Chapter 5.6 Summarizing a Continuous Random Variable

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Chapter 5 Continuous Random Variables

Introduction

- ▶ Want to summarize a continuous random variable.

- ▶ Natural summaries are given by the mean μ and the standard deviation σ

- ▶ They look like summaries for a discrete random variable, but sums are replaced by integrals.

The Mean

▶ The mean μ, or expected value of X, is given by

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx. \qquad (1)$$

▶ Interpretation: If one observes a large number of values of X, then μ is approximately equal to the sample mean $\bar{X}$ of these random values of X.

The Standard Deviation

▶ To define the spread of the values of X, one computes the average squared deviation about the mean, the variance,

$$\sigma^2 = Var(X) = E(X - \mu)^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx. \quad (2)$$

▶ The standard deviation of X, σ, is defined to be the square root of the variance.

Example

▶ Return to the bus waiting time problem.

▶ Using the definition of f, one gets that the mean is equal to

$$\mu = \int_0^{10} x \left(\frac{3x^2}{1000} \right) dx.$$

▶ Performing the integration, one gets

$$\mu = \int_0^{10} x \left(\frac{3x^2}{1000} \right) dx = \frac{3x^4}{4000} \bigg|_0^{10} = \frac{3(10)^4}{1000} = 7.5.$$

▶ On, the average, one expects the longest wait in a week to be 7.5 minutes.

Computing the standard deviation

▶ The computation of the variance is a bit more tedious.

$$\sigma^2 = \int_0^{10} (x - \mu)^2 \left(\frac{3x^2}{1000} \right) dx = 3.75.$$

▶ The standard deviation of X is $\sigma = \sqrt{3.75} = 1.94$.

A Percentile

► A useful summary of a continuous random variable

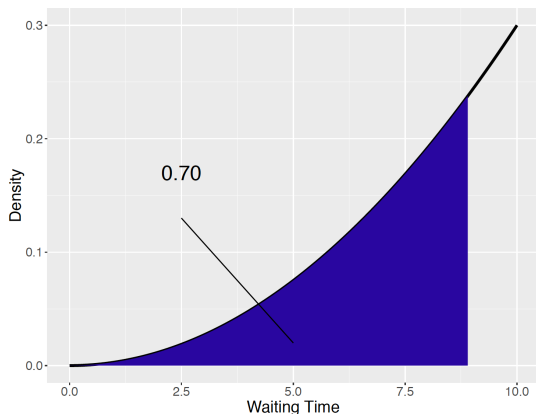► The 70th percentile is the value of X, call it x, such that 70% of the probability is to the left.

Figure 1: Illustration of the 70th percentile.

Finding a Percentile

- ▶ The 70th percentile, call it x_{70}, satisfies the equation

$$P(X \leq x_{70}) = 0.70.$$

- ▶ Since the left hand side of the equation is the cdf F, the equation is written as

$$F(x_{70}) = 0.70,$$

- that is,

$$\frac{x_{70}^3}{1000} = 0.70.$$

Finding a Percentile

▶ To find the 70th percentile, solve

$$\frac{x_{70}^3}{1000} = 0.70.$$

for x_{70}

▶ After some algebra, we get

$$x_{70} = \sqrt[3]{700} = 8.88.$$

▶ This means that if one waits many weeks for this bus, approximately 70% of the longest waiting times will be shorter than 8.88 minutes.