

Chapter 2.7 Playing Yahtzee

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Chapter 2 Counting Methods

Introduction

- ▶ Yahtzee is a popular game played with five dice.
- ▶ The game is similar to the card game poker – in both games, one is trying to achieve desirable patterns in the dice faces or cards.
- ▶ Some of the dice patterns in the first roll in Yahtzee are described .
- ▶ The problem of determining the chances of several of the patterns are considered.

Outcomes of one roll of five dice

- ▶ When a player rolls five dice in the game Yahtzee, the most valuable result is when all of the five dice show the same number such as

2, 2, 2, 2, 2.

- ▶ This is called a “Yahtzee” and the player scores 50 points with this pattern.
- ▶ A second valuable pattern is a “four-of-a-kind” where you observe one number appearing four times, such as

3, 4, 3, 3, 3.

Table gives all of the possible patterns when you roll five dice in Yahtzee.

Pattern	Sample of pattern	Point value
Yahtzee	4, 4, 4, 4, 4	50
Four-of-a-kind	6, 6, 6, 4, 6	
Large straight	2, 6, 4, 5, 3	40
Small straight	4, 2, 1, 3, 2	30
Full house	5, 1, 1, 5, 1	25
Three-of-a-kind	2, 2, 3, 4, 2	
Two pair	6, 3, 3, 6, 2	
One pair	4, 3, 4, 1, 5	
Nothing	1, 3, 2, 5, 6	

Total number of outcomes

- ▶ It is useful to distinguish the five dice when one counts outcomes.
- ▶ One can represent an outcome by placing a value of individual die rolls (1 through 6) in the six slots.

die 1	die 2	die 3	die 4	die 5
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- ▶ So two possible outcomes are

2, 3, 4, 5, 5 and 3, 2, 4, 5, 5.

Number of outcomes

- ▶ Each die has 6 possibilities and so, applying the multiplication rule, the total number of outcomes in the rolls of five dice is

$$6 \times 6 \times 6 \times 6 \times 6 = 7776.$$

- ▶ Since all of the outcomes are equally likely, we assign a probability of $1/7776$ to each outcome.

Probability of a Yahtzee

- ▶ One represents the Yahtzee roll as the outcome

$$x, x, x, x, x$$

where x denotes a roll of one die. There are six possible choices for x , and so the number of possible Yahtzees is 6.

- ▶ Since each outcome has probability $1/7776$, the probability of a Yahtzee is

$$\text{Prob}(\text{Yahtzee}) = \frac{6}{7776}.$$

Probability of four-of-a-kind

- ▶ In the pattern “four of a kind”, one wants to have one number appear four times and a second number appear once.
- ▶ One is interested in counting outcomes of the form

$$x, x, x, x, y$$

where the four x 's and the single y can be in different orders.

Representation of 4-of-a-kind

- ▶ To apply the multiplication rule, think of writing down a possible “four-of-a-kind” in three steps.
- ▶ Step 1: Choose the number for x (the number that appears four times).
- ▶ Step 2: Next choose the number for the singleton y .
- ▶ Step 3: Mix up the orders of the four x 's and the one y .
- ▶ For example, one chooses the outcome 5, 5, 5, 3, 5 by (1) choosing 5 to be the number that appears four times, (2) choosing 3 as the number that appears once, and then arranging the digits 5, 5, 5, 5, 3 to get 5, 5, 5, 3, 5.

Counting outcomes

- ▶ Step 1: There are 6 ways of choosing x .
- ▶ Step 2: Once x has been chosen, there are 5 ways of choosing the value for y .
- ▶ Step 3: Last, once x and y have been selected, there are $\binom{5}{4} = 5$ ways of mixing up the x 's and y 's.
- ▶ Apply the multiplication rule:

Number of ways $= 6 \times 5 \times 5 = 150$.

- ▶ The corresponding probability of four-of-a-kind is

$$Prob(\text{four-of-a-kind}) = \frac{150}{7776}.$$