Math 3904
lecture 6 2/13

$y \in \mathbb{R}$

$p = 1$

$\mathcal{H} = \{ \vec{w} \cdot \vec{x} : \vec{w} \in \mathbb{R}^{p+1} \}$ all linear models

$\vec{w} \cdot \vec{x} = w_0 + w_1 x_1$

$\vec{x} = [1 \ \vec{x}]$

$A_{b_0 b_1} = \underset{\vec{w} \in \mathbb{R}^{p+1}}{\operatorname{argmin}} \left\{ \sum_{i=1}^{n} (y_i - \vec{w} \cdot \vec{x}_i) \right\}$

using algorithms...

$b_0 = \bar{y} - b_1 \bar{x}$

$b_1 = \dfrac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$

var(X) estimated by s^2_x

$s^2_x = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$= \frac{1}{n-1} (\sum x_i^2 - n\bar{x}^2)$

For 2 r.v's X, Y

correlation: $\operatorname{cov}[X,Y] \Rightarrow \dfrac{\operatorname{cov}[X,Y]}{\sqrt{\operatorname{var}(X)\operatorname{var}(Y)}}$

covariance : $\operatorname{cov}(X,Y) \Rightarrow E[(X - m_x)(Y - m_y)]$

correlation : $\operatorname{corr}[X,Y] \Rightarrow r = \dfrac{S_{xy}}{\sqrt{S_x^2 S_y^2}} = \dfrac{S_{xy}}{S_x S_y}$

$S_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$

$= \frac{1}{n-1} (\sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y})$

$b_0 = \bar{y} - b_1 \bar{x}$

$b_1 = \dfrac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = r \dfrac{s_y}{s_x}$

$b_0 = \bar{y} - b_1 \bar{x}$

$\downarrow$

$\bar{y} - r \dfrac{s_y}{s_x} \bar{x}$

$A^{D} \Rightarrow g(x) = b_0 + b_1 x$ (how we predict)