Chapter 1.5 Sample Space

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Chapter 1 Probability, A Measure of Uncertainty

The Sample Space

- A sample space lists all possible outcomes of a random experiment.
- There are different ways to write down the sample space, depending on how one thinks about outcomes.
- Let's illustrate the variety of sample spaces by the simple experiment "roll two fair dice."

Rolling Two Dice

- Each die is the usual six-sided object that we are familiar with, with the numbers 1, 2, 3, 4, 5, 6 on each side.
- One is imagining that each die is constructed such that the six possible numbers are equally likely to come up when rolled.
- What can happen when you roll two dice? The collection of all outcomes that are possible is the sample space.

Sum of the Rolls

- Suppose one is interested in the sum of the numbers on the two dice. What are the possible sums?
- So the sample space, denoted by S, would be

$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

Record Roll on Each Die

- Suppose instead that one wishes to record the rolls on each of the two dice. One possible outcome would be

(4 on one die, 3 on the other die) or (4, 3).

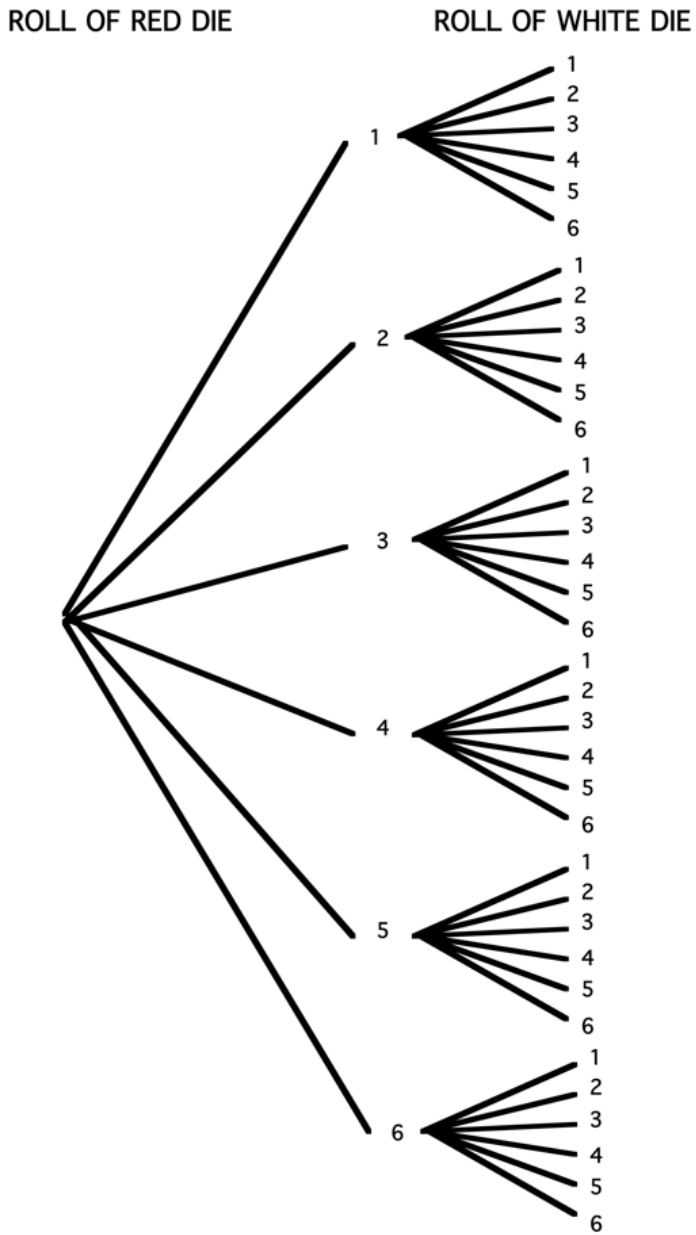
- There are twenty-one possibilities:

(1, 1)	(1, 2)	(1, 3)	(1, 4)
(1, 5)	(1, 6)	(2, 2)	(2, 3)
(2, 4)	(2, 5)	(2, 6)	(3, 3)
(3, 4)	(3, 5)	(3, 6)	(4, 4)
(4, 5)	(4, 6)	(5, 5)	(5, 6)
(6, 6)			

Distinguish the Dice

- What if we distinguished the two dice – perhaps one die is red and one die is white.
- One are considering all of the possible rolls of both dice.
- One way of representing possible rolls of two distinct dice is by a tree diagram shown in the Figure on the next slide.

Tree Diagram



Other Ways of Showing Sample Space

- Write down an outcome by the ordered pair

(roll on white die, roll on red die).

Then table shows the possible outcomes.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Three ways of representing the sample space

- These representations differ by how one records the outcome of rolling two dice.
- One either (1) records the sum of the two dice, (2) records the individual rolls, not distinguishing the two dice, or (3) records the individual rolls, distinguishing the two dice.

Which sample space is best?

- All of the sample spaces shown above are correct. Each sample space represents all possible outcomes of the experiment of rolling two dice.
- But some sample spaces are more convenient than other sample spaces when one wishes to assign probabilities.
- For rolling two fair dice, the sample space with distinguishable dice is desirable from the viewpoint of computing probabilities since the outcomes are equally likely.
- In writing down a sample space, one uses whatever method one likes. The important thing is that one has displayed all of the possible outcomes in S.