IO III Problem Set I

Haritz Garro and Nil Karacaoglu

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1 Exercise 3

First of all, we perform a logit with a as dependent variable and polynomials of x as independent variables (x, x^2 and x^3). We make sure that we do not include an intercept, since x never takes value zero in this setting. When we calculate the predicted probabilities of changing the engine given x, these consitute our estimators for the conditional choice probabilities $P(a = 1 \mid x)$.

Once we do this, we can leverage on the Hotz-Miller inversion to get the differences of the value functions in the following way:

$$\hat{v}(x, 0) - \hat{v}(x, 1) = log(\hat{p}(0 \mid x)) - log(\hat{p}(1 \mid x)) \tag{1}$$

where the second element of $v(\quad , \quad)$ refers to the action of replacing the engine a. We can calculate the LHS of equation 1 for every x.

The next step is to normalize the flow payoff, and we will, wlog, normalize the utility of changing the engine to zero, for every x, i.e. $u(x, 1) = 0$, $\forall$ x.

After this useful normalization, we can characterize the value function of changing the engine as:

$$v(x, 1) = \beta \left(v(x' = 1, 1) - log(\hat{p}(1 \mid x' = 1)) \right)$$

where we are making use of the fact that after changing the engine tomorrow x is equal to one.

First we solve for v(1, 1), and after that we can solve for v(x, 1), $\forall$ x.