Chapter 5.2 Uniform Distribution

Jim Albert and Monika Hu

Chapter 5 Continuous Random Variables

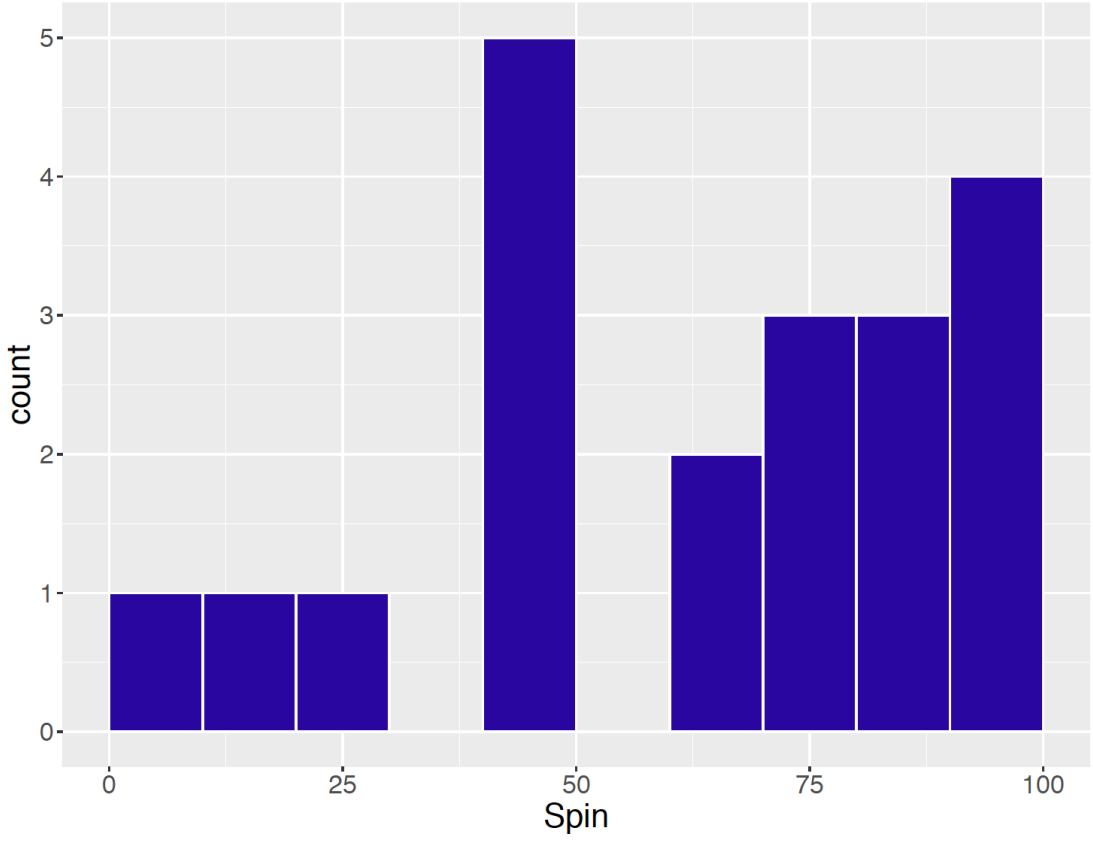
Spinner experiment

- Consider a spinner experiment described in Section 5.1 where the location of the spinner X can be any number between 0 and 100.
- Here are some spins.

95.0	23.1	60.7	48.6	89.1	76.2	45.6	1.9	93.5	91.7
82.1	44.5	61.5	79.2	92.2	73.8	17.6	40.6	41.0	89.4

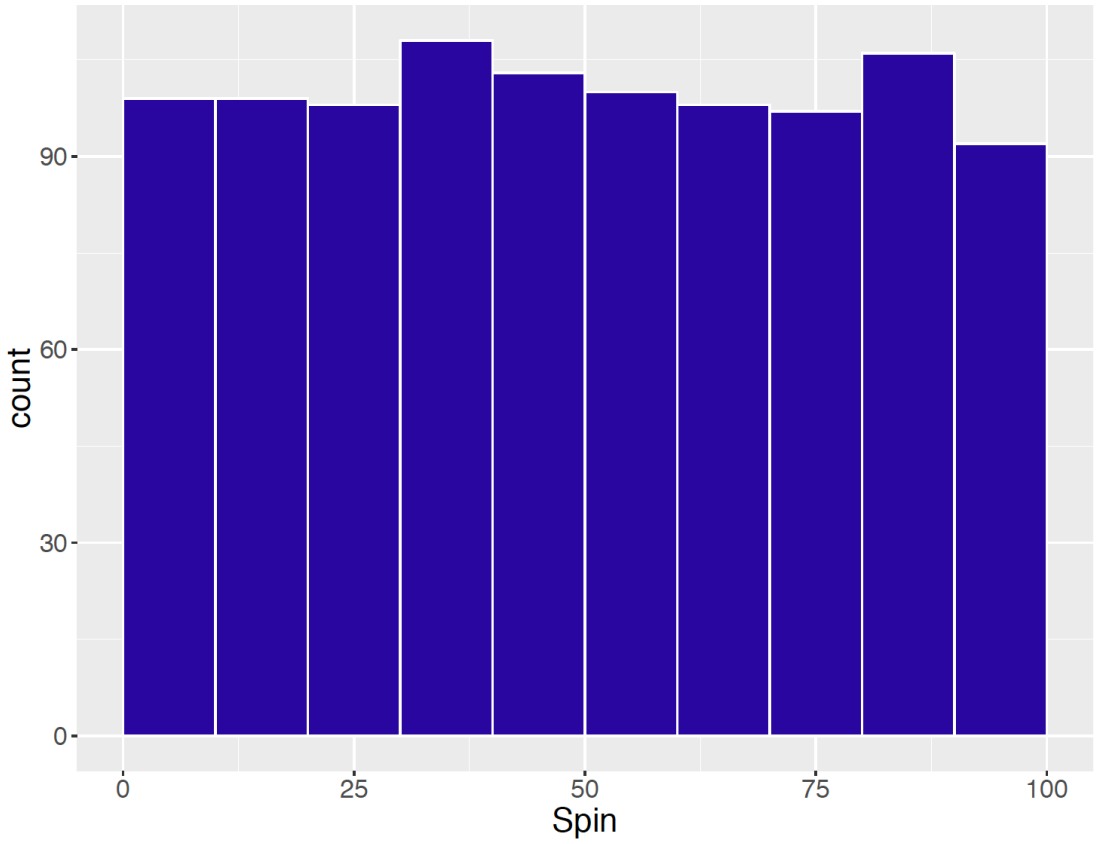
Graph of spins

- A histogram of these values of X is shown below.



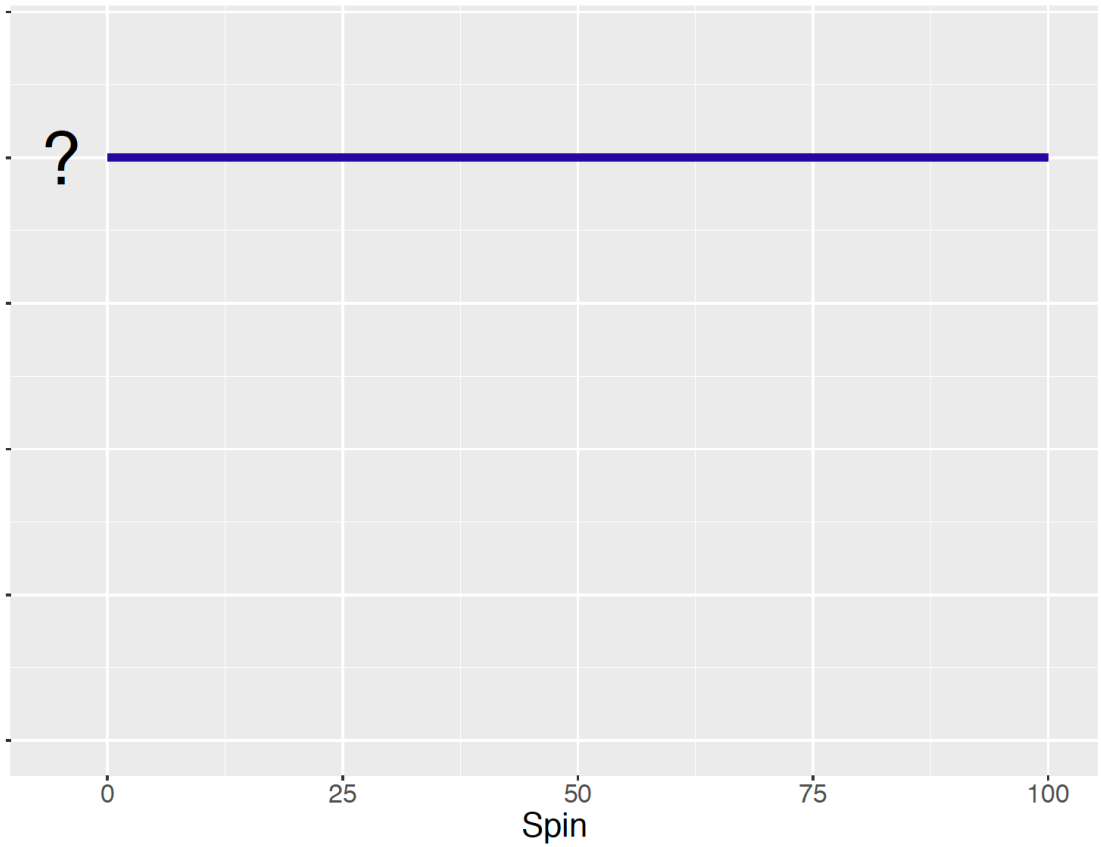
Many spins

- Let's try spinning 1000 times– a histogram of the spins is shown below.



General shape

- The general shape of the histogram seems to be pretty flat or Uniform over the entire interval of possible values of X between 0 and 100.
- If we spun the spinner a zillion times then the shape of the histogram looks close to the Uniform density.



Density curve

- For a continuous random variable X, such as the case of the spinner result here, one represents probabilities by means of a smooth curve that is called a density curve
- When X is continuous, then probabilities are represented by areas under the density curve.

Simple example of finding an area

- What is the chance that the spinner result falls between 0 and 100?
- This probability is represented by the total area under the flat line between 0 and 100.
- Since the area of this rectangle is given by height times base, and the base is equal to 100, the height of this density curve must be $1/100 = 0.01$.

Computing probabilities

- What is the probability the spin falls between 20 and 60?
- This is equal to the shaded area under the Uniform density between 20 and 60. The base is $60 - 20 = 40$ and the height is 0.01, so

$$P(20 < X < 60) = 40(0.01) = 0.4.$$

Here's the picture

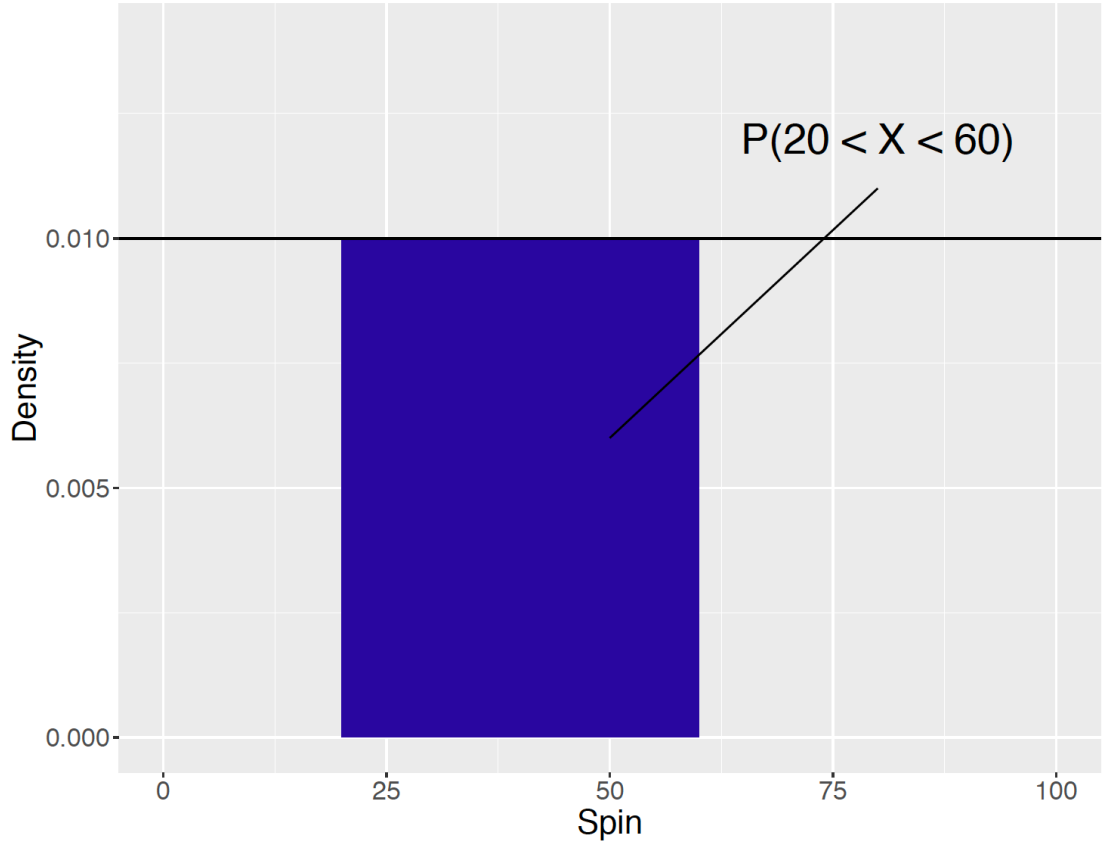


Figure 1: Illustration of finding the probability of $P(20 < X < 60)$.

What is the probability the spin is greater than 80?

By finding the area of the shaded rectangle, we see that $P(X > 80) = 20\ (0.01) = 0.2$.

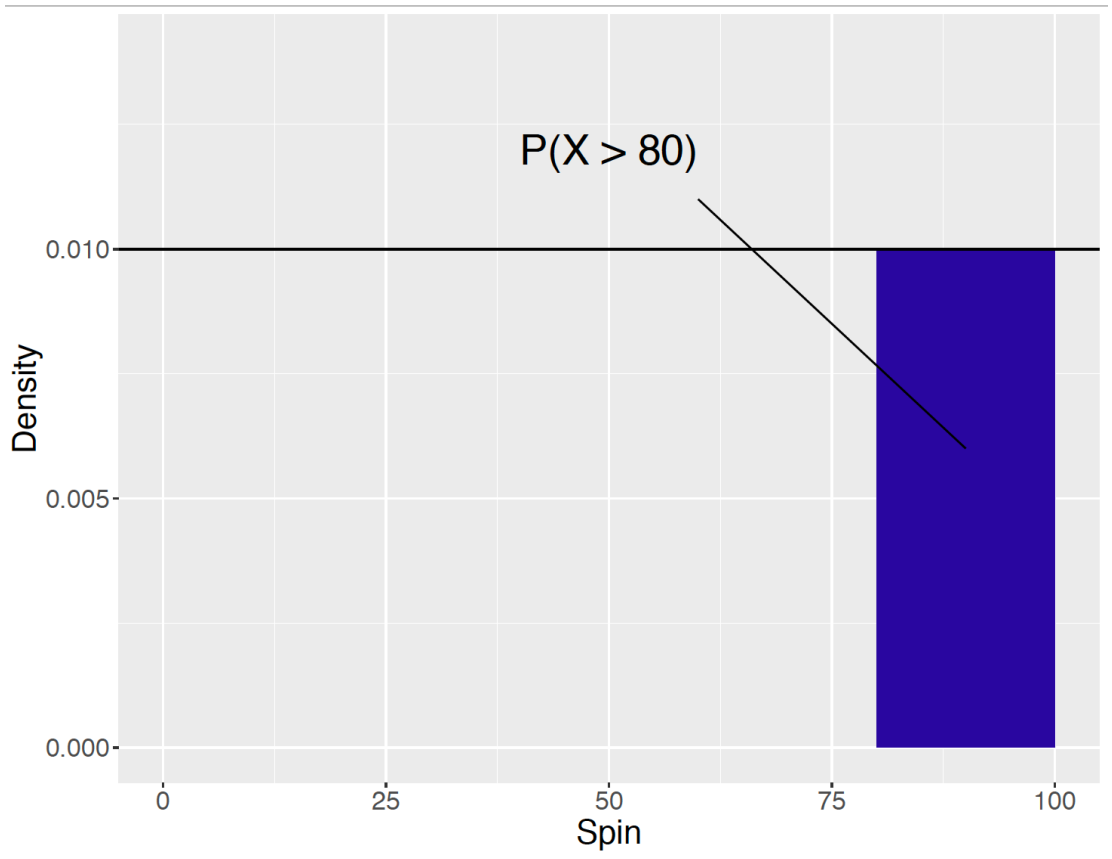


Figure 2: Illustration of finding the probability of $P(X > 80)$.

Summary

- A continuous random variable can take an infinite number of values.
- We represent probabilities by use of a density curve.
- Compute probabilities by finding areas under a density curve.