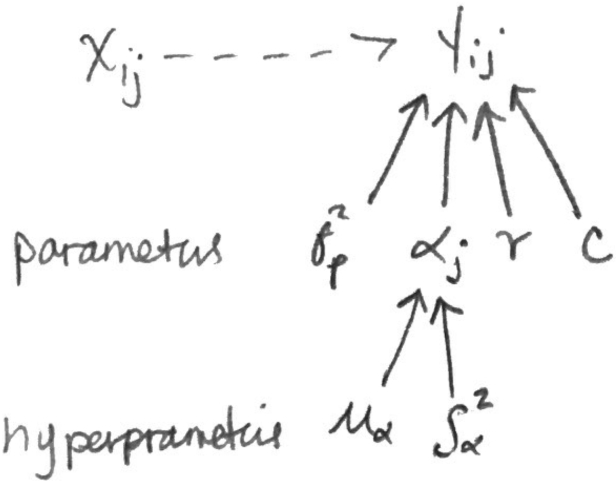
MULTILEVEL EXAMPLE

(Akin to "random effects")

$$\mu_{ij} = g(\alpha_j, \gamma, c, x_{ij}) = \frac{\alpha_j (x_{ij} c)}{\frac{\alpha_j}{\gamma} + (x_{ij} + c)}$$



collect information @ several sites $\therefore$ have site-level variation

$\alpha_j \sim \text{gamma}\left(\frac{\mu_\alpha^2}{\varsigma_\alpha^2}, \frac{\mu_\alpha}{\varsigma_\alpha^2}\right)$

$$[\sigma_p^2, \underline{\alpha}, \gamma, c, \mu_\alpha, \varsigma_\alpha^2 \mid \underline{y}] \propto [\sigma_p^2, \alpha_j, \gamma, c, \mu_\alpha, \varsigma_\alpha^2, y_{ij}]$$

$$\propto \prod_{i=1}^{n_j} \prod_{j=1}^{J} [y_{ij} \mid g(\alpha_j, \gamma, c, x_{ij}), \sigma_p^2]$$

$$\times [\alpha_j \mid \mu_\alpha, \varsigma_\alpha^2] [\sigma_p^2] [\gamma] [c]$$

$$\times [\mu_\alpha] [\varsigma_\alpha^2]$$

"Borrowing Strength" → for sites where there is a lot of data the influence on μ_α is greater

$$y_i \sim \text{normal}\left(g(\alpha_{ij}, r, c, X_{ij}), \sigma_P^2\right)$$

$$r \sim \text{uniform}(0.1, 200)$$

$$c \sim \text{normal}(0, 10000)$$

$$\sigma_P^2 \sim \text{inverse gamma}(.001, .001)$$

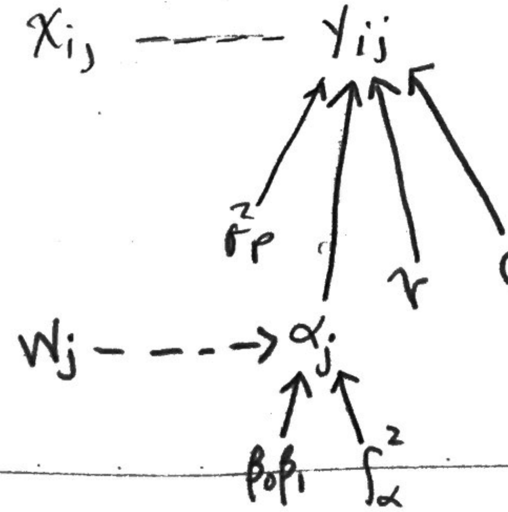
$$\alpha_j \sim \text{gamma}(\mu_\alpha, \varsigma_\alpha^2)$$

$$\mu_\alpha \sim \text{normal}(\ \ ,\ \ )$$

$$\varsigma_\alpha^2 \sim \text{inverse gamma}(.001, .001)$$

what if you have site-level covariates?

$$\mu_\alpha = \beta_0 + \beta_1 W_j = h(\beta_0, \beta_1, W_j)$$



$$[\sigma_P^2, \underline{\alpha}, r, c, \mu_\alpha, \varsigma_\alpha^2, \beta_0, \beta_1 \mid \underline{y}]$$

$$\propto \prod_{j=1}^{n_j} \prod_{j=1}^{J} [y_{ij} \mid g(\alpha_j, r, c, X_{ij}), \sigma_P^2]$$

$$\times [\alpha_j \mid h(\beta_0, \beta_1, W_j), \varsigma_\alpha^2]$$

$$\times [\sigma_P^2][\beta_0][\beta_1][r][c][\varsigma_\alpha^2]$$