

Chapter 2.4 Permutations

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Chapter 2 Counting Methods

Playing some music

- ▶ Suppose one places six songs, Song *A*, Song *B*, Song *C*, Song *D*, Song *E*, and Song *F* in one's playlist on the streaming service.
- ▶ The songs are played in a random order and one listens to the first three songs.
- ▶ How many different selections of three songs can one hear?

Permutations

- ▶ In this example, one is assuming that the order that the songs are played is important. So hearing the selections

Song *A*, Song *B*, Song *C*

in that order will be considered different from hearing the selections in the sequence

Song *C*, Song *B*, Song *A*.

- ▶ An outcome such as this is called a permutation or arrangement of 3 out of the 6 songs.

Representing outcomes

- ▶ One represents possible permutations by a set of three blanks, where songs are placed in the blanks.

1st Song	2nd Song	3rd Song
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Counting number of permutations

- ▶ First, it is known that 6 possible songs can be played first.

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6		
1st Song	2nd Song	3rd Song
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- ▶ If one places a particular song, say Song A, in the first slot, there are 5 possible songs in the second position.

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6	5	
1st Song	2nd Song	3rd Song
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- ▶ By use of the multiplication rule, there are $6 \times 5 = 30$ ways of placing two songs in the first two slots.

Counting permutations

- ▶ Continuing in the same way, one sees that there are 4 ways of putting a song in the third slot and completing the list of three songs.

6	5	4
1st Song	2nd Song	3rd Song

- ▶ Again using the multiplication rule, we see that the number of possible permutations of six songs in the three positions is

$$6 \times 5 \times 4 = 120.$$

Permutations rule

- ▶ If one has n objects (all distinguishable), then the number of ways to arrange r of them, called the number of permutations, is

$${}_nP_r = n \times (n - 1) \times \dots \times (n - r + 1).$$

- ▶ In this example, $n = 6$ and $r = 3$ and ${}_nP_r = 120$.

Computing probabilities

- ▶ If three songs are played in one's playlist, each of the 120 possible permutations will be equally likely to occur.
- ▶ So the probability of any single permutation, say Song *A*, Song *D*, Song *B*

is equal to $1/120$.

Playing all songs

- ▶ Suppose one listens to all six songs on the playlist. How many possible orders are there?
- ▶ One is finding the number of ways of arranging the entire set of 6 objects. Here $n = 6$ and $r = 6$ and, applying the permutation rule formula, the number of permutations is

$${}_6P_6 = n! = 6 \times 5 \times 4 \times \dots \times 1 = 720.$$

- ▶ One uses the special symbol $n!$ to denote the product of the integers from 1 to n . So the number of ways of arranging n distinct objects is

$${}_nP_n = n! = n \times (n - 1) \times (n - 2)$$