

Chapter 7.1 Introduction - Thinking About A Proportion Subjectively

Jim Albert and Monika Hu

Chapter 7 Learning About a Binomial Probability

Example: 60 balls in a box

- ▶ White balls and red balls
- ▶ Do not know the proportions of color balls
- ▶ Goal: learn the proportion, p , of red balls

Review: three views of a probability

- ▶ The classical view: one needs to write down the sample space where each outcome is equally likely
- ▶ The frequency view: one needs to repeat the random experiments many times under identical conditions
- ▶ The subjective view: one needs to express one's opinion about the likelihood of a one-time event
- ▶ Which one works, why and how?

Possible values of p

- ▶ Proportion can take any possible value between 0 and 1
- ▶ Example 1: one thinks that p is 0.5
 - ▶ i.e. the probability of $p = 0.5$ is 1
 - ▶ could be too strong
- ▶ Example 2: p can take 10 different values, denoted by set A :

$$A = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$$

Probability associated with each possible value of p

Though p can take the same 10 multiple values in both scenarios, we assign different probabilities to each possible value.

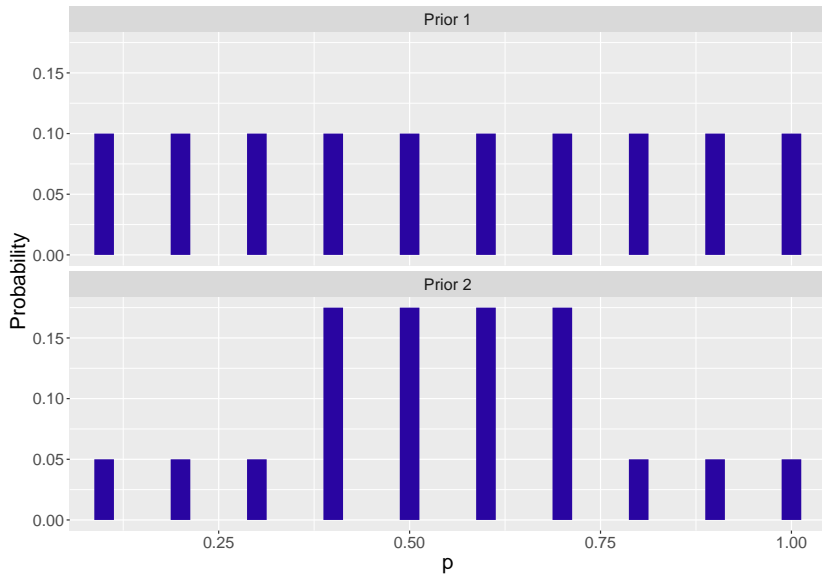
- Scenario 1:

$$f_1(A) = (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)$$

- Scenario 2:

$$f_2(A) = (0.05, 0.05, 0.05, 0.175, 0.175, 0.175, 0.175, 0.05, 0.05, 0.05)$$

Comparing $f_1(A)$ and $f_2(A)$



Comparing $f_1(A)$ and $f_2(A)$ cont'd

- ▶ The probability assignment in $f_1(A)$ is called a discrete uniform distribution
 - ▶ each possible value of the proportion p is equally likely
 - ▶ each value gets assigned a probability of $1/10 = 0.1$
- ▶ The probability assignment in $f_2(A)$ is also discrete, however, not a Uniform distribution
 - ▶ the probabilities of the first three values and last three values are each $1/3.5$ of that of the middle four values
 - ▶ the shape of the bins reflects the opinion that the middle values of p are 3.5 times as likely as the extreme values of p

Review: three probability axioms

- ▶ Both sets of probabilities follow the three probability axioms in Chapter 1.
- ▶ Within each set:
 - ▶ each probability is nonnegative;
 - ▶ the sum of the probabilities is 1;
 - ▶ the probability of mutually exclusive values is the sum of probability of each value

Bayesian inference

- ▶ Step 1: **Prior**: express an opinion about the location of the proportion p before sampling
- ▶ Step 2: **Data/Likelihood**: take the sample and record the observed proportion of red balls
- ▶ Step 3: **Posterior**: use Bayes' rule to sharpen and update the previous opinion about p given the information from the sample