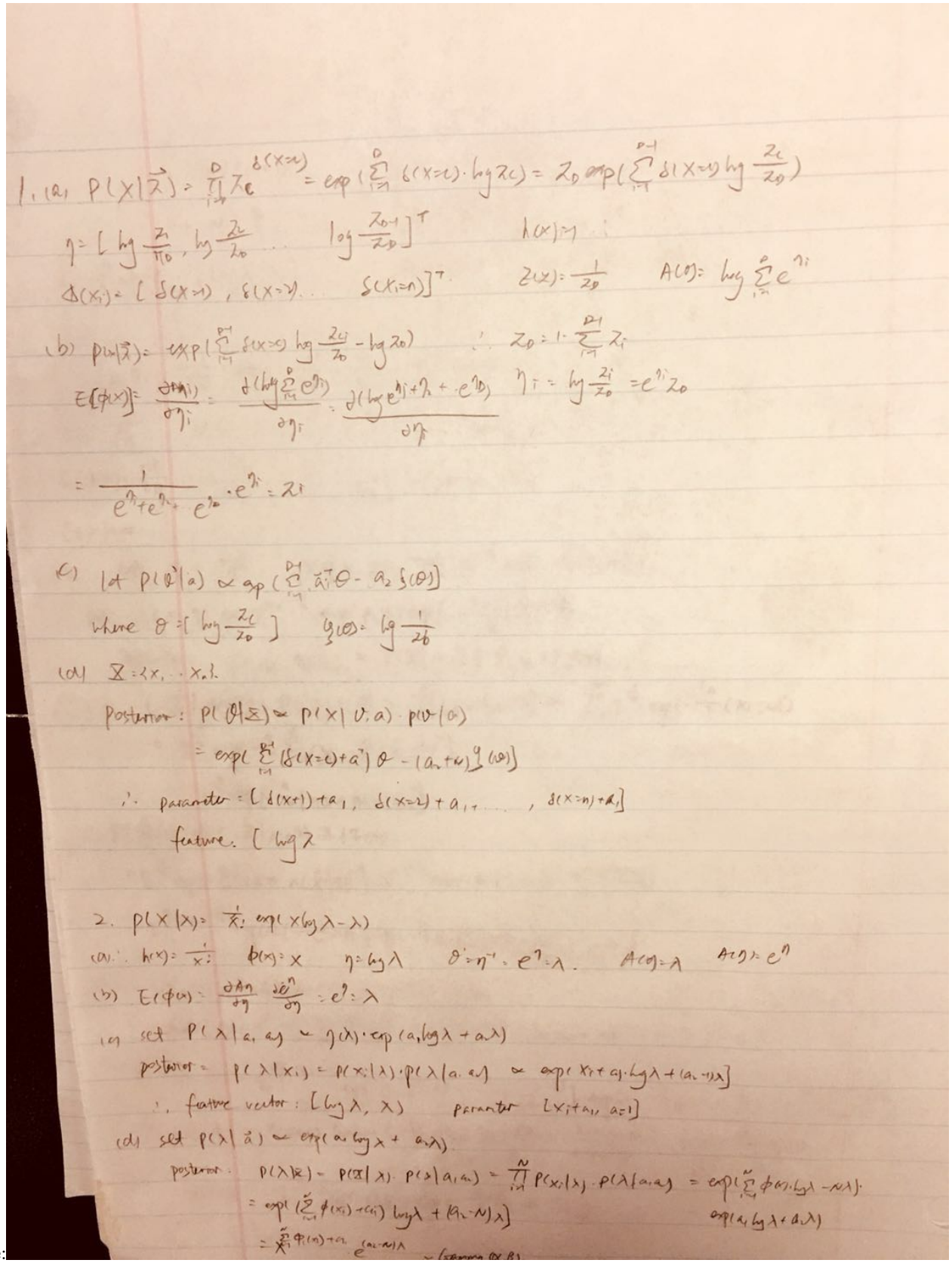
HW4

Tianyi Fang

October 9, 2017

1. Conjugate:

image:



$$\begin{cases} \sum_{i=1}^{N} \phi(x_i) + a = \alpha \\ a_2 - N = \beta \end{cases} \Rightarrow \begin{cases} a_1 = \alpha - \sum_{i=1}^{N} \phi(x_i) \\ a_2 = \beta + N \end{cases}$$

feature : $[\log\lambda, \lambda]$ parameter $[\alpha - \sum_{i=1}^{N} \phi(x_i), N+\beta]$

3. $p(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2} + \frac{\mu x}{\sigma^2} - \frac{\mu^2}{2\sigma^2} + \log\sigma\right)$

(a) $h(x) = \frac{1}{\sqrt{2\pi}}$ $\phi(x) = [x, x^2]^T$ $\eta = [\frac{\mu}{\sigma^2}, -\frac{1}{2\sigma^2}]^T$ $A(\eta) = -\frac{\mu^2}{2\sigma^2} - \log\sigma = \frac{\eta_1^2}{4\eta_2} - \frac{1}{2}\log(-2\eta_2)$

(b) $E(\phi(x)) = \frac{\partial A}{\partial \eta} = \left[-\frac{2\eta_1}{4\eta_2^2}, \frac{\eta_1^2}{4\eta_2^2} + \frac{1}{4\eta_2}\right] = [\mu, \mu^2+\sigma^2]$

$E[x] = \mu$

(Ctd) let $\tau = \frac{1}{\sigma^2}$ $p(x|\mu,\tau) = \frac{1}{\sqrt{2\pi}} \tau^{-\frac{1}{2}} \exp\left(-\frac{\tau}{2}(x-\mu)^2\right)$

let $\lambda(\mu,\tau) \propto \tau^{\alpha_0 - \frac{1}{2}} \exp(\beta_0\tau) \exp\left(-\frac{\lambda_0\tau(\mu-\mu_0)^2}{2}\right)$

posterior: $p(\mu,\tau|X) \propto p(X|\mu,\tau) \cdot p(\mu,\tau) \cdot p(\tau)$

(Post) $L(X|\mu,\tau) = p(X|\mu,\tau) = \prod_{i=1}^{n} l(x_i|\mu,\tau) \propto \prod_{i=1}^{n} \tau^{\frac{1}{2}} \cdot \exp\left(-\frac{n}{2}(x_i-\mu)^2\right)$

$= \tau^{\frac{n}{2}} \exp\left(-\frac{n}{2}\sum_{i=1}^{n}(x_i - \bar{x} + \bar{x} - \mu)^2\right]$

$= \tau^{\frac{n}{2}} \exp\left(-\frac{n}{2}(ns + n(\bar{x}-\mu)^2\right]$

$\therefore p(\mu,\tau|X) \propto L(X|\mu,\tau) \cdot \lambda(\tau,\mu)$

$= \tau^{\frac{n}{2}} \exp\left(-\frac{\tau}{2}(ns + n(\bar{x}-\mu)^2\right] \cdot \tau^{\alpha_0 - \frac{1}{2}} \exp(-\beta_0\tau) \cdot \exp\left(-\frac{\lambda_0\tau(\mu-\mu_0)^2}{2}\right)$

$= \tau^{\frac{n}{2}+\alpha_0-\frac{1}{2}} \exp(-\tau(\frac{ns}{2} + \beta_0)] \exp\left(-\frac{\tau}{2}[n(\bar{x}-\mu)^2 + \lambda_0(\mu-\mu_0)^2\right]$

(*) $= (\lambda_0+n)\mu^2 - 2(\lambda_0\mu_0 + n\bar{x})\mu + n\bar{x}^2 + \lambda_0\mu_0^2$

$= (\lambda_0+n)\left(\mu - \frac{\lambda_0\mu_0 + n\bar{x}}{2(\lambda_0+n)}\right)^2 - \frac{(\lambda\mu_0 + n\bar{x})^2}{\lambda_0+n} + \lambda_0\mu_0^2 + n\bar{x}^2$

$\therefore p(\tau,\mu|X) = \tau^{\frac{n}{2}+\alpha_0-\frac{1}{2}} \exp\left[-\tau\left(\frac{ns}{2} + \beta_0 + \frac{\lambda_0 n(\bar{x}-\mu_0)^2}{2(\lambda_0+n)}\right)\right] \exp\left(-\frac{\tau}{2}(\lambda_0+n)\cdot\left(\mu - \frac{\lambda_0\mu_0 + n\bar{x}}{\lambda_0+n}\right)^2\right]$

$p(\tau,\mu|X) \sim$ Normal Gamma $\left(\underbrace{\frac{n}{2}+\alpha_0}_{\vartheta_1}, \underbrace{\beta_0 + \frac{ns}{2} + \frac{\lambda_0 n(\bar{x}-\mu_0)^2}{2(\lambda_0+n)}}_{\vartheta_2}, \underbrace{\lambda_0+n}_{\vartheta_3}, \underbrace{\frac{\lambda_0\mu_0+n\bar{x}}{\lambda_0+n}}_{\vartheta_4}\right)$

$\therefore$

posterior $= \tau^{\vartheta_1 - \frac{1}{2}} \exp(-\vartheta_2\tau) \cdot \exp\left(-\frac{\tau}{2}\vartheta_3(\mu-\vartheta_4)^2\right]$

image:

2. EM

1~6

image:

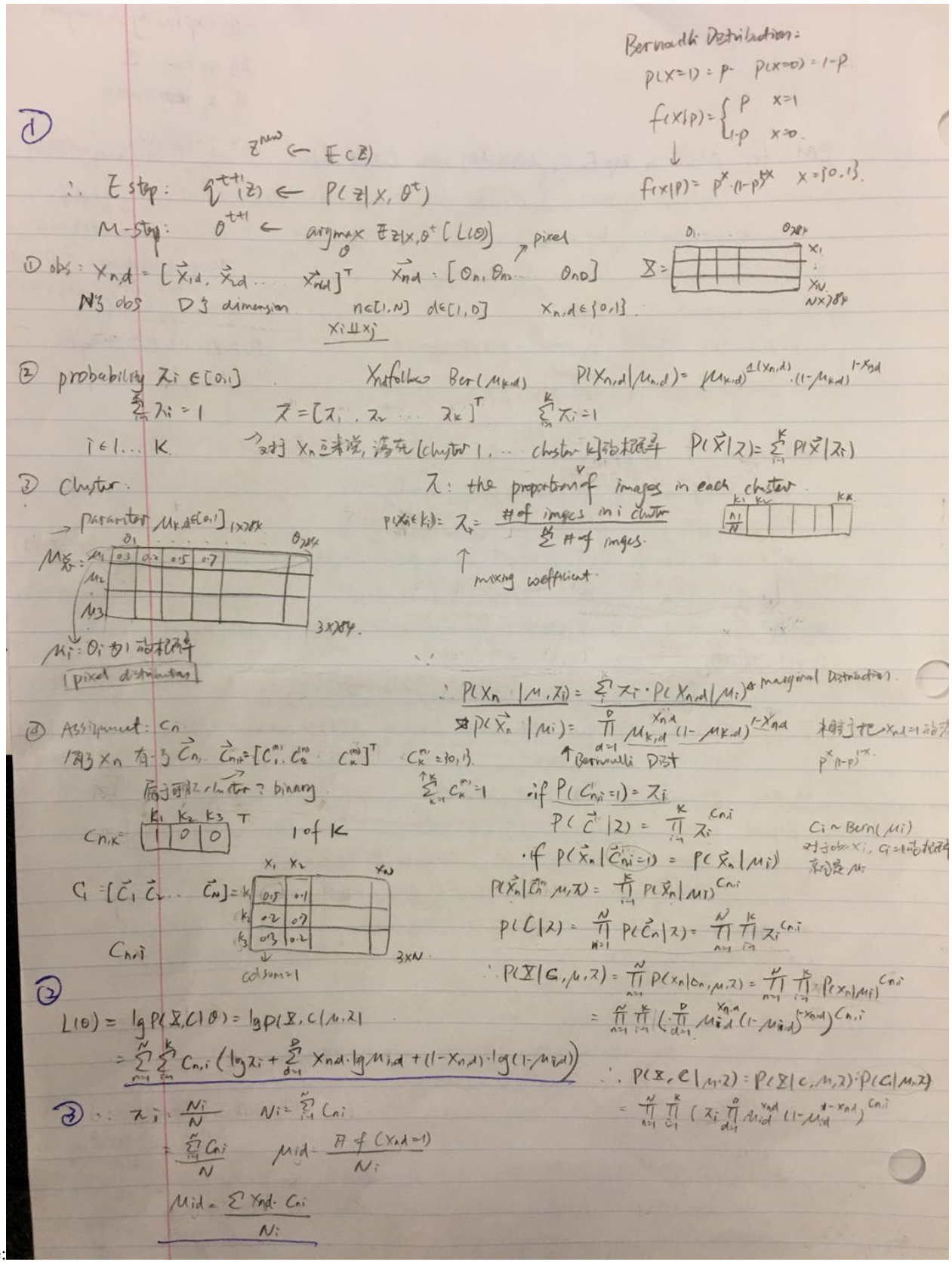
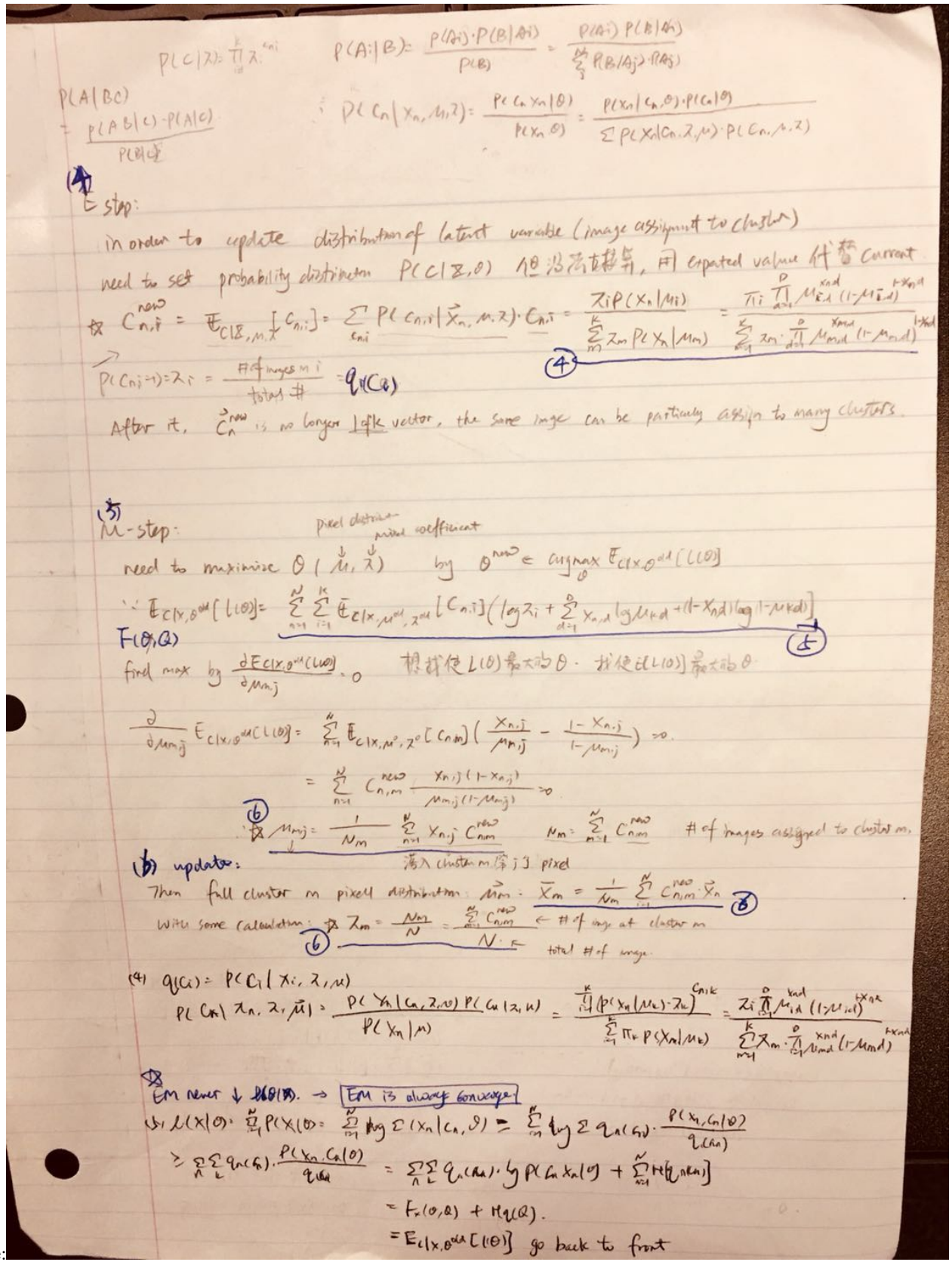

Bernoulli Distribution:
P(X=1) = p P(X=0) = 1-p
f(x|p) = { p x=1 ; 1-p x=0 }
f(x|p) = p^x (1-p)^(1-x) x={0,1}
E step: q^(t+1)(z) ← P(z|X, θ^t)
M-step: θ^(t+1) ← argmax_θ E_{z|X,θ^t}[L(θ)]
① obs: X_{n,d} = [x_{n1}, x_{n2} ... x_{nD}]^T x_{nd} = [0_{n1}, 0_{n2} ... 0_{nD}]
N obs D dimension n∈[1,N] d∈[1,D] X_{n,d}∈{0,1}
pixel
X_i ⊥⊥ X_j
② probability z_i ∈ [0,1] X_n follows Ber(μ_{k,d}) P(X_{n,d}|μ_{n,d}) = μ_{k,d}^{1(x_{n,d})} · (1-μ_{k,d})^{1-x_{nd}}
Σ_{i=1}^k z_i = 1 z = [z_1, z_2 ... z_k]^T Σ_{i=1}^k z_i = 1
i ∈ 1...K 对于 X_n 已经说, 满足 [cluster 1, ... cluster k] 的概率 P(x|z) = Σ_{i=1}^k P(x|z_i)
③ Cluster.
z: the proportion of images in each cluster
P(x_n ∈ k_i) = z_i = # of imges in i cluster / Σ # of imges
mixing coefficient
parameter μ_{k,d} ∈ [0,1] 1×D×K
μ_k:
μ_i: θ_i 为 1 的概率
[pixel distribution]
3×D×K
∴ P(X_n|μ, z_i) = Σ z_i · P(X_{n,d}|μ_i) marginal Distribution
P(x_n|μ_i) = Π_{d=1}^D μ_{k,d}^{x_{nd}} (1-μ_{k,d})^{1-x_{nd}} Bernoulli Dist
相对于把 x_nd=1 的概率 p^x (1-p)^{1-x}.
④ Assignment: c_n
对于 X_n 有了 c_n. c_n = [c_1^(n), c_2^(n) ... c_K^(n)]^T c_k^(n) = {0,1}
Σ_{k=1}^K c_k^(n) = 1 if P(c_{ni}=1) = z_i
P(c|z) = Π_{i=1}^K z_i^{c_{ni}}
C_i ~ Bern(μ_i)
对于 x_i, C_i=1 的概率是 μ_i
if P(x_n|c_{ni}=1) = P(x_n|μ_i)
P(x_n|c_n, μ, z) = Π_{i=1}^K P(x_n|μ_i)^{c_{ni}}
P(c|z) = Π_{n=1}^N P(c_n|z) = Π_{n=1}^N Π_{i=1}^K z_i^{c_{ni}}
P(X|C, μ, z) = Π_{n=1}^N P(x_n|c_n, μ, z) = Π_{n=1}^N Π_{i=1}^K P(x_n|μ_i)^{c_{ni}}
= Π_{n=1}^N Π_{i=1}^K (Π_{d=1}^D μ_{id}^{x_{nd}} (1-μ_{id})^{(1-x_{nd})})^{c_{ni}}
P(X, c|μ, z) = P(X|c, μ, z) · P(c|μ, z)
= Π_{n=1}^N Π_{i=1}^K (z_i Π_{d=1}^D μ_{id}^{x_{nd}} (1-μ_{id})^{1-x_{nd}})^{c_{ni}}
C_{n,i}
1 of K
C = [c_1 c_2 ... c_N]
k_1 k_2 k_3
x_1 x_2 ... x_N
0.5 0.1
0.2 0.7
0.3 0.2
col sum = 1
3×N
⑤
L(θ) = lg P(X, C|θ) = lg P(X, C|μ, z)
= Σ_{n=1}^N Σ_{i=1}^K c_{ni} (lg z_i + Σ_{d=1}^D x_{nd} · lg μ_{id} + (1-x_{nd}) · lg(1-μ_{id}))
⑥ ∴ z_i = N_i / N N_i = Σ_{i=1}^N c_{ni}
= Σ_{i=1}^N c_{ni} / N μ_{id} = # of (x_nd = 1) / N_i
μ_id = Σ x_nd · c_ni / N_i

image:


E step:
in order to update distribution of latent variable (image assignment to cluster)
need to set probability distribution P(C|X,θ) 但沒有直接算, 用 expected value 計算 current
M-step:
need to maximize θ (μ, π)
find max by
(a) update:
Then full cluster m pixel distribution:
With some calculation:
(4) q(C) = P(C_i | x_i, π, μ)
EM never ↓ L(θ). → EM is always convergent
= E_{C|X,θ^old}[L(θ)] go back to front

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.4.1
```

```
library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 3.4.1
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
load_mnist <- function() {
  # load image files
  load_image_file <- function(filename) {
    ret = list()
    f = file(filename,'rb')
    readBin(f, 'integer', n = 1, size = 4, endian = 'big')#magic number 2051
    n    = readBin(f, 'integer', n = 1, size = 4, endian = 'big')# number of images60000
    nrow = readBin(f, 'integer', n = 1, size = 4, endian = 'big')#number of rows 28
    ncol = readBin(f, 'integer', n = 1, size = 4, endian = 'big')#num of col 28
    x = readBin(f, 'integer', n = n * nrow * ncol, size = 1, signed = FALSE)
    ret$x = matrix(x, ncol=nrow*ncol, byrow=TRUE)
    close(f)
    ret
  }
  # load label files
  load_label_file <- function(filename) {
    f = file(filename,'rb')
    readBin(f,'integer',n=1,size=4,endian='big')
    n = readBin(f,'integer',n=1,size=4,endian='big')
    y = readBin(f,'integer',n=n,size=1,signed=F)
    close(f)
    y
  }
  # load images
  train <<- load_image_file('train-images-idx3-ubyte')
  ##test <<- load_image_file('t10k-images-idx3-ubyte')
  # load labels
  train$y <<- load_label_file('train-labels-idx1-ubyte')
  ##test$y <<- load_label_file('t10k-labels-idx1-ubyte')
}

# helper function for visualization
show_digit <- function(arr784, col=gray(12:1/12), ...) {
  image(matrix(arr784, nrow=28)[,28:1], col=col, ...)
}
load_mnist()
#find images with 2 and 3
```

```
in_digits_2 <- which(train$y==2)
sum(in_digits_2)
```

```
## [1] 180004248
```

```
in_digits_3 <- which(train$y==3)
sum(in_digits_3)
```

```
## [1] 183631823
```

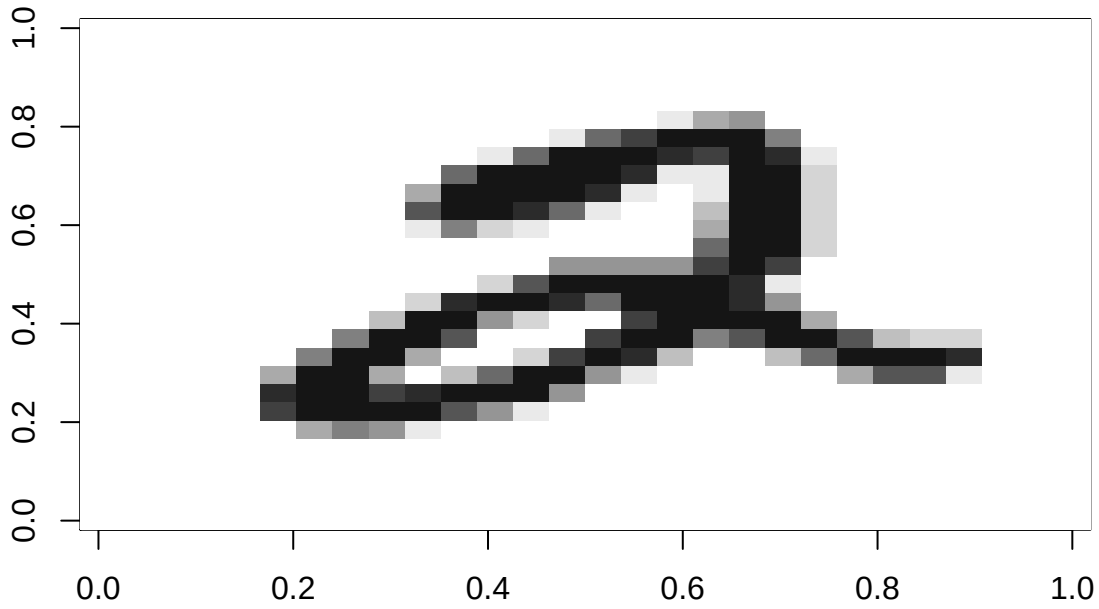
```
#select 1000 images
image_2<- train$x[in_digits_2, ][1:1000,]
image_3<- train$x[in_digits_3, ][1:1000,]
dim(train$x[in_digits_2, ])[1]
```

```
## [1] 5958
```

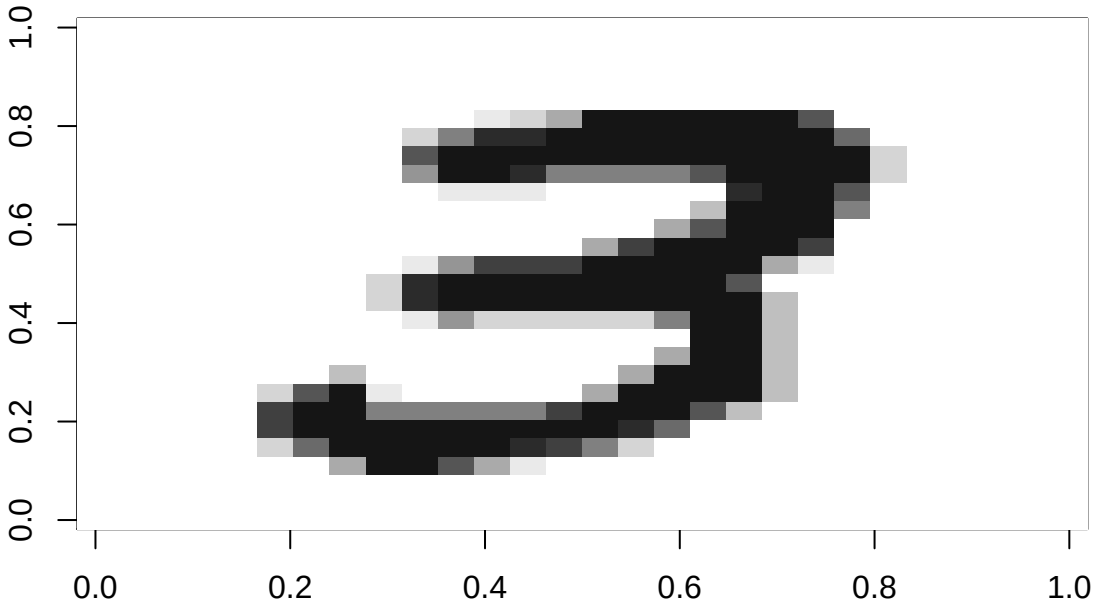
```
dim(train$x[in_digits_3, ])[1]
```

```
## [1] 6131
```

```
show_digit(image_2[1,])
```




```
show_digit(image_3[1,])
```



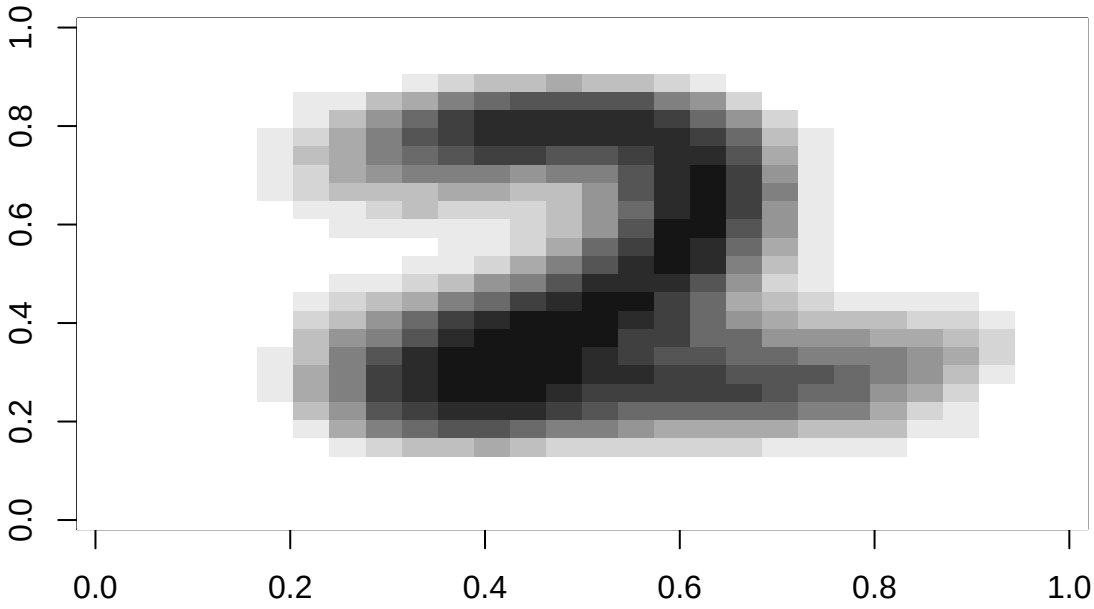

```
#set threshold as 4, <4 assign 0, >=4 assign =1
image_2[image_2 > 4] <- 1
image_3[image_3 > 4] <- 1
```

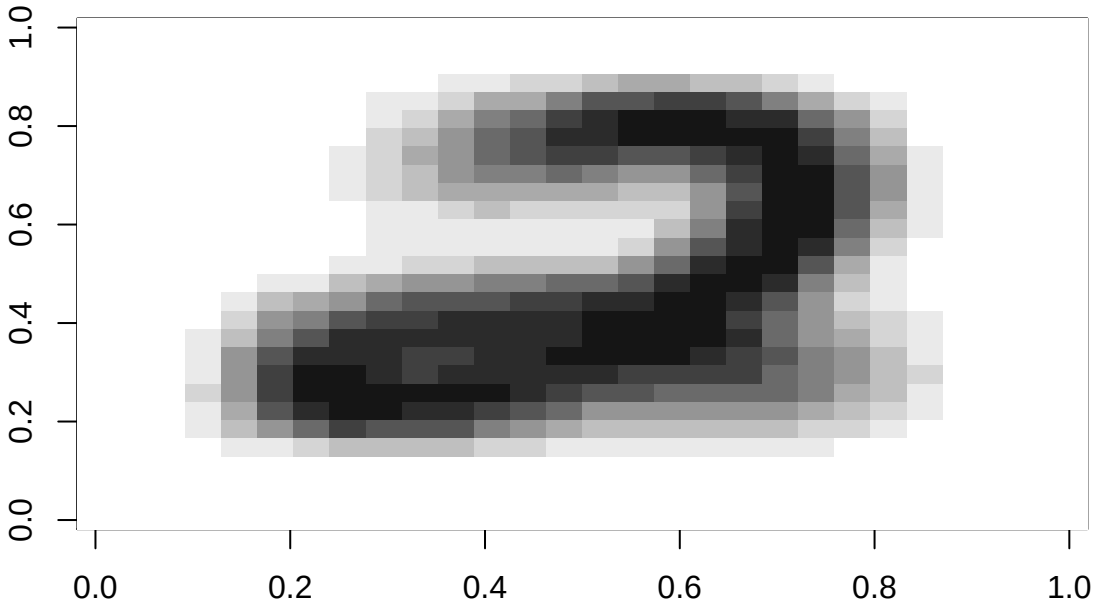
```
#input image dataset, K clusters
em <- function(image, K){
  #initialize:
  N = nrow(image)
  #D = ncol(image)784
  set.seed(1)
  #each cluster has random probability
  pi <- c(runif(K))/sum(c(runif(K)))
  #mu, c
  mu <- matrix(runif(K*784), ncol=784, nrow=K)
  c <- matrix(rep(0, N*K), ncol=K, nrow=N)
  #lower bound
  lower_bound <- rep(0)
  lower_bound_update <- 1
  m <- rep(0)
  #E step
  i <- 1
  logc <- matrix(rep(0, N*K), nrow=N, ncol=K)
  while(lower_bound_update>0.000001){
    i <- i + 1
    for(k in 1:K){
       for(nn in 1:N){
```

```
        logc[nn,k] <- log(pi[k])+sum(image[nn,]*log(mu[k,])+(1-image[nn,])*log(1-mu[k,]))
        m[nn] <- max(logc[nn,])
        c[nn,]<- exp(logc[nn,]-m[nn])/sum(exp(logc[nn,]-m[nn]))
      }
    }
    #M step
    for(k in 1:K){
      #update pi
       pi[k] <- sum(c[,k])/N
       for(d in 1:784){
         #update mu
         mu[k,d] <-sum(image[,d]*c[,k])/sum(c[,k])
        }
       }
    #lower bound f
    lower_bound[i] <- sum(c*logc)
    lower_bound_update <- abs(lower_bound[i]-lower_bound[i-1])
    #avoid NA in log(mu)
    mu[mu==0] <- 1e-100
    mu[mu==1] <- 0.999999999
    }
  entropy <- c*log(c)
  result <- list(pi, mu, lower_bound, entropy)
  return(result)
}
```

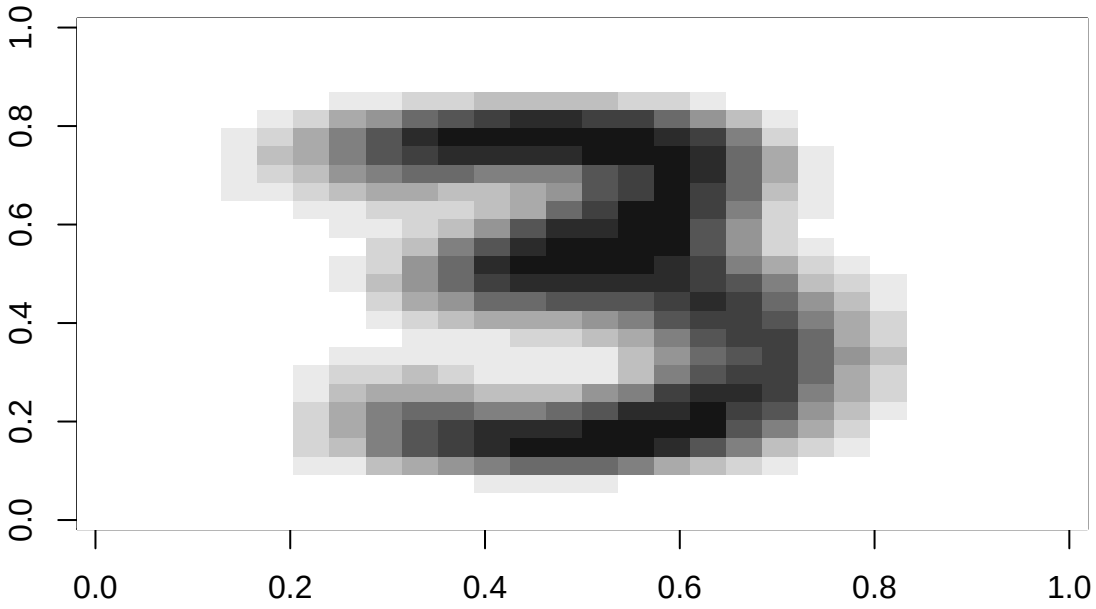
8 plot cluster parameter

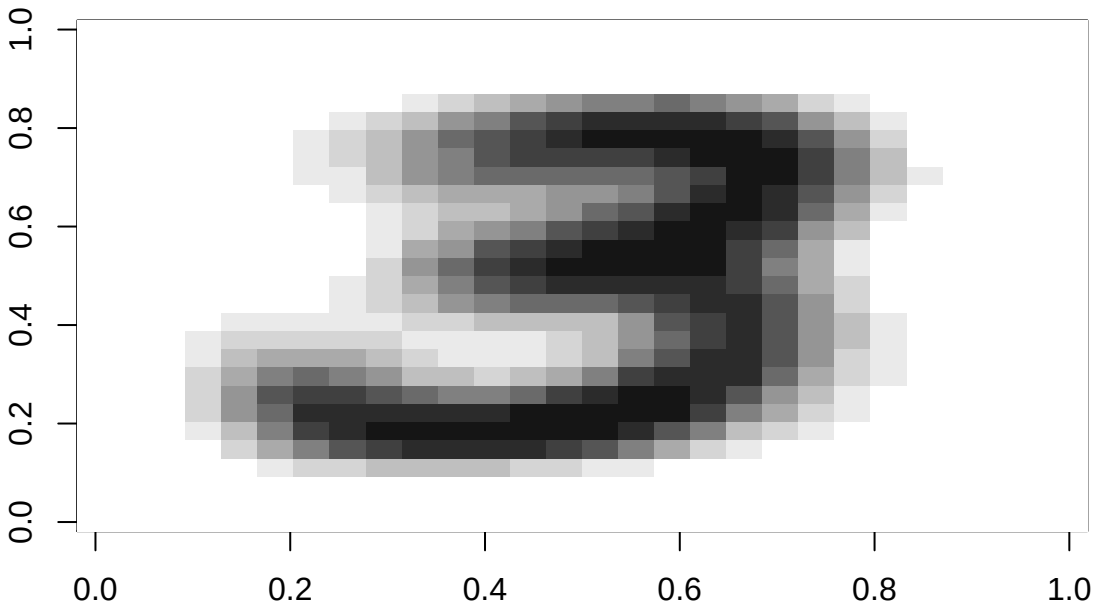
```
em_2 <- em(image_2,2)
em_3 <- em(image_3,2)
mu_2 <- em_2[[2]]
mu_3 <- em_3[[2]]
#show 2 cluster image
for(i in 1:2){
   show_digit(mu_2[i,])
}
```


0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0



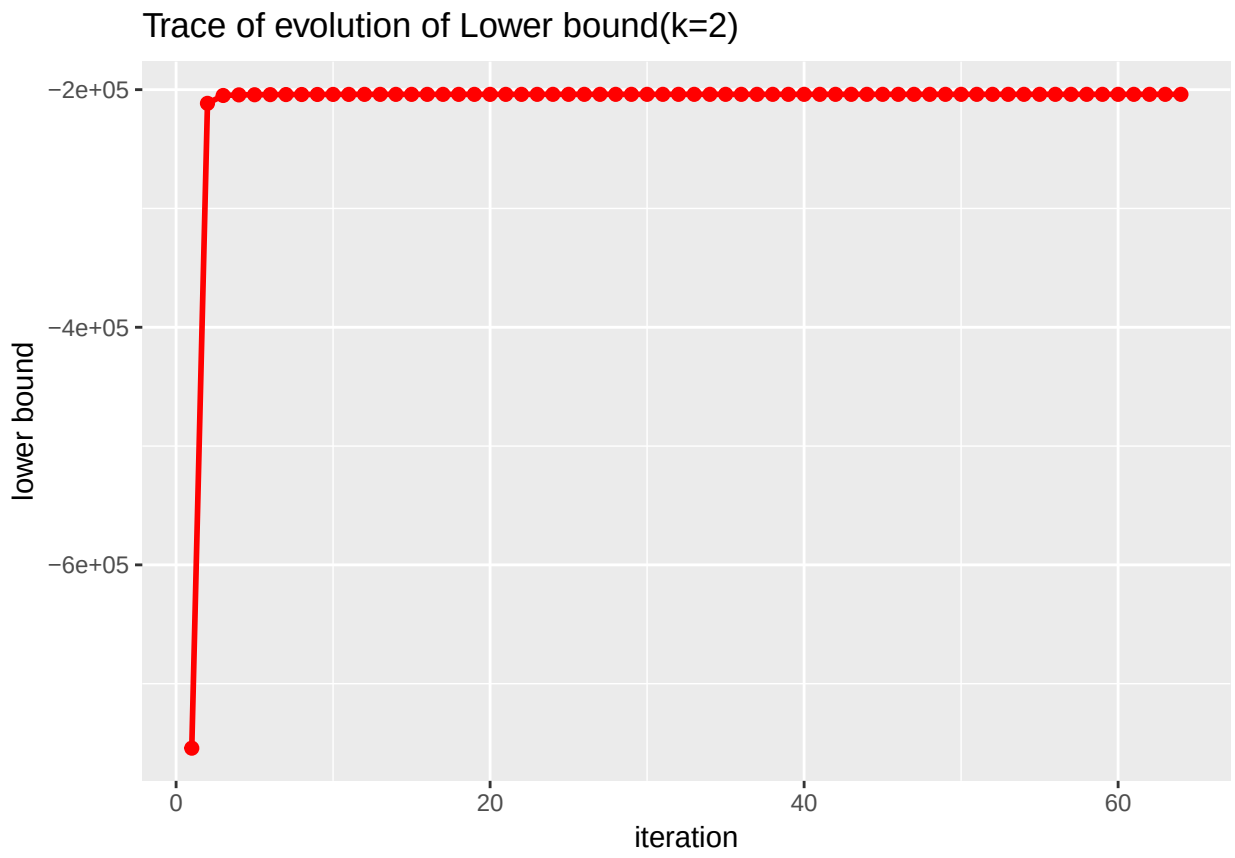

```
#show 3 cluster image
for(i in 1:2){
     show_digit(mu_3[i,])
}
```


0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0

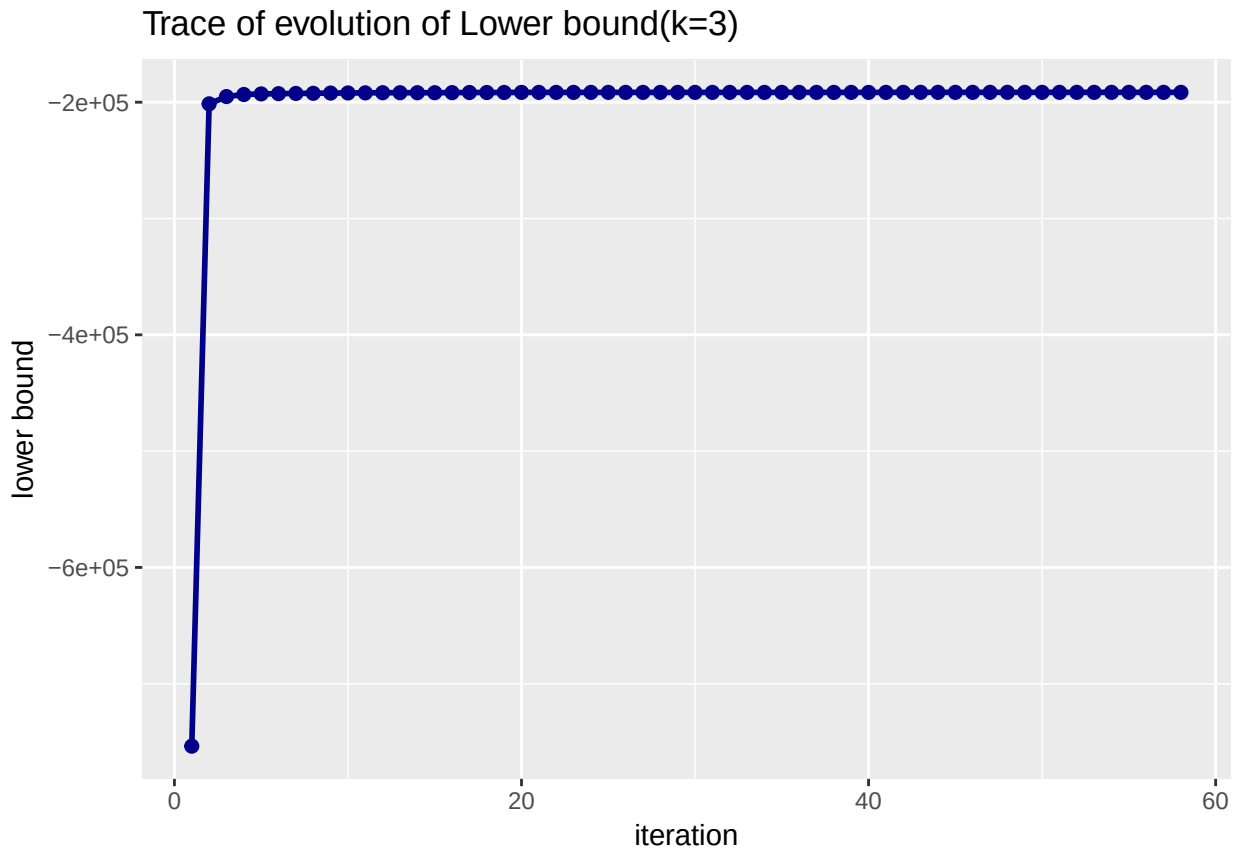


The trace of evolution of F

```
lower_bound_2 <- em_2[[3]]
lower_bound_3 <- em_3[[3]]
#F of image_2
lower_bound_2 <-lower_bound_2[-1]
index <- c(1:length(lower_bound_2))
plot <- as.data.frame(cbind(index, lower_bound_2))
plot %>% ggplot(aes(x=index, y=lower_bound_2))+geom_point(color = "red", size = 2) + geom_line(color =
```




```
#F of image_3
lower_bound_3 <-lower_bound_3[-1]
index_3 <- c(1:length(lower_bound_3))
plot_3 <- as.data.frame(cbind(index_3, lower_bound_3))
plot_3 %>% ggplot(aes(x=index_3, y=lower_bound_3))+geom_point(color = "darkblue", size = 2) + geom_line
```




```
#write down the value of pi, f
pi_2 <- em_2[[1]]
pi_3 <- em_3[[1]]
print(pi_2, pi_3)
```

```
## [1] 1 0
```

```
last_h <- c(lower_bound_2[length(lower_bound_2)], lower_bound_3[length(lower_bound_3)])
print(last_h)
```

```
## [1] -203956.7 -191515.0
```

9.Entropy

```
entropy_2 <- em_2[[4]]
entropy_3 <- em_3[[4]]
#K=2, calculate the entropy for final q of each digit
print(entropy_2)
```

```
##                   [,1]           [,2]
##    [1,]  -1.721762e-26   0.000000e+00
##    [2,]  -2.727729e-11  -6.635196e-10
##    [3,]            NaN   0.000000e+00
##    [4,] -1.119554e-270   0.000000e+00
##    [5,]   0.000000e+00  -8.601672e-73
##    [6,]  -1.824466e-14  -4.440892e-16
##    [7,]   0.000000e+00            NaN
```

```
##    [8,]  -7.899237e-12  -2.019386e-10
##    [9,]   0.000000e+00  -2.599797e-33
##   [10,]   0.000000e+00  -1.187582e-26
##   [11,]   0.000000e+00  -9.312730e-36
##   [12,] -5.164593e-279   0.000000e+00
##   [13,]           NaN   0.000000e+00
##   [14,]  -7.000502e-07  -9.921194e-06
##   [15,]           NaN   0.000000e+00
##   [16,]   0.000000e+00  -1.759085e-45
##   [17,]  -1.127987e-12  -3.103232e-11
##   [18,]   0.000000e+00  -8.317169e-38
##   [19,]   0.000000e+00  -6.095116e-24
##   [20,]  -3.665627e-02  -7.461577e-03
##   [21,]  -9.009320e-27   0.000000e+00
##   [22,]           NaN   0.000000e+00
##   [23,]           NaN   0.000000e+00
##   [24,]   0.000000e+00  -5.317556e-74
##   [25,]           NaN   0.000000e+00
##   [26,]  -2.042810e-14  -6.454448e-13
##   [27,]   0.000000e+00  -9.329122e-44
##   [28,]           NaN   0.000000e+00
##   [29,]           NaN   0.000000e+00
##   [30,]           NaN   0.000000e+00
##   [31,]   0.000000e+00 -8.383660e-286
##   [32,]           NaN   0.000000e+00
##   [33,]           NaN   0.000000e+00
##   [34,]           NaN   0.000000e+00
##   [35,]   0.000000e+00           NaN
##   [36,] -6.713292e-151   0.000000e+00
##   [37,]           NaN   0.000000e+00
##   [38,]   0.000000e+00  -2.201775e-17
##   [39,]           NaN   0.000000e+00
##   [40,]           NaN   0.000000e+00
##   [41,]   0.000000e+00  -3.180878e-40
##   [42,]           NaN   0.000000e+00
##   [43,]   0.000000e+00  -1.646353e-52
##   [44,]   0.000000e+00  -1.088795e-49
##   [45,]  -3.153949e-07  -1.766767e-08
##   [46,]   0.000000e+00  -3.661966e-59
##   [47,]           NaN   0.000000e+00
##   [48,]           NaN   0.000000e+00
##   [49,]   0.000000e+00  -2.069773e-57
##   [50,]   0.000000e+00  -9.707018e-16
##   [51,]           NaN   0.000000e+00
##   [52,]           NaN   0.000000e+00
##   [53,]           NaN   0.000000e+00
##   [54,]   0.000000e+00  -1.974018e-26
##   [55,] -2.765477e-152   0.000000e+00
##   [56,]           NaN   0.000000e+00
##   [57,]           NaN   0.000000e+00
##   [58,]   0.000000e+00  -5.292766e-48
##   [59,]           NaN   0.000000e+00
##   [60,]  -8.622759e-49   0.000000e+00
##   [61,]  -2.421343e-05  -2.573577e-04
```

```
##   [62,]  -9.547918e-15  -3.083468e-13
##   [63,]   0.000000e+00  -1.141872e-47
##   [64,]            NaN   0.000000e+00
##   [65,]            NaN   0.000000e+00
##   [66,]            NaN   0.000000e+00
##   [67,]   0.000000e+00  -3.410801e-55
##   [68,]   0.000000e+00  -4.720837e-65
##   [69,]            NaN   0.000000e+00
##   [70,]   0.000000e+00  -3.806532e-43
##   [71,]            NaN   0.000000e+00
##   [72,]   0.000000e+00  -7.648223e-53
##   [73,]  -1.204586e-02  -1.924888e-03
##   [74,]   0.000000e+00  -8.703787e-66
##   [75,]  -2.889586e-08  -1.418284e-09
##   [76,]  -6.661338e-16  -2.391239e-14
##   [77,]            NaN   0.000000e+00
##   [78,]   0.000000e+00  -1.420904e-34
##   [79,]  -6.180565e-28   0.000000e+00
##   [80,]   0.000000e+00  -3.270526e-40
##   [81,]            NaN   0.000000e+00
##   [82,]            NaN   0.000000e+00
##   [83,]  -1.926481e-11  -4.753155e-10
##   [84,] -1.133682e-127   0.000000e+00
##   [85,]   0.000000e+00  -4.063075e-27
##   [86,]   0.000000e+00  -4.202935e-62
##   [87,] -1.178739e-247   0.000000e+00
##   [88,]            NaN   0.000000e+00
##   [89,]   0.000000e+00  -4.748710e-56
##   [90,]   0.000000e+00  -1.416561e-16
##   [91,]  -2.065681e-12  -5.557820e-11
##   [92,]            NaN   0.000000e+00
##   [93,]            NaN   0.000000e+00
##   [94,]            NaN   0.000000e+00
##   [95,]            NaN   0.000000e+00
##   [96,]            NaN   0.000000e+00
##   [97,]   0.000000e+00  -4.869654e-43
##   [98,]            NaN   0.000000e+00
##   [99,]            NaN   0.000000e+00
##  [100,]            NaN   0.000000e+00
##  [101,]            NaN   0.000000e+00
##  [102,]            NaN   0.000000e+00
##  [103,]  -6.015182e-10  -2.462497e-11
##  [104,]            NaN   0.000000e+00
##  [105,]  -9.212778e-07  -5.512100e-08
##  [106,]            NaN   0.000000e+00
##  [107,]   0.000000e+00  -2.436710e-57
##  [108,]   0.000000e+00  -1.328452e-22
##  [109,] -1.964461e-165   0.000000e+00
##  [110,]  -1.375557e-79   0.000000e+00
##  [111,]            NaN   0.000000e+00
##  [112,]   0.000000e+00  -4.386631e-18
##  [113,]            NaN   0.000000e+00
##  [114,]   0.000000e+00  -3.036598e-44
##  [115,]            NaN   0.000000e+00
```

```
##  [116,]            NaN   0.000000e+00
##  [117,]  -1.869908e-17   0.000000e+00
##  [118,]   0.000000e+00  -4.932722e-43
##  [119,]  -3.469703e-08  -5.959775e-07
##  [120,]  -4.440892e-16  -1.898763e-14
##  [121,]   0.000000e+00  -1.231612e-36
##  [122,]   0.000000e+00  -1.819982e-40
##  [123,]   0.000000e+00  -3.847157e-15
##  [124,]  -1.092968e-11  -3.823608e-13
##  [125,]   0.000000e+00            NaN
##  [126,]   0.000000e+00  -1.055009e-59
##  [127,]  -3.230116e-07  -1.812000e-08
##  [128,]  -5.827194e-10  -1.239055e-08
##  [129,]   0.000000e+00  -7.132982e-22
##  [130,]   0.000000e+00  -4.254345e-70
##  [131,]  -3.876877e-10  -8.401510e-09
##  [132,]   0.000000e+00  -1.925690e-61
##  [133,]  -6.661338e-16  -2.467229e-14
##  [134,]   0.000000e+00  -1.737694e-65
##  [135,]  -6.094443e-14  -1.776357e-15
##  [136,]   0.000000e+00            NaN
##  [137,]   0.000000e+00            NaN
##  [138,]            NaN   0.000000e+00
##  [139,]            NaN   0.000000e+00
##  [140,]   0.000000e+00  -1.666596e-17
##  [141,]   0.000000e+00  -5.557799e-26
##  [142,]            NaN   0.000000e+00
##  [143,]  -7.204299e-54   0.000000e+00
##  [144,]   0.000000e+00  -1.403446e-39
##  [145,]   0.000000e+00  -3.223668e-20
##  [146,]  -5.017911e-04  -3.813102e-03
##  [147,]            NaN   0.000000e+00
##  [148,]            NaN   0.000000e+00
##  [149,]            NaN   0.000000e+00
##  [150,]  -7.006188e-31   0.000000e+00
##  [151,]            NaN   0.000000e+00
##  [152,]            NaN   0.000000e+00
##  [153,]   0.000000e+00  -9.334015e-60
##  [154,]  -2.246868e-03  -1.371480e-02
##  [155,]            NaN   0.000000e+00
##  [156,]            NaN   0.000000e+00
##  [157,]   0.000000e+00  -1.231509e-53
##  [158,]   0.000000e+00  -1.637107e-62
##  [159,]   0.000000e+00  -2.513507e-39
##  [160,]            NaN   0.000000e+00
##  [161,]   0.000000e+00  -9.398873e-34
##  [162,] -8.695536e-302   0.000000e+00
##  [163,]   0.000000e+00 -6.677129e-281
##  [164,]            NaN   0.000000e+00
##  [165,]            NaN   0.000000e+00
##  [166,]  -2.666984e-32   0.000000e+00
##  [167,]   0.000000e+00  -5.747537e-53
##  [168,]   0.000000e+00  -4.080010e-42
##  [169,] -1.433521e-134   0.000000e+00
```

```
##  [170,]            NaN   0.000000e+00
##  [171,]   0.000000e+00  -9.818235e-34
##  [172,]   0.000000e+00  -7.123087e-42
##  [173,]   0.000000e+00  -2.023755e-21
##  [174,]  -1.997059e-07  -3.080747e-06
##  [175,]   0.000000e+00  -7.210017e-30
##  [176,]  -1.159497e-14  -2.220446e-16
##  [177,]  -1.075951e-15   0.000000e+00
##  [178,]   0.000000e+00  -1.080378e-21
##  [179,]   0.000000e+00  -6.639325e-21
##  [180,]   0.000000e+00  -4.754270e-34
##  [181,]   0.000000e+00            NaN
##  [182,]   0.000000e+00  -3.353757e-35
##  [183,]   0.000000e+00  -6.203906e-65
##  [184,]   0.000000e+00  -2.728740e-46
##  [185,]   0.000000e+00            NaN
##  [186,] -1.716144e-308   0.000000e+00
##  [187,]   0.000000e+00  -2.807770e-40
##  [188,]  -3.695906e-11  -1.352474e-12
##  [189,]   0.000000e+00  -2.032985e-21
##  [190,]   0.000000e+00  -3.829768e-44
##  [191,]            NaN   0.000000e+00
##  [192,]   0.000000e+00  -5.273423e-70
##  [193,]  -1.852968e-32   0.000000e+00
##  [194,]   0.000000e+00 -1.897497e-171
##  [195,]   0.000000e+00  -3.417832e-22
##  [196,]   0.000000e+00  -8.361443e-51
##  [197,]   0.000000e+00  -9.262141e-64
##  [198,]   0.000000e+00  -7.981318e-32
##  [199,] -1.724961e-227   0.000000e+00
##  [200,]   0.000000e+00  -3.766075e-20
##  [201,]   0.000000e+00 -1.365132e-297
##  [202,]   0.000000e+00  -4.925434e-72
##  [203,]   0.000000e+00  -9.883503e-37
##  [204,]            NaN   0.000000e+00
##  [205,]  -3.589367e-22   0.000000e+00
##  [206,]   0.000000e+00  -2.715786e-35
##  [207,]   0.000000e+00  -9.514417e-35
##  [208,]   0.000000e+00  -2.205105e-50
##  [209,]  -8.656629e-08  -1.407772e-06
##  [210,]  -1.088814e-09  -4.573275e-11
##  [211,]  -3.673224e-21   0.000000e+00
##  [212,]   0.000000e+00            NaN
##  [213,] -4.186446e-157   0.000000e+00
##  [214,]   0.000000e+00  -2.839811e-43
##  [215,]  -5.411703e-14  -1.554312e-15
##  [216,]   0.000000e+00  -6.566000e-42
##  [217,]  -1.556278e-10  -6.023848e-12
##  [218,]   0.000000e+00  -2.998744e-40
##  [219,]  -2.053051e-44   0.000000e+00
##  [220,]   0.000000e+00            NaN
##  [221,]   0.000000e+00  -2.717205e-48
##  [222,]   0.000000e+00            NaN
##  [223,]   0.000000e+00  -3.529476e-31
```

```
##  [224,]  -1.153122e-11  -2.904273e-10
##  [225,]            NaN   0.000000e+00
##  [226,]   0.000000e+00 -3.159791e-258
##  [227,]  -7.406459e-06  -8.749397e-05
##  [228,]            NaN   0.000000e+00
##  [229,]   0.000000e+00  -3.945219e-28
##  [230,]   0.000000e+00  -7.045281e-31
##  [231,]            NaN   0.000000e+00
##  [232,]            NaN   0.000000e+00
##  [233,]   0.000000e+00  -8.055539e-47
##  [234,]  -4.773959e-14  -1.461320e-12
##  [235,]            NaN   0.000000e+00
##  [236,]            NaN   0.000000e+00
##  [237,]  -5.410641e-09  -2.444722e-10
##  [238,]            NaN   0.000000e+00
##  [239,]  -6.358980e-11  -1.492999e-09
##  [240,]            NaN   0.000000e+00
##  [241,]   0.000000e+00  -6.306744e-26
##  [242,]   0.000000e+00  -1.660737e-27
##  [243,]  -1.162617e-09  -4.897349e-11
##  [244,]            NaN   0.000000e+00
##  [245,] -1.045911e-130   0.000000e+00
##  [246,]   0.000000e+00  -1.913578e-71
##  [247,]            NaN   0.000000e+00
##  [248,]   0.000000e+00  -3.924130e-35
##  [249,]  -1.230242e-15   0.000000e+00
##  [250,]            NaN   0.000000e+00
##  [251,]            NaN   0.000000e+00
##  [252,]  -3.815526e-12  -1.285638e-13
##  [253,]   0.000000e+00  -3.562735e-60
##  [254,]   0.000000e+00  -1.652501e-56
##  [255,]  -2.123876e-33   0.000000e+00
##  [256,]            NaN   0.000000e+00
##  [257,]  -9.008713e-03  -1.364845e-03
##  [258,] -8.183586e-143   0.000000e+00
##  [259,]   0.000000e+00  -1.314177e-28
##  [260,]  -8.384972e-36   0.000000e+00
##  [261,]            NaN   0.000000e+00
##  [262,]  -4.348365e-02  -1.384711e-01
##  [263,]            NaN   0.000000e+00
##  [264,]   0.000000e+00            NaN
##  [265,]   0.000000e+00  -7.134156e-26
##  [266,]   0.000000e+00  -7.350072e-29
##  [267,]            NaN   0.000000e+00
##  [268,]   0.000000e+00  -1.187747e-57
##  [269,]   0.000000e+00  -5.043459e-21
##  [270,]   0.000000e+00  -1.522677e-35
##  [271,]   0.000000e+00            NaN
##  [272,]            NaN   0.000000e+00
##  [273,]  -3.438591e-19   0.000000e+00
##  [274,]   0.000000e+00  -1.252797e-23
##  [275,] -8.784634e-135   0.000000e+00
##  [276,]  -4.597499e-05  -4.591807e-04
##  [277,]   0.000000e+00  -6.328919e-57
```

```
##  [278,]   0.000000e+00  -2.588099e-55
##  [279,]   0.000000e+00  -8.600421e-46
##  [280,]             NaN   0.000000e+00
##  [281,]   0.000000e+00  -1.376906e-36
##  [282,]   0.000000e+00  -1.234116e-49
##  [283,]   0.000000e+00  -1.855868e-28
##  [284,]  -2.669709e-11  -6.499775e-10
##  [285,]             NaN   0.000000e+00
##  [286,]  -6.227805e-03  -8.856561e-04
##  [287,]             NaN   0.000000e+00
##  [288,]   0.000000e+00  -1.390460e-36
##  [289,]             NaN   0.000000e+00
##  [290,]             NaN   0.000000e+00
##  [291,]             NaN   0.000000e+00
##  [292,]             NaN   0.000000e+00
##  [293,]             NaN   0.000000e+00
##  [294,]   0.000000e+00  -1.406345e-63
##  [295,]  -2.220446e-16  -1.171032e-14
##  [296,]  -2.468101e-23   0.000000e+00
##  [297,]  -1.196852e-41   0.000000e+00
##  [298,]   0.000000e+00  -1.235995e-56
##  [299,]  -8.676939e-03  -1.305868e-03
##  [300,]  -4.009676e-26   0.000000e+00
##  [301,]  -4.143976e-20   0.000000e+00
##  [302,] -1.399856e-149   0.000000e+00
##  [303,]  -9.630383e-08  -1.555861e-06
##  [304,]   0.000000e+00  -3.664064e-28
##  [305,] -3.057543e-212   0.000000e+00
##  [306,]             NaN   0.000000e+00
##  [307,]  -5.689223e-42   0.000000e+00
##  [308,]  -2.379184e-01  -1.001872e-01
##  [309,]  -1.718134e-47   0.000000e+00
##  [310,]   0.000000e+00  -1.450453e-18
##  [311,]             NaN   0.000000e+00
##  [312,]   0.000000e+00  -7.720878e-40
##  [313,]   0.000000e+00  -4.376454e-49
##  [314,]             NaN   0.000000e+00
##  [315,]  -8.737766e-06  -6.106598e-07
##  [316,] -3.270065e-220   0.000000e+00
##  [317,]             NaN   0.000000e+00
##  [318,]   0.000000e+00             NaN
##  [319,]   0.000000e+00 -6.639461e-303
##  [320,]  -6.875323e-10  -1.450549e-08
##  [321,]  -2.859959e-27   0.000000e+00
##  [322,]   0.000000e+00  -3.253139e-77
##  [323,]   0.000000e+00  -1.882731e-56
##  [324,]   0.000000e+00             NaN
##  [325,]   0.000000e+00  -2.581133e-29
##  [326,]             NaN   0.000000e+00
##  [327,]   0.000000e+00  -1.491860e-19
##  [328,]             NaN   0.000000e+00
##  [329,]  -1.134998e-08  -2.076371e-07
##  [330,]  -2.571323e-32   0.000000e+00
##  [331,]  -8.411240e-03  -4.032557e-02
```

```
##  [332,]   0.000000e+00  -1.351751e-39
##  [333,]  -3.401086e-03  -4.399689e-04
##  [334,]  -4.486004e-11  -1.653566e-12
##  [335,]  -3.507670e-18   0.000000e+00
##  [336,]  -2.400148e-31   0.000000e+00
##  [337,]   0.000000e+00            NaN
##  [338,]  -1.065814e-14  -3.397084e-13
##  [339,]  -4.817213e-11  -1.780576e-12
##  [340,]   0.000000e+00  -1.803069e-47
##  [341,]  -3.428096e-01  -2.090859e-01
##  [342,]   0.000000e+00  -4.997417e-42
##  [343,]            NaN   0.000000e+00
##  [344,]  -3.316350e-37   0.000000e+00
##  [345,]   0.000000e+00  -3.564244e-34
##  [346,]  -2.761175e-52   0.000000e+00
##  [347,]   0.000000e+00  -3.477067e-24
##  [348,]  -5.968830e-12  -2.042810e-13
##  [349,]   0.000000e+00  -1.199121e-20
##  [350,] -1.496291e-175   0.000000e+00
##  [351,]   0.000000e+00 -2.059617e-312
##  [352,]            NaN   0.000000e+00
##  [353,]   0.000000e+00  -1.090200e-60
##  [354,]   0.000000e+00  -1.529877e-21
##  [355,]   0.000000e+00  -2.253654e-42
##  [356,]            NaN   0.000000e+00
##  [357,]            NaN   0.000000e+00
##  [358,]            NaN   0.000000e+00
##  [359,]   0.000000e+00  -3.941042e-36
##  [360,]            NaN   0.000000e+00
##  [361,]            NaN   0.000000e+00
##  [362,]  -6.404632e-47   0.000000e+00
##  [363,]   0.000000e+00  -2.821950e-28
##  [364,]   0.000000e+00            NaN
##  [365,]            NaN   0.000000e+00
##  [366,]   0.000000e+00  -4.265719e-52
##  [367,]   0.000000e+00  -6.281763e-65
##  [368,]   0.000000e+00  -1.103814e-50
##  [369,]            NaN   0.000000e+00
##  [370,]   0.000000e+00  -9.461136e-37
##  [371,]  -1.159744e-23   0.000000e+00
##  [372,]   0.000000e+00  -1.918283e-41
##  [373,]            NaN   0.000000e+00
##  [374,]            NaN   0.000000e+00
##  [375,]            NaN   0.000000e+00
##  [376,]  -1.237408e-03  -1.393639e-04
##  [377,]   0.000000e+00 -5.651088e-287
##  [378,]   0.000000e+00  -1.446786e-58
##  [379,]  -2.435244e-08  -4.269137e-07
##  [380,]  -6.740073e-41   0.000000e+00
##  [381,]  -1.848810e-26   0.000000e+00
##  [382,]  -3.120748e-13  -9.769963e-15
##  [383,]   0.000000e+00  -8.735431e-41
##  [384,]            NaN   0.000000e+00
##  [385,]   0.000000e+00  -1.985964e-36
```

```
##  [386,]  -4.348277e-11  -1.037440e-09
##  [387,]   0.000000e+00  -2.880667e-30
##  [388,]   0.000000e+00  -1.073228e-52
##  [389,]            NaN   0.000000e+00
##  [390,]  -1.220630e-10  -2.786271e-09
##  [391,] -6.666034e-217   0.000000e+00
##  [392,]   0.000000e+00  -1.646706e-48
##  [393,]  -4.324998e-19   0.000000e+00
##  [394,]  -1.550159e-05  -1.132218e-06
##  [395,]            NaN   0.000000e+00
##  [396,]   0.000000e+00  -7.897032e-22
##  [397,]  -3.291799e-02  -6.524232e-03
##  [398,]  -7.347174e-02  -1.821536e-02
##  [399,]   0.000000e+00  -1.986099e-34
##  [400,]   0.000000e+00  -7.286034e-48
##  [401,]            NaN   0.000000e+00
##  [402,]            NaN   0.000000e+00
##  [403,]            NaN   0.000000e+00
##  [404,]   0.000000e+00  -5.148845e-84
##  [405,]   0.000000e+00  -7.661442e-19
##  [406,]   0.000000e+00            NaN
##  [407,]  -6.134869e-09  -1.160059e-07
##  [408,]   0.000000e+00  -2.224611e-58
##  [409,]  -1.097519e-07  -1.758779e-06
##  [410,]            NaN   0.000000e+00
##  [411,]            NaN   0.000000e+00
##  [412,]   0.000000e+00  -8.922684e-60
##  [413,]  -4.440892e-16  -1.698529e-14
##  [414,] -5.232638e-172   0.000000e+00
##  [415,]  -1.488912e-27   0.000000e+00
##  [416,]   0.000000e+00  -3.720118e-55
##  [417,]   0.000000e+00  -1.344483e-40
##  [418,]   0.000000e+00  -2.898243e-70
##  [419,]   0.000000e+00  -7.434084e-31
##  [420,]  -1.211683e-15   0.000000e+00
##  [421,]  -4.132926e-22   0.000000e+00
##  [422,]  -1.794384e-12  -5.884182e-14
##  [423,]            NaN   0.000000e+00
##  [424,]   0.000000e+00  -1.743126e-34
##  [425,]  -9.543221e-04  -6.639557e-03
##  [426,]            NaN   0.000000e+00
##  [427,]  -1.119386e-02  -1.764275e-03
##  [428,]   0.000000e+00  -9.556068e-39
##  [429,]  -1.258683e-09  -5.320588e-11
##  [430,] -1.529678e-245   0.000000e+00
##  [431,]   0.000000e+00  -6.935881e-36
##  [432,]            NaN   0.000000e+00
##  [433,]  -4.711917e-18   0.000000e+00
##  [434,]            NaN   0.000000e+00
##  [435,]   0.000000e+00  -1.231436e-35
##  [436,]   0.000000e+00  -1.809610e-70
##  [437,]            NaN   0.000000e+00
##  [438,]  -1.777089e-39   0.000000e+00
##  [439,]   0.000000e+00            NaN
```

```
##  [440,]  -3.465446e-03  -4.495452e-04
##  [441,]   0.000000e+00  -5.665168e-34
##  [442,]  -2.511638e-09  -4.973629e-08
##  [443,]   0.000000e+00  -7.799948e-36
##  [444,]   0.000000e+00  -1.318914e-23
##  [445,]   0.000000e+00  -5.142468e-38
##  [446,]   0.000000e+00           NaN
##  [447,]  -6.661338e-16  -2.092559e-14
##  [448,]  -2.220446e-16  -8.785211e-15
##  [449,]           NaN   0.000000e+00
##  [450,]  -3.648254e-35   0.000000e+00
##  [451,]           NaN   0.000000e+00
##  [452,]  -2.368689e-53   0.000000e+00
##  [453,]   0.000000e+00  -1.967744e-48
##  [454,]   0.000000e+00  -2.149818e-27
##  [455,]  -3.193446e-12  -8.452945e-11
##  [456,]   0.000000e+00  -8.418379e-48
##  [457,]   0.000000e+00  -3.991552e-64
##  [458,]   0.000000e+00  -9.119796e-54
##  [459,]  -3.790383e-40   0.000000e+00
##  [460,]   0.000000e+00  -3.836632e-52
##  [461,]   0.000000e+00  -2.399832e-40
##  [462,]   0.000000e+00  -1.798918e-32
##  [463,]  -4.104634e-03  -2.259566e-02
##  [464,]   0.000000e+00  -1.495878e-37
##  [465,]   0.000000e+00  -6.305994e-49
##  [466,]   0.000000e+00  -2.477045e-29
##  [467,]   0.000000e+00  -1.130758e-44
##  [468,]   0.000000e+00  -5.442931e-32
##  [469,]  -2.307082e-04  -2.146262e-05
##  [470,]   0.000000e+00           NaN
##  [471,]   0.000000e+00  -5.852981e-38
##  [472,]   0.000000e+00 -2.067850e-285
##  [473,]   0.000000e+00  -2.098480e-25
##  [474,]   0.000000e+00  -1.577289e-22
##  [475,]  -9.473706e-03  -1.448268e-03
##  [476,]   0.000000e+00           NaN
##  [477,]   0.000000e+00  -3.323934e-48
##  [478,]   0.000000e+00  -1.426511e-40
##  [479,]   0.000000e+00  -6.028397e-23
##  [480,]   0.000000e+00           NaN
##  [481,]   0.000000e+00  -6.654161e-66
##  [482,]  -5.195844e-14  -1.587790e-12
##  [483,]  -9.110170e-17   0.000000e+00
##  [484,]           NaN   0.000000e+00
##  [485,]  -1.427261e-10  -5.505374e-12
##  [486,]   0.000000e+00  -1.206765e-54
##  [487,]  -3.035417e-19   0.000000e+00
##  [488,]   0.000000e+00  -2.950465e-30
##  [489,] -1.260235e-313   0.000000e+00
##  [490,]  -1.089783e-07  -1.747153e-06
##  [491,]           NaN   0.000000e+00
##  [492,]   0.000000e+00  -3.714372e-42
##  [493,]   0.000000e+00  -2.774660e-54
```

```
##  [494,] -2.309272e-189   0.000000e+00
##  [495,]  -6.218505e-19   0.000000e+00
##  [496,]  -6.877136e-09  -3.143019e-10
##  [497,]   0.000000e+00  -2.547933e-36
##  [498,]  -2.207147e-04  -1.858298e-03
##  [499,]            NaN   0.000000e+00
##  [500,]  -2.642278e-12  -8.792966e-14
##  [501,]            NaN   0.000000e+00
##  [502,]   0.000000e+00  -3.356412e-46
##  [503,]   0.000000e+00 -1.370781e-290
##  [504,]  -5.599051e-38   0.000000e+00
##  [505,]            NaN   0.000000e+00
##  [506,]            NaN   0.000000e+00
##  [507,]   0.000000e+00  -3.628160e-18
##  [508,]            NaN   0.000000e+00
##  [509,]   0.000000e+00  -7.133001e-38
##  [510,]   0.000000e+00  -1.086987e-27
##  [511,]            NaN   0.000000e+00
##  [512,]            NaN   0.000000e+00
##  [513,]            NaN   0.000000e+00
##  [514,]            NaN   0.000000e+00
##  [515,]            NaN   0.000000e+00
##  [516,]            NaN   0.000000e+00
##  [517,]            NaN   0.000000e+00
##  [518,]   0.000000e+00  -6.123503e-61
##  [519,]   0.000000e+00  -1.280782e-30
##  [520,]  -2.886580e-15  -9.765311e-14
##  [521,]  -2.220446e-13  -6.468082e-12
##  [522,]            NaN   0.000000e+00
##  [523,]  -5.124894e-03  -2.708336e-02
##  [524,]            NaN   0.000000e+00
##  [525,]   0.000000e+00  -4.079985e-39
##  [526,]            NaN   0.000000e+00
##  [527,]   0.000000e+00 -2.994387e-228
##  [528,]  -1.665135e-11  -4.132628e-10
##  [529,]   0.000000e+00  -1.230075e-53
##  [530,]  -9.694130e-02  -2.333173e-01
##  [531,]   0.000000e+00  -5.790551e-58
##  [532,]   0.000000e+00 -5.068961e-309
##  [533,]  -6.507683e-39   0.000000e+00
##  [534,]  -3.960471e-07  -5.838421e-06
##  [535,]  -7.932221e-50   0.000000e+00
##  [536,]   0.000000e+00  -3.442089e-46
##  [537,] -6.165438e-156   0.000000e+00
##  [538,]            NaN   0.000000e+00
##  [539,]            NaN   0.000000e+00
##  [540,]            NaN   0.000000e+00
##  [541,]   0.000000e+00  -1.936586e-43
##  [542,]  -9.641554e-14  -2.886580e-15
##  [543,]   0.000000e+00            NaN
##  [544,]            NaN   0.000000e+00
##  [545,]            NaN   0.000000e+00
##  [546,]  -2.871127e-06  -1.852130e-07
##  [547,]            NaN   0.000000e+00
```

```
##  [548,]            NaN   0.000000e+00
##  [549,]   0.000000e+00  -1.607200e-48
##  [550,]   0.000000e+00  -5.785412e-46
##  [551,]            NaN   0.000000e+00
##  [552,]            NaN   0.000000e+00
##  [553,]  -7.909007e-12  -2.021804e-10
##  [554,]   0.000000e+00  -8.039726e-66
##  [555,]            NaN   0.000000e+00
##  [556,]   0.000000e+00  -9.306459e-53
##  [557,]            NaN   0.000000e+00
##  [558,]   0.000000e+00 -3.891297e-296
##  [559,]   0.000000e+00  -8.854695e-37
##  [560,]   0.000000e+00  -1.104473e-42
##  [561,]            NaN   0.000000e+00
##  [562,]   0.000000e+00  -1.591784e-54
##  [563,]            NaN   0.000000e+00
##  [564,]            NaN   0.000000e+00
##  [565,]  -1.582879e-17   0.000000e+00
##  [566,]            NaN   0.000000e+00
##  [567,]            NaN   0.000000e+00
##  [568,]            NaN   0.000000e+00
##  [569,]   0.000000e+00  -2.386311e-26
##  [570,]            NaN   0.000000e+00
##  [571,]            NaN   0.000000e+00
##  [572,]   0.000000e+00  -1.624243e-39
##  [573,]  -1.151301e-30   0.000000e+00
##  [574,]   0.000000e+00  -4.860300e-47
##  [575,]            NaN   0.000000e+00
##  [576,]   0.000000e+00  -6.967140e-45
##  [577,]   0.000000e+00            NaN
##  [578,]   0.000000e+00            NaN
##  [579,]  -9.197803e-55   0.000000e+00
##  [580,]  -3.310255e-15   0.000000e+00
##  [581,]   0.000000e+00  -2.492141e-55
##  [582,] -1.801285e-227   0.000000e+00
##  [583,]   0.000000e+00  -1.289528e-43
##  [584,]            NaN   0.000000e+00
##  [585,]  -5.458326e-14  -1.554312e-15
##  [586,]            NaN   0.000000e+00
##  [587,]  -3.675940e-29   0.000000e+00
##  [588,]   0.000000e+00  -1.945738e-18
##  [589,]   0.000000e+00            NaN
##  [590,]            NaN   0.000000e+00
##  [591,]  -1.033037e-52   0.000000e+00
##  [592,]   0.000000e+00  -7.586599e-17
##  [593,]   0.000000e+00  -6.674737e-20
##  [594,]   0.000000e+00 -4.551235e-191
##  [595,]   0.000000e+00  -5.500933e-53
##  [596,]   0.000000e+00  -1.341244e-39
##  [597,]   0.000000e+00  -1.669470e-15
##  [598,]  -1.515544e-15   0.000000e+00
##  [599,]   0.000000e+00            NaN
##  [600,]  -1.555114e-25   0.000000e+00
##  [601,]   0.000000e+00  -5.483321e-66
```

```
##  [602,]            NaN   0.000000e+00
##  [603,]  -1.298542e-07  -2.059080e-06
##  [604,]  -9.484465e-03  -4.434612e-02
##  [605,]   0.000000e+00  -1.937471e-17
##  [606,]            NaN   0.000000e+00
##  [607,]   0.000000e+00  -7.248042e-42
##  [608,]   0.000000e+00  -9.088941e-67
##  [609,]  -1.656274e-05  -1.823299e-04
##  [610,]   0.000000e+00  -8.947271e-18
##  [611,]   0.000000e+00  -1.344478e-30
##  [612,]   0.000000e+00  -1.318878e-39
##  [613,]   0.000000e+00  -2.975323e-74
##  [614,]  -4.237821e-27   0.000000e+00
##  [615,]   0.000000e+00  -3.733882e-20
##  [616,]   0.000000e+00  -3.127567e-41
##  [617,]   0.000000e+00  -3.219489e-44
##  [618,]  -2.056181e-28   0.000000e+00
##  [619,]   0.000000e+00            NaN
##  [620,] -1.099829e-163   0.000000e+00
##  [621,]   0.000000e+00  -2.026660e-48
##  [622,]   0.000000e+00  -3.435937e-22
##  [623,]   0.000000e+00            NaN
##  [624,]   0.000000e+00  -2.258956e-48
##  [625,]  -2.222470e-09  -4.428193e-08
##  [626,]  -1.989147e-27   0.000000e+00
##  [627,]            NaN   0.000000e+00
##  [628,]            NaN   0.000000e+00
##  [629,]            NaN   0.000000e+00
##  [630,]            NaN   0.000000e+00
##  [631,]            NaN   0.000000e+00
##  [632,]   0.000000e+00  -4.929235e-41
##  [633,]   0.000000e+00  -6.917148e-65
##  [634,]   0.000000e+00  -1.192948e-34
##  [635,]            NaN   0.000000e+00
##  [636,]   0.000000e+00            NaN
##  [637,]   0.000000e+00  -1.470682e-42
##  [638,]            NaN   0.000000e+00
##  [639,]  -2.338398e-20   0.000000e+00
##  [640,]            NaN   0.000000e+00
##  [641,]  -1.982674e-23   0.000000e+00
##  [642,]   0.000000e+00  -1.468292e-19
##  [643,] -2.036274e-170   0.000000e+00
##  [644,]   0.000000e+00  -5.032798e-31
##  [645,]   0.000000e+00  -2.106499e-27
##  [646,]            NaN   0.000000e+00
##  [647,]  -3.497682e-03  -4.543531e-04
##  [648,]   0.000000e+00  -2.368863e-25
##  [649,]  -3.035636e-05  -3.157865e-04
##  [650,]  -1.554090e-12  -4.225694e-11
##  [651,]            NaN   0.000000e+00
##  [652,]  -9.013235e-12  -2.292287e-10
##  [653,]  -4.330568e-33   0.000000e+00
##  [654,]   0.000000e+00 -4.156306e-244
##  [655,]   0.000000e+00  -9.225208e-29
```

```
##  [656,]  -9.104627e-07  -1.266392e-05
##  [657,]   0.000000e+00            NaN
##  [658,]  -4.440892e-16  -1.319290e-14
##  [659,]   0.000000e+00  -1.602851e-15
##  [660,]  -1.054563e-15   0.000000e+00
##  [661,]  -1.776357e-15  -5.944069e-14
##  [662,]  -3.538993e-11  -1.292744e-12
##  [663,]   0.000000e+00  -1.681282e-25
##  [664,]   0.000000e+00  -5.626030e-23
##  [665,]   0.000000e+00  -7.213701e-32
##  [666,]  -6.160911e-45   0.000000e+00
##  [667,]   0.000000e+00  -2.660985e-26
##  [668,]   0.000000e+00  -5.412611e-19
##  [669,]   0.000000e+00 -1.346051e-302
##  [670,]   0.000000e+00  -5.612088e-29
##  [671,]            NaN   0.000000e+00
##  [672,]  -2.448570e-10  -9.653833e-12
##  [673,]   0.000000e+00  -2.981045e-65
##  [674,]   0.000000e+00  -7.510471e-25
##  [675,]   0.000000e+00  -1.422839e-28
##  [676,]  -1.439802e-07  -2.268205e-06
##  [677,]   0.000000e+00  -2.019545e-64
##  [678,]  -1.226614e-36   0.000000e+00
##  [679,]            NaN   0.000000e+00
##  [680,]  -5.828584e-20   0.000000e+00
##  [681,]            NaN   0.000000e+00
##  [682,]   0.000000e+00  -7.742668e-59
##  [683,]   0.000000e+00  -4.625236e-37
##  [684,]            NaN   0.000000e+00
##  [685,] -4.696443e-171   0.000000e+00
##  [686,]  -5.085936e-30   0.000000e+00
##  [687,]            NaN   0.000000e+00
##  [688,]  -1.783261e-06  -2.360516e-05
##  [689,] -1.166018e-173   0.000000e+00
##  [690,]  -2.545760e-10  -1.005307e-11
##  [691,]   0.000000e+00  -1.313149e-39
##  [692,]            NaN   0.000000e+00
##  [693,]  -1.883406e-08  -9.044536e-10
##  [694,]   0.000000e+00  -6.772849e-23
##  [695,]            NaN   0.000000e+00
##  [696,]            NaN   0.000000e+00
##  [697,]            NaN   0.000000e+00
##  [698,]            NaN   0.000000e+00
##  [699,]            NaN   0.000000e+00
##  [700,]   0.000000e+00  -1.807432e-31
##  [701,]            NaN   0.000000e+00
##  [702,]  -1.207696e-16   0.000000e+00
##  [703,]   0.000000e+00  -1.340573e-57
##  [704,]   0.000000e+00  -1.088489e-33
##  [705,]  -2.837116e-08  -4.930311e-07
##  [706,]  -2.019416e-21   0.000000e+00
##  [707,]  -1.249338e-10  -2.848895e-09
##  [708,]  -2.501675e-13  -7.771561e-15
##  [709,]            NaN   0.000000e+00
```

```
##  [710,]  -2.864818e-08  -4.975667e-07
##  [711,]            NaN   0.000000e+00
##  [712,]   0.000000e+00  -1.796398e-35
##  [713,]            NaN   0.000000e+00
##  [714,]            NaN   0.000000e+00
##  [715,]   0.000000e+00            NaN
##  [716,]   0.000000e+00  -1.244425e-40
##  [717,]   0.000000e+00  -3.755873e-16
##  [718,]   0.000000e+00  -1.061263e-22
##  [719,]   0.000000e+00  -7.239161e-53
##  [720,]   0.000000e+00  -2.515962e-19
##  [721,]  -6.619301e-10  -1.399046e-08
##  [722,]   0.000000e+00  -1.560186e-29
##  [723,]   0.000000e+00  -1.027106e-35
##  [724,]   0.000000e+00  -5.936644e-45
##  [725,]   0.000000e+00  -3.974742e-52
##  [726,] -4.417229e-137   0.000000e+00
##  [727,]   0.000000e+00  -3.499874e-61
##  [728,]  -5.506706e-14  -1.681813e-12
##  [729,]  -1.798561e-14  -5.698460e-13
##  [730,]   0.000000e+00  -1.793916e-62
##  [731,]   0.000000e+00  -3.666639e-40
##  [732,]   0.000000e+00  -1.173245e-61
##  [733,]   0.000000e+00  -3.595452e-52
##  [734,]   0.000000e+00  -6.640520e-29
##  [735,]   0.000000e+00  -2.253391e-29
##  [736,]   0.000000e+00  -6.031054e-48
##  [737,]   0.000000e+00  -1.233206e-41
##  [738,]   0.000000e+00  -8.474195e-28
##  [739,]  -6.515012e-21   0.000000e+00
##  [740,]   0.000000e+00  -5.333508e-44
##  [741,]   0.000000e+00  -9.961977e-66
##  [742,]   0.000000e+00  -6.516092e-66
##  [743,]   0.000000e+00  -2.414438e-61
##  [744,]   0.000000e+00 -3.443902e-309
##  [745,]   0.000000e+00  -2.676084e-64
##  [746,]   0.000000e+00  -1.038957e-47
##  [747,]  -2.398057e-18   0.000000e+00
##  [748,]   0.000000e+00  -1.195959e-31
##  [749,]            NaN   0.000000e+00
##  [750,]  -1.671071e-02  -6.881628e-02
##  [751,]            NaN   0.000000e+00
##  [752,]   0.000000e+00  -7.434220e-56
##  [753,]            NaN   0.000000e+00
##  [754,]   0.000000e+00  -1.359168e-23
##  [755,]            NaN   0.000000e+00
##  [756,]   0.000000e+00  -1.119264e-25
##  [757,]   0.000000e+00  -2.463909e-45
##  [758,]            NaN   0.000000e+00
##  [759,]   0.000000e+00  -1.477547e-62
##  [760,]   0.000000e+00  -6.943660e-53
##  [761,]   0.000000e+00  -7.076489e-41
##  [762,]   0.000000e+00  -3.184844e-17
##  [763,]   0.000000e+00  -4.419069e-16
```

```
##  [764,] -2.317234e-155   0.000000e+00
##  [765,] -7.927124e-246   0.000000e+00
##  [766,]   0.000000e+00            NaN
##  [767,]  -6.661338e-16  -2.035004e-14
##  [768,]  -1.376364e-06  -1.857551e-05
##  [769,]            NaN   0.000000e+00
##  [770,]  -2.824981e-02  -1.018194e-01
##  [771,]            NaN   0.000000e+00
##  [772,]            NaN   0.000000e+00
##  [773,]   0.000000e+00  -4.267626e-44
##  [774,]  -3.200190e-02  -6.299299e-03
##  [775,]   0.000000e+00  -1.024846e-41
##  [776,]   0.000000e+00  -9.227964e-36
##  [777,]  -3.197442e-13  -9.198877e-12
##  [778,]   0.000000e+00 -2.814608e-297
##  [779,]  -8.881784e-16  -2.933204e-14
##  [780,] -1.739956e-298   0.000000e+00
##  [781,]            NaN   0.000000e+00
##  [782,]   0.000000e+00  -4.300043e-39
##  [783,]   0.000000e+00  -4.999658e-27
##  [784,]   0.000000e+00  -1.003654e-49
##  [785,] -6.096300e-297   0.000000e+00
##  [786,]  -1.738109e-02  -2.985892e-03
##  [787,]  -1.875181e-07  -1.019018e-08
##  [788,]  -6.909854e-02  -1.680078e-02
##  [789,]   0.000000e+00  -4.194154e-50
##  [790,]            NaN   0.000000e+00
##  [791,] -3.096430e-184   0.000000e+00
##  [792,]   0.000000e+00            NaN
##  [793,]            NaN   0.000000e+00
##  [794,]   0.000000e+00  -3.619167e-30
##  [795,]  -1.115801e-22   0.000000e+00
##  [796,]  -4.164481e-06  -2.757227e-07
##  [797,]  -4.484523e-06  -5.522647e-05
##  [798,]   0.000000e+00            NaN
##  [799,]            NaN   0.000000e+00
##  [800,]   0.000000e+00  -1.770430e-26
##  [801,]   0.000000e+00  -7.411533e-38
##  [802,]            NaN   0.000000e+00
##  [803,]   0.000000e+00  -5.725725e-47
##  [804,]   0.000000e+00  -5.763229e-53
##  [805,]            NaN   0.000000e+00
##  [806,]   0.000000e+00  -5.397147e-52
##  [807,]   0.000000e+00  -3.675593e-41
##  [808,]            NaN   0.000000e+00
##  [809,]            NaN   0.000000e+00
##  [810,]            NaN   0.000000e+00
##  [811,]            NaN   0.000000e+00
##  [812,]  -4.504683e-06  -5.545453e-05
##  [813,]            NaN   0.000000e+00
##  [814,]   0.000000e+00  -1.280870e-23
##  [815,]   0.000000e+00  -2.030352e-58
##  [816,]   0.000000e+00  -2.028255e-27
##  [817,]  -8.997429e-24   0.000000e+00
```

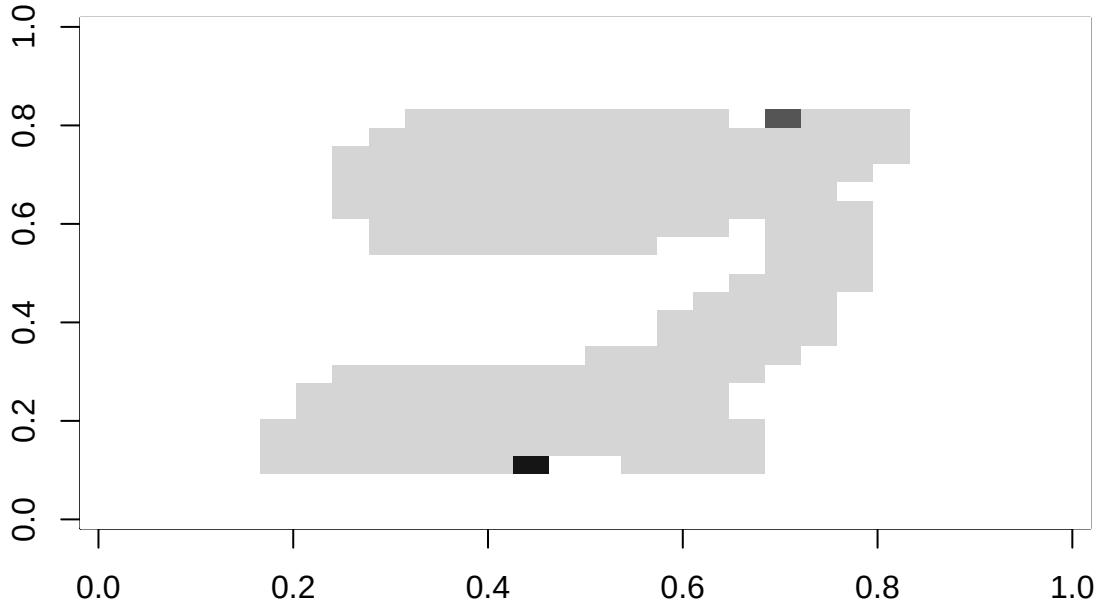
```
##  [818,]   0.000000e+00  -2.113396e-53
##  [819,]  -2.492360e-28   0.000000e+00
##  [820,]            NaN   0.000000e+00
##  [821,]            NaN   0.000000e+00
##  [822,]   0.000000e+00  -7.272001e-43
##  [823,]            NaN   0.000000e+00
##  [824,]            NaN   0.000000e+00
##  [825,]   0.000000e+00            NaN
##  [826,]   0.000000e+00            NaN
##  [827,]            NaN   0.000000e+00
##  [828,]   0.000000e+00  -1.551445e-40
##  [829,]  -1.379394e-20   0.000000e+00
##  [830,]  -2.373768e-06  -3.074277e-05
##  [831,]  -7.950715e-07  -4.712821e-08
##  [832,]  -2.607220e-29   0.000000e+00
##  [833,]            NaN   0.000000e+00
##  [834,]   0.000000e+00  -3.831237e-43
##  [835,]   0.000000e+00  -3.004057e-23
##  [836,]            NaN   0.000000e+00
##  [837,] -1.276250e-160   0.000000e+00
##  [838,]   0.000000e+00            NaN
##  [839,]            NaN   0.000000e+00
##  [840,]  -1.381423e-31   0.000000e+00
##  [841,]  -5.107026e-15  -1.714127e-13
##  [842,]   0.000000e+00  -3.835997e-45
##  [843,]  -1.036912e-27   0.000000e+00
##  [844,]            NaN   0.000000e+00
##  [845,]            NaN   0.000000e+00
##  [846,]   0.000000e+00            NaN
##  [847,]            NaN   0.000000e+00
##  [848,]   0.000000e+00  -1.193103e-35
##  [849,]   0.000000e+00  -2.488448e-18
##  [850,]   0.000000e+00  -4.838369e-37
##  [851,]            NaN   0.000000e+00
##  [852,]  -2.601030e-11  -9.392487e-13
##  [853,]  -7.991110e-09  -3.678586e-10
##  [854,]            NaN   0.000000e+00
##  [855,]   0.000000e+00  -5.206190e-29
##  [856,]   0.000000e+00  -7.201860e-40
##  [857,]  -2.584599e-12  -6.896061e-11
##  [858,]   0.000000e+00  -2.492842e-35
##  [859,]  -3.736510e-05  -3.809351e-04
##  [860,]  -5.634610e-04  -5.773194e-05
##  [861,]  -1.154041e-28   0.000000e+00
##  [862,]   0.000000e+00            NaN
##  [863,]            NaN   0.000000e+00
##  [864,]   0.000000e+00  -3.169011e-36
##  [865,]   0.000000e+00  -4.751138e-37
##  [866,]  -3.435316e-25   0.000000e+00
##  [867,]  -6.116099e-05  -5.934003e-04
##  [868,]   0.000000e+00  -7.303488e-58
##  [869,]   0.000000e+00  -2.627588e-58
##  [870,]            NaN   0.000000e+00
##  [871,]   0.000000e+00  -9.022003e-49
```

```
##  [872,]   0.000000e+00  -1.501822e-51
##  [873,]   0.000000e+00  -2.232081e-54
##  [874,]             NaN   0.000000e+00
##  [875,]   0.000000e+00  -4.744748e-16
##  [876,]   0.000000e+00  -1.201588e-48
##  [877,]   0.000000e+00  -8.835624e-27
##  [878,]   0.000000e+00  -3.556397e-60
##  [879,]   0.000000e+00  -9.079647e-46
##  [880,]   0.000000e+00  -3.872559e-37
##  [881,]   0.000000e+00  -1.928532e-18
##  [882,]   0.000000e+00  -1.280951e-46
##  [883,]   0.000000e+00  -3.867314e-52
##  [884,]  -8.331764e-05  -7.826193e-04
##  [885,]  -9.702828e-18   0.000000e+00
##  [886,]             NaN   0.000000e+00
##  [887,]  -2.847105e-05  -2.979995e-04
##  [888,]  -1.069819e-03  -7.321195e-03
##  [889,]  -9.214909e-10  -1.917164e-08
##  [890,]  -5.353565e-08  -2.714130e-09
##  [891,]   0.000000e+00             NaN
##  [892,]  -5.950924e-07  -8.530384e-06
##  [893,]   0.000000e+00  -5.150796e-36
##  [894,]             NaN   0.000000e+00
##  [895,]  -8.951412e-14  -2.664535e-15
##  [896,]  -3.911780e-15   0.000000e+00
##  [897,]   0.000000e+00  -1.079979e-58
##  [898,]   0.000000e+00  -1.069939e-24
##  [899,]  -2.807888e-30   0.000000e+00
##  [900,]  -4.192825e-19   0.000000e+00
##  [901,]             NaN   0.000000e+00
##  [902,]   0.000000e+00  -7.766535e-40
##  [903,]  -1.443621e-25   0.000000e+00
##  [904,]             NaN   0.000000e+00
##  [905,]   0.000000e+00             NaN
##  [906,]   0.000000e+00  -3.457211e-34
##  [907,]   0.000000e+00  -2.735256e-34
##  [908,]   0.000000e+00             NaN
##  [909,]             NaN   0.000000e+00
##  [910,]  -1.554312e-15  -5.554794e-14
##  [911,]             NaN   0.000000e+00
##  [912,]             NaN   0.000000e+00
##  [913,]  -4.712465e-33   0.000000e+00
##  [914,]             NaN   0.000000e+00
##  [915,]             NaN   0.000000e+00
##  [916,]   0.000000e+00  -6.301756e-27
##  [917,]   0.000000e+00  -2.946912e-28
##  [918,]             NaN   0.000000e+00
##  [919,]             NaN   0.000000e+00
##  [920,]   0.000000e+00  -4.548005e-55
##  [921,]             NaN   0.000000e+00
##  [922,]   0.000000e+00  -2.616500e-16
##  [923,]             NaN   0.000000e+00
##  [924,]   0.000000e+00             NaN
##  [925,]             NaN   0.000000e+00
```

```
##  [926,]            NaN   0.000000e+00
##  [927,]            NaN   0.000000e+00
##  [928,]   0.000000e+00  -9.981593e-73
##  [929,]  -6.647385e-06  -4.552230e-07
##  [930,]  -8.914921e-25   0.000000e+00
##  [931,]   0.000000e+00  -2.768156e-18
##  [932,]            NaN   0.000000e+00
##  [933,]            NaN   0.000000e+00
##  [934,]   0.000000e+00  -2.365440e-15
##  [935,]  -9.566776e-03  -1.465071e-03
##  [936,]            NaN   0.000000e+00
##  [937,]   0.000000e+00  -1.442698e-33
##  [938,]  -2.987879e-10  -1.187717e-11
##  [939,]  -3.634764e-10  -7.900272e-09
##  [940,]            NaN   0.000000e+00
##  [941,]   0.000000e+00  -3.152726e-65
##  [942,]  -1.149104e-03  -1.282151e-04
##  [943,]   0.000000e+00  -7.062467e-22
##  [944,]   0.000000e+00  -5.235922e-50
##  [945,]   0.000000e+00  -2.226449e-41
##  [946,]  -4.436384e-09  -8.532686e-08
##  [947,]   0.000000e+00  -6.506897e-38
##  [948,]  -6.661338e-15  -2.149535e-13
##  [949,]            NaN   0.000000e+00
##  [950,]  -1.607511e-05  -1.177479e-06
##  [951,]            NaN   0.000000e+00
##  [952,]  -5.157365e-09  -9.841718e-08
##  [953,]   0.000000e+00  -2.706535e-55
##  [954,]  -7.933181e-29   0.000000e+00
##  [955,]            NaN   0.000000e+00
##  [956,]  -2.667755e-08  -4.652416e-07
##  [957,]  -2.862703e-06  -1.846321e-07
##  [958,]   0.000000e+00  -5.841872e-29
##  [959,]   0.000000e+00  -4.287463e-38
##  [960,]            NaN   0.000000e+00
##  [961,]  -3.535385e-06  -4.437866e-05
##  [962,]   0.000000e+00  -4.663603e-65
##  [963,]  -6.114900e-20   0.000000e+00
##  [964,]   0.000000e+00  -4.373666e-45
##  [965,]   0.000000e+00  -2.438520e-39
##  [966,]            NaN   0.000000e+00
##  [967,]   0.000000e+00  -1.227502e-56
##  [968,]   0.000000e+00  -2.143250e-43
##  [969,]            NaN   0.000000e+00
##  [970,]  -7.382939e-09  -1.382388e-07
##  [971,]            NaN   0.000000e+00
##  [972,]  -3.077083e-17   0.000000e+00
##  [973,]  -3.429323e-06  -2.239664e-07
##  [974,]   0.000000e+00  -3.047744e-55
##  [975,]   0.000000e+00  -3.030900e-32
##  [976,]            NaN   0.000000e+00
##  [977,]   0.000000e+00 -4.039785e-281
##  [978,]            NaN   0.000000e+00
##  [979,]            NaN   0.000000e+00
```

```
##  [980,]   0.000000e+00  -2.947223e-36
##  [981,]  -1.577446e-14  -4.440892e-16
##  [982,]   0.000000e+00            NaN
##  [983,]   0.000000e+00  -8.646639e-41
##  [984,]            NaN   0.000000e+00
##  [985,] -1.663917e-276   0.000000e+00
##  [986,] -7.556341e-175   0.000000e+00
##  [987,]   0.000000e+00  -4.643367e-36
##  [988,]  -3.495710e-10  -7.611667e-09
##  [989,]   0.000000e+00  -8.650005e-52
##  [990,]   0.000000e+00  -5.918867e-25
##  [991,]  -6.742160e-12  -2.318146e-13
##  [992,]  -1.027311e-34   0.000000e+00
##  [993,]  -2.993165e-06  -3.807066e-05
##  [994,]   0.000000e+00 -3.316685e-306
##  [995,]            NaN   0.000000e+00
##  [996,]  -2.538871e-03  -1.518847e-02
##  [997,]  -2.411309e-19   0.000000e+00
##  [998,]  -4.678279e-31   0.000000e+00
##  [999,]   0.000000e+00            NaN
## [1000,]  -3.096412e-11  -7.492746e-10
```

```
#plot the digit with largest entrpopy
show_digit(image_2[which.max(rowSums(entropy_2)),])
```



```
show_digit(image_3[which.max(rowSums(entropy_3)),])
```