Chapter 6.1b Marginal PMFs

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Chapter 6 Joint Probability Distributions

Introduction

- Once a joint probability mass function for (X, Y) has been constructed, one can find probabilities for one of the two variables.
- In our balls example, suppose one wants to find the probability that exactly three red balls are chosen, that is $P(X = 3)$.
- Find this probability by summing values of the pmf $f(x, y)$ where $x = 3$ and y can be any possible value of the random variable Y.

$$\begin{aligned} P(X = 3) &= \sum_{y} f(3, y) \\ &= f(3, 0) + f(3, 1) + f(3, 2) \\ &= \frac{3}{252} + \frac{12}{252} + \frac{6}{252} \end{aligned}$$

Repeat this calculation for all X values

- Repeat this operation is done for each of the possible values of X.
- The *marginal* probability mass function of X, $f_X()$ is defined as follows:

$$f_X(x) = \sum_y f(x, y). \tag{1}$$

- One finds this marginal pmf of X from the joint pmf table by summing the joint probabilities for each row of the table.

The Marginal pmf for X

- One obtains ...

x	$f_X(x)$
0	21/252
1	105/252
2	105/252
3	21/252

- Note that the marginal pmf of X is a legitimate probability function in that the values are nonnegative and the probabilities sum to one.

Marginal pmf of Y

- One can also find the marginal pmf of Y, denoted by $f_Y()$, by a similar operation.
- For a fixed value of $Y = y$ one sums over all of the possible values of X.

$$f_Y(y) = \sum_x f(x, y).$$

Example

- To find $f_Y(2) = P(Y = 2)$ in our example, one sums the joint probabilities in the table over the rows in the column where $Y = 2$. One obtains the probability:

$$f_Y(2) = \sum_x f(x, 2)$$

$$= f(0,2) + f(1,2) + f(2,2) + f(3,2)$$

$$= \frac{6}{252} + \frac{54}{252} + \frac{54}{252} + \frac{6}{252}$$
$$= \frac{120}{252}.$$

Marginal pmf of Y

- By repeating this exercise for each value of Y, one obtains the marginal pmf displayed in Table 6.3.

y	$f_Y(y)$
0	$6/252$
1	$60/252$
2	$120/252$
3	$60/252$
4	$6/252$

- Given a table of a joint pmf of (X, Y), one can always find the marginal pmfs of X and Y.