9/2/20

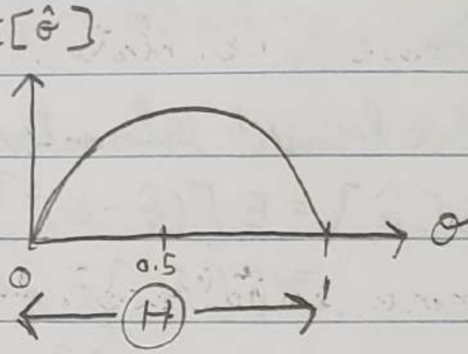
DGP: $X_1, \ldots, X_n \overset{iid}{\sim}$ with mean θ, variance σ^2

If $\hat{\theta} = \bar{X} \Rightarrow \hat{\theta}$ is unbiased $\quad R(\hat{\theta}, \theta) = \frac{\theta(1-\theta)}{n} = \text{MSE}$

$$SE[\hat{\theta}] = \sqrt{Var\left[\frac{1}{n}(x_1 + \ldots + x_n)\right]} = \sqrt{\frac{1}{n^2} \sum Var[X_i]} = \sqrt{\frac{1}{n^2} n \sigma^2} = \frac{\sigma}{\sqrt{n}}$$

$$= \sqrt{\frac{\theta(1-\theta)}{n}}$$

$x_1, \ldots x_n \overset{iid}{\sim} Bern(\theta)$



$\sup\limits_{\theta \in \Theta} R(\hat{\theta}, \theta) = \frac{1}{4n}$

Supreme (maximum)

Goal #3 of inference: theory testing (hypothesis testing) you have some well-specified mathematical theory about the DGP. For example, in the iphone survey, "I think the proportion of iphone users in the population is NOT 52.4%. I want to prove my theory to the world (using my sample).

Note: it is absolutely impossible to prove or disprove my theory (never going to have access to the whole pop.) because you cannot see the whole population (or go inside of the DGP). We must use inference which is always a guess.

Two ways to go about "proving" my theory:

1. I assume I'm right and wait for other people to show me data that contradicts my theory.
2. I assume my theory is wrong, then I adduce (bring) evidence (i.e. data) to the contrary until people are convinced my theory is right.

#2 is more intellectually honest and more likely to convince.

A hypothesis is a mathematical statement about the DGP e.g. theta = 0.9, theta > 0.9, theta is not equal to 0.9, or theta <= 0.9 or theta is in the set [0.89, 0.91], etc

The "alternative hypothesis" (H_a) is the theory you want to prove.

The "null hypothesis" H_0 is the opposite you assume in #2 for the purpose of contradicting it. Usual cases:

$H_0: \theta \leq \theta_0$, $H_a: \theta > \theta_0$ (right-tailed test)

$H_0: \theta \geq \theta_0$, $H_a: \theta < \theta_0$ (left-tailed test)

$H_0: \theta = \theta_0$, $H_a: \theta \neq \theta_0$ (two-tailed test)

How to perform this test? There are many, many options even for the same DGP. The protocol goes a follows

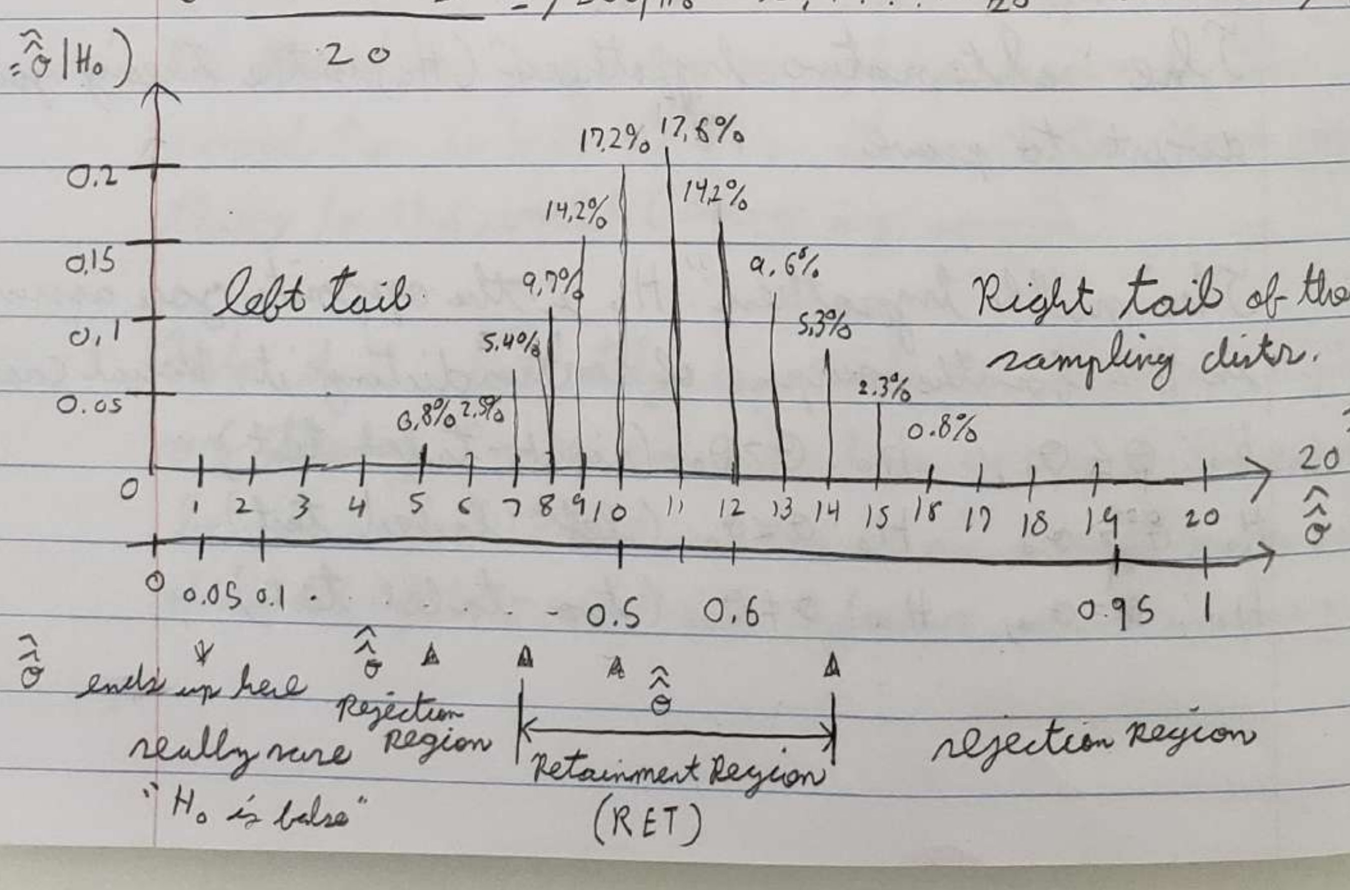
1. you think of a "test statistic" that could measure the departure away from H_0
2. Derive the statistical estimator's distribution under H_0
3. Gauge the departure ...

We begin with the DGP: iid Bern(theta) and the "binomial exact test"

$$H_a: \theta \underbrace{\neq .524}_{\neq \theta_0},\quad H_0: \theta \underbrace{= .524}_{= \theta_0}$$

1. My test stat. is ... $\hat{\hat{\theta}} = \bar{x}$, $\hat{\hat{\theta}}$ is a realization from $\hat{\theta}$
2. $\hat{\theta} \mid H_0 \sim ?$ $n = 20$

$$\hat{\theta} = \frac{x_1 + \ldots + x_{20}}{20} \Rightarrow 20\hat{\theta} \mid H_0 = x_1 + \ldots + x_{20} \sim \text{Binom}(20, \theta_0 = .524)$$



$\hat{\theta} \in RET \Rightarrow$ Retain H_0 (fail to reject H_0). Not enough evidence to reject H_0. Some authors say "accept H_0"

$\hat{\theta} \notin RET \Rightarrow$ Reject H_0 / accept H_a. My estimate is "stat significant".

Let's say we rejected H_0 but it was really was true. This is called a Type I error. Where is the P(Type I error) on our plot?

$$\alpha := P(\text{Type I error}) = P(\hat{\theta} \notin RET / H_0)$$

Then in a 2-tailed test, I'll apportion about alpha/2 to the left tail and about alpha/2 to the right tail.

In my RET

$\alpha = P(\hat{\theta} = 0 / H_0) + \ldots + P(\hat{\theta} = 0.3 / H_0) + P(\hat{\theta} = 0.75 / H_0) + \ldots +$
$P(\hat{\theta} = 1 / H_0) = 7.06\%$

The choice of alpha is up to you. The scientific community standard is 5% and sometimes 1%

If you would like to prove your theory, you have to accept a positive probability of a Type I error.

If I fail to reject H_0 when H_a is true that's a different error, a "type II error". Failure to prove your theory

The smaller the alpha, the larger the P(Type II error)

Truth	retain H_0	Reject H_0
H_0	✓	Type I error
H_a	Type II error	✓

Decision

as of now, we cannot calculate the P(Type II error)

Enter in R: dbinom(0:20, 20, .524)