Chapter 2.2 Equally Likely Outcomes

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Chapter 2 Counting Methods

Introduction

- Assume one writes the sample space in such a way that the outcomes are equally likely.
- Applying the classical interpretation, the probability of each outcome will be

$$\mathit{Prob}(\text{Outcome}) = \frac{1}{\text{Number of outcomes}}. \tag{1}$$

- If one is interested in the probability of some event, then the probability is given by

$$\mathit{Prob}(\text{Event}) = \frac{\text{Number of outcomes in event}}{\text{Number of outcomes}}. \tag{2}$$

An example

- To illustrate the use (and misuse) of this formula, suppose one has a box containing five balls of which three are red, one is blue, and one is white.
- One selects three balls without replacement from the box – what is the probability that all red balls are chosen?
- Let's consider two representations of the sample space of this experiment.

Sample space 1:

- Suppose one does not distinguish between balls of the same color and does not care about the order in which the balls are selected.
- Then if R, B, W denote choosing a red, blue, and white ball respectively, then there are four possible outcomes:

$$S_1 = \{(R, R, R), (R, R, B), (R, R, W), (R, B, W)\}.$$

- If these outcomes in S_1 are assumed equally likely, then the probability of choosing all red balls is

$$Prob(\text{all reds}) = \frac{1}{4}.$$

Sample space 2

- Suppose instead that one distinguishes the balls of the same color, so the balls in the box are denoted by $R1, R2, R3, B, W$.
- Then one writes down ten possible outcomes

$S_2 = \{(R1, R2, R3), (R1, R2, B), (R1, R2, W), (R1, R3, B), (R1, R3, W), (R2, R3, B), (R2, R3, W), (R1, B, W), (R2, B, W), (R3, B, W)\}$.

- If one assumes these outcomes are equally likely, then the probability of choosing all reds is

$$Prob(\text{all reds}) = \frac{1}{10}.$$

Two different answers?

- The problem is that the outcomes in the first sample space S_1 are not equally likely.
- The chance of choosing three reds (R, R, R) is smaller than the chance of choosing a red, blue and white (R, B, W)
- There is only one way of selecting three reds, but there are three ways of selecting exactly one red.
- The outcomes in sample space S_2 are equally likely since the balls were distinguished and it is reasonable that any three of the five balls has the same chance of being selected.

What have we learned?

- When one writes down a sample space, one should think carefully about the assumption that outcomes are equally likely.
- When one has an experiment with duplicate items (like three red balls), it may be preferable to distinguish the items when one writes down the sample space and computes probabilities.