Multilevel Model

Jason Qiang Guo, New York University

Multilevel Model

Multilevel model is all about shrinkage.

- Suppose we have a hierarchical structure:

$$y_i \sim N(\alpha_{j[i]} + \beta x_i, \sigma^2), \text{ for } i = 1, \ldots, n,$$

$$\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2) \text{ for } j = 1, \ldots, J$$

- Exactly the same as what we have done in the section of random effects model,
  - For any unit i, $var(\epsilon_i^{all}) = \sigma^2 + \sigma_\alpha^2$
  - For any units i and k in the same group j, $cov(\epsilon_i^{all}, \epsilon_k^{all}) = \sigma_\alpha^2$
  - For any units i and k in different groups, their covariance is 0
- The shrinkage:

$$\text{estimate of } \alpha_j = \frac{\frac{n_j}{\sigma^2}}{\frac{n_j}{\sigma^2} + \frac{1}{\sigma_\alpha^2}} \underbrace{(\bar{y}_j - \beta \bar{x}_j)}_{\text{no-pooling estimate for group j}} + \frac{\frac{1}{\sigma_\alpha^2}}{\frac{n_j}{\sigma^2} + \frac{1}{\sigma_\alpha^2}} \underbrace{\mu_\alpha}_{\text{regression prediction } \hat{\alpha}}$$

Multilevel Model

Several implications we can draw from the estimate of group coefficients α_j:

- $\sigma_\alpha^2 \downarrow$ causes more pooling, $\sigma_\alpha^2 = 0$ means complete pooling. As the distribution of group-level parameter is more accurate, each group can borrow more information from groups that are closer than when σ_α^2 is large.
- When $\sigma^2 \uparrow$ and $\sigma_\alpha^2 \downarrow$, effective sample size is n, as the influence of intra-group correlation goes down
- When $\sigma^2 \downarrow$ and $\sigma_\alpha^2 \uparrow$, effective sample size is j
- When $n_j \uparrow$, the estimate is pooled closer to the non-pooling estimate for its own group.