Chapter 11.4 A Weakly Informative Prior

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Chapter 11 Simple Linear Regression

Priors: review

- What does prior distribution do?
- What is the relationship between the prior and the posterior?
- What kinds of prior distributions we have used before?
- What prior distribution do you have in mind for the parameters in the simple linear regression model: $(\beta_0, \beta_1, \sigma)$?

When limited prior information is known

- Limited prior information about: the location of the regression parameters or the standard deviation
- To implement the Bayesian approach, we need to assign a prior distribution
- When limited prior information is known, we can assign a prior that has little impact on the posterior distribution: a weakly informative prior

A weakly informative prior: jointly and marginally

- Assume independence among the parameters $(\beta_0, \beta_1, \sigma)$

$$\pi(\beta_0, \beta_1, \sigma) = \pi(\beta_0, \beta_1)\pi(\sigma) \tag{1}$$

- A weakly informative prior on each parameter

Prior on the intercept β_0 and slope β_1

- Assume independence between β_0 and β_1:

$$\pi(\beta_0, \beta_1) = \pi(\beta_0)\pi(\beta_1) \tag{2}$$

- Use a normal prior for each

$$\begin{aligned} \beta_0 &\sim \text{Normal}(\mu_0, s_0) && (3) \\ \beta_1 &\sim \text{Normal}(\mu_1, s_1) && (4) \end{aligned}$$

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- The choice of the standard deviation s_j in the normal prior reflects how confident the person believes in a prior guess of β_j
- If we have little information about the location of a regression parameter: choose a large value for the prior standard deviation s_j, e.g. Normal(0, 100)

Prior on sampling standard deviation σ

$$Y_i \mid \mu_i, \sigma \overset{ind}{\sim} \text{Normal}(\mu_i, \sigma) \tag{5}$$

- σ represents the variability of the house price about the regression line
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- In Chapter 9 and Chapter 10, we a weakly informative prior for the standard deviation σ
- Here, we can also do

$$\phi = 1/\sigma^2 \sim \text{Gamma}(1, 1) \tag{6}$$