PLSC 504 – Autumn 2017
GLS-ARMA

October 24, 2017

For:

$$Y_{it} = \mathbf{X}_{it}\beta + u_{it}$$

i.i.d. u_{it}s require:

$$\begin{aligned}
\mathbf{u}\mathbf{u}' \equiv \boldsymbol{\Omega} &= \sigma^2\mathbf{I} \\
&= \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{pmatrix}
\end{aligned}$$

That is, within units:

- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \; \forall \; t \neq s$ (temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{is}) = 0 \; \forall \; t \neq s$ (no within-unit autocorrelation)

and between units:

- $\text{Var}(u_{it}) = \text{Var}(u_{jt}) \; \forall \; i \neq j$ (cross-unit homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = 0 \; \forall \; i \neq j$ (no between-unit / spatial correlation)

The Key: $\mathbf{\Omega}$

Estimator:

$$\hat{\beta}_{GLS} = (\mathbf{X}'\mathbf{\Omega}^{-\mathbf{1}}\mathbf{X})^{-1}\mathbf{X}'\mathbf{\Omega}^{-\mathbf{1}}\mathbf{Y}$$

with:

$$\widehat{\mathrm{V}(\beta_{GLS})} = (\mathbf{X}'\mathbf{\Omega}^{-\mathbf{1}}\mathbf{X})^{-1}$$

Two approaches:

- Use OLS $\hat{u}_{it}$s to get $\hat{\mathbf{\Omega}}$ ("feasible GLS")
- Use substantive knowledge about the data to structure $\mathbf{\Omega}$

Parks' Approach

Assume:

- $E(u_{it}^2) = E(u_{is}^2) \; \forall \; t \neq s$
- $E(u_{it}, u_{jt}) = \sigma_{ij} \; \forall \; i \neq j,$
- $E(u_{it}, u_{js}) = 0 \; \forall \; i \neq j, t \neq s$
- $E(u_{it}, u_{is}) = \rho$ or ρ_i

(B&K: "panel error assumptions").

Then

1. Use OLS to generate $\hat{u}$s $\rightarrow \hat{\rho}$ ($\rightarrow \hat{\mathbf{\Omega}}$),
2. Use $\hat{\rho}$ for Prais-Winsten.

This method was widely used prior to B&K (1995)

Parks' Problems

$$\mathbf{\Omega} = \begin{pmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{pmatrix} = \Sigma \otimes \mathbf{I}_N$$

where

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}$$

Means:

- $\frac{N(N-1)}{2}$ distinct contemporaneous correlations,
- NT observations,
- $\rightarrow 2T/(N+1)$ observations per $\hat{\sigma}$

Panel-Corrected Standard Errors

Key to PCSEs:

$$\hat{\sigma}_{ij} = \frac{\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt}}{T}$$

Define:

$$\underset{T\times N}{\mathbf{U}} = \begin{pmatrix} \hat{u}_{11} & \hat{u}_{21} & \cdots & \hat{u}_{N1} \\ \hat{u}_{12} & \hat{u}_{22} & \cdots & \hat{u}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{1T} & \hat{u}_{2T} & \cdots & \hat{u}_{NT} \end{pmatrix}$$

$$\hat{\mathbf{\Sigma}} = \frac{(\mathbf{U}'\mathbf{U})}{T}$$

$$\hat{\mathbf{\Omega}}_{PCSE} = \frac{(\mathbf{U}'\mathbf{U})}{T} \otimes \mathbf{I}_T$$

Correct formula:

$$\mathrm{Cov}(\hat{\beta}_{PCSE}) = (\mathbf{X}'\mathbf{X})^{-1}[\mathbf{X}'\boldsymbol{\Omega}\mathbf{X}](\mathbf{X}'\mathbf{X})^{-1}$$

PCSEs:

- Do not fix unit-level heterogeneity (a la “fixed” / “random” effects)
- Do not deal with dynamics
- Depend critically on the “panel data assumptions” of Park / B&K

Panel Assumptions and Numbers of Parameters to be Estimated

Panel Assumptions	No AR(1)	Common $\hat{\rho}$	Separate $\hat{\rho}_i$s
$\sigma_i^2 = \sigma^2$, $Cov(\sigma_{it}, \sigma_{jt}) = 0$	$k+1$	$k+2$	$k+N+1$
$\sigma_i^2 \neq \sigma^2$, $Cov(\sigma_{it}, \sigma_{jt}) = 0$	$k+N$	$k+N+1$	$k+2N$
$\sigma_i^2 \neq \sigma^2$, $Cov(\sigma_{it}, \sigma_{jt}) \neq 0$	$\frac{N(N-1)}{2} + k + N$	$\frac{N(N-1)}{2} + k + N + 1$	$\frac{N(N-1)}{2} + k + 2N$

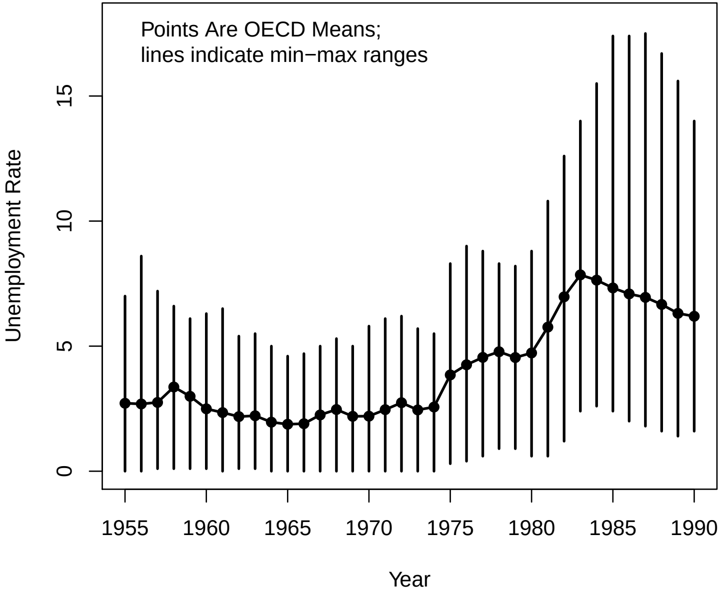
Example: Central Banks, Unions, Unemployment

- Hall and Franzese (1998 *IO*)
- 18 OECD countries, 1955-1990 ($N = 18,\ T = 36,\ NT = 648$)
- $Y =$ unemployment
- Covariates: GDP, openness, union density, left cabinets, central bank independence, coordinated wage bargaining, interaction

Example: Data

```
> summary(HF)
    country          year            ue              inf              cbi       
 Min.   : 1.0   Min.   :1955   Min.   : 0.0   Min.   :-1.7   Min.   :0.12  
 1st Qu.: 5.0   1st Qu.:1964   1st Qu.: 1.6   1st Qu.: 3.2   1st Qu.:0.41  
 Median : 9.5   Median :1972   Median : 3.0   Median : 4.9   Median :0.47  
 Mean   :10.3   Mean   :1972   Mean   : 4.0   Mean   : 6.0   Mean   :0.50  
 3rd Qu.:15.0   3rd Qu.:1981   3rd Qu.: 5.7   3rd Qu.: 7.7   3rd Qu.:0.61  
 Max.   :21.0   Max.   :1990   Max.   :17.5   Max.   :27.2   Max.   :0.93  
    cwagebrg         GDP_PC         open            uden            lcab      
 Min.   :0.00   Min.   :7.6   Min.   :0.07   Min.   :0.10   Min.   :0.00  
 1st Qu.:0.25   1st Qu.:8.9   1st Qu.:0.31   1st Qu.:0.32   1st Qu.:0.00  
 Median :0.50   Median :9.2   Median :0.43   Median :0.41   Median :0.07  
 Mean   :0.49   Mean   :9.1   Mean   :0.46   Mean   :0.44   Mean   :0.31  
 3rd Qu.:0.75   3rd Qu.:9.4   3rd Qu.:0.54   3rd Qu.:0.56   3rd Qu.:0.58  
 Max.   :1.00   Max.   :9.8   Max.   :1.40   Max.   :0.85   Max.   :1.00  
    wagexcbi        HasLCAB 
 Min.   :0.00   Min.   :1  
 1st Qu.:0.04   1st Qu.:1  
 Median :0.21   Median :1  
 Mean   :0.25   Mean   :1  
 3rd Qu.:0.37   3rd Qu.:1  
 Max.   :0.70   Max.   :1  
```

Unemployment in 18 Nations, 1955-1990



Example: OLS

```
> summary(HF.OLS)
Oneway (individual) effect Pooling Model

Balanced Panel: n=18, T=36, N=648

Residuals :
   Min. 1st Qu.  Median 3rd Qu.    Max.
 -5.120  -1.500  -0.241   1.230   9.290

Coefficients :
            Estimate Std. Error t-value     Pr(>|t|)
(Intercept)  -13.579      2.328   -5.83 0.0000000086 ***
GDP_PC         1.603      0.263    6.09 0.0000000020 ***
open           5.119      0.418   12.24      < 2e-16 ***
uden           0.709      0.808    0.88         0.38
lcab           0.236      0.293    0.81         0.42
cbi            5.169      1.097    4.71 0.0000030150 ***
cwagebrg      -1.292      0.792   -1.63         0.10
wagexcbi      -7.030      1.505   -4.67 0.0000036327 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Total Sum of Squares:    6500
Residual Sum of Squares: 3730
R-Squared:      0.426
Adj. R-Squared: 0.421
F-statistic: 67.9634 on 7 and 640 DF, p-value: <2e-16
```

Example: Prais-Winsten

```
> HF.prais <- prais.winsten(ue~GDP_PC+open+uden+lcab+cbi+cwagebrg+wagexcbi,
                           data=HF,iter=100)
> HF.prais

Residuals:
   Min     1Q Median     3Q    Max
-8.456 -0.431 -0.144  0.314  4.615

Coefficients:
          Estimate Std. Error t value Pr(>|t|)
Intercept -15.2783     2.2128   -6.90  1.2e-11 ***
GDP_PC      2.3415     0.2515    9.31  < 2e-16 ***
open       -0.3491     0.7950   -0.44   0.6608
uden        5.5466     1.1492    4.83  1.7e-06 ***
lcab       -0.0593     0.1861   -0.32   0.7501
cbi        -3.4801     2.4753   -1.41   0.1602
cwagebrg  -10.5954     2.0019   -5.29  1.7e-07 ***
wagexcbi   10.6805     3.4942    3.06   0.0023 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.95 on 640 degrees of freedom
Multiple R-squared:  0.279,Adjusted R-squared:  0.27
F-statistic:   31 on 8 and 640 DF,  p-value: <2e-16

  Rho Rho.t.statistic Iterations
 0.94              73         10
```

Example: GLS with Homoscedastic AR(1) Errors

```
> HF.GLS <- gls(ue~GDPPC+open+uden+lcab+cbi+cwagebrg+wagexcbi,
                             HF,correlation=corAR1(form=~1|country))
> summary(HF.GLS)
Generalized least squares fit by REML
  Model: ue ~ GDPPC + open + uden + lcab + cbi + cwagebrg + wagexcbi
  Data: HF
   AIC  BIC logLik
  1484 1529   -732

Correlation Structure: AR(1)
 Formula: ~1 | country
 Parameter estimate(s):
 Phi
0.99

Coefficients:
            Value Std.Error t-value p-value
(Intercept)    43       7.3     5.8   0.000
GDPPC          -4       0.7    -5.5   0.000
open           -1       0.8    -1.8   0.072
uden            1       2.2     0.3   0.792
lcab            0       0.1    -0.7   0.473
cbi            -1       7.5    -0.2   0.848
cwagebrg       -5       6.4    -0.8   0.402
wagexcbi        3      11.7     0.3   0.770

 Correlation:
         (Intr) GDPPC  open   uden   lcab   cbi    cwgbrg
GDPPC    -0.827
open      0.124 -0.207
uden     -0.145  0.017 -0.048
lcab      0.041 -0.054  0.005 -0.003
cbi      -0.421 -0.100  0.033  0.069  0.009
cwagebrg -0.371 -0.065  0.018 -0.084 -0.014  0.721
wagexcbi  0.334  0.082 -0.028  0.017  0.004 -0.813 -0.905

Standardized residuals:
  Min    Q1   Med    Q3   Max
-1.49 -0.64 -0.20  0.46  2.55
```

More GLS: Unit-Wise Heteroscedisticity

```
> HF.GLS2 <- gls(ue~GDPPC+open+uden+lcab+cbi+cwagebrg+wagexcbi,
+               HF,correlation=corAR1(form=~1|country),
+               weights = varIdent(form = ~1|country))
> summary(HF.GLS2)
Generalized least squares fit by REML
   AIC  BIC logLik
  1326 1446   -636

Correlation Structure: AR(1)
 Formula: ~1 | country
 Parameter estimate(s):
 Phi
0.98

Variance function:
 Structure: Different standard deviations per stratum
 Formula: ~1 | country
 Parameter estimates:
   1    2    3    4    5    6    7    8    9   10   11   13   14   15   18   19   20   21
1.00 0.19 0.75 0.54 0.60 1.00 0.95 0.32 0.88 0.89 0.78 1.09 0.92 0.57 0.33 0.38 0.86 0.60

Coefficients:
            Value Std.Error t-value p-value
(Intercept)  21.1       4.7     4.5  0.0000
GDPPC        -1.6       0.4    -4.3  0.0000
open         -2.2       0.6    -3.4  0.0008
uden          0.9       1.5     0.6  0.5415
lcab         -0.1       0.1    -1.2  0.2206
cbi          -1.9       7.3    -0.3  0.7984
cwagebrg     -6.3       4.7    -1.3  0.1794
wagexcbi      5.0       9.5     0.5  0.5996
.
.
.
```

Example: PCSEs

```
> library(lmtest)
> coeftest(HF.OLS,vcov=vcovBK)

t test of coefficients:

            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -13.579      7.320   -1.86    0.064 .
GDP_PC         1.603      0.821    1.95    0.051 .
open           5.119      1.304    3.93 0.000096 ***
uden           0.709      2.518    0.28    0.778
lcab           0.236      0.668    0.35    0.724
cbi            5.169      3.439    1.50    0.133
cwagebrg      -1.292      2.478   -0.52    0.602
wagexcbi      -7.030      4.726   -1.49    0.137
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Alternative Approach: pcse

```
> HF.lm<-lm(ue~GDP_PC+open+uden+lcab+cbi+cwagebrg+wagexcbi,data=HF)
> HF.pcse<-pcse(HF.lm,groupN = HF$country, groupT = HF$year)
> summary(HF.pcse)

 Results:

           Estimate PCSE t value Pr(>|t|)
(Intercept)  -13.58 4.74   -2.87  4.3e-03
GDP_PC         1.60 0.53    3.01  2.7e-03
open           5.12 0.53    9.71  7.0e-21
uden           0.71 0.53    1.35  1.8e-01
lcab           0.24 0.27    0.88  3.8e-01
cbi            5.17 0.85    6.10  1.9e-09
cwagebrg      -1.29 0.77   -1.67  9.6e-02
wagexcbi      -7.03 1.05   -6.68  5.3e-11

 ---------------------------------------------

# Valid Obs = 648; # Missing Obs = 0; Degrees of Freedom = 640.
```

General advice...