PLSC 504 – Autumn 2017
GLS-ARMA

October 26, 2017

Lagged: Y?

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

If ϵ_{it} is perfect...

- $\hat{\beta}_{LDV}$ is biased (but consistent),
- O(bias) $= \frac{-1+3\boldsymbol{\beta}_{LDV}}{T}$

If ϵ_{it} is autocorrelated...

- $\hat{\beta}_{LDV}$ is biased and <u>inconsistent</u>
- IV is one (bad) option...

Lagged Ys and GLS-ARMA

Can rewrite:

$$
\begin{aligned}
Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\
u_{it} &= \phi u_{it-1} + \eta_{it}
\end{aligned}
$$

as

$$
\begin{aligned}
Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\
&= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\
&= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it}
\end{aligned}
$$

where $\psi = \phi\beta_{AR}$ and $\psi = 0$ (by assumption).

Lagged Ys and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{LDV} + \epsilon_{it}$$

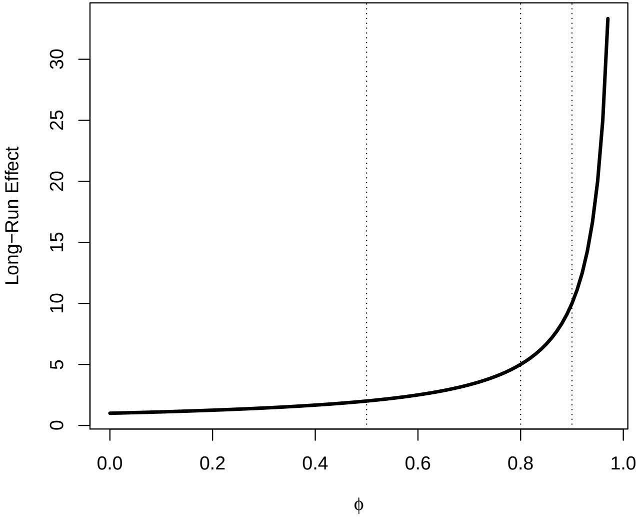
Achen: Bias "deflates" $\hat{\boldsymbol{\beta}}$ relative to $\hat{\phi}$, "suppress" the effects of **X**...

Keele & Kelly (2006):

- Contingent on ϵs having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is*:

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

Long-Run Impact for $\hat{\beta} = 1$



Lagged Ys and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}^*$$

with

$$u_{it}^* = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u_{it}^*) \neq 0.$$

“Nickell” Bias

Bias in $\hat{\phi}$ is

- toward zero when $\phi > 0$,
- increasing in ϕ.

Including unit effects still yields bias in $\hat{\phi}$ of $O(\frac{1}{T})$, and bias in $\hat{\boldsymbol{\beta}}$.

Solutions:

- Difference/GMM estimation
- Bias correction approaches

First Difference Estimation

$$
\begin{aligned}
Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\boldsymbol{\beta} + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\
\Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it}\boldsymbol{\beta} + \Delta u_{it}
\end{aligned}
$$

Anderson/Hsiao: If $\nexists$ autocorrelation, then use ΔY_{it-2} or Y_{it-2} as instruments for ΔY_{it-1}...

- Consistent in theory,
- in practice, the former is preferred, and both have issues if ϕ is high;
- both are inefficient.

A&H $\rightarrow$ A&B

Arellano & Bond (also Wawro): Use *all* lags of Y_{it} and $\mathbf{X}_{it}$ from $t-2$ and before.

- "Good" estimates, better as $T \rightarrow \infty$,
- Easy to handle higher-order lags of Y,
- Easy software (`plm` in R , `xtabond` in Stata).
- Model *is* fixed effects...
- $\mathbf{Z}_i$ has $T-p-1$ rows, $\sum_{i=p}^{T-2} i$ columns $\rightarrow$ difficulty of estimation declines in p, grows in T.

(See notes...)

Bias-Correction Models

Kiviet (1995, 1999; Bun and Kiviet 2003; Bruno 2005a,b): Derive the bias in $\hat{\phi}$ and $\hat{\beta}$, then correct it...

- More accurate than the instrumental-variables/GMM estimators of A&H/A&B...
- ...especially when T is small; but not as T gets reasonably large ($T \approx 20$)

Stationarity: Quick Intro

Mean stationarity:

$$E(Y_t) = \mu \; \forall \; t$$

Variance stationarity:

$$\text{Var}(Y_t) = E[(Y_t - \mu)^2] \equiv \sigma_Y^2 \; \forall \; t$$

Covariance stationarity:

$$\text{Cov}(Y_t, Y_{t-s}) = E[(Y_t - \mu)(Y_{t-s} - \mu)] = \gamma_s \; \forall \; s$$

I(1) Series and Unit Roots

I(1) ("integrated") series:

$$Y_t = Y_{t-1} + u_t$$

vs. AR(1) series:

$$Y_t = \rho Y_{t-1} + u_t$$

or *trending* series:

$$Y_t = \beta t + u_t$$

Differencing:

$$\begin{aligned} \Delta Y_t \equiv Y_t - Y_{t-1} &= Y_t + u_t - Y_{t-1} \\ &= u_t \end{aligned}$$

and

$$\begin{aligned} \Delta Y_t \equiv Y_t - Y_{t-1} &= \beta t + u_t - (\beta(t-1) + u_{t-1}) \\ &= \beta t + u_t - \beta t + \beta - u_{t-1} \\ &= u_t - u_{t-1} + \beta \end{aligned}$$

I(1) series (continued)

More generally:

- $|\rho| > 1$
  - Series is nonstationary / *explosive*
  - Past shocks have a greater impact than current ones
  - Uncommon
- $|\rho| < 1$
  - *Stationary* series
  - Effects of shocks die out exponentially according to ρ
  - Is mean-reverting
- $|\rho| = 1$
  - Nonstationary series
  - Shocks persist at full force
  - Not mean-reverting; variance increases with t

Unit Root Tests: Dickey-Fuller

Two steps:

- Estimate $Y_t = \rho Y_{t-1} + u_t$,
- test the hypothesis that $\hat{\rho} = 0$, *but*
- this requires that the *u*s are uncorrelated.

But suppose:

$$\Delta Y_t = \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

which yields

$$Y_t = Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t.$$

D.F. tests will be incorrect.

Augmented Dickey-Fuller Tests:

- Estimate

$$\Delta Y_t = Y_{t-1} + \sum_{i=1}^{p} d_i \Delta Y_{t-i} + u_t$$

- Test $\hat{\rho} = 0$

Phillips-Perron Tests:

- Estimate:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + u_t$$

- Calculate modified test statistics (Z_ρ and Z_t)
- Test $\hat{\rho} = 0$

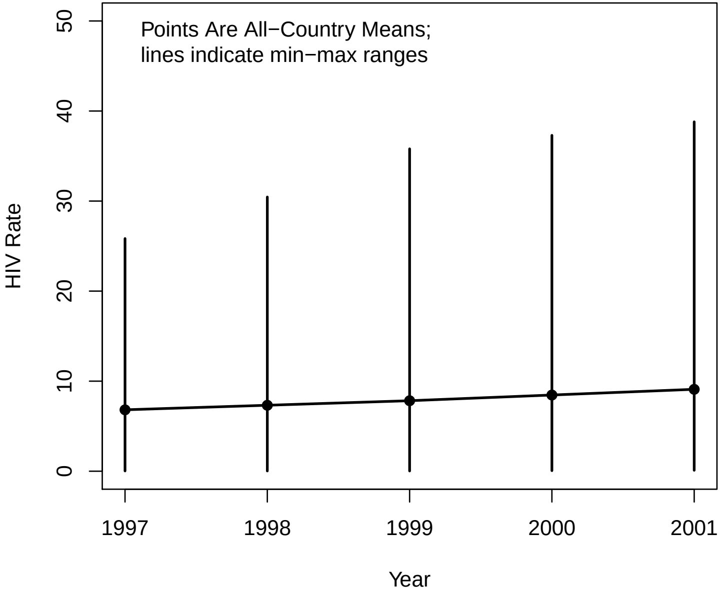
Issues with Unit Roots in Panel Data

- Short series + Asymptotic tests $\rightarrow$ "borrow strength"
- Requires uniform unit roots across *i*s
- Various alternatives:
    - Maddala and Wu (1999)
    - Hadri (2000)
    - Levin, Lin and Chu (2002)
- What to do?
    - Difference the data...
    - Error-correction models

Example: HIV/AIDS in Africa, 1997-2001

```
> summary(AIDS)
     ccode           year          lnAIDS         lnAIDSlag       warlag         popden
 Min.   :404   Min.   :1997   Min.   :-3.8   Min.   :-4   Min.   :0.00   Min.   :0.00
 1st Qu.:451   1st Qu.:1998   1st Qu.: 0.6   1st Qu.: 1   1st Qu.:0.00   1st Qu.:0.01
 Median :506   Median :1999   Median : 1.6   Median : 2   Median :0.00   Median :0.03
 Mean   :510   Mean   :1999   Mean   : 1.2   Mean   : 1   Mean   :0.14   Mean   :0.06
 3rd Qu.:560   3rd Qu.:2000   3rd Qu.: 2.4   3rd Qu.: 2   3rd Qu.:0.00   3rd Qu.:0.07
 Max.   :651   Max.   :2001   Max.   : 3.7   Max.   : 4   Max.   :1.00   Max.   :0.57
                                             NA's   :46
     refsin
 Min.   :  0
 1st Qu.:  1
 Median : 10
 Mean   : 56
 3rd Qu.: 46
 Max.   :543
```

HIV/AIDS in Africa, 1997-2001



Panel Unit Root Tests: R

```
> lnAIDS<-cbind(AIDS$ccode,AIDS$year,AIDS$lnAIDS)
> purtest(lnAIDS,exo="trend",test=c("levinlin"))

Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  lnAIDS
z.x1 = 3e+12, p-value <2e-16
alternative hypothesis: stationarity

> purtest(lnAIDS,exo="trend",test=c("hadri"))

Hadri Test (ex. var.: Individual Intercepts and Trend)

data:  lnAIDS
z = 60, p-value <2e-16
alternative hypothesis: at least one series has a unit root

> purtest(lnAIDS,exo="trend",test=c("ips"))

Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts and Trend)

data:  lnAIDS
z = 4, p-value = 0.0002
alternative hypothesis: stationarity
```

Final Thoughts: Dynamic Panel Models

- N vs. T...
- Are dynamics nuisance or substance?
- What problem(s) do you *really* care about?