

PLSC 504 – Autumn 2017

Dynamic Models

October 26, 2017

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\beta_{LDV} + \epsilon_{it}$$

If ϵ_{it} is perfect...

- $\hat{\beta}_{LDV}$ is biased (but consistent),
- $O(\text{bias}) = \frac{-1+3\beta_{LDV}}{T}$

If ϵ_{it} is autocorrelated...

- $\hat{\beta}_{LDV}$ is biased and inconsistent
- IV is one (bad) option...

Lagged Y s and GLS-ARMA

Can rewrite:

$$\begin{aligned}Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\u_{it} &= \phi u_{it-1} + \eta_{it}\end{aligned}$$

as

$$\begin{aligned}Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\&= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\&= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it}\end{aligned}$$

where $\psi = \phi\boldsymbol{\beta}_{AR}$ and $\psi = 0$ (by assumption).

Lagged Y s and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\beta_{LDV} + \epsilon_{it}$$

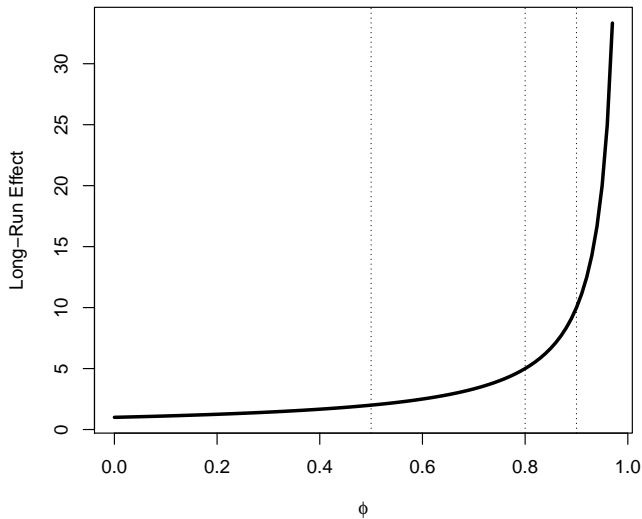
Achen: Bias “deflates” $\hat{\beta}$ relative to $\hat{\phi}$, “suppress” the effects of \mathbf{X} ...

Keele & Kelly (2006):

- Contingent on ϵ s having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is:*

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

Long-Run Impact for $\hat{\beta} = 1$



Lagged Y s and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}^*$$

with

$$u_{it}^* = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u_{it}^*) \neq 0.$$

Bias in $\hat{\phi}$ is

- toward zero when $\phi > 0$,
- increasing in ϕ .

Including unit effects still yields bias in $\hat{\phi}$ of $O(\frac{1}{T})$, and bias in $\hat{\beta}$.

Solutions:

- Difference/GMM estimation
- Bias correction approaches

First Difference Estimation

$$\begin{aligned}Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\beta + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\ \Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it} \beta + \Delta u_{it}\end{aligned}$$

Anderson/Hsiao: If \nexists autocorrelation, then use ΔY_{it-2} or Y_{it-2} as instruments for ΔY_{it-1} ...

- Consistent in theory,
- in practice, the former is preferred, and both have issues if ϕ is high;
- both are inefficient.

Arellano & Bond (also Wawro): Use *all* lags of Y_{it} and \mathbf{X}_{it} from $t - 2$ and before.

- “Good” estimates, better as $T \rightarrow \infty$,
- Easy to handle higher-order lags of Y ,
- Easy software (plm in R , xtabond in Stata).
- Model *is* fixed effects...
- \mathbf{Z}_i has $T - p - 1$ rows, $\sum_{i=p}^{T-2} i$ columns \rightarrow difficulty of estimation declines in p , grows in T .

(See notes...)

Kiviet (1995, 1999; Bun and Kiviet 2003; Bruno 2005a,b): Derive the bias in $\hat{\phi}$ and $\hat{\beta}$, then correct it...

- More accurate than the instrumental-variables/GMM estimators of A&H/A&B...
- ...especially when T is small; but not as T gets reasonably large ($T \approx 20$)

Stationarity: Quick Intro

Mean stationarity:

$$E(Y_t) = \mu \forall t$$

Variance stationarity:

$$\text{Var}(Y_t) = E[(Y_t - \mu)^2] \equiv \sigma_Y^2 \forall t$$

Covariance stationarity:

$$\text{Cov}(Y_t, Y_{t-s}) = E[(Y_t - \mu)(Y_{t-s} - \mu)] = \gamma_s \forall s$$

I(1) Series and Unit Roots

I(1) (“integrated”) series:

$$Y_t = Y_{t-1} + u_t$$

vs. AR(1) series:

$$Y_t = \rho Y_{t-1} + u_t$$

or *trending* series:

$$Y_t = \beta t + u_t$$

Differencing:

$$\begin{aligned}\Delta Y_t \equiv Y_t - Y_{t-1} &= Y_t + u_t - Y_{t-1} \\ &= u_t\end{aligned}$$

and

$$\begin{aligned}\Delta Y_t \equiv Y_t - Y_{t-1} &= \beta t + u_t - (\beta(t-1) + u_{t-1}) \\ &= \beta t + u_t - \beta t + \beta - u_{t-1} \\ &= u_t - u_{t-1} + \beta\end{aligned}$$

I(1) series (continued)

More generally:

- $|\rho| > 1$
 - Series is nonstationary / *explosive*
 - Past shocks have a greater impact than current ones
 - Uncommon
- $|\rho| < 1$
 - *Stationary* series
 - Effects of shocks die out exponentially according to ρ
 - Is mean-reverting
- $|\rho| = 1$
 - Nonstationary series
 - Shocks persist at full force
 - Not mean-reverting; variance increases with t

Unit Root Tests: Dickey-Fuller

Two steps:

- Estimate $Y_t = \rho Y_{t-1} + u_t$,
- test the hypothesis that $\hat{\rho} = 0$, *but*
- this requires that the u s are uncorrelated.

But suppose:

$$\Delta Y_t = \sum_{i=1}^p d_i \Delta Y_{t-i} + u_t$$

which yields

$$Y_t = Y_{t-1} + \sum_{i=1}^p d_i \Delta Y_{t-i} + u_t.$$

D.F. tests will be incorrect.

Augmented Dickey-Fuller Tests:

- Estimate

$$\Delta Y_t = Y_{t-1} + \sum_{i=1}^p d_i \Delta Y_{t-i} + u_t$$

- Test $\hat{\rho} = 0$

Phillips-Perron Tests:

- Estimate:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + u_t$$

- Calculate modified test statistics (Z_ρ and Z_t)
- Test $\hat{\rho} = 0$

Issues with Unit Roots in Panel Data

- Short series + Asymptotic tests \rightarrow “borrow strength”
- Requires uniform unit roots across i s
- Various alternatives:
 - Maddala and Wu (1999)
 - Hadri (2000)
 - Levin, Lin and Chu (2002)
- What to do?
 - Difference the data...
 - Error-correction models

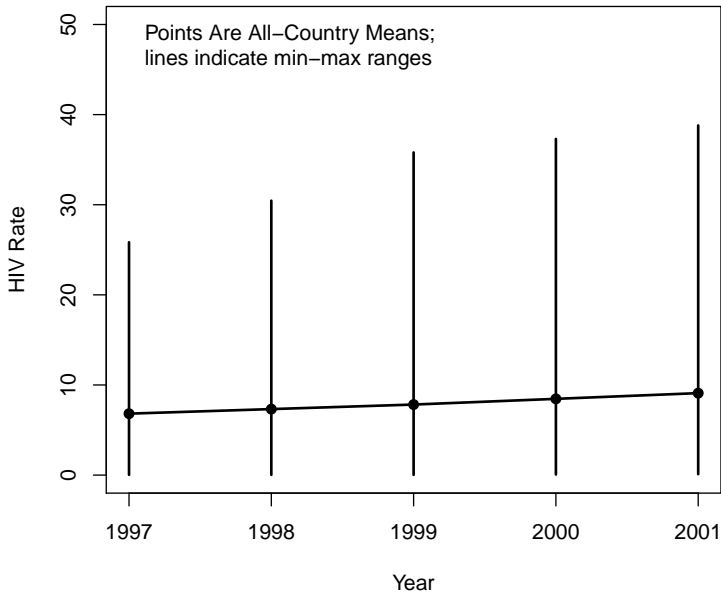
Example: HIV/AIDS in Africa, 1997-2001

```
> summary(AIDS)
```

ccode	year	lnAIDS	lnAIDSlag	warlag	popden
Min. :404	Min. :1997	Min. : -3.8	Min. : -4	Min. :0.00	Min. :0.00
1st Qu.:451	1st Qu.:1998	1st Qu.: 0.6	1st Qu.: 1	1st Qu.:0.00	1st Qu.:0.01
Median :506	Median :1999	Median : 1.6	Median : 2	Median :0.00	Median :0.03
Mean :510	Mean :1999	Mean : 1.2	Mean : 1	Mean :0.14	Mean :0.06
3rd Qu.:560	3rd Qu.:2000	3rd Qu.: 2.4	3rd Qu.: 2	3rd Qu.:0.00	3rd Qu.:0.07
Max. :651	Max. :2001	Max. : 3.7	Max. : 4	Max. :1.00	Max. :0.57

refsin
Min. : 0
1st Qu.: 1
Median : 10
Mean : 56
3rd Qu.: 46
Max. :543

HIV/AIDS in Africa, 1997-2001



Panel Unit Root Tests: R

```
> lnAIDS<-cbind(AIDS$ccode,AIDS$year,AIDS$lnAIDS)
> purtest(lnAIDS,exo="trend",test=c("levinlin"))
```

Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

```
data: lnAIDS
z.x1 = 3e+12, p-value <2e-16
alternative hypothesis: stationarity
```

```
> purtest(lnAIDS,exo="trend",test=c("hadri"))
```

Hadri Test (ex. var.: Individual Intercepts and Trend)

```
data: lnAIDS
z = 60, p-value <2e-16
alternative hypothesis: at least one series has a unit root
```

```
> purtest(lnAIDS,exo="trend",test=c("ips"))
```

Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts and Trend)

```
data: lnAIDS
z = 4, p-value = 0.0002
alternative hypothesis: stationarity
```

Final Thoughts: Dynamic Panel Models

- N vs. T ...
- Are dynamics nuisance or substance?
- What problem(s) do you *really* care about?