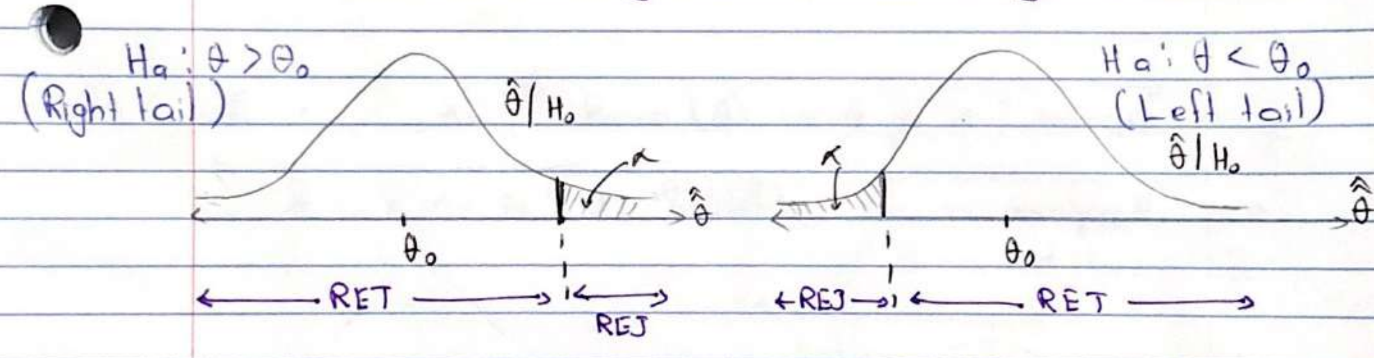
Lecture - 04 09/09/2020.

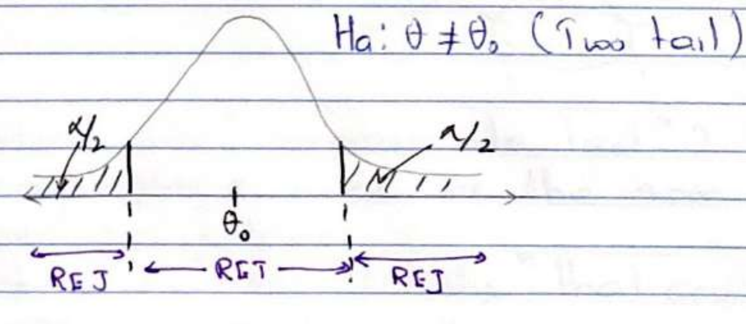
I don't think I'll give you exam questions on this:
1) "level of a test" α is defined as $P(\text{Type I error}) \geq \alpha$
2) "size of a test" is exactly $P(\text{Type I error})$

In our example the level was 5% but the size was 7.06% since $\alpha = 5\%$ was "unattainable"

If $\hat{\theta}/H_0$ is continuous, then level = size = α. If its discrete, some sizes won't be attainable.

If I want a level of $\alpha = 5\%$ and the size is lower, then I'm "cheating" (We'll see why next class).





What we did in the previous lecture was called a "binomial exact test of one proportion". Downsides:
I You need a binomial PMF calculator and it's a lot of work to get the retainment region

② Not all sizes are attainable. This is the recommended test.

Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim}$ some distribution with mean μ (mu) and variance σ^2 (sigsq). The central limit theorem (CLT) Shows that:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0,1)$$

"Convergence in distribution". It means as n gets large, the CDF of the left hand side (lhs) looks more and more like the CDF of the right hand side (rhs).

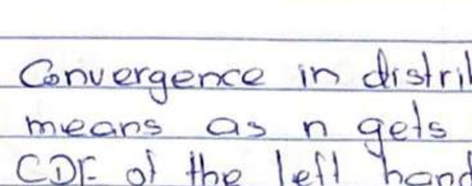
* approx distr.

$\Rightarrow \bar{X} \dot{\sim} N(\mu, \sigma^2/n)$ and $T = X_1 + \ldots + X_n \dot{\sim} N(n\mu, n\sigma^2)$

If $X_1, \ldots, X_n \overset{iid}{\sim}$ Bern(θ) and n is "large" then:

$\hat{\theta} = \bar{x} \dot{\sim} N\left(\theta, \frac{\theta(1-\theta)}{n}\right)$ this is a pretty good approx if θ is not too close to 0 or 1.

Ex:



How to perform an "approximate test"? There are many, many options even for the same DGP. The protocol goes as follows:

① You think of a "test statistic" that could measure the departure away from H_0.

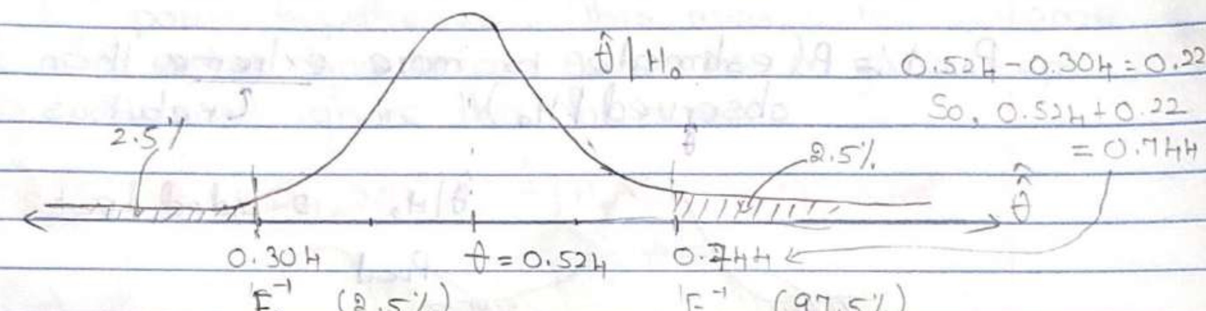
② Derive the statistical estimator's * approx * distribution under H_0, $\hat{\theta} | H_0$.

③ Gauge the departure of $\hat{\hat{\theta}}$ from the bulk of the distribution $\hat{\theta} | H_0$ at level α.

$H_0: \theta = 0.524$, $H_a: \theta \neq 0.524$, $n = 20$, $\hat{\hat{\theta}} = 0.6$ (same as last class)

$$\hat{\theta} | H_0 \sim N\left(0.524, \frac{0.524(1-0.524)}{20}\right) = N\left(0.524, \overset{\sigma^2}{\underset{\sigma}{0.112^2}}\right)$$

set $\alpha = 5\% \Longrightarrow \alpha/2 = 2.5\%$



$P(\hat{\theta} | H_0 \leq \hat{\hat{\theta}}) = 2.5\%$: solve for $\hat{\hat{\theta}}$.

$$\Longrightarrow P\Bigg(\underbrace{\frac{\hat{\theta} | H_0 - 0.524}{0.112}}_{N(0,1)} \leq \frac{\hat{\hat{\theta}} - 0.524}{0.112}\Bigg) = 2.5\%$$

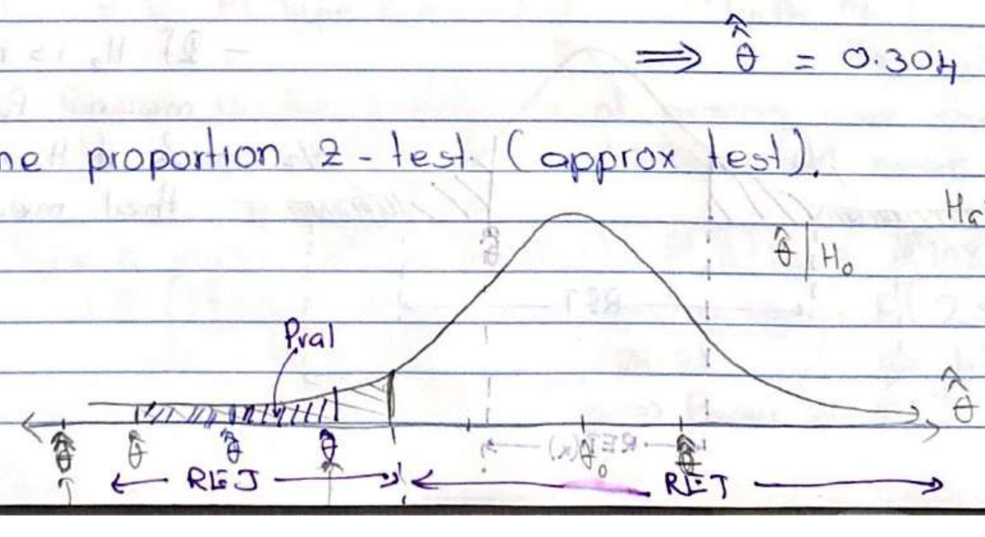
look at table or use computer

$$\Longrightarrow P\left(Z \leq \frac{\hat{\hat{\theta}} - 0.524}{0.112}\right) = 2.5\% \Longrightarrow \frac{\hat{\hat{\theta}} - 0.524}{0.112} \approx 1.96 \approx 2$$

$$\Longrightarrow \hat{\hat{\theta}} = 0.304$$

One proportion z - test (approx test).

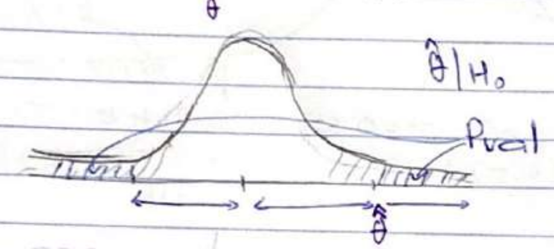
$H_a: \theta < \theta_0$



this estimate should imply a "stronger" rejection than this estimate.

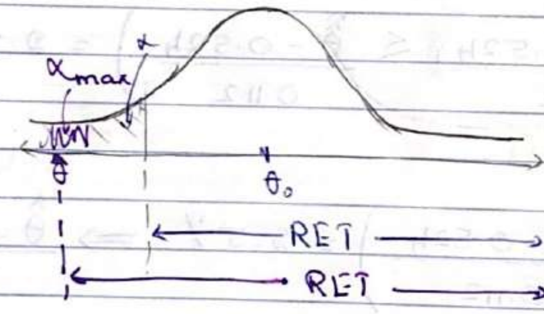
To measure the "strength" of a rejection (or "weakness" of a retainment), Fisher introduced the "P-value" also called the level of statistical significance as:

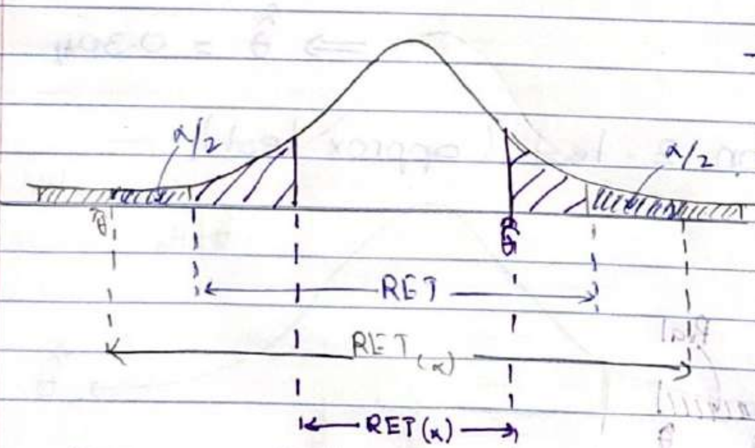
$P_{val} := P($estimate is more extreme than the one observed $| H_0)$ ← nebulous.



Real definition:

$$P_{val} = \max\{\alpha : \hat{\theta} \in RET(\alpha)\}$$





- If H_0 is retained, that means $P_{val} \geq$ alpha
- & if H_0 is rejected, that means $P_{val} < \alpha$

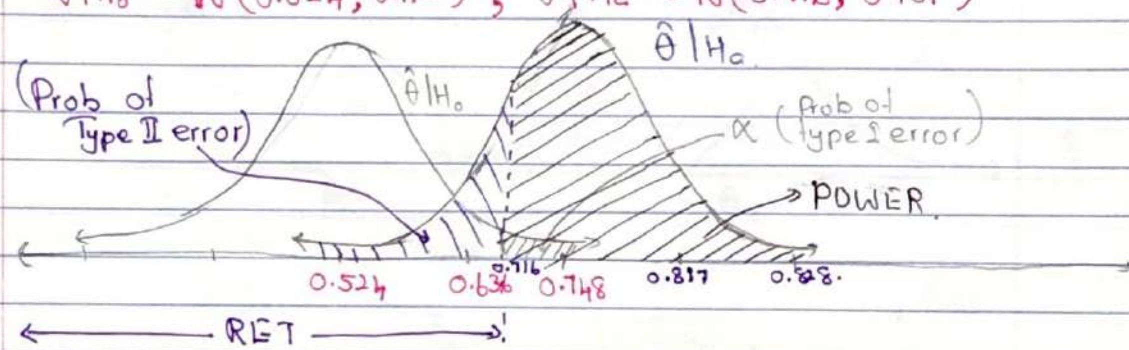
Type II errors and POWER

DGP: $X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(\theta)$

$H_0: \theta = 0.524 = \theta_0$ but $H_a: \theta = 0.716 = \theta_a$

This is a non-standard setup since both H_0 and H_a are "point hypotheses". This makes the outcome weird: either you retain $\theta = 0.524$ or you accept $\theta = 0.716$. But ignore this for now.

$\hat{\theta} \mid H_0 \sim N(0.524, 0.112^2)$, $\hat{\theta} \mid H_a \sim N(0.716, 0.101^2)$



At $\alpha = 5\%$, the z value is 1.645 which means the rejection region ends at $\hat{\hat{\theta}} = 0.524 + 1.645 * 0.112 = \underline{0.708}$

$$\text{POWER} = P(\text{Rejecting } H_0 \mid H_a)$$
$$= 1 - P(\text{Retaining } H_0 \mid H_a)$$
$$= 1 - P(\text{Type II error})$$

Errors

		Decision: RET	Decision: REJ
Truth	H_0		Type I
	H_a	Type II	

POWER is the probability of proving your theory is true!! You want POWER to be LARGE i.e. near 100%

$$P(\text{Type II error}) = P(\hat{\theta} \mid H_a \in \text{RET}) = P(\hat{\theta} \mid H_a \leq 0.708)$$
$$= P\left(\frac{(\hat{\theta} \mid H_a) - 0.716}{0.101} \leq \frac{0.708 - 0.716}{0.101}\right) = P(Z \leq -0.079) \approx 47\%$$

$\Longrightarrow$ Power $\approx 53\%$