Chapter 2.5 Combinations

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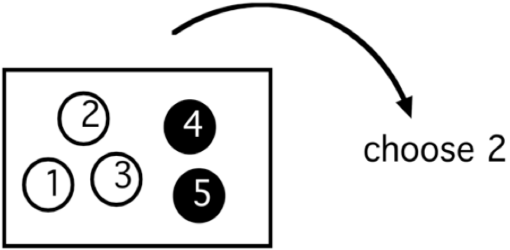
Chapter 2 Counting Methods

Balls in box example

- Suppose one has a box with five balls – three are white and two are black
- One chooses two balls out without replacement from the box

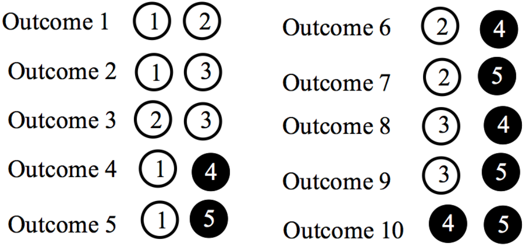


Outcomes

- The five balls have been labeled from 1 to 5 and an outcome is the numbers of the two balls that one selects.
- It is common practice not to consider the order of the selection. Then all that matters is the collection of two balls that we select.
- In this case, one calls the resulting outcome a combination.

Sample space

- When order does not matter, there are 10 possible pairs of balls that one can select. These outcomes or combinations are written below – this list represents a sample space.



Combinations Rule

- Suppose one has n objects and one wishes to take a subset of size r from the group of objects without regards to order.
- Then the number of subsets or combinations is given by the formula

$$\text{number of combinations} = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

- $\binom{n}{r}$ is pronounced "n choose r"

Example

- In our setting, one has $n = 5$ balls and one is selecting a subset of size $r = 2$ from the box of balls.
- Using $n = 5$ and $r = 2$ in the formula, one obtains

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{[2 \times 1] \times [3 \times 2 \times 1]} = \frac{120}{12} = 10.$$

Number of subsets

- Suppose one has a group of n objects and one is interested in the total number of subsets of this group.
- This total number is

$$2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}.$$

- The formula 2^n is found by noticing there are two possibilities for each object – either the object is in the subset or it is not – and then applying the multiplication rule.
- The right hand side of the equation is derived by first counting the number of subsets of size 0, of size 1, of size 2, and so on, and then adding all of these subset numbers.

Counting the number of pizzas

- One is interested in ordering a pizza and there are six possible toppings. How many toppings can there be in the pizza?
- Since there are six possible toppings, one can either have 0, 1, 2, 3, 4, 5, or 6 toppings on our pizza.
- Using combinations rule formula,
- There are $\binom{6}{0}$ pizzas that have no toppings.
- There are $\binom{6}{1}$ pizzas that have exactly one topping.
- There are $\binom{6}{2}$ pizzas that have two toppings.

Counting the number of pizzas

- To compute the total number of different pizzas, one continues in this fashion and the total number of possible pizzas is

$$N = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}.$$

- The reader can confirm that $N = 2^6 = 64$.