Chapter 11.5 Posterior Analysis

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Chapter 11 Simple Linear Regression

The linear regression model and priors

- Sampling model:

$$Y_i \mid \mu_i, \sigma \overset{ind}{\sim} \mathrm{Normal}(\mu_i, \sigma) \tag{1}$$

- Weakly informative priors:

$$\beta_0 \sim \mathrm{Normal}(\mu_0, s_0) \tag{2}$$
$$\beta_1 \sim \mathrm{Normal}(\mu_1, s_1) \tag{3}$$

$$\phi = 1/\sigma^2 \sim \mathrm{Gamma}(1, 1) \tag{4}$$

The likelihood function

- The likelihood is the joint density of these observations viewed as a function of $(\beta_0, \beta_1, \sigma)$
- For convenience, the standard deviation σ is reexpressed as the precision $\phi = 1/\sigma^2$ (Chapter 8)

$$
\begin{aligned}
L(\beta_0, \beta_1, \phi) &= \prod_{i=1}^{n} \left[\frac{\sqrt{\phi}}{\sqrt{2\pi}} \exp\left\{ -\frac{\phi}{2}(y_i - \beta_0 - \beta_1 x_i)^2 \right\} \right] \\
&\propto \phi^{\frac{n}{2}} \exp\left\{ -\frac{\phi}{2} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \right\} \qquad (5)
\end{aligned}
$$

The joint posterior

- By multiplying the likelihood by the prior for (β_0, β_1, ϕ), one obtains an expression for the posterior density

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$$
\begin{aligned}
\pi(\beta_0, \beta_1, \phi \mid y_1, \cdots, y_n) \quad &\propto \quad \phi^{\frac{n}{2}} \exp\left\{-\frac{\phi}{2}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2\right\} \\
&\times \quad \exp\left\{-\frac{1}{2s_0^2}(\beta_0 - \mu_0)^2\right\} \\
&\quad\quad \exp\left\{-\frac{1}{2s_1^2}(\beta_1 - \mu_1)^2\right\} \\
&\times \quad \phi^{a-1}\exp(-b\phi) \qquad (6)
\end{aligned}
$$

The joint posterior cont'd

- Since this is not a familiar probability distribution, one needs to use an MCMC algorithm to obtain simulated draws from the posterior
- We will use JAGS