Chapter 12.3b Priors

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Chapter 12 Bayesian Multiple Regression and Logistic Models

Priors

- When little prior information is available, can construct a weakly informative prior for the regression coefficients as was done in Chapter 11.
- But when prior information is available, a conditional means prior can be constructed for logistic regression with a single predictor.
- This type of prior was previously constructed in Chapter 11 for a Normal regression problem in the gas bill example.

Conditional means prior

- It is generally difficult to think directly about plausible values of the intercept β_0 and slope β_1 since they are not linearly related to the probabilities.
- A conditional means prior indirectly specifies a prior by constructing priors on the probability values p_1 and p_2 corresponding to two predictor values x_1^* and x_2^*.
- By assuming independence, this implies a prior on the probability vector (p_1^*, p_2^*).
- Since the regression coefficients β_0 and β_1 are functions of the probability values, this process specifies a prior on the vector β.

Conditional means prior

- Consider two values of the predictor x_1^* and x_2^*.
- For the first predictor value x_1^*, construct a Beta prior for the probability p_1^* with shape parameters a_1 and b_1.
- Similarly, for the second predictor value x_2^*, construct a Beta prior for the probability p_2^* with shape parameters a_2 and b_2.
- The joint prior for the vector (p_1^*, p_2^*) has the form

$$\pi(p_1^*, p_2^*) = \pi(p_1^*)\pi(p_2^*).$$

Relationship of p_1 and p_2 with β

- Prior on (p_1^*, p_2^*) implies a prior on the regression coefficient vector (β_0, β_1).
- Write the two conditional probabilities p_1^* and p_2^* as function of the regression coefficient parameters β_0 and β_1 and solve these equations for the β's.

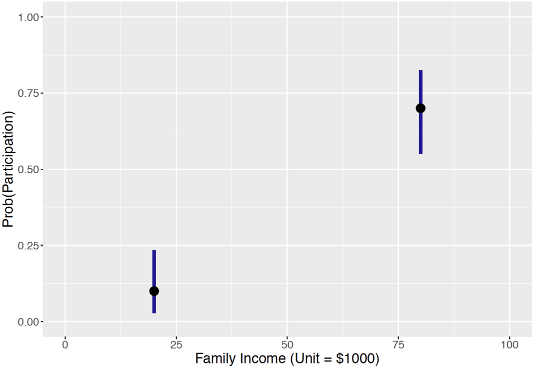
$$\beta_1 = \frac{\mathrm{logit}(p_1^*) - \mathrm{logit}(p_2^*)}{x_1^* - x_2^*}, \beta_0 = \log\left(\frac{p_1^*}{1 - p_1^*}\right) - \beta_1 x_1^*.$$

Example

- Consider two different family incomes (exclusive of the wife's income), say \$20,000 and \$80,000 (predictor is in \$1000 units).
- Consider the labor participation probability p_1^* for the value $x = 20$, corresponding to a \$20,000 family income. Suppose one believes the median of this probability is 0.10 and the 90th percentile is equal to 0.2. This belief is matched to a Beta prior with shape parameters 2.52 and 20.08.
- Next the participation probability p_2^* for the value $x = 80$, corresponding to a \$80,000 family income. The median and 90th percentile of this probability are thought to be 0.7 and 0.8, respectively, and this information is matched to a Beta prior with shape parameters 20.59 and 9.01.

Graph of Conditional Means Prior

- Each bar displays the 90% interval estimate for the participation probability for a particular value of the family income.



Conditional Means Prior

Assuming independence of the prior beliefs about the two probabilities, one represents the joint prior density function for (p_1^*, p_2^*) as the product of densities

$$\pi(p_1^*, p_2^*) = \pi_B(p_1^*, 2.52, 20.08)\pi_B(p_2^*, 20.59, 9.01),$$

where $\pi_B(y, a, b)$ denotes the Beta density with shape parameters a and b.

Corresponding Prior for β

- To simulate pairs (β_0, β_1) from the prior distribution, one simulates values of the means p_1^* and p_2^* from independent Beta distributions and apply the expressions in the previous slide.

- Figure displays a scatterplot of the simulated pairs (β_0, β_1) from the prior.