Chapter 3.2 Independence

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Chapter 3 Conditional Probability

Independent Events

- One says that events A and B are independent if the knowledge of event A does not change the probability of the event B.
- Using symbols

$$P(B \mid A) = P(B).$$

Rolls of Two Dice

- Consider an example where one rolls a red die and a white die – the 36 possible outcomes of rolling the two dice are shown below.

		Red Die					
		One	Two	Three	Four	Five	Six
	One	x	x	x	x	x	x
	Two	x	x	x	x	x	x
White Die	Three	x	x	x	x	x	x
	Four	x	x	x	x	x	x
	Five	x	x	x	x	x	x
	Six	x	x	x	x	x	x

Consider the following three events:

- $S =$ the sum of the two rolls is 7
- $E =$ the red die is an even number
- $D =$ the rolls of the two dice are different

Are events S and E independent?

1. First one finds the probability one rolls a sum equal to 7. There are 36 outcomes and 6 outcomes results in a sum of 7, so $P(S) = 6/36$.
2. Next find $P(S \mid E)$. Given that the red die is an even number (event E), note that there are 18 outcomes where E occurs. Of these 18 outcomes, there are 3 outcomes where the sum is equal to 7. So $P(S \mid E) = 3/18$.
3. Note $P(S \mid E) = P(S)$, so events S and E are independent. Knowing the red die is even does not change one's probability of rolling a 7.

Are events S and D independent?

- To see if these two events are independent, one computes $P(S \mid D)$ and checks if $P(S \mid D) = P(S)$.
- One can show that $P(S \mid D) = 6/30$.
- This probability is not equal to $P(S)$ so S and D are not independent events.