R Computing Midterm Exam: In Class

Instructor: Dr. Howard Hao-Chun Chuang

9:10AM – 11:50AM April 24 (Thu), 2014

Student ID:

Student Name:

The in-class part of midterm exam has 70 points, while the take-home part has 40 points. This makes a total of 110 points for the midterm exam. The exam is *open-notes* and *open-homework* (Except a calculator, NO electronic devices can be used). Don't be panic and just show me what you have learnt so far. Take it easy.

Q1. (10%) For the Poisson distribution $P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}$, $x=0,1,2,\ldots,\infty$.

(a) Derive the ratio $P(X=k) / P(X=k-1)$

(b) Use the ratio in part (a), write an *R* function that uses "recursion" to return $P(X=k)$ and $P(X\le k)$ given $P(X=0)$ as the initial.

Q2. (20%) (a) NCCU issues professors identical-looking keys to various doors. A professor has 5 keys on his key ring and only one will open the door to his office. He tries a key selected at random to see if it unlocks the door. If unsuccessful, he always drops the key ring so that he cannot identify the key that he tried before and must again try a key at random from the key ring. Let a random variable X denote the number of attempts that it takes for him to successfully unlock his door. Show me the probability density function of X as well as the possible range of x.

(b) A couple has three children. Births are independent with the probability of a boy being 0.5 for each birth. First, obtain the probability that all three children are boys if you know that the number of boys is odd (奇數). Second, obtain the probability that the first-born child is a boy if you know that the family has at least one boy (Hint: Write out the sample space would help).

(c) A statistics class has 16 students of various majors: 9 students are statistics majors, 4 are industrial engineering majors, and 3 are computer science majors. Three students are assigned at random to work on a project. What is the probability that the three students have three different majors? What is the probability that all three students have the same majors?

Q3. (10%) For the exponential random variable X~exp(λ) with its pdf $f(x) = \lambda e^{-\lambda x}$, $x > 0$, you have proven the memoryless property $P(X > s+t \mid X > s) = P(X > t)$ in HW3. Now, we want to verify the property by simulating 5000 exponential random numbers from *rexp*(5000, rate) where rate is a positive number. Show me the *R* codes you will write to verify the idea.

Q4. (10%) For the Benford's law $X \sim f(x) = \log_{10}(1+\frac{1}{x}),\ x = 1, 2, ..., 9$. Finish the table below and compute the following: $E(X)$, $E(X^2)$, and $Var(X)$.

x	1	2	3	4	5	6	7	8	9
$f(x)$	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
$F(x)$									

Q5. (10%) A random variable X has its pdf $f(x) = \theta x^{\theta-1}$, $0 \le x \le 1$ & θ>1. Write an R function with two inputs x and θ. The function will compute $f(x)$ and return "Not Feasible" when either x or θ is not in the feasible range. Also, given the cdf $F(x) = x^{\theta}$, how to simulate random numbers using inverse transformation given a uniform random number u?

Q6. (10%) Suppose T is a random variable such that $E(T) = 3\theta$ and $Var(T) = 6\theta^2$. Consider the following two estimators $\hat{\theta}_1 = \frac{T}{3}$ and $\hat{\theta}_2 = \frac{T}{4}$. Derive $E(\hat{\theta}_1)$, $E(\hat{\theta}_2)$, $Var(\hat{\theta}_1)$, and $Var(\hat{\theta}_2)$.

Once that is done, derive $MSE(\hat{\theta}_1) = E(\hat{\theta}_1) - \theta + Var(\hat{\theta}_1)$ and $MSE(\hat{\theta}_2) = E(\hat{\theta}_2) - \theta + Var(\hat{\theta}_2)$.

Show me which estimator has the smaller MSE (mean squared error)?