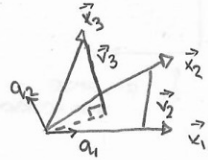
Math 390.4
lecture 12 3/5

Q-R decomposition

$X = QR$

1) Gram Schmidt
2) compute R from X & Q




let $X = [\vec{x}_1 \mid \vec{x}_2 \mid \vec{x}_3 \ldots ]$

we first must find an orthogonal basis for colsp $[X]$

$\vec{v}_1 = \vec{x}_1$

$\vec{v}_2 = \vec{x}_2 - \text{proj}_{\vec{v}_1}(\vec{x}_2)$

$\vec{v}_3 = \vec{x}_3 - \text{proj}_{[\vec{v}_1 | \vec{v}_2]}(\vec{x}_3)$ OR $\vec{v}_3 = \vec{x}_3 - (\text{proj}_{\vec{v}_1}(x_3) + (\text{proj}_{\vec{v}_2}(x_3))$ *normalize: divide by the norm (gives orthonormal basis)

$\vdots \qquad \vdots$

$\vec{v}_j = \vec{x}_j - \sum_{i=1}^{j-1} \text{proj}_{v_i}(x_j)$

* $Q_i$ must span $X_i$

$X = QR$

$[\vec{x}_1 | \vec{x}_2 | \vec{x}_3 \ldots ] = [\vec{Q}_1 | \vec{Q}_2 | \vec{Q}_3 \ldots ] \begin{bmatrix} a & b & c & \ldots \\ 0 & d & e & \ldots \\ 0 & 0 & f & \ldots \\ 0 & 0 & 0 & \ldots \\ & & & \ldots \end{bmatrix}$

$\vec{x}_1 = a\vec{Q}_1 = \|v_1\|$

$\vec{x}_2 = b\vec{Q}_1 + d\vec{Q}_2$ * $\vec{x}_2 \in \text{span}\{\vec{Q}_1, \vec{Q}_2\}$

$\vec{x}_2 = \text{proj}_{[q_1|q_2]}(\vec{x}_2) = \vec{x}_2 = \text{proj}_{\vec{Q}_1}(\vec{x}_2) + \text{proj}_{\vec{Q}_2}(\vec{x}_2)$

$H_1\vec{x}_2 + H_2\vec{x}_2$

$\underbrace{q_1 q_1^T \vec{x}_2}_{} + q_2 \underbrace{q_2^T \vec{x}_2}_{}$

$b\vec{q}_1 + d\vec{q}_2$

$\|v_1\| = Q_1^T x_1$

$\begin{bmatrix} Q_1^T x_1 & Q_1^T x_2 & \\ 0 & Q_2^T x_2 & \ldots \\ \vdots & 0 & \\ 0 & \vdots & \\ & 0 & \end{bmatrix}$

$X = QR$, $R = \begin{cases} \vec{q}_i^{\,T} \vec{x}_i & \text{if } i \le j \\ 0 & \text{o/w} \end{cases}$

dimension reduction
chance capitalization
capitalizing on a non 90° angle


SSR : size of proj. onto the colsp

$= \|\text{proj}_X(y)\|^2 - n\bar{y}^2$

∴ X expressed as Q

$= \|\text{proj}_Q(y)\|^2 - n\bar{y}^2$

$H = X(X^TX)^{-1}X^T$

$= QQ^T$

$\hat{\vec{y}} = H\vec{y} \approx QQ^T\vec{y}$

$\approx X(X^TX)^{-1}X^T\vec{y}$

$b = (X^TX)^{-1}X^T\vec{y}$

$= (Q^TQ)^{-1}Q^T\vec{y}$

$= Q^T\vec{y}$

Generalization error:
error when using your model on
$\vec{x} \notin D$
ie. in the formula

can't trust $R^2$ or RMSE (nor SSE/SST/SSR)

$D \xrightarrow{A} g \xrightarrow{D} R^2$, RMSE, etc (not trustworthy)

$\hookrightarrow^{D^*}$ $R^2$, RMSE (trust worthy)

$D^*$: future data

$H = X(X^TX)^{-1}X \approx I$
iff rank$(X) = n$

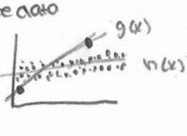
$\frac{D \text{ past}}{D_* \text{ future}} \downarrow$ time

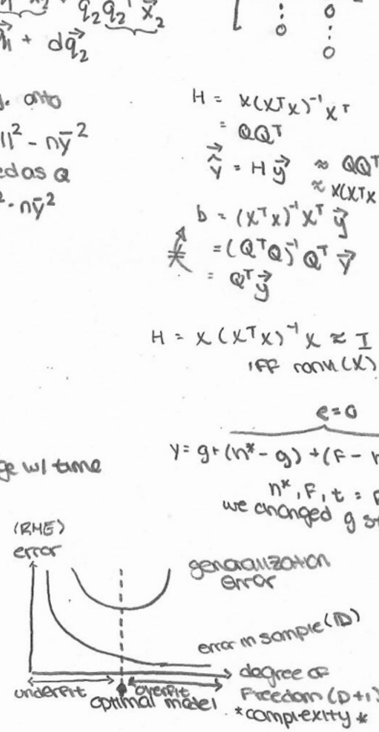
we assumed "stationary":
$z$'s, $t$, $f$, $h^*$, $y$, $\vec{x}$'s do not change w/ time

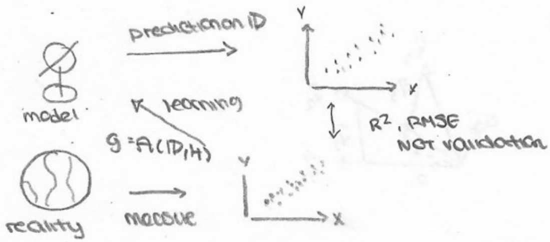
$y = g + \overbrace{(h^* - g) + (f - h^*) + (t - f)}^{e=0}$

$h^*$, $f$, $t$ = fixed
we changed $g$ st $e$ is 0


future data



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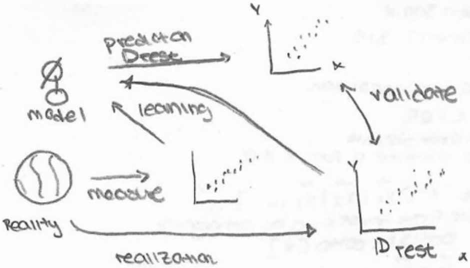


out of sample validation

OOS $R^2$ & RMSE

$$\mathbb{D} = \underbrace{\mathbb{D}_{train}}_{g = \mathcal{A}(\mathbb{D}_{train}, \mathcal{H})} \cup \mathbb{D}_{test}$$

$\mathbb{D}_{test}$ → stationary future

Don't let A peek inside set

use it as validation → generalization error