

Statistical Analysis of Repeated Measurements Data

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March 27 – 31, 2017

Contents

1	Motivating Data Sets	1
1.1	Motivating Longitudinal Studies	2
1.2	Features of Longitudinal Data	14
1.3	Review of Key Points	27
2	Marginal Models for Continuous Data	28
2.1	Simple Methods	29
2.2	Review of Linear Regression	39
2.3	Marginal Models	48

2.4 Interpretation	54
2.5 Estimation	67
2.6 Fitting Marginal Models in R	73
2.7 Covariance Matrix	77
2.8 Model Building	88
2.9 Hypothesis Testing	91
2.10 Confidence Intervals	115
2.11 Design Considerations - Sample Size	117
2.12 Residuals	122
2.13 Review of Key Points	138

3	The Linear Mixed Effects Model	140
3.1	The Linear Mixed Model	141
3.2	Interpretation	147
3.3	Hierarchical vs Marginal	155
3.4	Estimation	165
3.5	Mixed-Effects Models in R	175
3.6	Nested and Crossed Random Effects*	183
3.7	Mixed Models with Correlated Errors	194
3.8	Time-Varying Covariates*	200
3.9	Model Building	210
3.10	Hypothesis Testing	213

3.11 Residuals	236
3.12 Review of Key Points	246

4 Marginal Models for Discrete Data 249

4.1 Review of Generalized Linear Models	250
4.2 Generalized Estimating Equations	263
4.3 Interpretation	271
4.4 Generalized Estimating Equations in R	278
4.5 Working Correlation Matrix	281
4.6 Hypothesis Testing	292
4.7 Review of Key Points	301

5	Mixed Models for Discrete Data	303
5.1	Generalized Linear Mixed Models	304
5.2	Interpretation	311
5.3	Estimation	339
5.4	GLMMs in R	351
5.5	Model Building	355
5.6	Hypothesis Testing	357
5.7	Review of Key Points	362
6	Statistical Analysis with Incomplete Grouped Data	364
6.1	Missing Data in Longitudinal Studies	365

6.2 Missing Data Mechanisms	370
6.3 Analysis with Incomplete Data	385
6.4 Summary	407
6.5 Review of Key Points	409

7 Closing 411

7.1 Concluding Remarks	412
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Practicals 416

Practical 1: Marginal Models Continuous	417
Practical 2: Mixed Models Continuous	427
Practical 3: Marginal Models Discrete	436

Practical 4: Mixed Models Discrete	444
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What is this Course About

Grouped data arise in a wide range of disciplines

- Typical examples of grouped data
 - ▷ *repeated measurements*: measuring the same outcome multiple times on the same sample unit (e.g., biomarkers in patients)
 - ▷ *multilevel data*: outcomes measured on sample units that are organized in different levels (e.g., patients in medical centers or students in schools)

What is this Course About (cont'd)

- Statistical analysis of clustered/grouped data
 - ▷ Features of grouped data
 - ▷ describe their distribution
 - ▷ inference using suitable regression models

Lexical convention

- The following terms are used interchangeably to denote multivariate outcomes
 - ▷ clustered data
 - ▷ repeated measurements data
 - ▷ multilevel data
 - ▷ grouped data

Learning Objectives

- Goals: After this course participants will be able to
 - ▷ identify settings in which a repeated measurements model is required,
 - ▷ construct and fit an appropriate model to the data at hand, and
 - ▷ correctly interpret the results
- Even though the course will be primarily explanatory
 - ▷ sufficient mathematical detail will be provided in order participants to obtain a clear view on the different modeling approaches, and how they should be used in practice

Agenda

- **Chapter 1:** Motivating Data Sets

- ▷ Data sets that we will use throughout the course
- ▷ General repeated measurements settings
- ▷ Formulation of possible research questions

- **Chapter 2:** Marginal Models for Continuous Data

- ▷ Features of repeated measurements data
- ▷ Naive approaches
- ▷ Review linear regression
- ▷ Marginal models

Agenda (cont'd)

- **Chapter 3:** The Linear Mixed Effects Model

- ▷ Intuition behind mixed models
- ▷ Mixed models with correlated errors
- ▷ Nested and cross random effects
- ▷ Time-varying covariates

- **Chapter 4:** Marginal Models for Discrete Data

- ▷ Review generalized linear models
- ▷ Generalized estimating equations

Agenda (cont'd)

- **Chapter 5:** Mixed Models for Discrete Data

- ▷ Generalized linear mixed effects models
- ▷ interpretation of parameters
- ▷ approximations of the integrand & integral

- **Chapter 6:** Statistical Analysis with Incomplete Grouped Data

- ▷ Problems with incomplete data
- ▷ Missing data mechanisms
- ▷ Valid inferential approaches

Structure of the Course & Material

- Lectures & software practicals using R
- Material:
 - ▷ Course Notes
 - ▷ R code in soft format
- Within the course notes there are several examples of R syntax – these are denoted by the symbol 'R> '

Software Requirements

- The up-to-date versions of R and Rstudio; downloadable from
 - ▷ <http://cran.r-project.org/>
 - ▷ <http://www.rstudio.com/>
- Additional required packages
 - ▷ **nlme, lme4, MCMCglmm, geepack,**
 - ▷ **MASS, lattice, shiny, corrplot**

Software Requirements

- Up-to-date versions of these packages and their dependencies can be installed using the command

```
install.packages(c("shiny", "nlme", "lattice", "lme4",  
                  "MCMCglmm", "geepack", "MASS", "corrplot"),  
                dependencies = TRUE)
```

- Up-to-date version of a modern web browser, e.g.,
 - ▷ Mozilla Firefox (<https://www.mozilla.org/firefox/>)
 - ▷ Google Chrome (<http://www.google.com/chrome/>)

Software Requirements

- We will use a [shiny](#) web app that replicates all analyses in the course including also some additional illustrations
- The app is available on GitHub and can be invoked using the following two-step procedure (assuming internet connection is available and you have installed the aforementioned packages)
 1. Start R
 2. Run the command

```
shiny::runGitHub("Repeated_Measurements", "drizopoulos")
```

this will open a new web browser window (or tab) with the app

- Note: in order the app to be functional you should **not** close R

References

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 - ▷ Fitzmaurice, G., Laird, N., and Ware, J. (2011). *Applied Longitudinal Analysis*, 2nd Ed. Hoboken: John Wiley & Sons.
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 - ▷ Lindsey, J. (1993). *Models for Repeated Measurements*. Oxford: Oxford University Press.
 - ▷ Pinheiro, J. and Bates, D. (2000). *Mixed Effects Models in S and S-plus*. New York: Springer-Verlag.
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Use of Statistical Models

... the megalomaniacal strategy of fitting a grand unified model, supposedly capable of answering any conceivable question that might be posed, is, in our view, dangerous, unnecessary and counterproductive.

Drum and McCullach (1993, *Statistical Science* **8**, 300–301)