

# Statistical Analysis of Repeated Measurements Data

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## Contents

<b>1</b>	<b>Motivating Data Sets</b>	<b>1</b>
1.1	Motivating Longitudinal Studies . . . . .	2
1.2	Features of Longitudinal Data . . . . .	14
1.3	Review of Key Points . . . . .	26
<b>2</b>	<b>Marginal Models for Continuous Data</b>	<b>27</b>
2.1	Simple Methods . . . . .	28
2.2	Review of Linear Regression . . . . .	37
2.3	Marginal Models . . . . .	46

2.4 Interpretation . . . . .	52
2.5 Estimation . . . . .	63
2.6 Fitting Marginal Models in R . . . . .	69
2.7 Covariance Matrix . . . . .	73
2.8 Model Building . . . . .	84
2.9 Hypothesis Testing . . . . .	87
2.10 Confidence Intervals . . . . .	111
2.11 Residuals . . . . .	113
2.12 Review of Key Points . . . . .	129

<b>3</b>	<b>The Linear Mixed Effects Model</b>	<b>131</b>
3.1	The Linear Mixed Model . . . . .	132
3.2	Interpretation . . . . .	138
3.3	Hierarchical vs Marginal . . . . .	146
3.4	Estimation . . . . .	156
3.5	Mixed-Effects Models in R . . . . .	166
3.6	Nested and Crossed Random Effects* . . . . .	174
3.7	Mixed Models with Correlated Errors . . . . .	185
3.8	Time-Varying Covariates* . . . . .	191
3.9	Model Building . . . . .	201
3.10	Hypothesis Testing . . . . .	204

3.11 Residuals . . . . .	205
3.12 Review of Key Points . . . . .	215

## 4 Marginal Models for Discrete Data 218

4.1 Review of Generalized Linear Models . . . . .	219
4.2 Generalized Estimating Equations . . . . .	232
4.3 Interpretation . . . . .	240
4.4 Generalized Estimating Equations in R . . . . .	247
4.5 Working Correlation Matrix . . . . .	250
4.6 Hypothesis Testing . . . . .	261
4.7 Review of Key Points . . . . .	270

<b>5</b>	<b>Mixed Models for Discrete Data</b>	<b>272</b>
5.1	Generalized Linear Mixed Models . . . . .	273
5.2	Interpretation . . . . .	275
5.3	Estimation . . . . .	277
5.4	GLMMs in R . . . . .	279
5.5	Model Building . . . . .	281
5.6	Hypothesis Testing . . . . .	283
5.7	Review of Key Points . . . . .	285
<b>6</b>	<b>Statistical Analysis with Incomplete Grouped Data</b>	<b>287</b>
6.1	Missing Data in Longitudinal Studies . . . . .	288

6.2 Missing Data Mechanisms . . . . .	293
6.3 Analysis with Incomplete Data . . . . .	308
6.4 Summary . . . . .	330
6.5 Review of Key Points . . . . .	332

## **7 Closing 334**

7.1 Concluding Remarks . . . . .	335
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## **Practicals 339**

Practical 1: Marginal Models Continuous . . . . .	340
Practical 2: Mixed Models Continuous . . . . .	350
Practical 3: Marginal Models Discrete . . . . .	359

Practical 4: Mixed Models Discrete . . . . .	367
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# What is this Course About

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*Grouped data* arise in a wide range of disciplines

- Typical examples of grouped data
  - ▷ *repeated measurements*: measuring the same outcome multiple times on the same sample unit (e.g., biomarkers in patients)
  - ▷ *multilevel data*: outcomes measured on sample units that are organized in different levels (e.g., patients in medical centers or students in schools)

# What is this Course About (cont'd)

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- Statistical analysis of grouped data
  - ▷ Features of grouped data
  - ▷ describe their distribution
  - ▷ inference using suitable regression models

# Learning Objectives

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- Goals: After this course participants will be able to
  - ▷ identify settings in which a repeated measurements model is required,
  - ▷ construct and fit an appropriate model to the data at hand, and
  - ▷ correctly interpret the results
- Even though the course will be primarily explanatory
  - ▷ sufficient mathematical detail will be provided in order participants to obtain a clear view on the different modeling approaches, and how they should be used in practice

# Agenda

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- **Chapter 1:** Motivating Data Sets

- ▷ Data sets that we will use throughout the course
- ▷ General repeated measurements settings
- ▷ Research questions

- **Chapter 2:** Marginal Models for Continuous Data

- ▷ Features of repeated measurements data
- ▷ Naive approaches
- ▷ Review linear regression
- ▷ Marginal models

# Agenda (cont'd)

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- **Chapter 3:** The Linear Mixed Effects Model
  - ▷ Intuition behind mixed models
  - ▷ nested and cross random effects
- **Chapter 4:** Marginal Models for Discrete Data
  - ▷ Review generalized linear models
  - ▷ Generalized estimating equations

# Agenda (cont'd)

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- **Chapter 5:** Mixed Models for Discrete Data

- ▷ Generalized linear mixed effects models
- ▷ approximations of the integrand & integral
- ▷ interpretation of parameters

- **Chapter 6:** Statistical Analysis with Incomplete Grouped Data

- ▷ Problems with incomplete data
- ▷ Missing data mechanisms
- ▷ Valid inferential approaches

# Structure of the Course & Material

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- Lectures & software practicals using R
- Material:
  - ▷ Course Notes
  - ▷ R code in soft format
- Within the course notes there are several examples of R syntax – these are denoted by the symbol 'R> '

# Software Requirements

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- The up-to-date versions of R and Rstudio; downloadable from
  - ▷ <http://cran.r-project.org/>
  - ▷ <http://www.rstudio.com/>
- Additional required packages
  - ▷ **nlme, lme4, MCMCglmm, geepack,**
  - ▷ **MASS, lattice, shiny, corrplot**



# Software Requirements

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- Up-to-date versions of these packages and their dependencies can be installed using the command

```
install.packages(c("shiny", "nlme", "lattice", "lme4",  
                  "MCMCglmm", "geepack", "MASS", "corrplot"),  
                dependencies = TRUE)
```

- Up-to-date version of a modern web browser, e.g.,
  - ▷ Mozilla Firefox (<https://www.mozilla.org/firefox/>)
  - ▷ Google Chrome (<http://www.google.com/chrome/>)

# Software Requirements

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- A [shiny](#) web app that replicates all analyses in the course including also some additional illustrations
- The app is available on GitHub and can be invoked using the following two-step procedure (assuming internet connection is available)

1. Start R

2. Run the command

```
shiny::runGitHub("Repeated_Measurements", "drizopoulos")
```

this will open a new web browser window (or tab) with the app

- Note: in order the app to be functional you should **not** close R

# References

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