PLSC 504 – Autumn 2017
Panel Models for Binary Responses

October 31, 2016

Start with:

$$
\begin{aligned}
Y_{it}^{*} &= \mathbf{X}_{it}\beta + u_{it} \\
Y_{it} &= \begin{cases} 0 \text{ if } Y_{it}^{*} \leq 0 \\ 1 \text{ if } Y_{it}^{*} > 0 \end{cases}
\end{aligned}
$$

which is generically:

$$
Y_{it} = f(\mathbf{X}_{it}\beta + u_{it})
$$

What Can Go Wrong?

Suppose:

$$
\begin{aligned}
X_{it} &= \rho_X \mathbf{X}_{it-1} + \nu_{it} \\
u_{it} &= \rho_u u_{it-1} + \epsilon_{it}
\end{aligned}
$$

For high values of ρ, logit/probit:

- $\hat{\beta}$s are consistent, but s.e.s are biased, inefficient (Poirier and Ruud 1988);
- underestimate Var(β) by up to 50 percent (Beck and Katz 1997).

One-way unit effects:

$$Y_{it} = f(\mathbf{X}_{it}\beta + \alpha_i + u_{it})$$

for logit only, so:

$$\Pr(Y_{it} = 1) = \frac{\exp(\mathbf{X}_{it}\beta + \alpha_i)}{1 + \exp(\mathbf{X}_{it}\beta + \alpha_i)} \equiv \Lambda(\mathbf{X}_{it}\beta + \alpha_i)$$

- Nonlinearity $\rightarrow$ inconsistency in both $\hat{\alpha}$s *and* $\hat{\boldsymbol{\beta}}$.
- Anderson:

$$L^U = \prod_{i=1}^{N} \prod_{t=1}^{T} \Lambda(\mathbf{X}_{it} + \alpha_i)^{Y_{it}} [1 - \Lambda(\mathbf{X}_{it} + \alpha_i)]^{1-Y_{it}}$$

- Chamberlain:

$$L^C = \prod_{i=1}^{N} \Pr\left(Y_{i1} = y_{i1}, Y_{i2} = y_{i2}, ... Y_{iT} = y_{iT} \mid \sum_{t=1}^{T} Y_{it} \right)$$

Intuition:

- $\Pr(Y_{i1} = 0 \text{ and } Y_{i2} = 0 \mid \sum_T Y_{it} = 0) = 1.0$
- $\Pr(Y_{i1} = 1 \text{ and } Y_{i2} = 1 \mid \sum_T Y_{it} = 2) = 1.0$

Fixed-Effects (continued)

More intuition:

$$\Pr\left(Y_{i1} = 0 \text{ and } Y_{i2} = 1 \mid \sum_T Y_{it} = 1\right) = \frac{\Pr(0,1)}{\Pr(0,1) + \Pr(1,0)}$$

with a similar statement for $\Pr(Y_{i1} = 0 \text{ and } Y_{i2} = 1 \mid \sum_T Y_{it} = 1)$.

Points:

- Fixed effects = no estimates for $\boldsymbol{\beta}_b$
- Interpretation: per logit, but $\mid \hat{\alpha}_i$.
- BTSCS in IR: Green et al. (2001) v. B&K (2001).

Random Effects

Model is:

$$\begin{aligned} Y_{it}^* &= \mathbf{X}_{it}\beta + u_{it} \\ Y_{it} &= 0 \text{ if } Y_{it}^* \leq 0 \ ; \\ &= 1 \text{ if } Y_{it}^* > 0 \end{aligned}$$

with:

$$u_{it} = \alpha_i + \eta_{it}$$

with $\eta_{it} \sim$ i.i.d. N(0,1), and $\alpha_i \sim N(0, \sigma_\alpha^2)$.

Random Effects (continued)

Implies:

$$\text{Var}(u_{it}) = 1 + \sigma_\alpha^2$$

and so:

$$\text{Corr}(u_{it}, u_{is},\ t \neq s) \equiv \rho = \frac{\sigma_\alpha^2}{1 + \sigma_\alpha^2}$$

which means that we can write $\sigma_\alpha^2 = \left(\frac{\rho}{1-\rho}\right)$.

Random Effects Variants

Probit:

$$
\begin{aligned}
L_i &= \text{Prob}(Y_{i1} = y_{i1},\ Y_{i2} = y_{i2},\ ...Y_{iT} = y_{iT}) \\
&= \int_{-\infty}^{X_{i1}\beta} \int_{-\infty}^{X_{i2}\beta} ... \int_{-\infty}^{X_{iT}\beta} \phi(u_{i1}, u_{i2}...u_{iT}) du_{iT}...du_{i2}\ du_{i1}
\end{aligned}
$$

Logit:

$$
\begin{aligned}
L_i &= \text{Prob}(Y_{i1} = y_{i1},\ Y_{i2} = y_{i2},\ ...Y_{iT} = y_{iT}) \\
&= \int_{-\infty}^{X_{i1}\beta} \int_{-\infty}^{X_{i2}\beta} ... \int_{-\infty}^{X_{iT}\beta} \lambda(u_{i1}, u_{i2}...u_{iT}) du_{iT}...du_{i2}\ du_{i1}
\end{aligned}
$$

Solution?

$$
\phi(u_{i1}, u_{i2}, ...u_{iT}) = \int_{-\infty}^{\infty} \phi(u_{i1}, u_{i2}, ...u_{iT} \mid \alpha_i)\phi(\alpha_i) d\alpha_i
$$

Practical Things

- $\hat{\rho} =$ proportion of the variance due to the α_is.
- Implementation: Gauss-Hermite quadrature or MCMC.
- Best with N large and T small.
- Critically requires $\text{Cov}(\mathbf{X}, \alpha) = 0$ (see notes re: Chamberlain's CRE Estimator).

Software

R

- `glmmML` (Gauss-Hermite quadrature)
- `pglm` (panel GLMs) (maximum likelihood + quadrature)
- `MCMCpack` (`MCMChlogit`)
- Various user-generated functions (e.g., here).

Stata

- `xtprobit`, `xtlogit`, `xtcloglog`
- Plus `xttrans` (transition probabilities), `quadchk` (quadrature checking), `xtrho` / `xtrhoi` (estimation of within-unit covariances)

Example: Segal (1986) Search & Seizure Cases

$Y = 1$ (search allowed)

- `warrant`: Whether (=1) or not (=0) a warrant was issued,
- `house`: Whether (=1) or not (=0) the search was of a private home,
- `person`: Whether (=1) or not (=0) the search was of a person,
- `business`: Whether (=1) or not (=0) the search was of a business,
- `car`: Whether (=1) or not (=0) the search was of an automobile,
- `us`: Whether (=1) or not (=0) the U.S. government was the petitioner,
- `except`: The number of "exceptions" outlined by the Court under which the search fell, and
- `justideo`: The justice's Segal-Cover (1989) ideology score, ranging from zero (most conservative) to 1 (most liberal).

$N = 14, \ \bar{T} = 74.1.$

Data

```
> summary(Segal)
    justid          caseid           year           vote           warrant
 Min.   : 1.0   Min.   :  1   Min.   :63   Min.   :0.00   Min.   :0.00
 1st Qu.: 6.0   1st Qu.: 34   1st Qu.:69   1st Qu.:0.00   1st Qu.:0.00
 Median : 8.0   Median : 64   Median :73   Median :1.00   Median :0.00
 Mean   : 8.1   Mean   : 64   Mean   :73   Mean   :0.53   Mean   :0.15
 3rd Qu.:11.0   3rd Qu.: 94   3rd Qu.:78   3rd Qu.:1.00   3rd Qu.:0.00
 Max.   :14.0   Max.   :123   Max.   :81   Max.   :1.00   Max.   :1.00
    house          person          business          car            us
 Min.   :0.00   Min.   :0.00   Min.   :0.00   Min.   :0.0   Min.   :0.00
 1st Qu.:0.00   1st Qu.:0.00   1st Qu.:0.00   1st Qu.:0.0   1st Qu.:0.00
 Median :0.00   Median :0.00   Median :0.00   Median :0.0   Median :0.00
 Mean   :0.23   Mean   :0.31   Mean   :0.15   Mean   :0.2   Mean   :0.45
 3rd Qu.:0.00   3rd Qu.:1.00   3rd Qu.:0.00   3rd Qu.:0.0   3rd Qu.:1.00
 Max.   :1.00   Max.   :1.00   Max.   :1.00   Max.   :1.0   Max.   :1.00
    except        justideo
 Min.   :0.00   Min.   :0.05
 1st Qu.:0.00   1st Qu.:0.17
 Median :0.00   Median :0.73
 Mean   :0.35   Mean   :0.59
 3rd Qu.:1.00   3rd Qu.:0.88
 Max.   :3.00   Max.   :1.00
```

```
> SegalLogit<-glm(vote~warrant+house+person+business+car+us+
                   except,data=Segal,family="binomial")
> summary(SegalLogit)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.9802  -1.0391   0.6581   1.1220   1.5798

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)   0.4164     0.2246   1.854  0.06369 .
warrant       0.4818     0.1932   2.493  0.01265 *
house        -1.1742     0.2575  -4.560 5.12e-06 ***
person       -0.7509     0.2387  -3.146  0.00166 **
business     -1.3258     0.2758  -4.807 1.53e-06 ***
car          -0.7541     0.2609  -2.890  0.00385 **
us            0.4077     0.1377   2.961  0.00307 **
except        0.8679     0.1307   6.639 3.17e-11 ***
---
Signif. codes:
0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1434.9  on 1036  degrees of freedom
Residual deviance: 1338.7  on 1029  degrees of freedom
AIC: 1354.7

Number of Fisher Scoring iterations: 4
```

Fixed Effects

```
> library(glmmML)
> SegalFE<-glmmboot(vote~warrant+house+person+business+car+us+
           except,data=Segal,family="binomial",
           cluster=justid)
> summary(SegalFE)

Call:  glmmboot(formula = vote ~ warrant + house + person + business +
        car + us + except, family = "binomial", data = Segal, cluster = justid)


           coef se(coef)     z Pr(>|z|)
warrant   0.599    0.228  2.63  8.7e-03
house    -1.473    0.305 -4.82  1.4e-06
person   -1.124    0.282 -3.99  6.7e-05
business -1.837    0.326 -5.63  1.8e-08
car      -1.202    0.308 -3.90  9.6e-05
us        0.537    0.162  3.32  9.1e-04
except    1.093    0.155  7.03  2.1e-12

Residual deviance: 1050 on 1016 degrees of freedom  AIC: 1090
```

Random Effects

```
> SegalRE<-glmmML(vote~warrant+house+person+business+car+us+
                  except+justideo,data=Segal,family="binomial",
                  cluster=justid)
> summary(SegalRE)

Call:  glmmML(formula = vote ~ warrant + house + person + business +
car + us + except + justideo, family = "binomial", data = Segal, cluster = justid)

              coef se(coef)     z Pr(>|z|)
(Intercept)  2.016    0.565  3.57  3.6e-04
warrant      0.594    0.226  2.63  8.5e-03
house       -1.434    0.303 -4.73  2.2e-06
person      -1.104    0.280 -3.95  7.9e-05
business    -1.799    0.324 -5.56  2.7e-08
car         -1.181    0.306 -3.86  1.1e-04
us           0.531    0.160  3.31  9.3e-04
except       1.070    0.154  6.95  3.6e-12
justideo    -2.344    0.737 -3.18  1.5e-03

Scale parameter in mixing distribution:  0.926 gaussian
Std. Error:                              0.195

        LR p-value for H_0: sigma = 0:  4.63e-24

Residual deviance: 1100 on 1027 degrees of freedom  AIC: 1120
```