PLSC 504

Introduction to Item Response Theory Models - II

November 30, 2017

IRT Models in R

- Library `ltm` (marginal estimation)
    - `rasch` (1PLM)
    - `ltm` (2PLM)
    - `tpl` (3PLM)
- Library `MCMCpack` (Bayesian estimation)
    - 1 and 2PLM
    - Standard, hierarchical, dyamic, multidimensional
- `ideal` (in library `pscl`) (Bayesian estimation)
    - 1 and 2PLM
    - *k*-dimensional
    - takes a `rollcall` object
- Other packages: `eRm`, `irtoys`, `irtProb`, `MiscPsycho`, etc.

Example: SCOTUS Voting, 1994-2004

```
> summary(SCOTUS)

       id          Rehnquist      Stevens        OConnor        Scalia
 Min.   :   1   Min.   :0     Min.   :0     Min.   :0     Min.   :0
 1st Qu.: 377   1st Qu.:0     1st Qu.:0     1st Qu.:0     1st Qu.:0
 Median : 753   Median :0     Median :1     Median :0     Median :0
 Mean   : 753   Mean   :0     Mean   :1     Mean   :0     Mean   :0
 3rd Qu.:1129   3rd Qu.:1     3rd Qu.:1     3rd Qu.:1     3rd Qu.:1
 Max.   :1505   Max.   :1     Max.   :1     Max.   :1     Max.   :1
                NA's   :49    NA's   :51    NA's   :55    NA's   :41
    Kennedy         Souter         Thomas        Ginsburg       Breyer
 Min.   :0     Min.   :0     Min.   :0     Min.   :0     Min.   :0
 1st Qu.:0     1st Qu.:0     1st Qu.:0     1st Qu.:0     1st Qu.:0
 Median :0     Median :1     Median :0     Median :1     Median :1
 Mean   :0     Mean   :1     Mean   :0     Mean   :1     Mean   :1
 3rd Qu.:1     3rd Qu.:1     3rd Qu.:0     3rd Qu.:1     3rd Qu.:1
 Max.   :1     Max.   :1     Max.   :1     Max.   :1     Max.   :1
 NA's   :32    NA's   :37    NA's   :44    NA's   :39    NA's   :61
```

1PLM Using rasch

```
> # 1PLM / Rasch Model:
> require(ltm)
> OnePLM<-rasch(SCOTUS[c(2:10)])
> summary(OnePLM)

Model Summary:
 log.Lik   AIC   BIC
   -5529 11079 11132

Coefficients:
                 value std.err z.vals
Dffclt.Rehnquist  0.46   0.040   11.5
Dffclt.Stevens   -0.59   0.030  -19.8
Dffclt.OConnor    0.14   0.030    4.6
Dffclt.Scalia     0.52   0.041   12.5
Dffclt.Kennedy    0.21   0.032    6.5
Dffclt.Souter    -0.36   0.027  -13.1
Dffclt.Thomas     0.60   0.043   13.8
Dffclt.Ginsburg  -0.37   0.027  -13.4
Dffclt.Breyer    -0.26   0.027   -9.9
Dscrmn            3.74   0.130   28.9

Integration:
method: Gauss-Hermite
quadrature points: 21

Optimization:
Convergence: 0
max(|grad|): 0.0027
quasi-Newton: BFGS
```

Converted to $\widehat{\Pr(Y_i = 1|\theta_i = 0)}$

```
> # Convert to probabilities given theta=0
>
> coef(OnePLM, prob=TRUE, order=TRUE)
          Dffclt Dscrmn P(x=1|z=0)
Stevens    -0.59    3.7      0.900
Ginsburg   -0.37    3.7      0.797
Souter     -0.36    3.7      0.791
Breyer     -0.26    3.7      0.729
OConnor     0.14    3.7      0.373
Kennedy     0.21    3.7      0.311
Rehnquist   0.46    3.7      0.151
Scalia      0.52    3.7      0.126
Thomas      0.60    3.7      0.096
```

Alternative Model Constraining $\alpha = 1.0$

```
> AltOnePLM<-rasch(IRTData, constraint=cbind(length(IRTData)+1,1))
> summary(AltOnePLM)

Model Summary:
 log.Lik   AIC   BIC
   -6452 12923 12971

Coefficients:
                 value std.err z.vals
Dffclt.Rehnquist  1.26   0.073   17.3
Dffclt.Stevens   -1.07   0.071  -15.1
Dffclt.OConnor    0.56   0.069    8.1
Dffclt.Scalia     1.37   0.074   18.6
Dffclt.Kennedy    0.72   0.069   10.4
Dffclt.Souter    -0.58   0.068   -8.6
Dffclt.Thomas     1.53   0.075   20.3
Dffclt.Ginsburg  -0.61   0.068   -8.9
Dffclt.Breyer    -0.40   0.068   -5.9
Dscrmn            1.00      NA     NA
```

2PLM

```
> TwoPLM<-ltm(IRTData ~ z1)
> summary(TwoPLM)

Coefficients:
                  value std.err z.vals
Dffclt.Rehnquist  0.44   0.035   12.3
Dffclt.Stevens   -0.63   0.038  -16.7
Dffclt.OConnor    0.14   0.026    5.6
Dffclt.Scalia     0.59   0.042   14.1
Dffclt.Kennedy    0.20   0.028    7.2
Dffclt.Souter    -0.27   0.025  -10.7
Dffclt.Thomas     0.68   0.044   15.2
Dffclt.Ginsburg  -0.29   0.025  -11.8
Dffclt.Breyer    -0.24   0.025   -9.6
Dscrmn.Rehnquist  4.77   0.377   12.7
Dscrmn.Stevens    2.46   0.165   14.9
Dscrmn.OConnor    4.14   0.341   12.1
Dscrmn.Scalia     2.82   0.188   15.0
Dscrmn.Kennedy    4.74   0.448   10.6
Dscrmn.Souter     6.69   0.535   12.5
Dscrmn.Thomas     2.84   0.190   14.9
Dscrmn.Ginsburg   5.83   0.439   13.3
Dscrmn.Breyer     3.76   0.253   14.9
```

2PLM: Probabilities and Testing

```
> coef(TwoPLM, prob=TRUE, order=TRUE)

          Dffclt Dscrmn P(x=1|z=0)
Stevens    -0.63    2.5       0.82
Ginsburg   -0.29    5.8       0.85
Souter     -0.27    6.7       0.86
Breyer     -0.24    3.8       0.71
OConnor     0.14    4.1       0.35
Kennedy     0.20    4.7       0.28
Rehnquist   0.44    4.8       0.11
Scalia      0.59    2.8       0.16
Thomas      0.68    2.8       0.13
```

```
> anova(OnePLM, TwoPLM)

 Likelihood Ratio Table
         AIC   BIC log.Lik   LRT df p.value
OnePLM 11079 11132   -5529
TwoPLM 10882 10978   -5423 212.7  8  <0.001
```

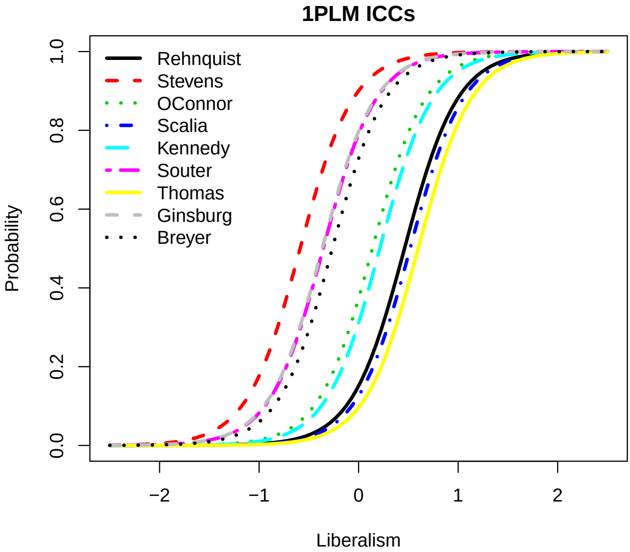
3PLM

```
> ThreePLM<-tpm(IRTData)
> summary(ThreePLM)

Coefficients:
                   value  std.err  z.vals
Gussng.Rehnquist  0.049    0.008   6.260
Gussng.Stevens    0.000    0.001   0.018
Gussng.OConnor    0.043    0.013   3.415
Gussng.Scalia     0.097    0.011   9.119
Gussng.Kennedy    0.071    0.014   5.162
Gussng.Souter     0.011    0.029   0.386
Gussng.Thomas     0.087    0.010   8.900
Gussng.Ginsburg   0.000    0.000   0.009
Gussng.Breyer     0.000    0.000   0.004
Dffclt.Rehnquist  0.716    0.030  23.511
Dffclt.Stevens   -0.630    0.038 -16.434
Dffclt.OConnor    0.340    0.040   8.537
Dffclt.Scalia     0.759    1.766   0.430
Dffclt.Kennedy    0.500    0.041  12.170
Dffclt.Souter    -0.294    0.063  -4.642
Dffclt.Thomas     0.808   10.610   0.076
Dffclt.Ginsburg  -0.329    0.030 -10.970
Dffclt.Breyer    -0.232    0.031  -7.439
Dscrmn.Rehnquist  8.735    4.259   2.051
Dscrmn.Stevens    2.577    0.181  14.214
Dscrmn.OConnor    3.979    0.439   9.068
Dscrmn.Scalia    26.537  578.889   0.046
Dscrmn.Kennedy    4.408    0.588   7.498
Dscrmn.Souter     6.698    1.416   4.731
Dscrmn.Thomas    34.074 2779.161   0.012
Dscrmn.Ginsburg   5.800    0.509  11.394
Dscrmn.Breyer     3.538    0.231  15.335
```

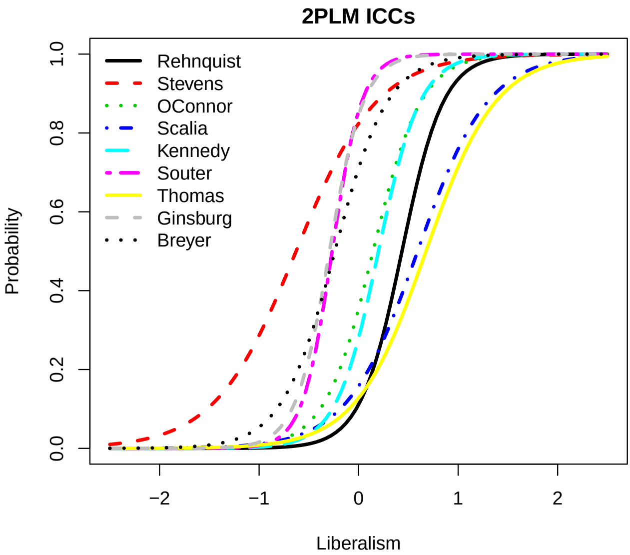
3PLM: Testing

```
> anova(TwoPLM, ThreePLM)

 Likelihood Ratio Table
           AIC   BIC log.Lik    LRT df p.value
TwoPLM   10882 10978   -5423
ThreePLM 10737 10881   -5342 162.94  9  <0.001
```

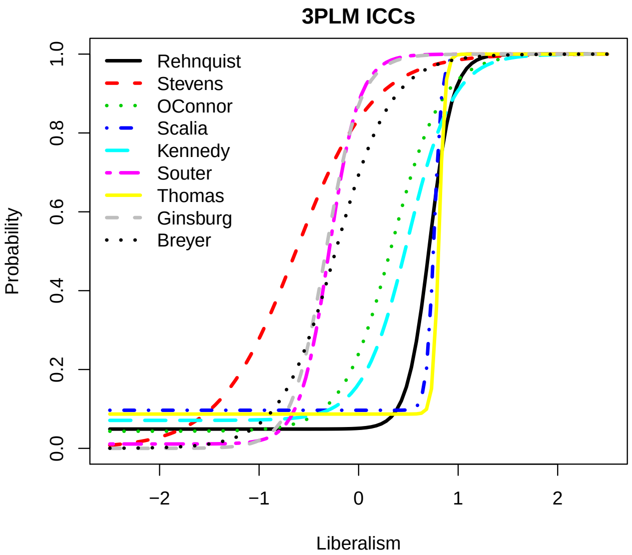
Cool Plots, I


1PLM ICCs
Rehnquist
Stevens
OConnor
Scalia
Kennedy
Souter
Thomas
Ginsburg
Breyer
Probability
0.0
0.2
0.4
0.6
0.8
1.0
−2
−1
0
1
2
Liberalism

Cool Plots, II



Cool Plots, III



Miscellaneous Things, I: Dimensionality

- Usually, *unidimensional*
- Sometimes, *two-dimensional*
- Tests:
    - Tetrachoric correlations among items
    - DIMTEST (Stout & Zhang, etc.)
    - Yen's Q_3
    - 1-D vs. 2-D comparisons (LR tests, etc.)

Miscellaneous Things, II: "DIF"

- *Differential item functioning*
- Formally,

$$\Pr(Y_{ij} = 1) = \Lambda[\alpha_j(\theta_i - \mathbf{X}_i\beta_j)].$$

- $\rightarrow$ violates *local item independence*

Extensions

- Nominal/Multinomial Y
- Ordinal Y:
    - *Graded response model* ("GRM") (Samejima 1969)
    - *Partial credit model* (Masters 1982)
    - *Generalized partial credit model* (Muraki 1992)
- Models for mixed response types (Thissen and Wainer 2001, 2003)
- Hierarchical IRT models (e.g. Bolt and Kim 2005)
- Models with covariates (e.g., DeBoeck and Wilson 2004)

Further Reading / Useful References

Hambleton, Ronald K., H. Swaminathan, and H. Jane Rogers. 1991. *Fundamentals of Item Response Theory*. Newbury Park CA: Sage Publications.

de Ayala, R. J. 2008. *The Theory and Practice of Item Response Theory*. New York: The Guilford Press.

Fahrmeier, L., and G. Tutz. 2000. *Multivariate Statistical Modelling Based on Generalized Linear Models*. Berlin: Springer-Verlag.

De Boeck, Paul, and Mark Wilson, Eds. 2004. *Explanatory Item Response Models: A Generalized Linear and Nonlinear Approach.* New York: Springer.