Chapter 8.4 Continuous Priors

Jim Albert and Monika Hu

Chapter 8 Modeling Measurement and Count Data

The Normal prior

- One is interested in learning about the time-to-serve for the tennis player Roger Federer.
- His serving times are Normally distributed with unknown mean μ and known standard deviation $\sigma = 4$.
- One standard approach for representing one's belief about a Normal mean is based on a Normal prior density with mean μ_0 and standard deviation σ_0, that is

$$\mu \sim \mathrm{Normal}(\mu_0, \sigma_0).$$

- The value μ_0 represents one's "best guess" at the mean time-to-serve μ and σ_0 indicates how sure one thinks about the guess.

Two priors

- Consider the opinion of Joe who has strong prior information about the mean.
- His best guess at Federer's mean time-to-serve is 18 seconds so he lets $\mu_0 = 18$.
- He is very sure of this guess and so he chooses σ_0 to be the relatively small value of 0.4.
- In contrast, Kate also thinks that Federer's mean time-to-serve is 18 seconds, but does not have a strong belief in this guess and chooses the large value 2 of the standard deviation σ_0.

Plot of two Normal priors for the mean time-to-serve μ.

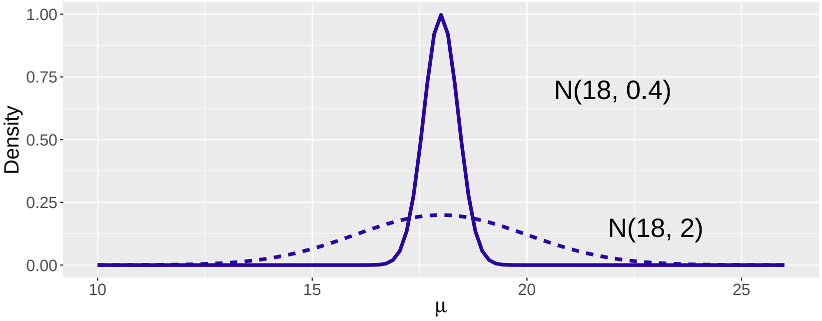

Figure 1: Two priors for the Normal mean μ.

Choosing a informative Normal prior

- Want to choose a Normal prior for μ that reflects prior beliefs about the location of this parameter
- One indirect strategy for choosing for selecting values μ_0 and σ_0 is based on the specification of quantiles.
- On the basis of one's prior beliefs, one specifies two quantiles of the Normal density.
- Then the Normal parameters are found by matching these two quantiles to a particular Normal curve.

Construction of a prior

- Suppose one specifies the 0.5 quantile to be 18 seconds — this means that μ is equally likely to be smaller or larger than 18 seconds.
- Next, one decides that the 0.9 quantile is 20 seconds. This means that one's probability that μ is smaller than 20 seconds is 90%.
- Given values of these two quantiles, the unique Normal curve is found that matches this information.

Normal prior

- The Normal curve with mean $\mu_0 = 18$ and $\sigma_0 = 1.56$, displayed in Figure 8.5, matches the prior information stated by the two quantiles.

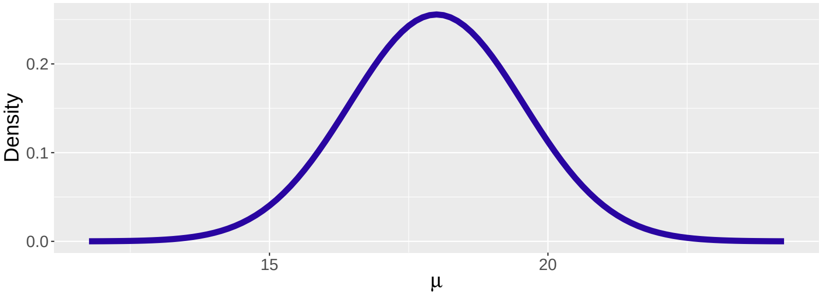

Figure 2: A person's Normal prior for Federer's mean time-to-serve μ.

This prior is just an approximation to one's beliefs about μ

- Recommend in practice that one perform several checks to see if this Normal prior makes sense.
- For example, one finds the 0.25 quantile of our prior using the `qnorm()` function.

```
qnorm(0.25, 18, 1.56)
```

```
## [1] 16.9478
```

- The prior probability that μ is smaller than 16.95 is 25%.
- If this does not seem reasonable, make adjustments the values of the Normal mean and standard deviation until a reasonable Normal prior is found.

Weekly informative prior

- What would a user do in the situation where little is known about the location on μ?
- If one is really unsure about any guess at μ, then one assigns the standard deviation σ_0 a large value.
- Then the choice of the prior mean will not matter, so we suggest using a Normal(0, σ_0) with a large value for σ_0.
- This prior indicates that μ may plausibly range over a large interval and represents weakly informative prior belief about the parameter.
- The posterior inference for μ will largely be driven by the data.