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Lec #04

I don't think I'll give you exam questions on this:

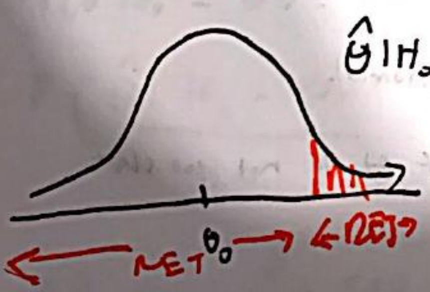
"Level of a test": alpha is defined as p(Type I error) $\geq$ alpha. "size of a test" is exactly p(Type I error).

In our example the level of alpha was 5%, but the size was 7.06% since alpha = 5% was "unattainable"!

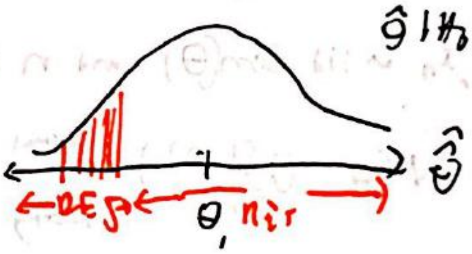
If $\hat{\theta} \mid H_0$ is continuous, then level = size = alpha. If it's discrete, some sizes won't attainable.

If I want a level of $\alpha = 5\%$ and the size is lower, then I'm "cheating"

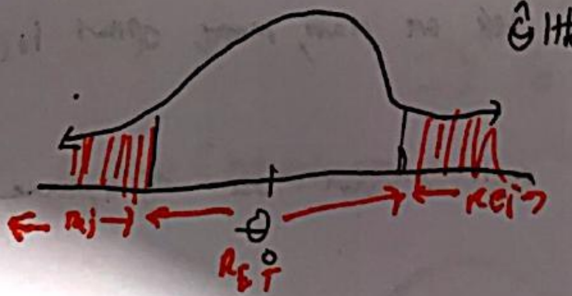
$H_a: \theta > \theta_0$



$H_a: \theta < \theta_0$



$H_a: \theta \neq \theta_0$



What we did in the previous lecture was called a "binomial exact test" of one proportion. Downsides: (1) you need a binomial pmf calculator and it's a lot of work to get the retainment region (2) not all sizes are attainable.

Let $X_1, X_2, \ldots, X_n \sim$ iid some distribution with mean μ and variance σ^2. The CLT shows that:

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{d} N(0,1)$$

"convergence in distribution". As n gets large, CDF looks more like bell curve; LHS CDF looks more and more the RHS CDF.

$$\Rightarrow \bar{X} \mathrel{\dot\sim} N\left(\mu, \frac{\sigma^2}{n}\right) \text{ and } T = X_1 + \ldots + X_n \mathrel{\dot\sim} N(n\mu, n\sigma^2).$$

If $X_1, \ldots, X_n \sim$ iid Bern(θ) and n is "large" then:

$$\hat{\theta} = \bar{X} \mathrel{\dot\sim} N\left(\theta, \frac{\theta(1-\theta)}{n}\right)$$

normal distribution ~~approx~~ to binomial. Pretty good approximation if θ is not too close to 0 or 1.

How to perform an "approximate test"? There are many, many, options for the same DGP. The protocol goes as follows:

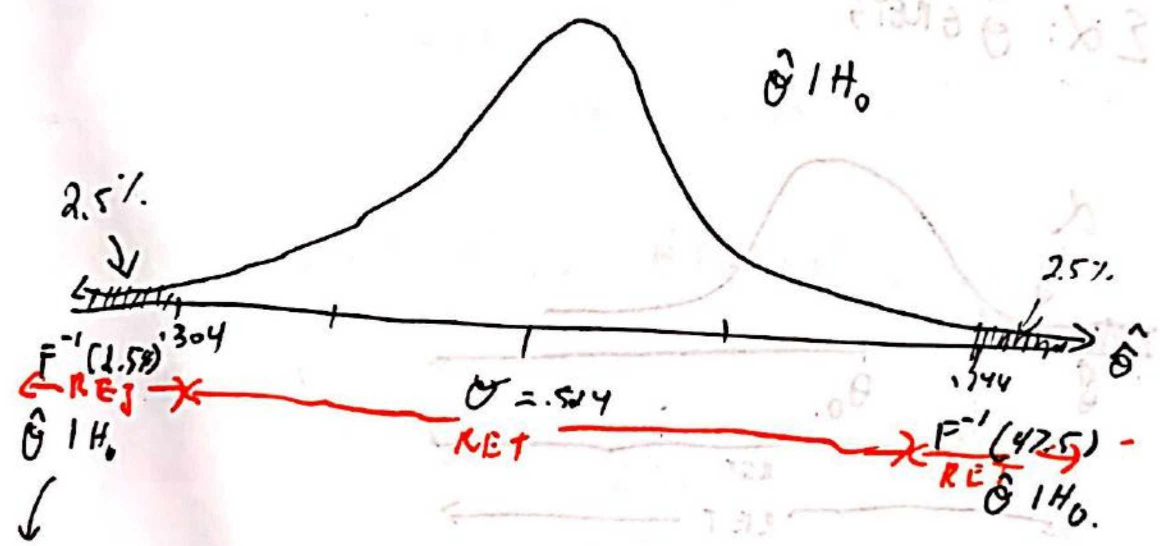
(1) you think of a "test statistic" that could measure the departure away from H_0.

(2) Derive the statistic/estimator's "approx" distribution under H_0.

(3) Gauge the departure of $\hat{\theta}$ from the scale of the distribution of H_0 at level ~~of~~ α.

$H_0: \theta = .524$, $H_a: \theta \neq .524$, $n = 20$, $\hat{\theta} = 0.16$ (same as last class)

$$\hat{\theta} \mid H_0 \sim N\left(.524, \frac{.524(1-.524)}{20}\right) = N(.524, .112^2)$$



$P(\hat{\theta} \mid H_0 \leq \hat{\theta}) = 2.5\%$. Solve for $\hat{\theta}$.

$$P\left(\frac{\hat{\theta} \mid H_0 - .524}{.112} \leq \frac{\hat{\theta} - .524}{.112}\right) = 2.5\%$$

two tailed test, so 5%/2 = 2.5%. 100 - 2.5 = 97.5%. Find the corresponding z-score for 97.5%.

If our α is 5%. Numbers in the middle of the chart correspond to the REJ. ~~[illegible]~~

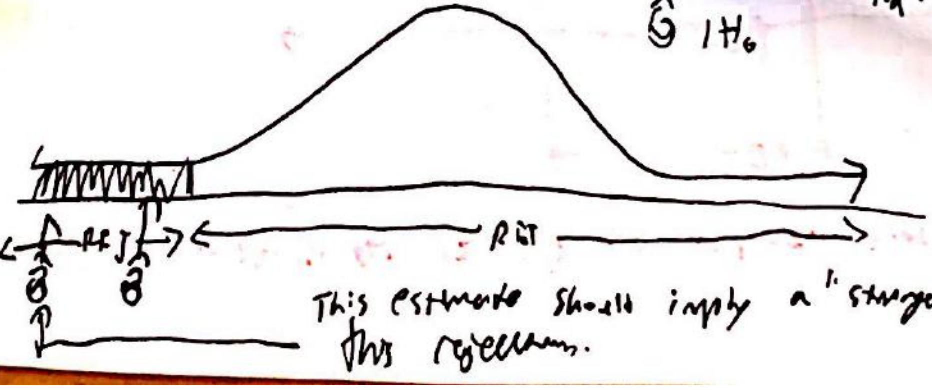
$$P\left(Z \leq \frac{\hat{\theta} - .524}{.112}\right) = 2.5\% \Rightarrow \frac{\hat{\theta} - .524}{.112} = 1.96 = 2.$$

Look at table

$$\Rightarrow \hat{\theta} = .304.$$

"One proportion Z-test (approximate test).

$H_a: \theta < \theta_0$



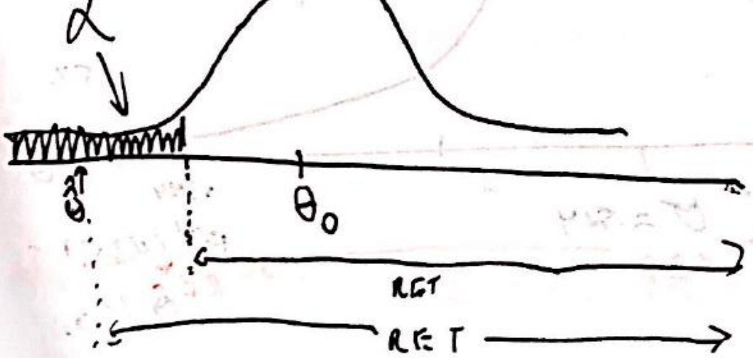
This estimate should imply a "stronger" rejection than this rejection.

To measure the "strength" of a rejection (or "weakness" of a retainment), Fisher introduces the "p-value" also called the level of statistical significance as:

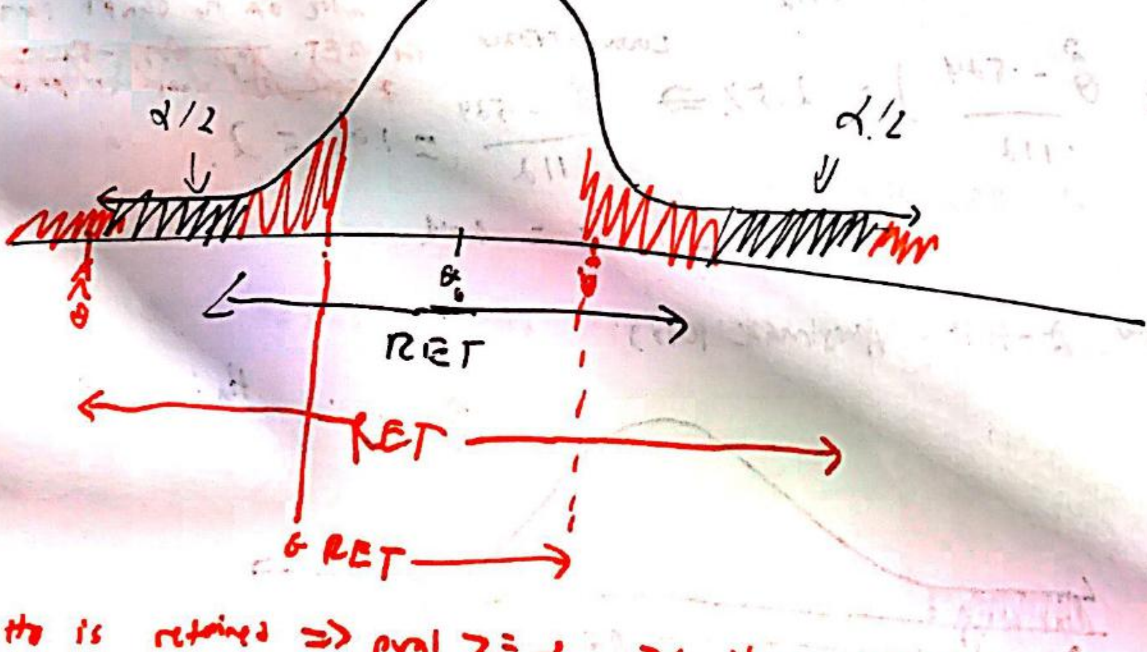
pvalue := p(estimate is more extreme than the one observed $\hat{\theta}$ | H_0)

real definition:

$$p_{value} = \max \{\alpha : \hat{\theta} \in RET\}$$



α max: The largest p-value such that α is still inside in RET.



If H_0 is retained $\Rightarrow$ pval $>= \alpha$. If H_0 is rejected $\Rightarrow$ pval $< \alpha$.

Type II errors and power

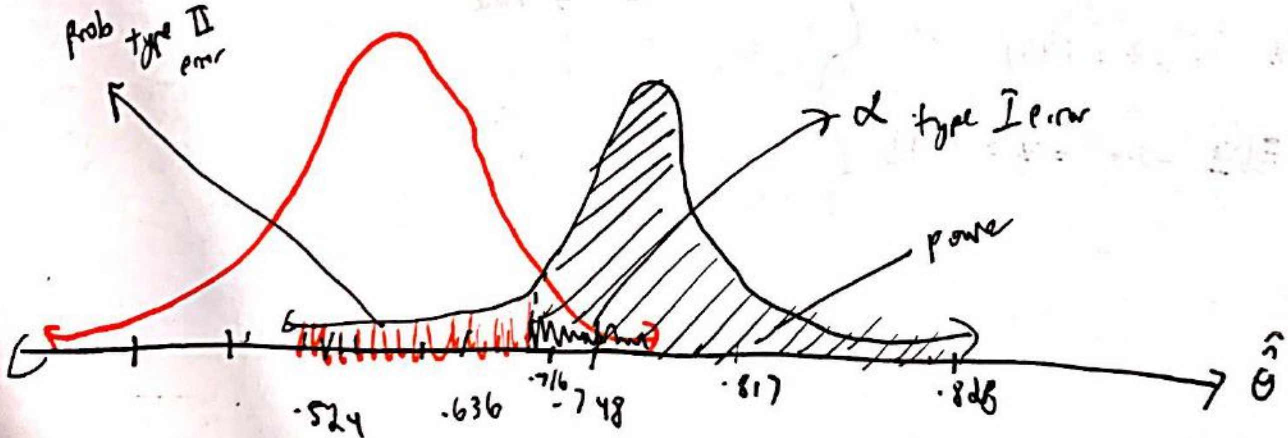
DGP: $X_1, \dots, X_n \sim$ iid Ben(θ)

$H_0: \theta = .524 = \theta_0$ but $\theta = .716 = \theta_a$.

Min standard

either you retain $\theta = .524$ or accept $\theta = .716$.

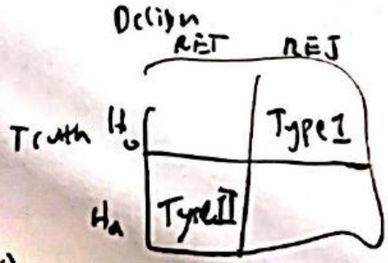
$\hat{\theta} \mid H_0 \sim N(.524, .112^2)$, $\hat{\theta} \mid H_a \sim N(.716, .101^2)$



At $\alpha = 5\%$, the z value is 1.645 which means the rejection region ends at

$$\hat{\theta} = .524 + 1.645 * .112 = .708$$

$$\theta_0 + z * SD$$



power = p(rejecting H_0 | H_a)

= 1 − p(retaining H_0 | H_a) = 1 − (type II error)

power is the prob of proving your theory is true!!

you want power to be near 100!

P(Type II error) = P($\hat{\theta}$ | H_a ∈ RET)

= P($\hat{\theta}$ | H_a ≤ .708)

⟹

$$P\left(\frac{\hat{\theta}\,|\,H_a - .716}{.101} \le \frac{.708 - .716}{.101}\right) = P(Z \le -.079) \approx 47\%$$

$.5 - .0319 \approx$ [illegible],

$1 - 47 = \boxed{53}$

Bad.

~~last step~~

$P(Z \le -0.79)$

$P(~~\text{RANGE}~~\ 0 \le Z < 1.40)$

$P(~~\text{RANGE}~~\ -0.6 \le Z \le 1.40)$

use "half" Z table.