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MATH 369

9/2/20

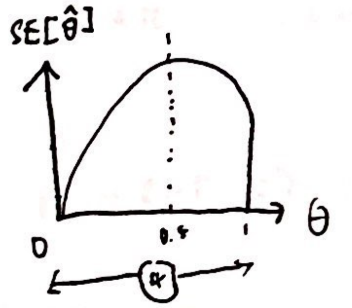
Lecture #3

DGP: $X_1, \ldots, X_n \overset{iid}{\sim}$ with mean θ, variance σ^2

$$SE[\hat\theta] = \sqrt{var[\tfrac{1}{n}(X_1 + \ldots + X_n)]} = \sqrt{\tfrac{1}{n^2}\sum var[X_i]} = \sqrt{\tfrac{1}{n^2} n \sigma^2} = \frac{\sigma}{\sqrt{n}}$$

$$= \sqrt{\frac{\theta(1-\theta)}{n}}$$

$X_1, \ldots, X_n \overset{iid}{\sim} Bern(\theta)$



$\sup_{\theta \in \Theta} R(\hat\theta, \theta) = \frac{1}{4n}$

supremum (maximum)

Goal #3 of inference: theory testing (hypothesis testing).

You have some well-specified mathematical theory about the DGP. For ex. in the Iphone Survey, "I think the proportion of Iphone users in the pop is not 52.4%." I want to prove my theory to the world (using my sample).

Note: it is absolutely impossible to prove or disprove my theory. No way to see the whole population. We must use inference which is always a guess.

Two ways to go about "proving" my theory:

(1) I assume I'm right and wait for other people to show me data that contradicts my theory.

(2) I assume my theory is wrong. Then I bring evidence (i.e. data) to the contrary until people are convinced my theory is right.

#d is more intellectually honest and more likely to convince.

A "hypothesis" is a mathematical statement about the DGP e.g. $\theta = 0.9$,

$\theta > 0.9$, $\theta \neq 0.9$, $\theta \leq 0.9$, $\theta \in [0.89, 0.91]$, etc.

The "alternative hypothesis" (H_a) is the theory you want to prove. The "null hypothesis" (H_0) is the opposite you assume in #d for the purpose of contradicting it. Usual cases:

$H_0: \theta \leq \theta_0$ $\quad H_a: \theta > \theta_0$ right tailed test

$H_0: \theta \geq \theta_0$, $\quad H_a: \theta < \theta_0$ left-tailed test

$H_0: \theta = \theta_0$, $\quad H_a: \theta \neq \theta_0$ (two-tailed ~~test~~)

How to perform this test? There are many, many options even for the same DGP. The protocol goes as follows

(1) you think of a "~~test~~ stat" that could measure the departure away from H_0.

(2) Derive ~~that~~ the statistical estimator's distribution under H_0.

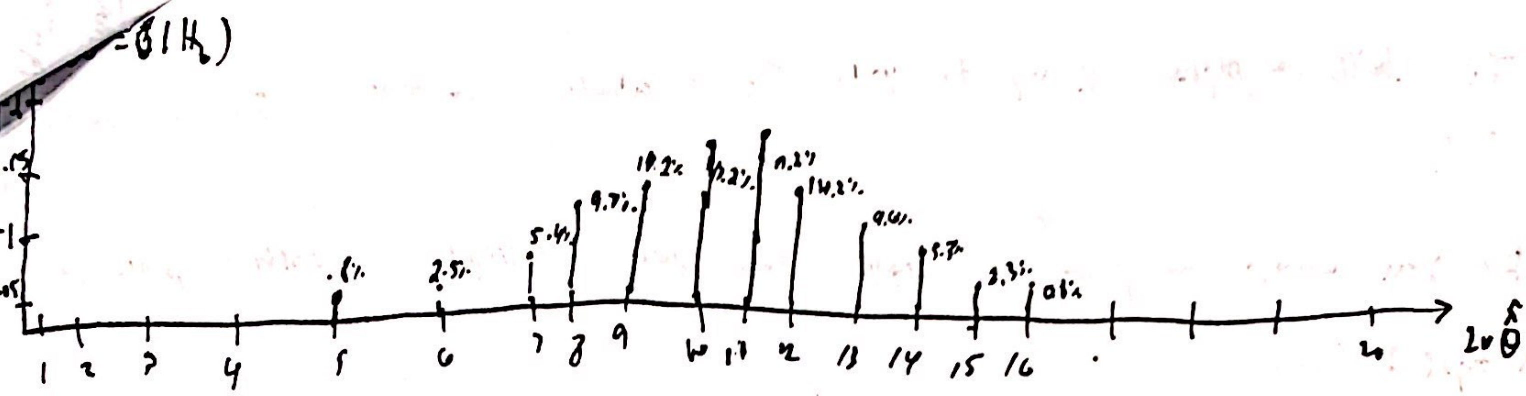
(3) Gauge the departure.

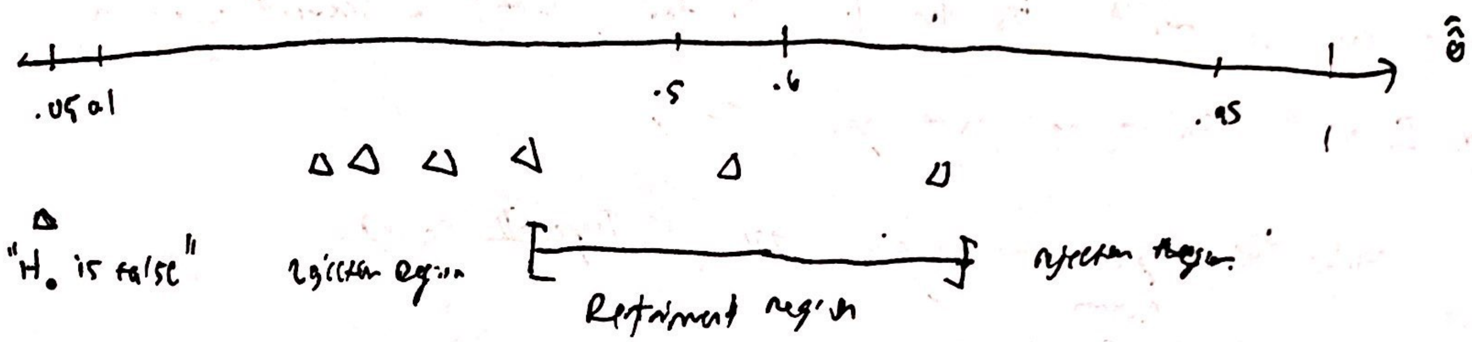
We begin with DGP: iid Bern(θ) and the "binomial exact test"

$H_a: \theta \neq .524$ $\quad H_0: \theta .524$

(1) my test stat is ... $\hat{\theta} = \bar{x}$. $\hat{\theta}$ is a realization from $\hat{\Theta}$.

(2) $\hat{\Theta} | H_0 \sim ?$ $n = 20$ $\quad \hat{\Theta} = \dfrac{X_1 + \ldots + X_{20}}{20} \Rightarrow 20 \hat{\Theta} | H_0 = X_1 + \ldots + X_{20} \sim Bin(20, \theta = .524)$





$\hat\theta \in RET \Rightarrow$ Retain H_0 (fail to reject H_0). Not enough evidence to reject H_0.

$\hat\theta \in RET \Rightarrow$ reject H_0. My estimate is statistically significant.

Let's say we rejected the H_0, but it really was true. This is called a type I error.

Where is the P(Type I error) on our plot?

$$\alpha := P(\text{Type I error}) = P(\hat\theta \notin RET \mid H_0)$$

Then in a 2-tailed test, I apportion $\alpha/2$ to the left tail and about $\alpha/2$ to the right tail.

$$\alpha = P(\hat\theta = 0 \mid H_0) + \dots + P(\hat\theta = 0.3 \mid H_0) + P(\hat\theta = 0.75) + \dots + P(\hat\theta = 1 \mid H_0) = 7.06\%$$

The choice of Alpha is up to you. The scientific community is 5% and sometimes 1%.

If you would to prove your theory, you have to accept a positive prob. of a type I error.

If I fail to reject H_0. then H_a is true, that's a different error, a "type II error". Failure to prove your theory.

The smaller the alpha, the larger the P(Type II error).

		Decision	
		Retain H_0	Reject H_0
Truth	H_0	✓	Type I
	H_a	Type II	✓