If DGP is iid and finite variance sigsq and thetahat = Xbar

If thetahat is an MLE

$SE[\hat\theta] = \frac{\sigma}{\sqrt{n}}$, $\hat{SE}[\hat\theta] = \frac{s}{\sqrt{n}}$

CLT implies asymptotic normality

asymptotic normality and efficiency of MLE thm. $SE[\hat\theta] = \sqrt{\frac{I(\theta)^{-1}}{n}}$, $\hat{SE}[\hat\theta] = \sqrt{\frac{I(\hat\theta^{MLE})^{-1}}{n}}$

$$\frac{\hat\theta - \theta}{SE[\hat\theta]} \xrightarrow{d} N(0,1)$$

def of $\xrightarrow{d}$

$$\hat\theta \overset{\cdot}{\sim} N(\theta, SE[\hat\theta]^2)$$

$$RET \approx [\theta \pm z_{1-\frac{\alpha}{2}} SE[\hat\theta]]$$

continuous mapping thm + Slutsky's thm.

$$\frac{\hat\theta - \theta}{\hat{SE}[\hat\theta]} \xrightarrow{d} N(0,1)$$

def of $\xrightarrow{d}$ and more

$$\hat\theta \overset{\cdot}{\sim} N(\hat\theta, \hat{SE}[\hat\theta]^2)$$

$$CI_{\theta,1-\alpha} \approx [\hat\theta \pm z_{1-\frac{\alpha}{2}} \hat{SE}[\hat\theta]]$$

first order Taylor series approx of the function g (the " delta method")

$$\frac{g(\hat\theta) - g(\theta)}{|g'(\theta)|\, SE[\hat\theta]} \xrightarrow{d} N(0,1)$$

$$g(\hat\theta) \overset{\cdot}{\sim} N(g(\theta), g'(\theta)^2 SE[\hat\theta]^2)$$

$$RET \approx [g(\theta) \pm z_{1-\frac{\alpha}{2}} |g'(\theta)|\, SE[\hat\theta]]$$

continuous mapping thm + Slutsky's thm.

$$\frac{g(\hat\theta) - g(\theta)}{|g'(\hat\theta)|\, \hat{SE}[\hat\theta]} \xrightarrow{d} N(0,1)$$

def of $\xrightarrow{d}$ and more

$$g(\hat\theta) \overset{\cdot}{\sim} N(g(\hat\theta), g'(\hat\theta)^2 \cdot \hat{SE}[\hat\theta]^2)$$

$$CI_{g(\theta),1-\alpha} \approx [g(\hat\theta) \pm z_{1-\frac{\alpha}{2}} |g'(\hat\theta)| \cdot \hat{SE}[\hat\theta]]$$

HW 5 7(a)

$\theta_{YW} := P(\text{yellow \& wrinkled})$, $\theta_{YR} = P(\text{yellow \& round})$, θ_{GW}, θ_{GR}

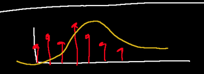
$\to H_0: \theta_{YR} = 9/16,\ \theta_{YW} = 3/16,\ \theta_{GR} = 3/16,\ \theta_{GW} = 1/16$

H_a: at least one of the equalities above is wrong

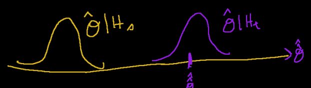
$H_0: \theta_{YR} = \theta_Y\theta_R,\ \theta_{YW} = \theta_Y\theta_W,\ \theta_{GR} = \theta_G\theta_R,\ \theta_{GW} = \theta_G\theta_W$

H_a: at least one of the equalities above is wrong

$H_0: F = G\frac{m_1 m_2}{r^2}$



N is a cont rv $\quad Supp[N] = \{0, 1, 2, \ldots\} \neq \mathbb{R}$



$\Rightarrow$ p value infinitesimal

(h)

$$CI_{\theta_1-\theta_2,1-\alpha} \approx \left[\hat\theta_1 - \hat\theta_2 \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$$

(f)

$$CI_{\theta_1-\theta_2,1-\alpha} = \left[\hat\theta_1 - \hat\theta_2 \pm t_{1-\frac{\alpha}{2}, n_1+n_2-2}\, s_{pooled} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right]$$

(g)

$$CI_{\theta_1-\theta_2,1-\alpha} \approx \left[\hat\theta_1 - \hat\theta_2 \pm t_{1-\frac{\alpha}{2}, df} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$$

Satterthwaite's approx.

5j $\quad \phi = g(\theta_1, \theta_2) = \frac{\theta_1}{\theta_2} \Rightarrow \frac{\partial g}{\partial \theta_1} = \frac{1}{\theta_2},\ \frac{\partial g}{\partial \theta_2} = -\frac{\theta_1}{\theta_2^2}$

$$\frac{g(\hat\theta_1, \hat\theta_2) - g(\theta_1, \theta_2)}{\sqrt{\left(\frac{\partial g}{\partial \theta_1}\right)^2 Var[\hat\theta_1] + \left(\frac{\partial g}{\partial \theta_2}\right)^2 Var[\theta_2]}} \xrightarrow{d} N(0,1)$$

$\hat\theta_1 = \bar X_1$, $\hat\theta_2 = \bar X_2$

$$\frac{\frac{\hat\theta_1}{\hat\theta_2} - \frac{\theta_1}{\theta_2}}{\sqrt{\frac{1}{\theta_2^2}\frac{\sigma_1^2}{n_1} + \frac{\theta_1^2}{\theta_2^4}\frac{\sigma_2^2}{n_2}}} \xrightarrow{d} N(0,1)$$

Thm in 2(a) &

Thm 5.5.4 & Slutsky's

$$\frac{\frac{\hat\theta_1}{\hat\theta_2} - \frac{\theta_1}{\theta_2}}{\sqrt{\frac{1}{\hat\theta_2^2}\frac{s_1^2}{n_1} + \frac{\hat\theta_1^2}{\hat\theta_2^4}\frac{s_2^2}{n_2}}} \xrightarrow{d} N(0,1)$$

$$CI_{\frac{\theta_1}{\theta_2},1-\alpha} \approx \left[\frac{\hat\theta_1}{\hat\theta_2} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{1}{\hat\theta_2^2}\frac{s_1^2}{n_1} + \frac{\hat\theta_1^2}{\hat\theta_2^4}\frac{s_2^2}{n_2}}\right]$$

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iid $N(0, \theta)$. Find $\hat\theta^{MLE}$ $\quad \boxed{\text{HW 4 2(d)}}$

$$\mathcal{L}(\theta; x_1, \ldots x_n) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{1}{2\theta}x_i^2} = (2\pi\theta)^{-n/2} e^{-\frac{1}{2\theta}\underbrace{\sum x_i^2}_{Q}}$$

$$\ell(\theta; x_1, \ldots, x_n) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\theta) - \frac{1}{2\theta}Q$$

$$\ell' = -\frac{n}{\cancel{2}}\frac{1}{\cancel{\theta}} + \frac{1}{\cancel{2}\theta^{\cancel{2}}}Q \overset{set}{=} 0 \Rightarrow \frac{Q}{\theta} = n \Rightarrow \hat\theta^{MLE} = \frac{\sum x_i^2}{n}$$

$\boxed{2(e)}$ $\quad Var[\hat\theta^{MLE}] = \frac{I(\theta)^{-1}}{n}$ if $\hat\theta^{MLE}$ is UMVUE

$$\ell'' = \frac{n}{2}\frac{1}{\theta^2} - \frac{1}{\theta^3}Q$$

$$I(\theta) = E[-\ell''] = E\left[-\frac{1}{2}\cdot\frac{1}{\theta^2} + \frac{1}{\theta^3}X^2\right] = -\frac{1}{2}\frac{1}{\theta^2} + \frac{1}{\theta^3}E[X^2]$$

$E[X^2] = \theta$

$$= -\frac{1}{2}\frac{1}{\theta^2} + \frac{\cancel{\theta}^{1}}{\cancel{\theta^3}^{2}}$$

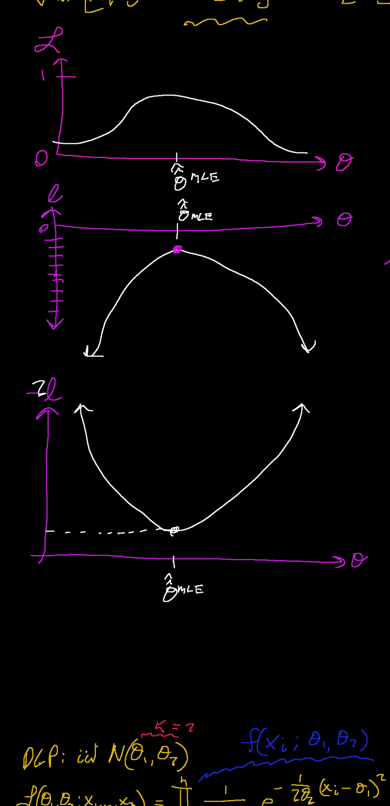
$$= \frac{1}{\theta^2}\left(1 - \frac{1}{2}\right) = \frac{1}{2\theta^2}$$

CRLB variance = $\frac{2\theta^2}{n}$

$$Var[\hat\theta^{MLE}] = Var\left[\frac{1}{n}\sum X_i^2\right] = \frac{1}{n^2}\sum Var[X_i^2]$$

$$= \frac{1}{n^2}\sum 2\theta^2 = \frac{2n\theta^2}{n^2} = \frac{2\theta^2}{n} \Rightarrow \hat\theta^{MLE} \text{ is the UMVUE}$$

$$Var[X_i^2] = E[X_i^4] - E[X_i^2]^2 = 3\theta^2 - \theta^2 = 2\theta^2$$



$\ell(\hat\theta^{MLE}; x_1, \ldots, x_n)$

$AIC = -2\ell + K$

DGP: iid $N(\underbrace{\theta_1, \theta_2}_{K=2})$

$$\mathcal{L}(\theta_1, \theta_2; x_1, \ldots, x_n) = \prod_{i=1}^n \underbrace{\frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2\theta_2}(x_i - \theta_1)^2}}_{f(x_i; \theta_1, \theta_2)} = (2\pi\theta_2)^{-n/2} e^{-\frac{1}{2\theta_2}\sum_{i=1}^n (x_i - \theta_1)^2}$$

$$\ell(\theta_1, \theta_2; x_1, \ldots, x_n) = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^n (x_i - \theta_1)^2$$

$$\ell(\hat\theta_1^{MLE}, \hat\theta_2^{MLE}; x_1, \ldots, x_n) =$$

$$AIC = -2\hat\ell + 2K \Rightarrow AIC - 2K = -2\ell \Rightarrow \ell = K - AIC/2$$

N: # of CI's that cover

$N \sim Bin(m, 1-\alpha)$ $\quad P(N = 0) = \binom{m}{0}(1-\alpha)^0 \alpha^m = \alpha^m$

$= 0.05^{37}$