## Statistical Analysis of Repeated Measurements Data

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April 18 – 22, 2016

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### What is this Course About



Grouped data arise in a wide range of disciplines

- Typical examples of grouped data
  - > repeated measurements: measuring the same outcome multiple times on the same sample unit (e.g., biomarkers in patients)

## What is this Course About (cont'd)



- Statistical analysis of grouped data

  - ▷ describe their distribution

## **Learning Objectives**



- Goals: After this course participants will be able to
  - ▷ identify settings in which a repeated measurements model is required,
  - > construct and fit an appropriate model to the data at hand, and
  - > correctly interpret the results
- Even though the course will be primarily explanatory
  - > sufficient mathematical detail will be provided in order participants to obtain a clear view on the different modeling approaches, and how they should be used in practice

## **Agenda**



## • Chapter 1: Motivating Data Sets

- Data sets that we will use throughout the course
- □ General repeated measurements settings

## • Chapter 2: Marginal Models for Continuous Data

- > Features of repeated measurements data
- Naive approaches

# Agenda (cont'd)



- Chapter 3: The Linear Mixed Effects Model
  - > Intuition behind mixed models
  - □ nested and cross random effects
- Chapter 4: Marginal Models for Discrete Data

# Agenda (cont'd)



- Chapter 5: Mixed Models for Discrete Data
  - □ Generalized linear mixed effects models
  - □ approximations of the integrand & integral
  - > interpretation of parameters
- Chapter 6: Statistical Analysis with Incomplete Grouped Data
  - > Problems with incomplete data

  - > Valid inferential approaches

### Structure of the Course & Material



• Lectures & software practicals using R

- Material:

 Within the course notes there are several examples of R syntax – these are denoted by the symbol 'R> '

## **Software Requirements**



• The up-to-date versions of R and Rstudio; downloadable from

```
> http://cran.r-project.org/
> http://www.rstudio.com/
```

- Additional required packages

  - **▷ MASS**, lattice, shiny, corrplot

## **Software Requirements**



 Up-to-date versions of these packages and their dependencies can be installed using the command

- Up-to-date version of a modern web browser, e.g.,

  - ▷ Google Chrome (http://www.google.com/chrome/)

## **Software Requirements**



- A shiny web app that replicates all analyses in the course including also some additional illustrations
- The app is available on GitHub and can be invoked using the following two-step procedure (assuming internet connection is available)
  - 1. Start R
  - 2. Run the command

```
shiny::runGitHub("Repeated_Measurements", "drizopoulos")
```

this will open a new web browser window (or tab) with the app

• Note: in order the app to be functional you should not close R

### References



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