More about priors

Models for Socio-Environmental Data

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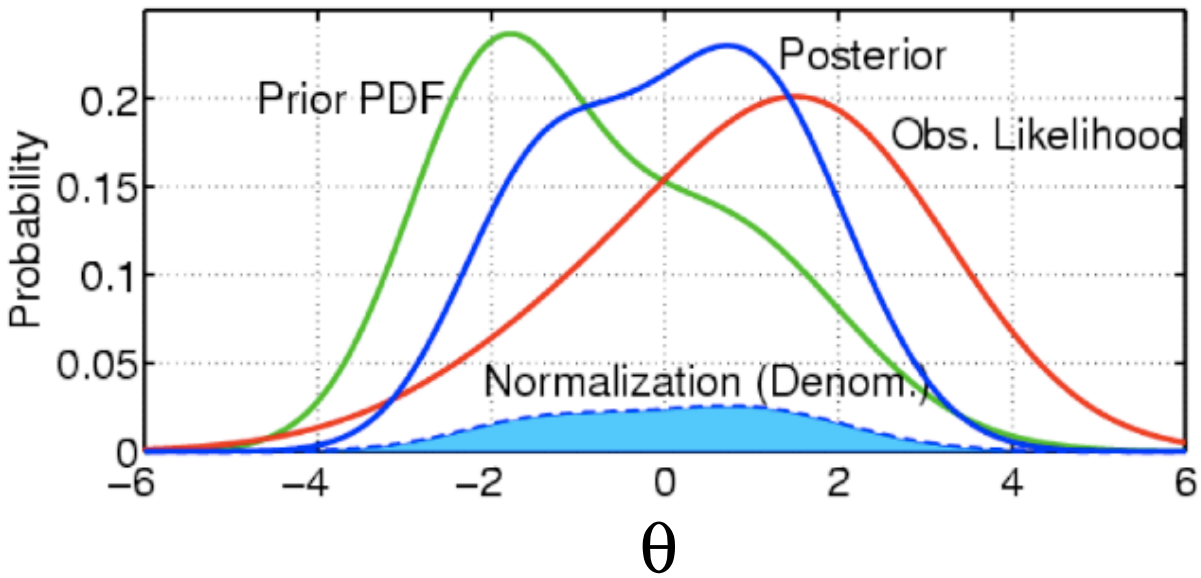
References for this lecture

- Hobbs and Hooten 2015, Section 5.4
- Seaman III, J. W. and Seaman Jr., J. W. and Stamey, J. D. 2012 Hidden dangers of specifying noninformative priors, The American Statistician 66, 77-84 (2012)

Recall that the posterior distribution represents a balance between the information contained in the likelihood and the information contained in the prior distribution.



An informative prior influences the posterior distribution. A vague prior exerts minimal influence.

Influence of data and prior information

$$\text{beta}\left(\phi|y\right) = \frac{\text{binomial}\left(y|\phi, n\right)\text{beta}\left(\phi|\alpha_{prior}, \beta_{prior}\right)}{[y]}$$

$$\alpha_{posterior} = \alpha_{prior} + y$$
$$\beta_{posterior} = \beta_{prior} + n - y$$

Influence of data and prior information

$$\text{gamma}\left(\lambda|\mathbf{y}\right) = \frac{\prod_{i=1}^{4} \text{Poisson}\left(y_i|\lambda\right) \text{gamma}\left(\lambda|\alpha_{prior}, \beta_{prior}\right)}{[\mathbf{y}]}$$

$$\alpha_{posterior} = \alpha_{prior} + \sum_{i=1}^{4} y_i$$
$$\beta_{posterior} = \beta_{prior} + n$$

A vague prior is a distribution with a range of uncertainty that is clearly wider than the range of reasonable values for the parameter (Gelman and Hill 2007:347).

Also called: diffuse, flat, automatic, nonsubjective, locally uniform, objective, and, incorrectly, "non-informative."

Vague priors are *provisional* in two ways:

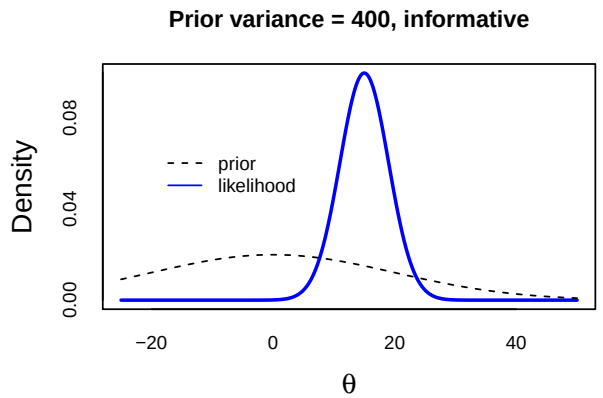
1. Operationally provisional: We try one. Does the output make sense? Are the posteriors sensitive to changes in parameters? Are there values in the posterior that are simply unreasonable? We may need to try another type of prior.
2. Strategically provisional: We use vague priors until we can get informative ones, which we prefer to use.

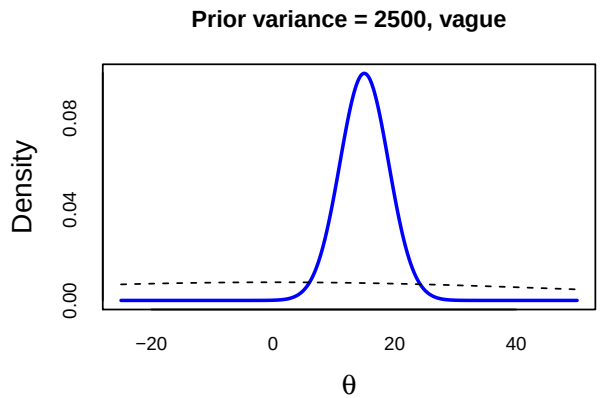
Scaling

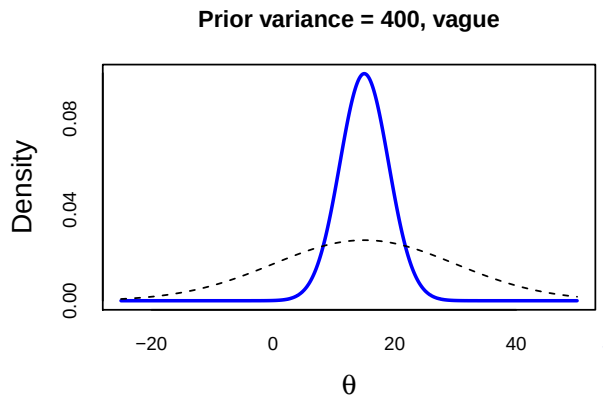
Vague priors need to be scaled properly.

Suppose you specify a prior on a parameter, $\theta \sim \text{normal}(\mu = 0, \sigma^2 = 1000)$. Will this prior influence the posterior distribution?

Scaling vague priors





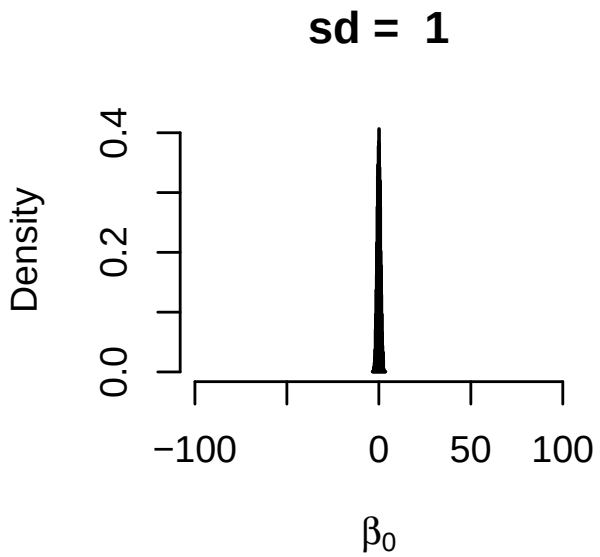


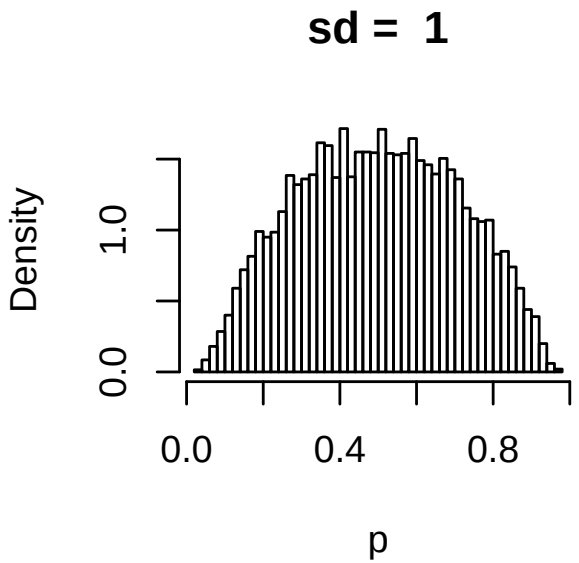
Problems with excessively vague priors

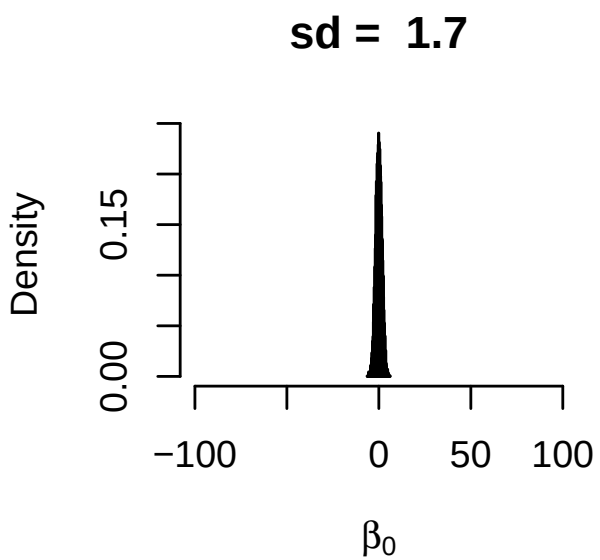
- Computational: failure to converge, slicer errors, failure to calculate log density, etc.
- Cause pathological behavior in posterior distribution, i.e, values are included that are unreasonable.
- Sensitivity: changing the parameters of "vague" priors meaningfully changes the posterior.
- Non-linear functions of parameters with vague priors have informative priors.

"Priors" on nonlinear functions of parameters

$$
\begin{aligned}
p_i = g(\boldsymbol{\beta}, x_i) &= \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \\
[\boldsymbol{\beta}|\mathbf{y}] &\propto \prod_{i=1}^{n} \text{Bernoulli}(y_i | g(\boldsymbol{\beta}, x_i)) \times \\
&\quad \text{normal}(\beta_0 | 0, 10000) \text{normal}(\beta_1 | 0, 10000)
\end{aligned}
$$


sd = 1
Density
0.0
0.2
0.4
−100
0
50
100
β0


sd = 1
Density
0.0
1.0
0.0
0.4
0.8
p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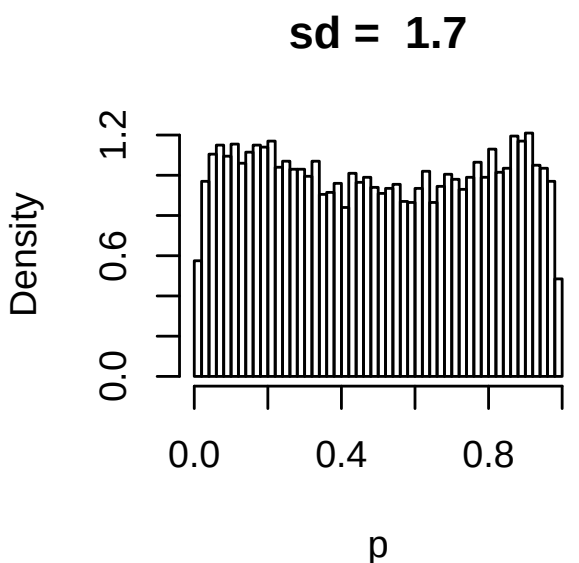
0.00
0.15
−100
0
50
100
β0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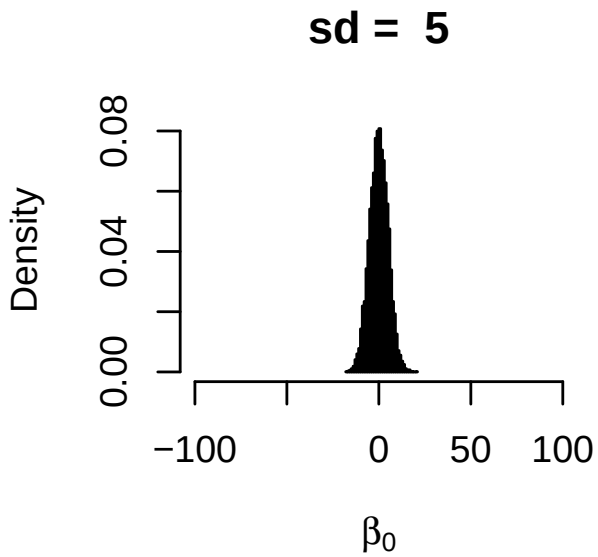
0.0
0.6
1.2
0.0
0.4
0.8
p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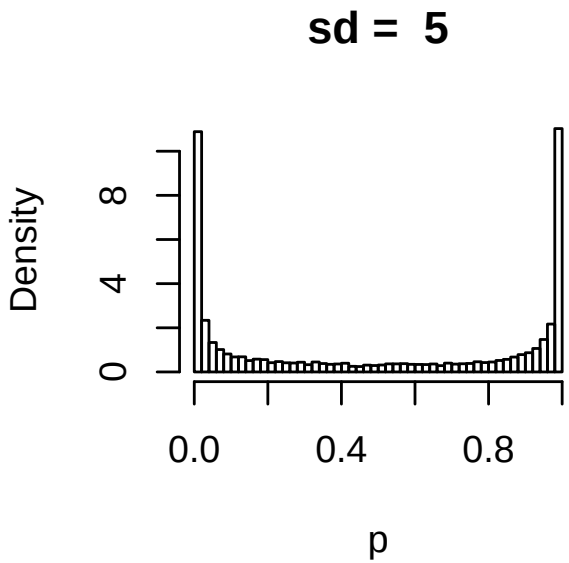
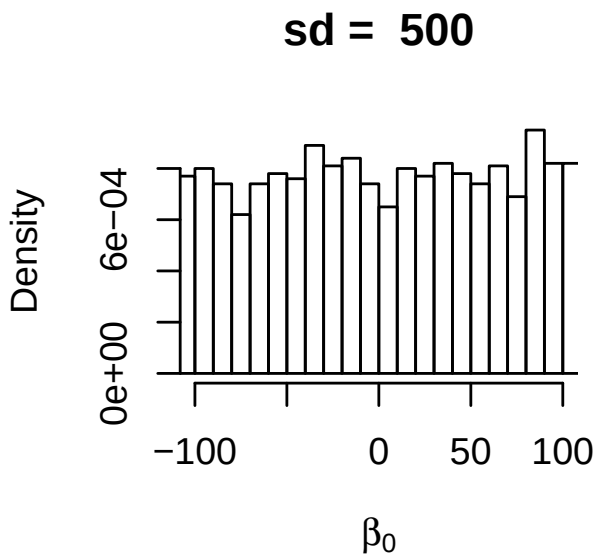
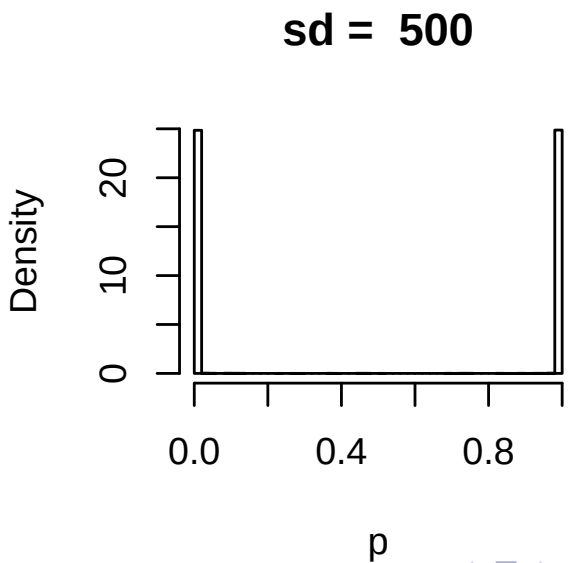
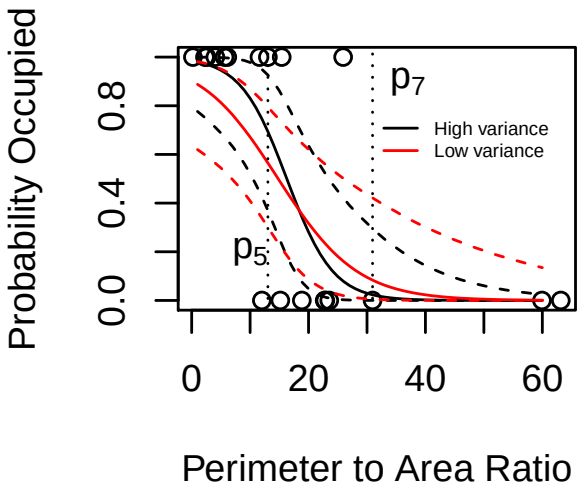
Density
0.00
0.04
0.08
−100
0
50
100
β0

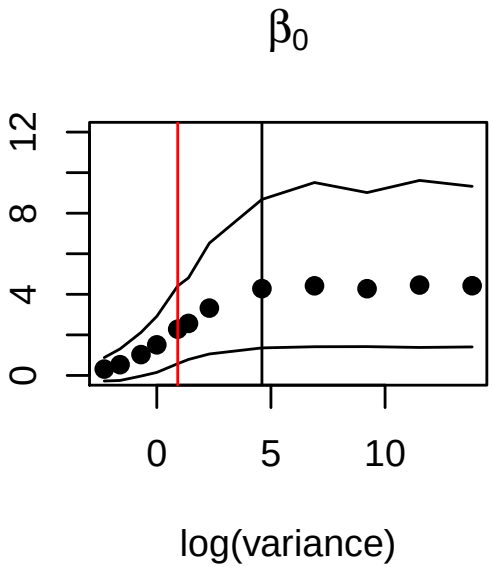

sd = 5
Density
0
4
8
0.0
0.4
0.8
p

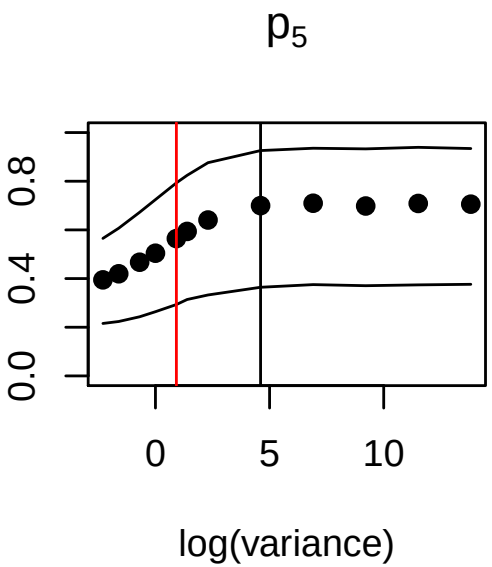

sd = 500
Density
0e+00
6e−04
−100
0
50
100
β0

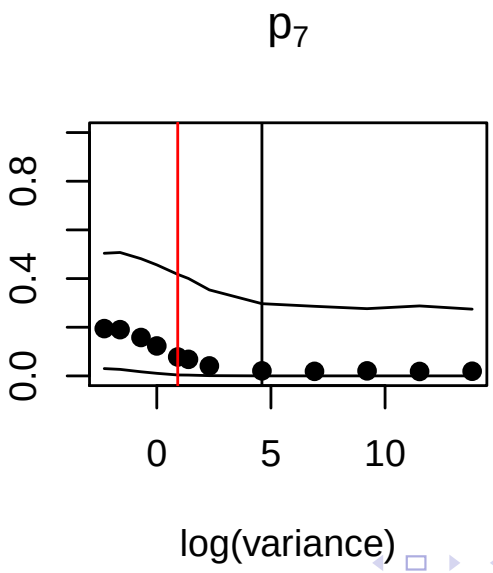

sd = 500
Density
0
10
20
0.0
0.4
0.8
p

Islands data

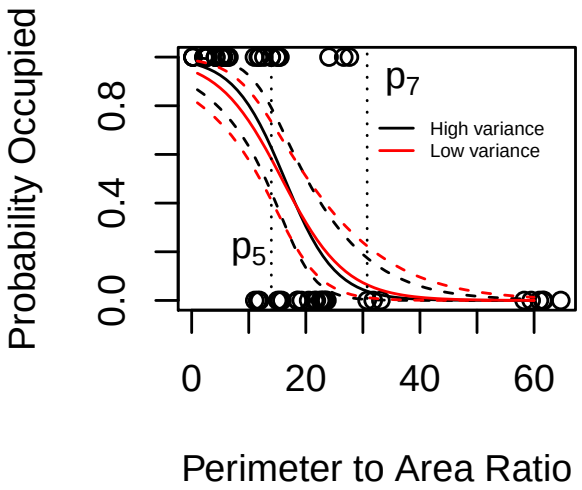


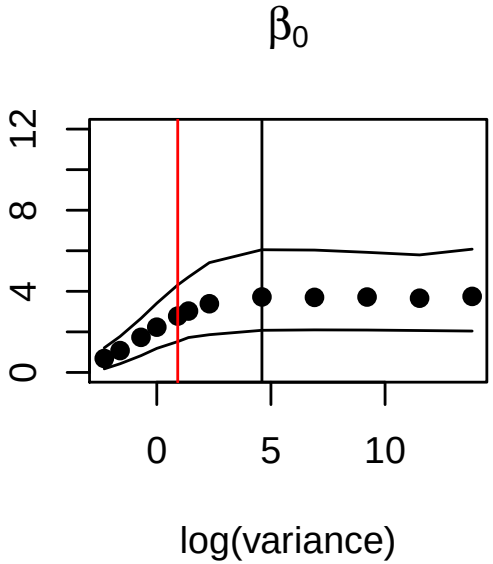


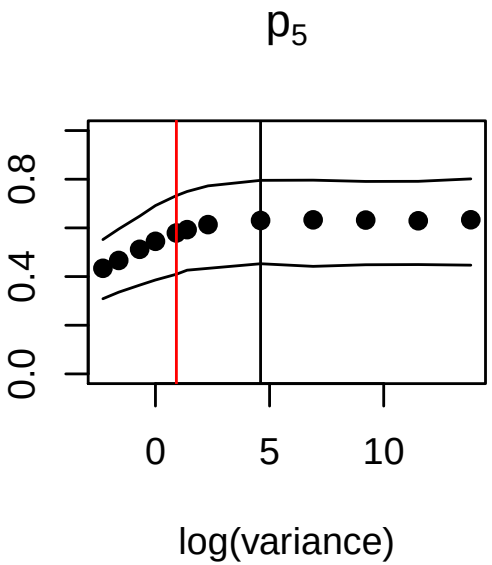


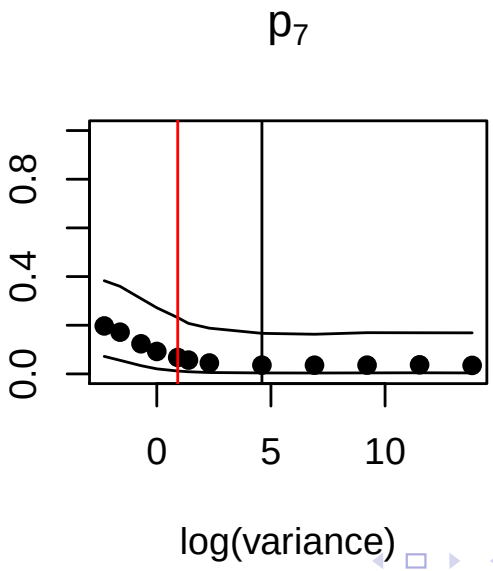


Islands data x 3







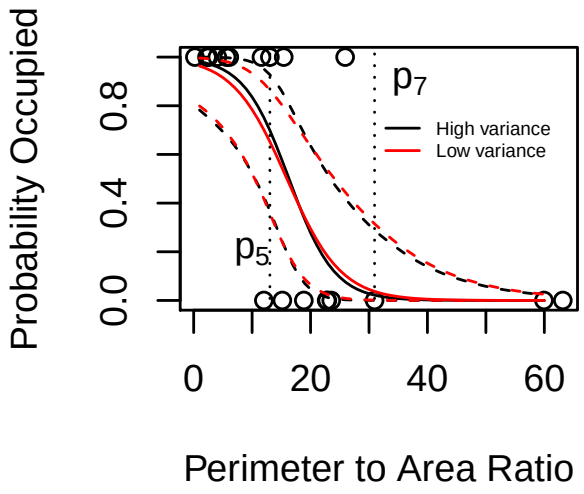


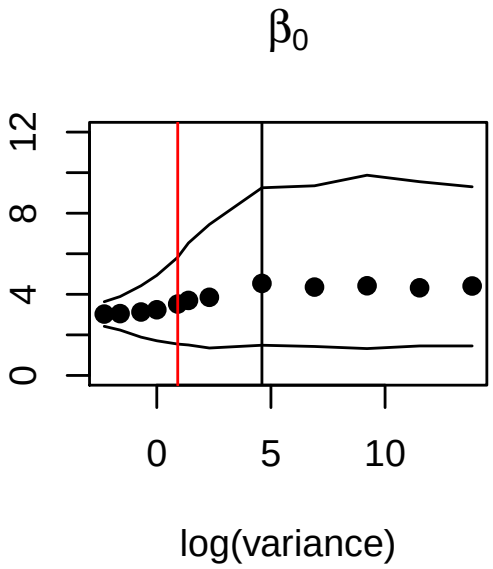
Slightly more informed priors with original data

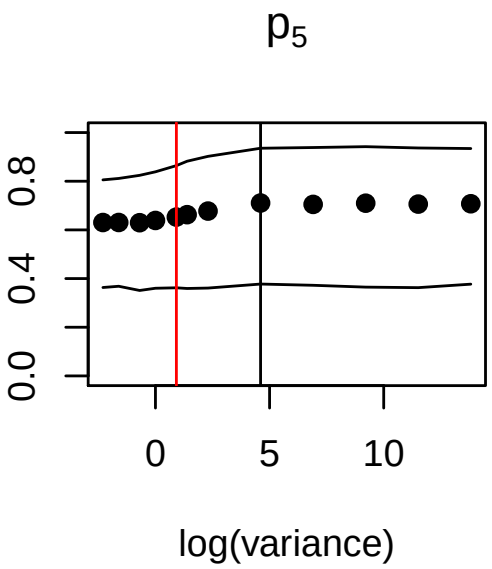
$$\begin{aligned} \beta_0 &\sim \text{normal}(3, \sigma^2) \\ \beta_1 &\sim \text{normal}(-1, \sigma^2) \end{aligned}$$

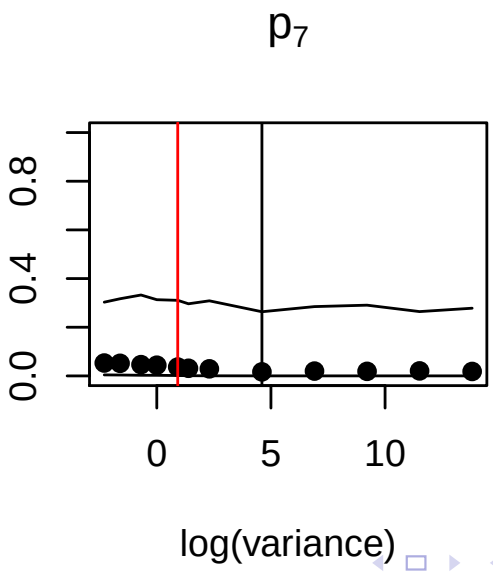
We center β_0 on 3 using the reasoning that large islands are almost certainly (p=.95 at PA = 0) occupied. Choosing a negative value for the slope make sense because we *know* the probability of occupancy goes down as islands get smaller.

Voila


Probability Occupied
0.0
0.4
0.8
0
20
40
60
p7
p5
High variance
Low variance
Perimeter to Area Ratio


β0
0
4
8
12
0
5
10
log(variance)


p5
0.0
0.4
0.8
0
5
10
log(variance)


p7
0.0
0.4
0.8
0
5
10
log(variance)

More guidance

- Know that priors that are vague for parameters can influence non-linear functions of parameters. This influence can be minimized if vague priors are centered in the vicinity of the central tendency of the posteriors of parameters.
- Always use informative priors when you can. You know more than you think you do.