Chapter 1.3 Frequency Viewpoint

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Chapter 1 Probability, A Measure of Uncertainty

Introduction

- The classical view of probability is helpful when we can construct a list of outcomes of the experiment in such a way where the outcomes are equally likely.
- The frequency interpretation of probability can be used in cases where outcomes are equally likely or not equally likely.
- This view of probability is appropriate when one is able to repeat the random experiment many times under the same conditions.

Getting out of jail in Monopoly

- Suppose someone is playing the popular game Monopoly and she lands in jail.
- To get out of jail on the next turn, she either pays $50 or roll “doubles” when she rolls two fair dice.
- What is the probability of rolling doubles when she rolls two dice?

Applying the frequency viewpoint

- The frequency notion can be applied to approximate the probability of rolling doubles.
- Imagine rolling two dice many times under similar conditions.
- Each time two dice are rolled, one observes if she get doubles or not.
- The probability of doubles is approximated by the relative frequency

$$\mathit{Prob}(\text{doubles}) \approx \frac{\text{Number of doubles}}{\text{Number of experiments}}.$$

Rolling two dice using R

- The `two_rolls()` function simulates rolls of a pair of dice and the `replicate()` function repeats this process 1000 times and stores the outcomes in the variable `many_rolls`.

```
two_rolls <- function(){
  sample(1:6, size = 2, replace = TRUE)
}
many_rolls <- replicate(1000, two_rolls())
```

Results of 10 experiments

- The results of the first 10 experiments are shown below. For each experiment, one records if there is a match (YES) or no match (NO) in the two numbers that are rolled.

	Die_1 <int>	Die_2 <int>	Match <chr>
1	5	2	No
2	1	4	No
3	4	6	No
4	4	2	No
5	3	6	No
6	5	4	No
7	4	2	No
8	4	5	No
9	4	5	No
10	5	6	No

After 50 rolls ..

- In the first 50 rolls we observed a match 7 times, so
- $Prob(match) \approx 7/50 = 0.14$.

After 10,000 rolls . . .

- Let's now roll the two dice 10,000 times with R – this time, 1662 matches are observed, so

$$Prob(match) \approx 1662/10000 = 0.1662.$$

- Is 0.1662 the actual probability of getting doubles?
- No, it is still only an approximation to the actual probability. However, as one continues to roll dice, the relative frequency will approach the actual probability of $1/6$.

Obtaining the exact probability

- In this example, one can show that are $6 \times 6 = 36$ equally likely ways of rolling two distinguishable dice
- There are exactly six ways of rolling doubles.
- So using the classical viewpoint, the probability of doubles is $6/36 = 1/6$.