OR<-ddply(OR,"dyadid",mutate,eventno=cumsum(dispute)+1,
          altstart=cumsum(one)-1,altstop=cumsum(one))
```

	dyadid	year	start	stop	altstart	altstop	dispute	eventno
461	2130	1951	0	1	0	1	0	1
462	2130	1952	1	2	1	2	1	1
463	2130	1953	0	1	2	3	0	2
464	2130	1954	1	2	3	4	1	2
465	2130	1956	0	1	4	5	0	3
466	2130	1957	1	2	5	6	0	3
467	2130	1958	2	3	6	7	0	3
468	2130	1959	3	4	7	8	0	3
469	2130	1960	4	5	8	9	0	3
470	2130	1961	5	6	9	10	0	3
471	2130	1962	6	7	10	11	0	3
472	2130	1963	7	8	11	12	1	3
473	2130	1964	0	1	12	13	0	4
474	2130	1965	1	2	13	14	0	4

.
.
.

First Events

```
> OR1st<-OR[OR$eventno==1,]
> OR.1st<-Surv(OR1st$altstart,OR1st$altstop,OR1st$dispute)
> OR.Cox.1st<-coxph(OR.1st~allies+contig+capratio+growth+democracy+
                    trade+cluster(dyadid),data=OR1st,method="efron")

> OR.Cox.1st
Call:
coxph(formula = OR.1st ~ allies + contig + capratio + growth +
    democracy + trade + cluster(dyadid), data = OR1st, method = "efron")


             coef exp(coef) se(coef) robust se      z           p
allies    -0.448    0.6389   0.1585    0.1640 -2.732 0.0063000000
contig     1.070    2.9167   0.1681    0.1767  6.059 0.0000000014
capratio  -0.196    0.8223   0.0603    0.0779 -2.510 0.0120000000
growth    -2.198    0.1110   1.7195    1.9005 -1.157 0.2500000000
democracy -0.424    0.6547   0.1298    0.1259 -3.365 0.0007600000
trade     -6.728    0.0012  12.3255   13.9025 -0.484 0.6300000000

Likelihood ratio test=121  on 6 df, p=0  n= 17158, number of events= 205
```

Andersen-Gill

```
> OR.AGS<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.AG<-coxph(OR.AGS~allies+contig+capratio+growth+democracy+
                     trade+cluster(dyadid),data=OR,method="efron")

> OR.Cox.AG
Call:
coxph(formula = OR.AGS ~ allies + contig + capratio + growth +
    democracy + trade + cluster(dyadid), data = OR, method = "efron")


              coef  exp(coef) se(coef) robust se      z       p
allies     -0.414 0.66090755   0.1107    0.1703 -2.431 1.5e-02
contig      1.213 3.36515975   0.1209    0.1782  6.811 9.7e-12
capratio   -0.214 0.80717357   0.0514    0.0817 -2.620 8.8e-03
growth     -3.227 0.03967003   1.2279    1.3169 -2.451 1.4e-02
democracy  -0.439 0.64437744   0.0998    0.1231 -3.571 3.6e-04
trade     -13.162 0.00000192  10.3266   13.8188 -0.953 3.4e-01

Likelihood ratio test=272  on 6 df, p=0  n= 20448, number of events= 405
```

Prentice et al.: Elapsed Time

```
> OR.PWPES<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.PWPE<-coxph(OR.PWPES~allies+contig+capratio+growth+democracy+
                     trade+strata(eventno)+cluster(dyadid),data=OR,
                     method="efron")

> OR.Cox.PWPE
Call:
coxph(formula = OR.PWPES ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
    method = "efron")


              coef exp(coef) se(coef) robust se      z       p
allies    -0.240    0.7865   0.1122    0.1283 -1.872 6.1e-02
contig     0.868    2.3811   0.1223    0.1329  6.526 6.8e-11
capratio  -0.162    0.8506   0.0472    0.0618 -2.618 8.8e-03
growth    -3.625    0.0266   1.2371    1.2032 -3.013 2.6e-03
democracy -0.273    0.7612   0.1036    0.1074 -2.541 1.1e-02
trade     -2.514    0.0810   9.2934    9.9432 -0.253 8.0e-01

Likelihood ratio test=133  on 6 df, p=0  n= 20448, number of events= 405
```

Prentice et al.: Gap Time

```
> OR.PWPGS<-Surv(OR$start,OR$stop,OR$dispute)
> OR.Cox.PWPG<-coxph(OR.PWPGS~allies+contig+capratio+growth+democracy+
                     trade+strata(eventno)+cluster(dyadid),data=OR,
                     method="efron")

> OR.Cox.PWPG
Call:
coxph(formula = OR.PWPGS ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
    method = "efron")


             coef exp(coef) se(coef) robust se     z       p
allies    -0.329    0.7193   0.1119    0.1229 -2.68 7.3e-03
contig     0.885    2.4232   0.1222    0.1285  6.89 5.6e-12
capratio  -0.171    0.8431   0.0481    0.0636 -2.68 7.3e-03
growth    -3.459    0.0315   1.2189    1.2102 -2.86 4.3e-03
democracy -0.284    0.7530   0.1028    0.1016 -2.79 5.2e-03
trade     -4.287    0.0137   9.9352   10.4592 -0.41 6.8e-01

Likelihood ratio test=139  on 6 df, p=0  n= 20448, number of events= 405
```

WLW: Data Organization

```
> OR.expand<-OR[rep(1:nrow(OR),each=max(OR$eventno)),]
> OR.expand<-ddply(OR.expand,c("dyadid","year"),mutate,
                   eventrisk=cumsum(one))
> OR.expand$dispute<-ifelse(OR.expand$eventno==OR.expand$eventrisk
                            & OR.expand$dispute==1,1,0)
> dim(OR.expand)
[1] 163584     17

> head(OR.expand,9)
  dyadid year start stop futime dispute allies contig trade growth
1   2020 1951     0    1     35       0      1      1 0.014 0.0085
2   2020 1951     0    1     35       0      1      1 0.014 0.0085
3   2020 1951     0    1     35       0      1      1 0.014 0.0085
4   2020 1951     0    1     35       0      1      1 0.014 0.0085
5   2020 1951     0    1     35       0      1      1 0.014 0.0085
6   2020 1951     0    1     35       0      1      1 0.014 0.0085
7   2020 1951     0    1     35       0      1      1 0.014 0.0085
8   2020 1951     0    1     35       0      1      1 0.014 0.0085
9   2020 1952     1    2     35       0      1      1 0.015 0.0259
  democracy capratio one eventno altstart altstop eventrisk
1         1     0.20   1       1        0       1         1
2         1     0.20   1       1        0       1         2
3         1     0.20   1       1        0       1         3
4         1     0.20   1       1        0       1         4
5         1     0.20   1       1        0       1         5
6         1     0.20   1       1        0       1         6
7         1     0.20   1       1        0       1         7
8         1     0.20   1       1        0       1         8
9         1     0.19   1       1        1       2         1
```

WLW Model

```
> OR.expand.S<-Surv(OR.expand$altstart,OR.expand$altstop,
                    OR.expand$dispute)
> OR.Cox.WLW<-coxph(OR.expand.S~allies+contig+capratio+growth+
                      democracy+trade+strata(eventno)+
                      cluster(dyadid),data=OR.expand,
                      method="efron")

> OR.Cox.WLW
Call:
coxph(formula = OR.expand.S ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR.expand,
    method = "efron")


             coef exp(coef) se(coef) robust se      z       p
allies    -0.230    0.7947   0.1122    0.1248 -1.841 6.6e-02
contig     0.852    2.3435   0.1223    0.1297  6.568 5.1e-11
capratio  -0.160    0.8524   0.0471    0.0609 -2.621 8.8e-03
growth    -3.508    0.0300   1.2370    1.1671 -3.005 2.7e-03
democracy -0.271    0.7625   0.1037    0.1055 -2.570 1.0e-02
trade     -2.656    0.0702   9.2807    9.6144 -0.276 7.8e-01

Likelihood ratio test=129  on 6 df, p=0  n= 163584, number of events= 405
```

Models of Repeated Events

	First	AG	PWP-E	PWP-G	WLW
Allies	−0.45	−0.41	−0.24	−0.33	−0.23
	(0.16)	(0.17)	(0.13)	(0.12)	(0.12)
Contiguity	1.07	1.21	0.87	0.89	0.85
	(0.18)	(0.18)	(0.13)	(0.13)	(0.13)
Capability Ratio	−0.20	−0.21	−0.16	−0.17	−0.16
	(0.08)	(0.08)	(0.06)	(0.06)	(0.06)
Growth	−2.20	−3.23	−3.63	−3.46	−3.51
	(1.90)	(1.32)	(1.20)	(1.21)	(1.17)
Democracy	−0.42	−0.44	−0.27	−0.28	−0.27
	(0.13)	(0.12)	(0.11)	(0.10)	(0.11)
Trade	−6.73	−13.16	−2.51	−4.29	−2.66
	(13.90)	(13.82)	(9.94)	(10.46)	(9.61)
AIC	2538.02	5015.77	3892.77	4103.47	5597.54
Num. events	205	405	405	405	405

Parameter Change Across Events

- Values of β differ from k to $k+1$
- Again: Institutionalization, learning, etc.
- Addressed using *strata by covariate interactions*

Parameter Change Example

```
> OR$capXevent<-OR$capratio*OR$eventno
> OR.Cox.BVary<-coxph(OR.PWPGS~allies+contig+growth+democracy+
                      trade+capratio+capXevent+strata(eventno)+
                      cluster(dyadid),data=OR,
                      method="efron")

> OR.Cox.BVary
Call:
coxph(formula = OR.PWPGS ~ allies + contig + growth + democracy +
    trade + capratio + capXevent + strata(eventno) + cluster(dyadid),
    data = OR, method = "efron")


              coef exp(coef) se(coef) robust se      z       p
allies    -0.349    0.7053   0.1120    0.1177 -2.967 3.0e-03
contig     0.897    2.4517   0.1221    0.1254  7.150 8.7e-13
growth    -3.519    0.0296   1.2196    1.2129 -2.901 3.7e-03
democracy -0.305    0.7374   0.1037    0.0972 -3.135 1.7e-03
trade     -3.297    0.0370   9.7624   10.1869 -0.324 7.5e-01
capratio  -0.340    0.7117   0.0997    0.1054 -3.227 1.2e-03
capXevent  0.135    1.1443   0.0631    0.0581  2.321 2.0e-02

Likelihood ratio test=143  on 7 df, p=0  n= 20448, number of events= 405
```

Conclusions / Recommendations

As a practical matter, estimating these models is simply a function of:

- Setting up the data correctly (so as to define the right risk sets),
- Stratifying when appropriate, and
- Calculating / using robust standard errors...