Problem Set 1[1] - Econ-450-3

Due date: May 1, 2016. Send to mar.reguant@northwestern.edu.
Northwestern University, Spring 2016

Single-Agent Dynamic Models

Consider the dynamic optimization problem of Harold Jr., who manages a larger fleet of buses and has to deal with a more volatile market than his father. For each bus i in each month t, Harold Jr. must decide whether to replace the bus's engine ($a_{it} = 1$) or perform regular maintenance ($a_i t = 0$).

He has the following current-period utility function:

$$u(a_{it}, x_{it}, \epsilon_{it}) = \begin{cases} -\theta_1 x_{it} + \epsilon_{0it} & \text{if } a_{it} = 0 \\ -\theta_2 RC_t + \epsilon_{1it} & \text{if } a_{it} = 1 \end{cases}$$

where $x_t \in \{1, 2, ..., 7\}$ is the bus's mileage, RC_t is the price of a replacement bus engine, and the ϵs are i.i.d. with a type-1 extreme value distribution.

Mileage evolves according to the following process:

$$x_{it} = \begin{cases} 1 & \text{if } a_{it} = 1 \\ \min(x_{it} + 1, 7) & \text{if } a_{it} = 0 \end{cases}$$

From Harold Jr.'s point of view, RC_t evolves exogenously according to the following process:

$$RC_t = \rho_0 + \rho_1 RC_{t-1} + e_t,$$

where e_t is normally distributed with standard deviation σ_ρ.

Harold Jr. has a discount factor $\beta = .95$ and acts to maximize expected discounted utility with each bus, $E[\sum_{t=1}^{\infty} u(a_{it}, x_{it}, \epsilon_{it})]$. He has rational expectations.

Exercises

1. Estimate $(\theta, \rho, \sigma_\rho)$ using a nested fixed point algorithm.

2. Estimate $(\theta, \rho, \sigma_\rho)$ without solving the value function in the estimation algorithm but relying on the Hotz-Miller inversion.

3. Estimate θ without solving the value function in the estimation algorithm and without estimating the process on the replacement costs explicitly, using the linear regression approach.

Notes

The (simulated) data set includes decisions for 1000 buses observed over 100 months. The variables in the data should be self-explanatory: i, t, a, x, and RC.

[1] Thanks to Paul Scott at TSE for this problem set.

Some hints:

- Please, discuss with each other as you see fit. The goal of this exercise is to learn how to implement thes algorithms in practice, taking advantage that you are all on the same boat.
- You should discretize the state space for RC to make solving the value function in 1 tractable, and to facilitate the computation of an integral based on the process for RC in 1-2.
- For 2-3, you will need (smoothed) estimates of choice probabilities. For 2, you can estimate choice probabilities as a flexible function of RC and x. For 3, you should estimate choice probabilities for each t as a flexible function of x.
- Code the exercises in whatever order you would like. The literature has in a sense gone from harder to easier, so if it helps to begin with Hotz-Miller or the regression to fix ideas, start there.
- Use whatever software you like.
- Some links:
  - `https://editorialexpress.com/jrust/nfxp.html` (Rust's website on NFXP)
  - `http://ddc.abbring.org/dynamicDiscreteChoice.m.html` (Jaap Abbring and Tobias Klein's Matlab guide, much more detail than what we could cover)

Handing your solution in

Please turn in the following via email to mar.reguant@northwestern.edu:

- A brief explanation of the steps involved in each estimation algorithm for 1-3, emphasizing the differences between each. Be clear about what objective function you are using.
- A table listing the point estimates for θ_1 and θ_2 from each estimation 1-3, and the length of time your algorithm for each estimation took.
- Your code.