Fixed Effects vs. Random Effects

Seminar in Criminology, Research and Analysis– Crim 7301
Week 7, 10/6/16
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Class Overview

- Motivation for fixed effects
- Change score oddities
- Regression to the mean
- Clustered standard errors vs fixed effects
- When to use Random effects

Motivation for fixed effects

- If the micro level model is (where unit *i* is nested within *j*):

$$Y_{ij} = \beta_1(X_{ij}) + \gamma(Z_j)$$

- For 1 unit imagine summing up all the micro level equations:

$$Y_{1j} = \beta_1(X_{1j}) + \gamma(Z_j)$$
$$Y_{2j} = \beta_1(X_{2j}) + \gamma(Z_j)$$
$$+\, Y_{3j} = \beta_1(X_{3j}) + \gamma(Z_j)$$

$$\sum Y_{ij} = \beta_1\left(\sum X_{ij}\right) + 3 \cdot \gamma(Z_j)$$

Motivation for fixed effects

- Divide that last equation by the total number of units

- $\sum Y_{ij}/3 = \overline{\mathrm{Y}}; \sum X_{ij}/3 = \bar{X}$

$$\bar{Y} = \beta_1(\bar{X}) + \gamma(Z_j)$$

- This is the *between* effects estimator. Subtract this from the micro level equation, and you get:

$$(Y_{ij} - \bar{Y}) = \beta_1(X_{ij} - \bar{X}) + [\gamma\left(Z_j\right) - \gamma(Z_j)]$$

- So the last term cancels out

Motivation for fixed effects

- Examples, comparing individuals only to themselves over time

- Micro level units nested within schools & you think teachers make a big deal

- For non-linear models, mostly equivalent to include a dummy variable for each unit

Change Score Oddities

- Change scores equivalently difference out fixed effects

$$Y_{i1} = \beta_1(X_{i1}) + \gamma(Z_i)$$
$$Y_{i2} = \beta_1(X_{i2}) + \gamma(Z_i)$$

$$(Y_{i2} - Y_{i1}) = \Delta Y = \beta_1(\mathrm{X_{i2}} - \mathrm{X_{i1}}) + [\gamma(Z_i) - \gamma(Z_i)]$$

Example: If these are people measured at multiple time points, a fixed effect would control for gender (and any stable genetic effect)

Change Score Oddities

- Do not include levels on the right hand side if using change scores:

$$(Y_{i2} - Y_{i1}) = \beta_1(T) + \beta_2(Y_{i1}) + \epsilon$$

- For linear models will estimate the same treatment effect as:

$$Y_{i2} = \beta_1(T) + (\beta_2 + 1)Y_{i1} + \epsilon$$

Regression to the Mean

- The correlation between change scores is often negative

- Why? (Assume x is mean centered for simplicity)

$$
\begin{aligned}
\mathbb{C}\text{ov}(x_1, x_2 - x_1) = {} & \mathbb{E}[x_1 \cdot (x_2 - x_1)] \\
& \mathbb{E}[(x_1 \cdot x_2) - (x_1 \cdot x_1)] \\
& \mathbb{E}[(x_1 \cdot x_2)] - \mathbb{E}[(x_1 \cdot x_1)] \\
& \mathbb{C}\text{ov}(x_1, x_2) - \mathbb{V}(x_1)
\end{aligned}
$$

- Even if the levels are random (ie no autocorrelation) the differences will have negative autocorrelation!

Clustered standard errors vs. fixed effects

- Accounts for *any* inter-dependence within clusters
- Needs many clusters!
- Makes a bigger difference if N per cluster gets higher, and/or intra-correlation within cluster is large
- Will make standard errors larger, effect estimates should be the same

When to use random effects

- When you care about estimating them!

$$Y_{ij} = \beta_0 + \beta_1\left(X_{ij}\right) + \gamma_j$$

- Shrinkage compared to fixed effects
- Can predict for new aggregate level units

- Can also allow effects to vary per aggregate level unit

$$Y_{ij} = \beta_{00} + \beta_{01}\left(X_{ij}\right) + \gamma_j$$
$$\gamma_j = \beta_{0j} + \beta_{1j}(X_{ij})$$

Homework & Next Weeks Class

Lab Assignment

Fixed effects and random effects in Stata, R and SPSS

For Next Week – Group Based Trajectory Models

- Skardhamar, T. (2010). Distinguishing facts and artifacts in group-based modeling. *Criminology*, 48(1):295-320.
- Weisburd, D., Bushway, S. D., Lum, C., and Yang, S.-M. (2004). Trajectories of crime at places: A longitudinal study of street segments in the city of Seattle. *Criminology*, 42(2):283-322.
- Erosheva, E. A., Matsueda, R. L., and Telesca, D. (2014). Breaking bad: Two decades of Life-Course data analysis in criminology, developmental psychology, and beyond. *Annual Review of Statistics*, 1(1):301-332.
- Nagin, D. and Odgers, C. (2010). Group-Based trajectory modeling (nearly) two decades later. *Journal of Quantitative Criminology*, 26(4):445-453.