Chapter 4.2 Random Variables

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Chapter 4 Discrete Distributions

A Coin Flipping Game

- Suppose that Peter and Paul play a simple coin game. A coin is tossed. If the coin lands heads, then Peter receives \$2 from Paul; otherwise Peter has to pay \$2 to Paul.
- The game is played for a total of five coin flips. After the five flips, what is Peter's net gain (in dollars)?
- There are 32 possibilities for the five flips.

HHHHH	*HTHHH*	*THHHH*	*TTHHH*
HHHHT	*HTHHT*	*THHHT*	*TTHHT*
HHHTH	*HTHTH*	*THHTH*	*TTHTH*
HHHTT	*HTHTT*	*THHTT*	*TTHTT*
HHTHH	*HTTHH*	*THTHH*	*TTTHH*
HHTHT	*HTTHT*	*THTHT*	*TTTHT*
HHTTH	*HTTTH*	*THTTH*	*TTTTH*
HHTTT	*HTTTT*	*THTTT*	*TTTTT*

A Random Variable

- For each possible outcome of the flips, say $HTHHT$, there will be a corresponding net gain for Peter.
- For this outcome, Peter won three times and lost twice, so his net gain is $3(2) - 2(2) = 2$ dollars.
- The net gain is an example of a **random variable**, a number that is assigned to each outcome of the random experiment.
- Use a capital letter G to denote Peter's gain in this experiment.

Assign Random Variable to All 32 Outcomes

$HHHHH, G = 10$	$HTHHH, G = 6$	$THHHH, G = 6$	$TTHHH$
$HHHHT, G = 6$	$HTHHT, G = 2$	$THHHT, G = 2$	$TTHHT$
$HHHTH, G = 6$	$HTHTH, G = 2$	$THHTH, G = 2$	$TTHTH$
$HHHTT, G = 2$	$HTHTT, G = -2$	$THHTT, G = -2$	$TTHTT$
$HHTHH, G = 6$	$HTTHH, G = 2$	$THTHH, G = 2$	$TTTHH$
$HHTHT, G = 2$	$HTTHT, G = -2$	$THTHT, G = -2$	$TTTHT$
$HHTTH, G = 2$	$HTTTH, G = -2$	$THTTH, G = -2$	$TTTTH$
$HHTTT, G = -2$	$HTTTT, G = -6$	$THTTT, G = -6$	$TTTTT$

Comments

- Possible gains for Peter are -10, -6, -2, 2, 6, and 10 dollars.
- Table gives all possible values of G.

Gain g (dollars)	Number of outcomes	$P(G = g)$
-10		
-6		
-2		
2		
6		
10		

Finding Probabilities

- What is the probability that Peter gains \$6 in this game?
- Looking at the table of outcomes, one sees that Peter won \$6 in five of the outcomes. Since each outcome has probability 1/32, the probability of Peter winning \$6 is 5/32.
- This process is continued for all of the possible values of G – obtain a probability distribution for G.

Gain g (dollars)	Number of outcomes	$P(G = g)$
-10	1	1/32
-6	5	5/32
-2	10	10/32
2	10	10/32
6	5	5/32
10	1	1/32