Group based trajectory models

Seminar in Criminology, Research and Analysis– Crim 7301
Week 8, 10/13/16
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Class Overview

- The basic model
- Selecting the number of groups
- Absolute fit criteria
- Other extensions
  - Covariates to predict group membership
  - Covariates to adjust the trajectories
  - Dual Trajectory Models
- Software implementations

The basic model

- Outcome for individual i is a polynomial function of time t

$$y_i = \beta_0 + \beta_1(t) + \beta_2(t^2) + \beta_3(t^3)$$

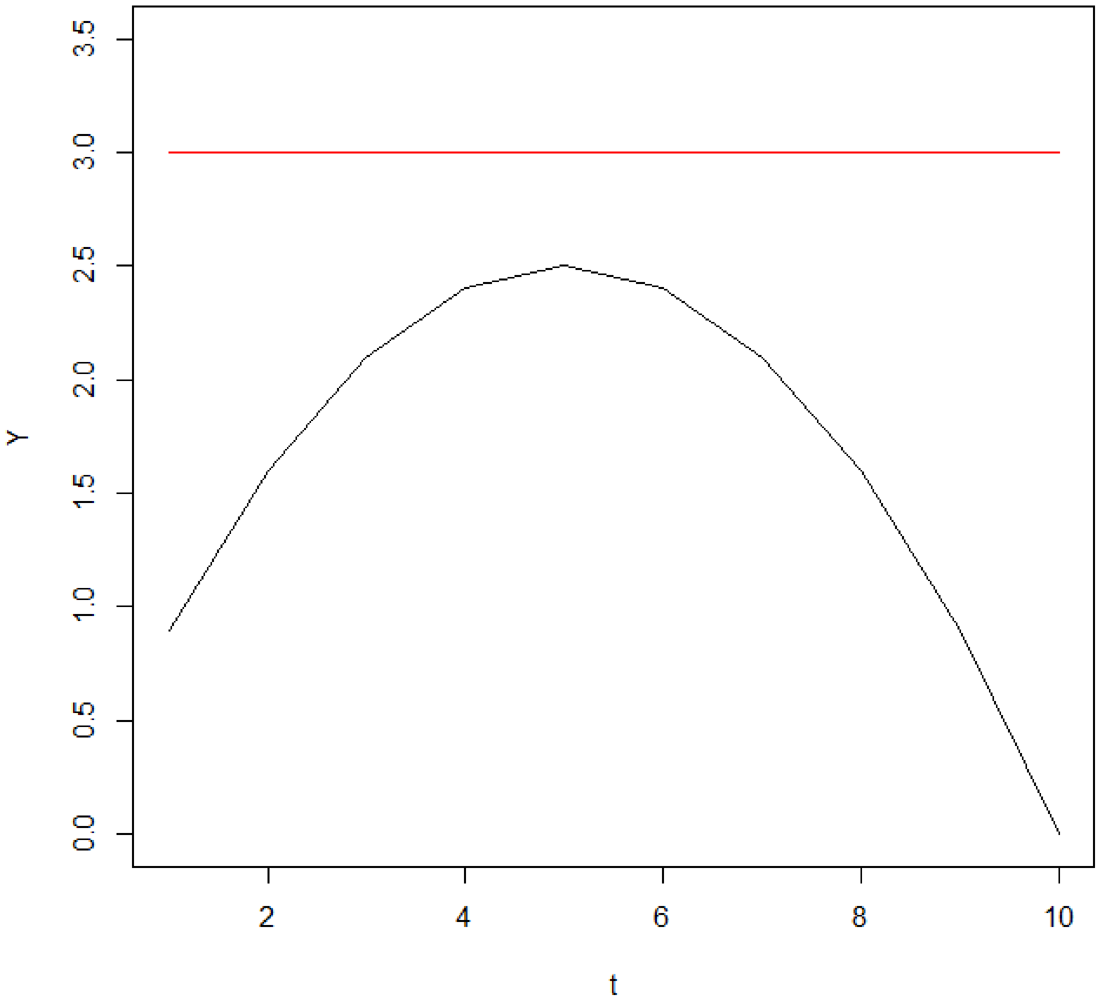
- Group based trajectory models try to group individuals who have similar functions over time, so for example:

Group 1: $y_i = 3$
Group 2: $y_i = t - 1/10 \cdot t^2$

The basic model

Group 1: $y_i = 3$

Group 2: $y_i = t - 1/10 \cdot t^2$



The basic model

- Can write generally as:

$$y_i = \beta_0^j + \beta_1^j(t) + \beta_2^j(t^2) + \beta_3^j(t^3)$$

$$\text{Prob}(j) = f(y_i, X)$$

- You need to select the number of groups before estimating the model though....

Selecting the Number of Groups

- Typically test a range of models, then use AIC, BIC, or cross-validation statistic to choose the model

Number of Groups	Log-Likelihood	AIC	BIC	Cross-Validation Error
2	-5978	11973	12030	1.22
3	-5722	11472	11560	1.02
4	-5632	11302	11422	0.97
5	-5559	**11167**	**11318**	0.93
6	-5579	11216	11399	0.92
7	-5558	11183	11397	**0.91**

Selecting the Number of Groups

- What is a big change in BIC?

- Approximate Bayes Factor between two models

$$\exp\left(\mathrm{BIC}_i - \mathrm{BIC}_j\right) \approx \text{Bayes Factor}$$

  - 1-3 weak evidence for model *i*
  - 3-10 moderate evidence for model *i*
  - > 10, strong evidence for model *i*

- So model 5 versus model 7: exp(11318 – 11397) = e^-79

Selecting the Number of Groups

- Extension for multiple models

$$P(\text{Model with j groups}) = \frac{\exp\left(\mathrm{BIC}_j - \mathrm{BIC}_{\min}\right)}{\sum_j \exp\left(\mathrm{BIC}_j - \mathrm{BIC}_{\min}\right)}$$

- In general for mixture models, more data (both in the cross section and in the temporal periods) results in more groups selected

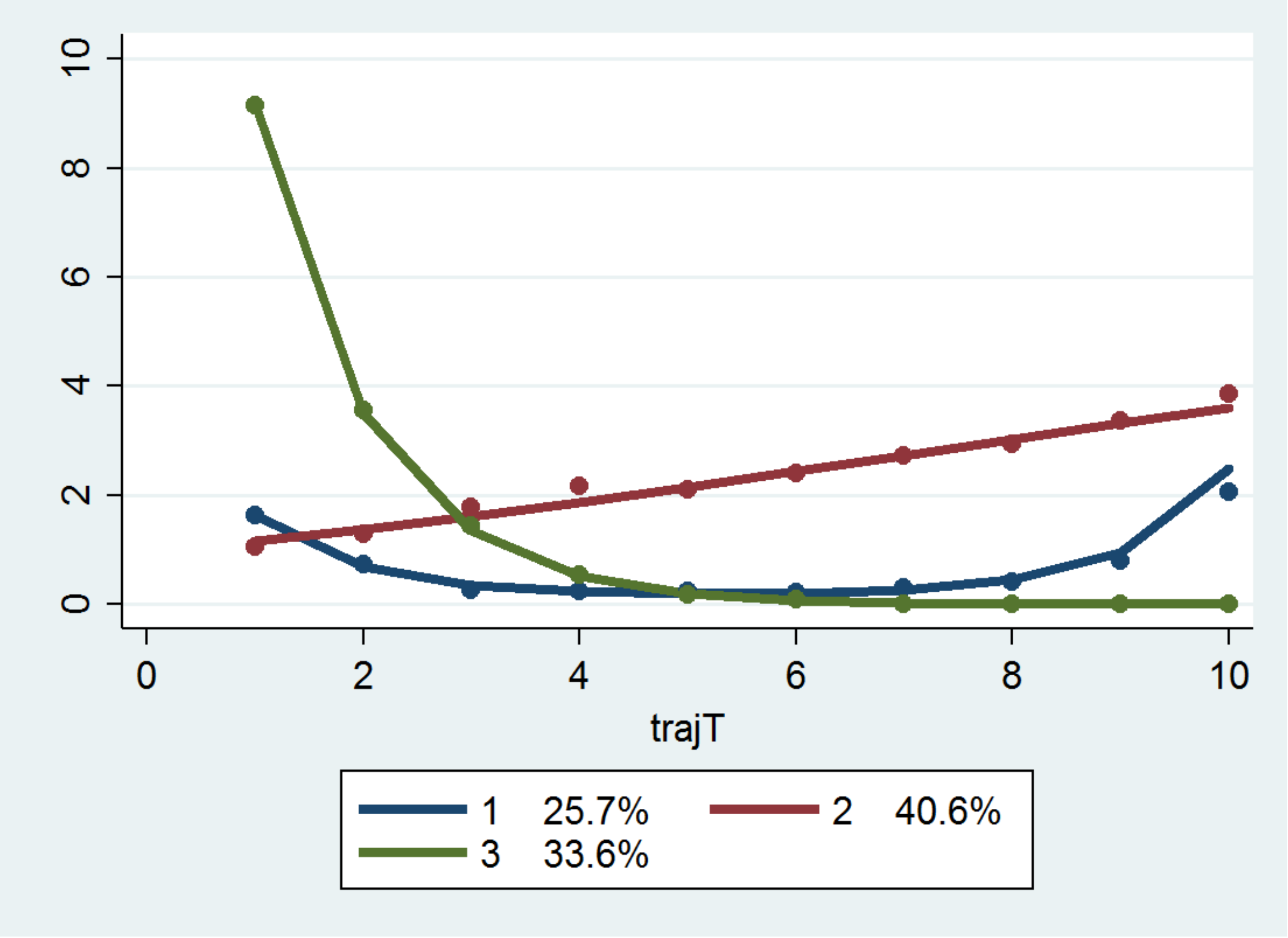
Absolute Fit Criteria

- For a model, every observation has a probability of belonging to a particular group, so a three group solution will have:

Person	Group 1	Group 2	Group 3	Group Assigned
1	0.80	0.15	0.05	1
2	0.01	0.03	0.96	3
3	0.40	0.59	0.01	2

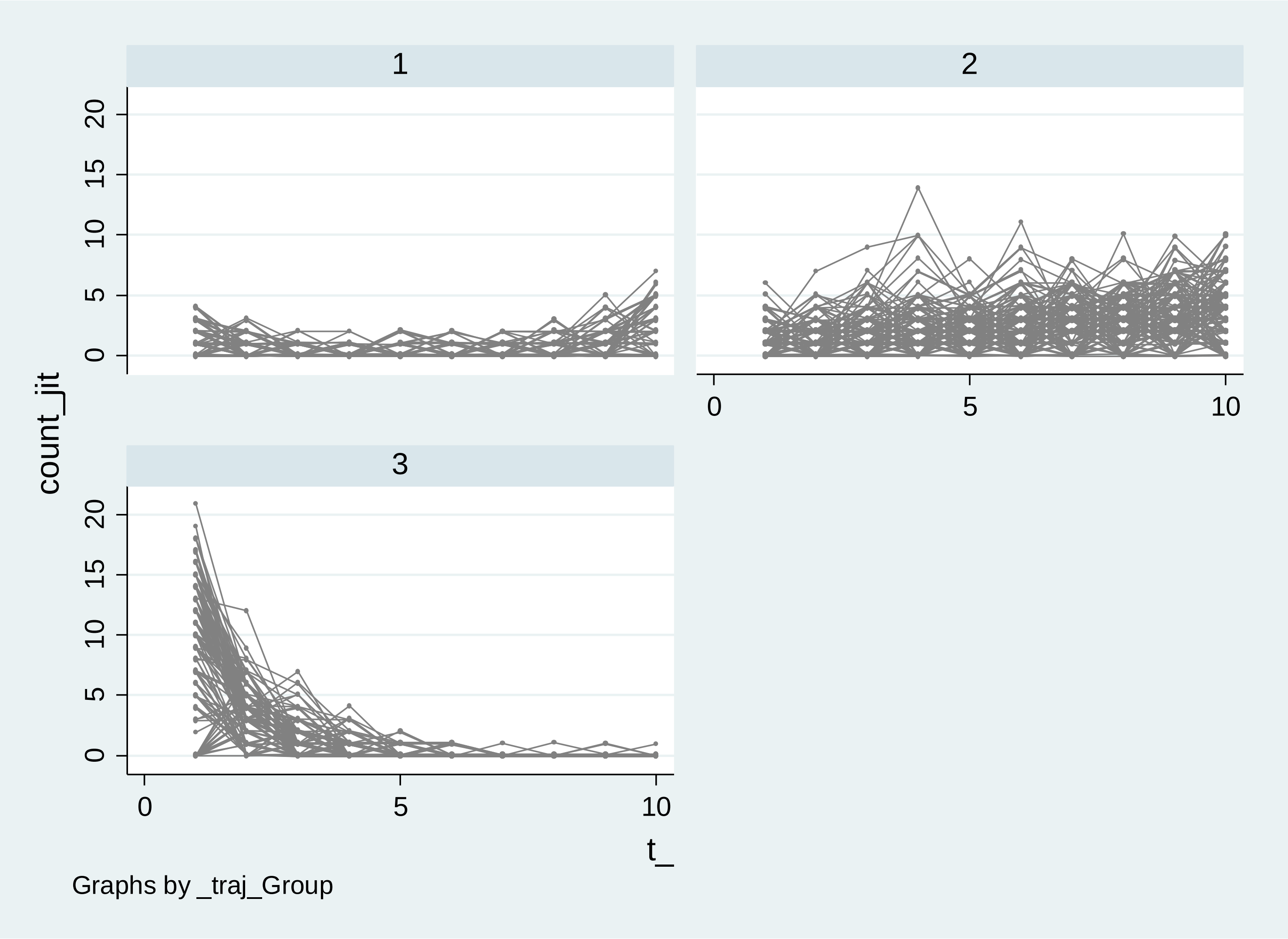
Absolute Fit Criteria

- Plots for predicted versus observed trajectories (weighted means)



Absolute Fit Criteria

- Plots of individual trajectories



Absolute Fit Criteria

- Average Posterior Probability (Nagin suggests above 0.7)

$$\text{AvePP}_j = \frac{1}{n}\sum p_j$$

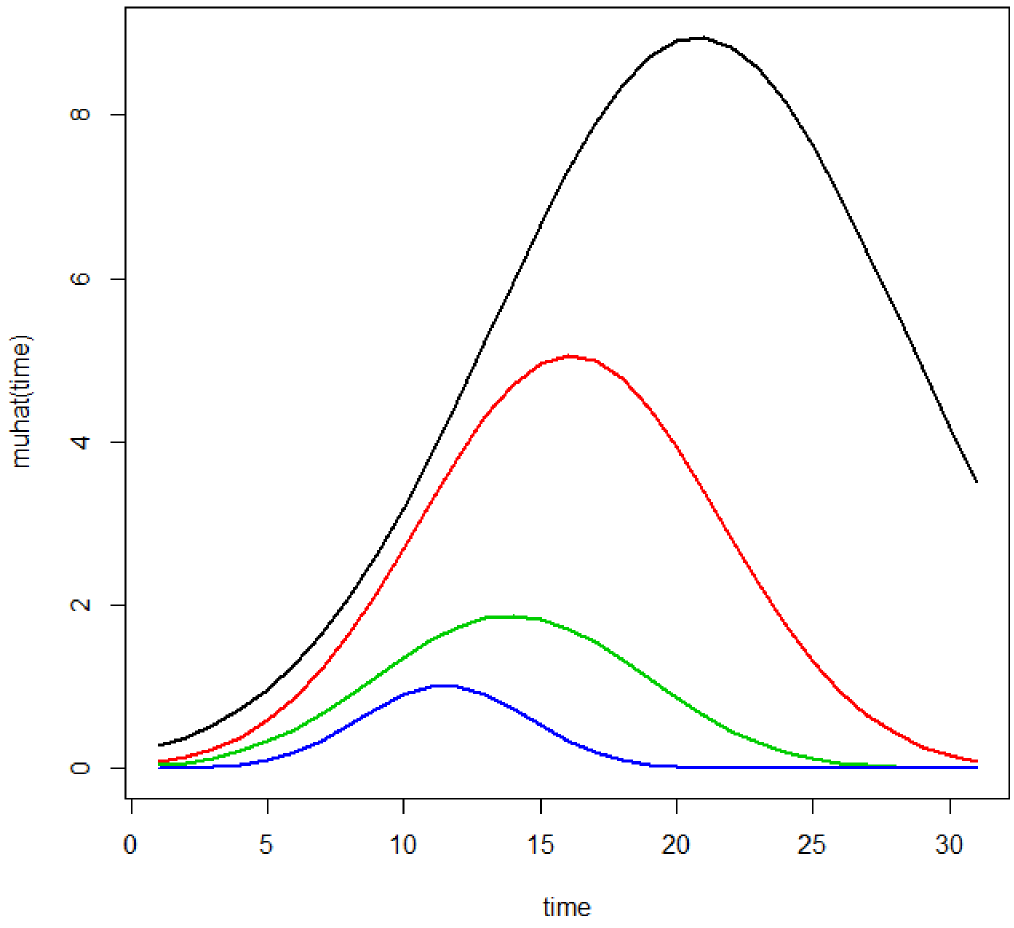
- Odds of Correct Classification (Nagin suggests above 5)

$$\frac{\text{AvePP}_j / \left(1 - \text{AvePP}_j\right)}{\hat{\pi}/(1 - \hat{\pi})}$$

- $\hat{\pi}$ = probability of being in that group based on the maximum posterior probability assignments

Absolute Fit Criteria

- You can have good separation between classes, but fit is still bad



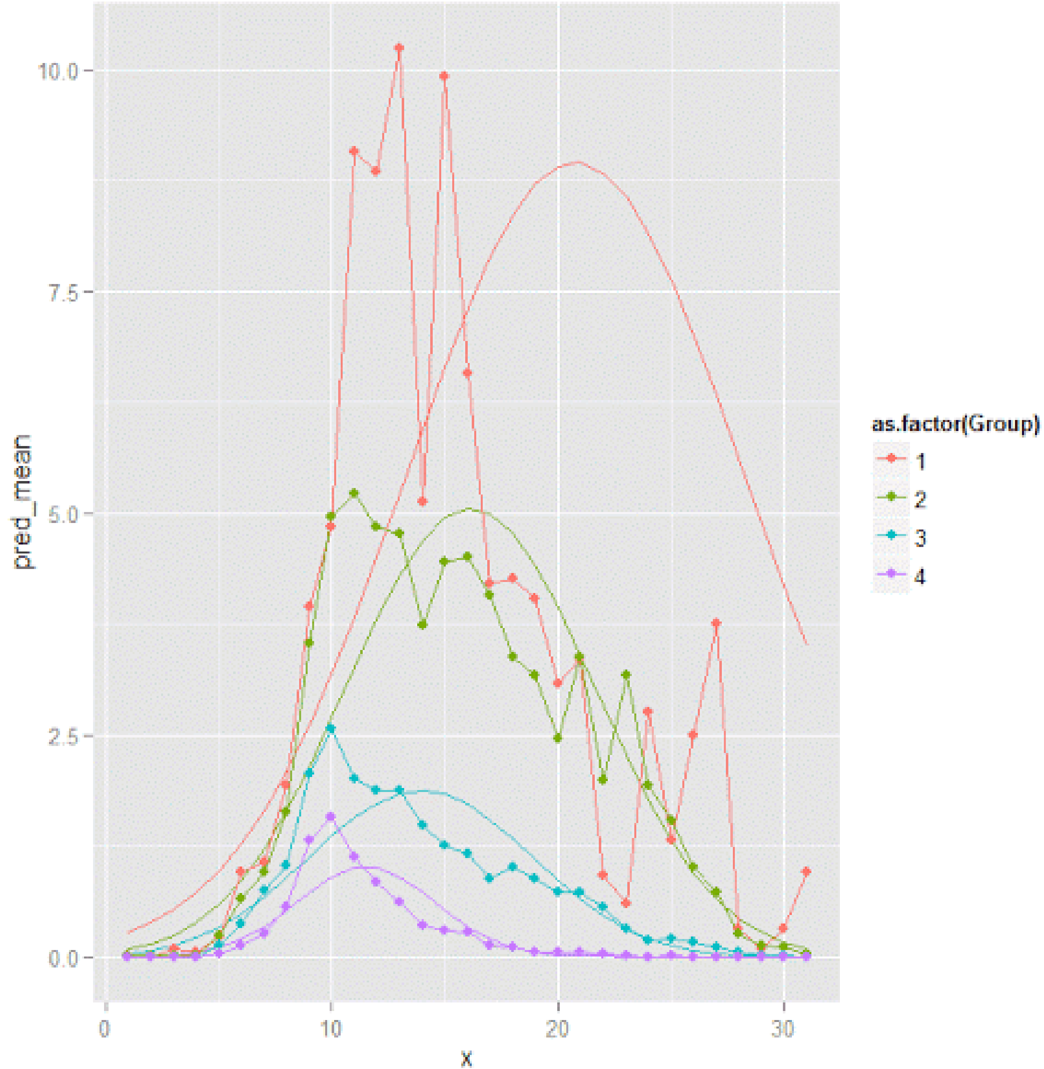
Absolute Fit Criteria

- You can have good separation between classes, but fit is still bad

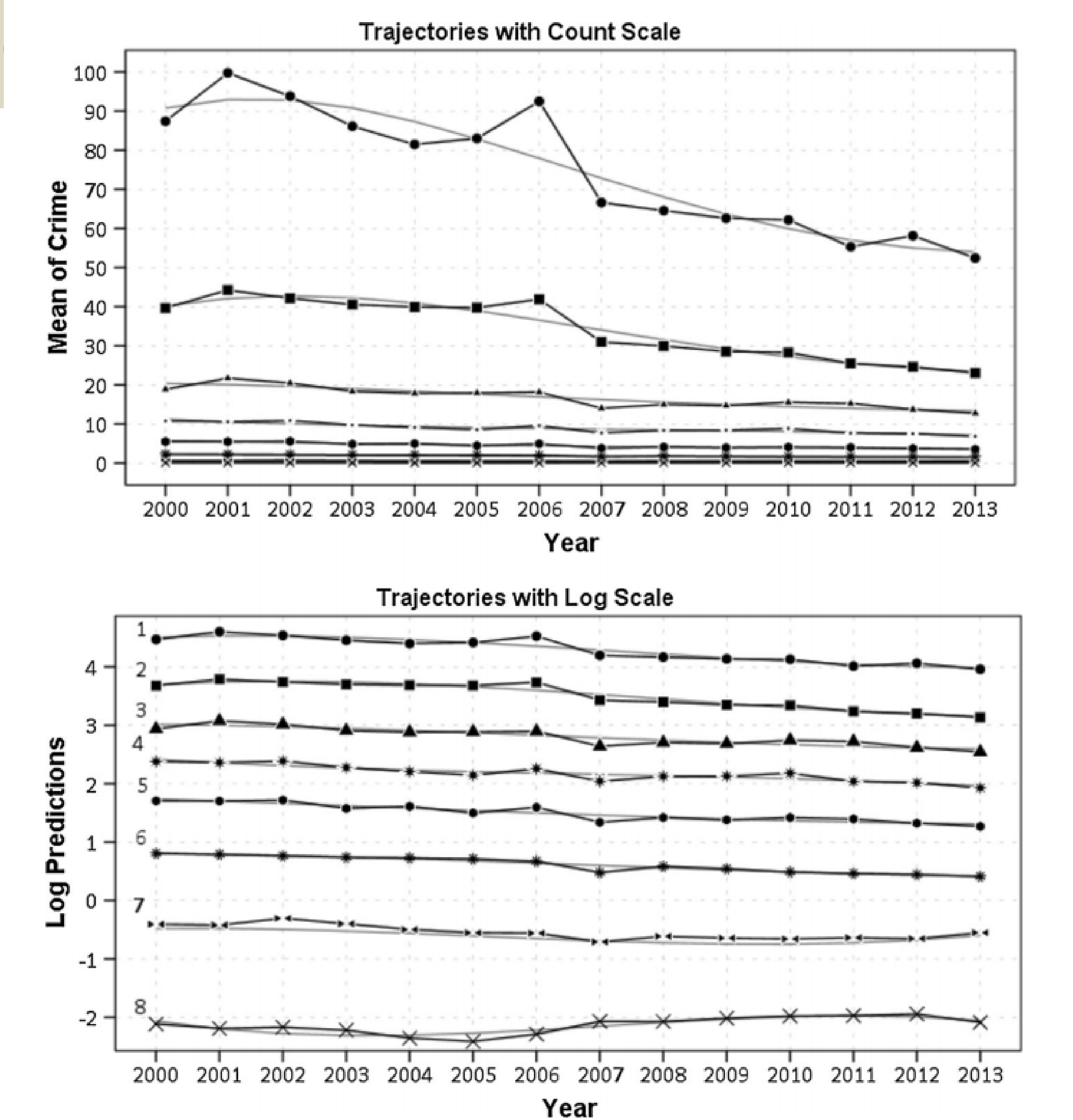
Group	N	AvePP	OCC	Class Prop.	Pred. Prop.
1	22	1.00	6667	0.06	0.06
2	55	0.98	304	0.15	0.15
3	134	0.94	31	0.35	0.35
4	167	0.97	35	0.44	0.44

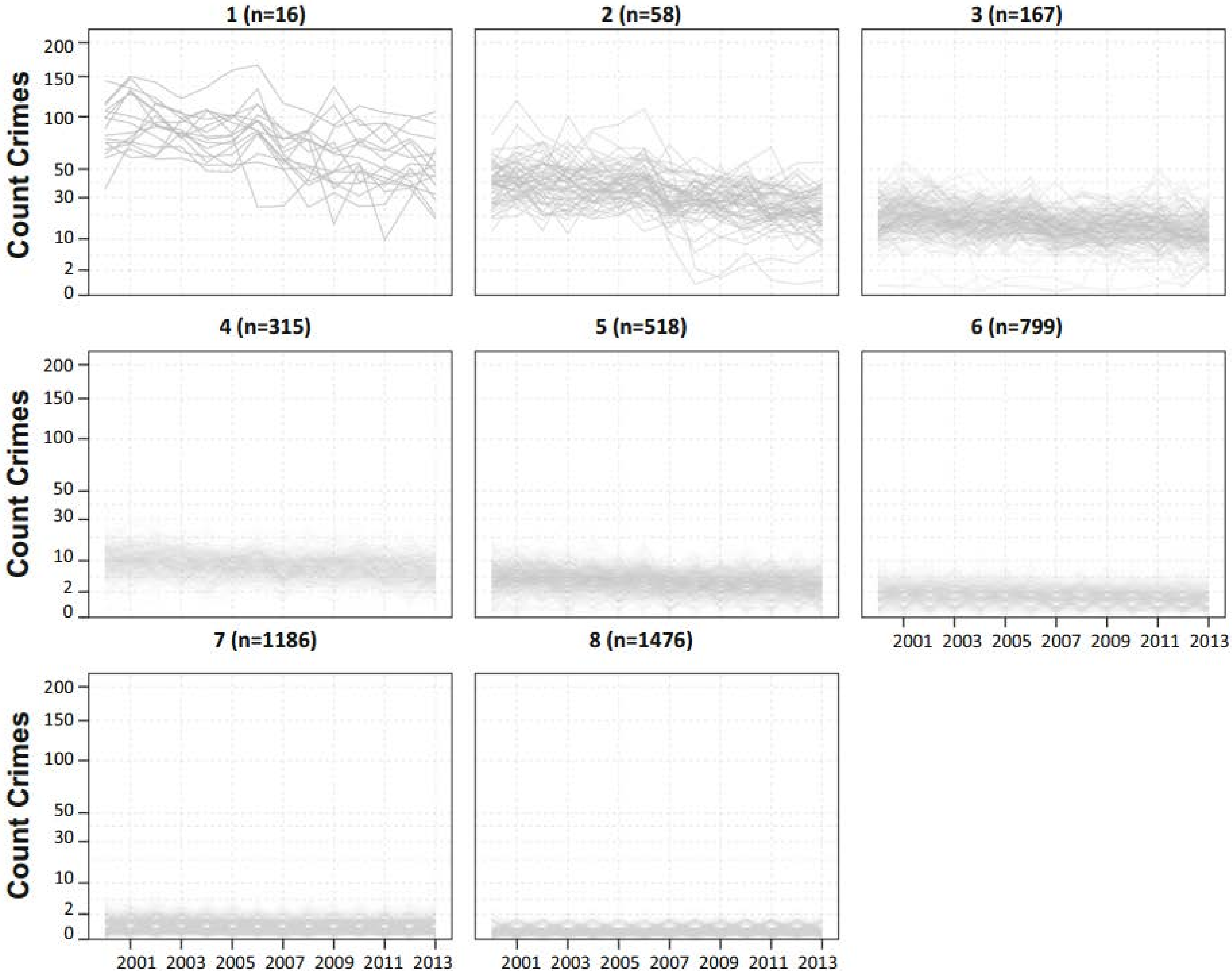
Absolute Fit Criteria

- You can have good separation between classes, but fit is still bad

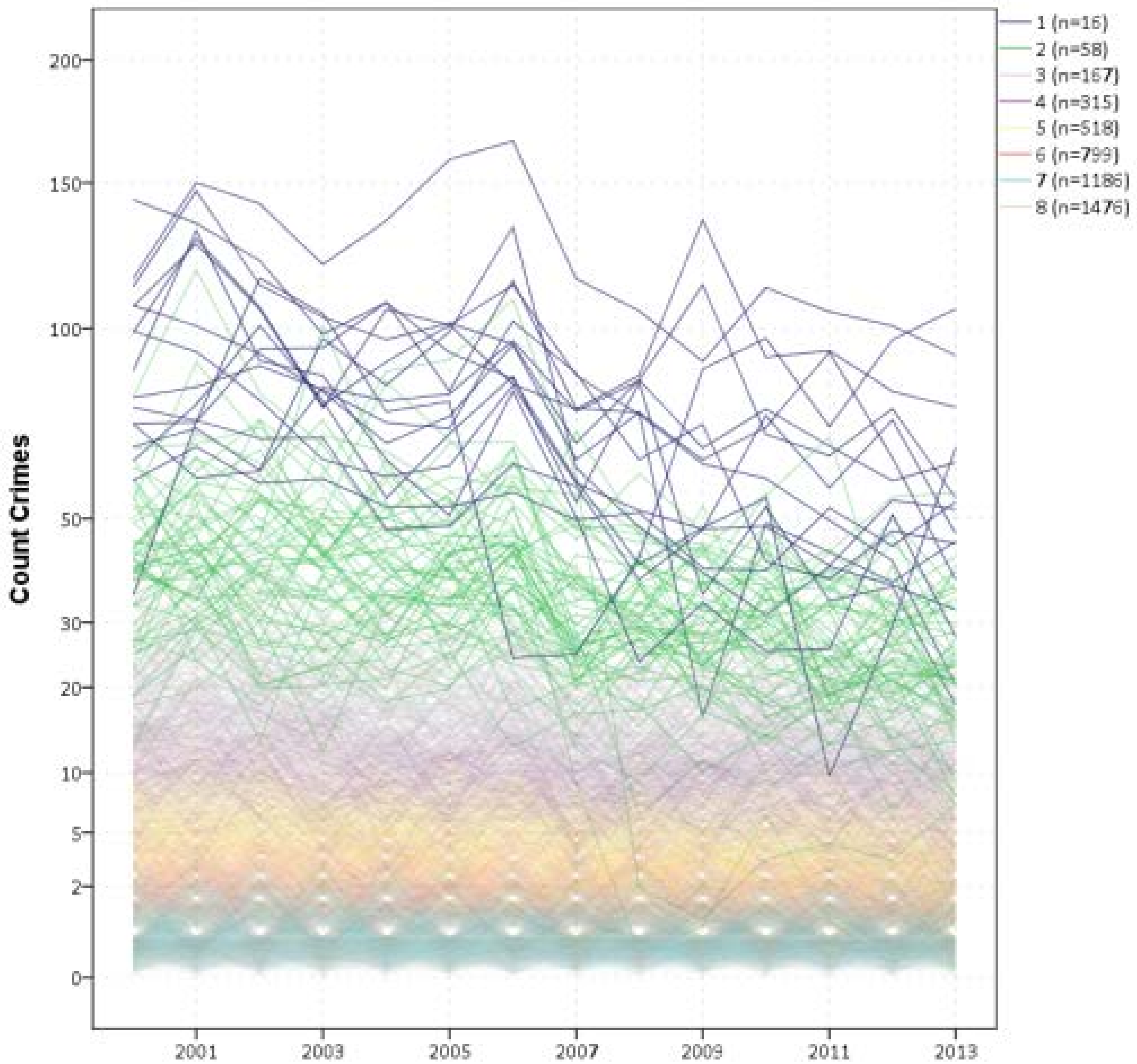


Absolut


Trajectories with Count Scale
Mean of Crime
100
90
80
70
60
50
40
30
20
10
0
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
Year
Trajectories with Log Scale
Log Predictions
4
3
2
1
0
-1
-2
1
2
3
4
5
6
7
8
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
Year


1 (n=16)
2 (n=58)
3 (n=167)
4 (n=315)
5 (n=518)
6 (n=799)
7 (n=1186)
8 (n=1476)
Count Crimes
200
150
100
50
30
10
2
0
2001
2003
2005
2007
2009
2011
2013

Absolute Fit Criteria



Other Extensions

- Covariates to predict group membership, e.g.:

$$\text{Logit}[\text{Prob}(j)] = \beta_0 + \beta_1(\text{Male})$$

- Additional Covariates in the trajectories:

$$y_i = \beta_o^j + \beta_1^j(t) + \beta_2^j(t^2) + \beta_3^j(T)$$

- Estimate dual trajectories:

$$\text{violent crime}_i = \beta_o^j + \beta_1^j(t) + \beta_2^j(t^2)$$
$$\text{property crime}_i = \beta_o^k + \beta_1^k(t) + \beta_2^k(t^2)$$

  – Then see how much j and k groups overlap

Software

- SAS (Proc Traj) and Stata plug-in have the most extensive set of potential models
  - Can have different time support
  - Censored normal, Logit, and Zero-Inflated Poisson models
  - Trajectories can follow different polynomial degrees
  - Estimates of the group proportions
  - Dual trajectory models

- Any software that can estimate latent classes (MPlus, gllamm for Stata) can be used to the same ends (also see finite mixture models that can include covariates)

- R, crimCV package has zero inflated Poisson models & estimates cross-validation scores (various other packages can fit mixtures, have not tried out though)

- I have examples of calling R code within SPSS, https://andrewpwheeler.wordpress.com/2014/08/12/estimating-group-based-trajectory-models-using-spss-and-r/.

Homework & Next Weeks Class

Lab Assignment

Estimate group based trajectory models in R or Stata for a simulated dataset. For your homework decide on the number of groups, and report model fit statistics and graphs.

For Next Week – Missing Data

- *Allison, Paul. 2002. Missing Data. Quantitative Applications in the Social Sciences. Sage University Papers. Thousand Oaks, CA.* [Sage Green book, available at library and online.]
- Fox, J. and Swatt, M. (2009). Multiple imputation of the supplementary homicide reports, 1976-2005. *Journal of Quantitative Criminology* 25(1): 51-77.
- Brame, R. and Paternoster, R. (2003). Missing data problems in criminological research: Two case studies. *Journal of Quantitative Criminology* 19(1): 55-78.