Introduction to Bayesian Statistics with R

Day 5

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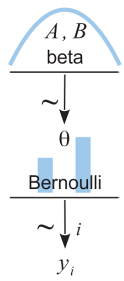
Bayes models

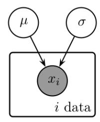
```
# The JAGS model
model
{
  # Likelihood
  for (i in 1:N)
  {
     y[i] ~  dbern( theta )
  }

  # Prior of theta
  theta ~ dbeta(a, b)
  a = 1
  b = 1
}
```



One Sample t-test

One Sample t-test

Model of estimating mean



$\mu \sim \mathrm{Gaussian}(0, 0.001)$
$\sigma \sim \mathrm{Uniform}(0, 10)$
$x_i \sim \mathrm{Gaussian}(\mu, \frac{1}{\sigma^2})$

One sample t-test

Is the mean of x_i different from zero?

$$\mathcal{H}_0 : \mu = 0 \qquad \mathcal{H}_1 : \mu \neq 0$$

or expressed in terms of effect sizes

$$\mathcal{H}_0 : \delta = 0 \qquad \mathcal{H}_1 : \delta \neq 0$$

One Sample t-test

The effect δ in a one sample t-test:

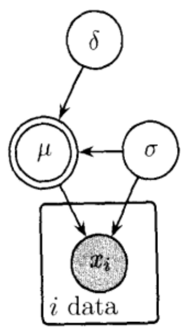
$$\delta = \frac{\mu}{\sigma}$$

thus,

$$\mu = \delta\sigma$$

$\mathcal{H}_0 : \delta = 0$
$\mathcal{H}_1 : \delta \neq 0$

One Sample t-test

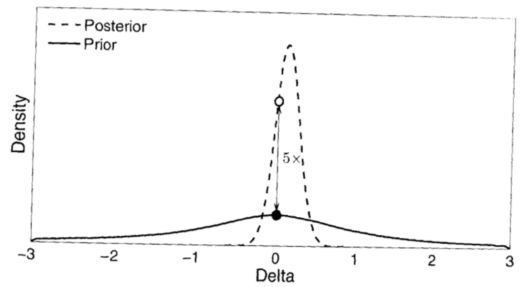


$$\delta \sim \mathrm{Cauchy}(0, 1)$$
$$\sigma \sim \mathrm{Cauchy}(0, 1)_{\mathcal{I}(0,\infty)}$$
$$\mu \leftarrow \delta\sigma$$
$$x_i \sim \mathrm{Gaussian}(\mu, 1/\sigma^2)$$

Estimating Mean

```
model{
  # Data Come From A Gaussian
  for (i in 1:n){
    x[i] ~ dnorm(mu, lambda)
  }

  # Priors
  mu ~ dnorm(100,.001) # plausible for IQ data
  sigma ~ dunif(0,100)
  lambda <- 1/pow(sigma,2)
}
```


Illustration results: Example

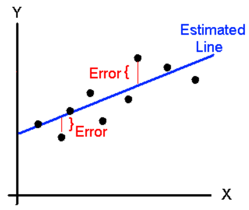


Prior and posterior distributions on effect size δ for the summer and winter data. Markers show the height of the prior and posterior distributions at $\delta = 0$ needed to estimate the Bayes factor between $\mathcal{H}_0 : \delta = 0$ and $\mathcal{H}_1 : \delta \sim \text{Cauchy}(0,1)$ using the Savage–Dickey method.

Linear Regression

Simple Linear Regression

$$\mu_i = \alpha + \beta x_i$$



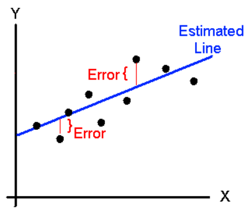
$$y_i = \alpha + \beta x_i + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

or

$$\mu_i = \alpha + \beta x_i$$
$$y_i \sim \mathcal{N}(\mu_i, \sigma_\epsilon^2)$$

Simple Linear Regression

$$\mu_i = \alpha + \beta x_i$$

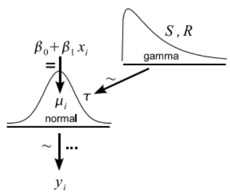


$$y_i = \alpha + \beta x_i + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Bayesian Model: (Likelihood)

$$\mu_i = \alpha + \beta x_i$$
$$y_i \sim \mathcal{N}(\mu_i, \sigma_\epsilon^2)$$

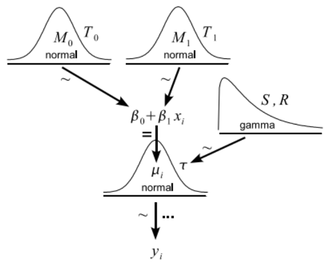
Bayesian Regression Model



JAGS Model: Simple Linear Regression (Likelihood)

```
....

# likelihood
for(i in 1:N)
{
    mu[i] <- alpha + beta * x[i]
    y[i] ~ dnorm(mu[i], lambda)
}

lambda ~ dgamma(0.01, 0.01)

    ....
```

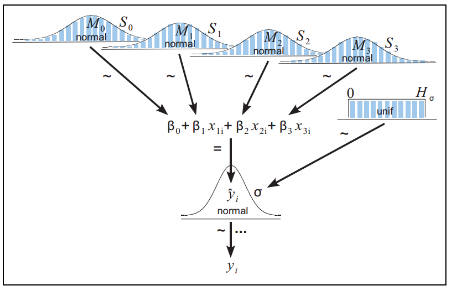

Bayesian Regression Model



JAGS Model: Simple Linear Regression

```
model
{
    # likelihood
    for(i in 1:N)
    {
        mu[i] <- alpha + beta * x[i]
        y[i] ~ dnorm(mu[i], lambda)
    }

    lambda ~ dgamma(0.01, 0.01)

    # priors
    alpha ~ dnorm(0, 0.001)
    beta ~ dnorm(0, 0.001)

    sigma <- 1/sqrt(lambda)
}
```


Multiple Regression



Thank you