More about priors

Models for Socio-Environmental Data

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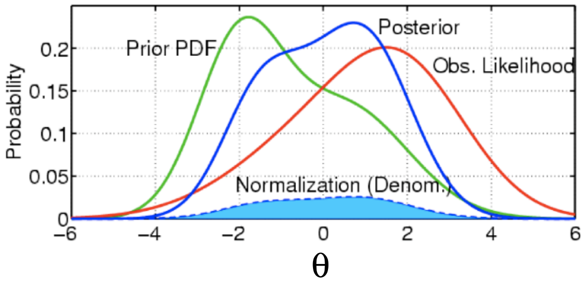
SESYNC

References for this lecture

- Hobbs and Hooten 2015, Section 5.4
- Seaman III, J. W. and Seaman Jr., J. W. and Stamey, J. D. 2012 Hidden dangers of specifying noninformative priors, The American Statistician 66, 77-84
- Northrup, J. M., and B. D. Gerber. 2018. A comment on priors for Bayesian occupancy models. PLoS ONE 13.

Recall that the posterior distribution represents a balance between the information contained in the likelihood and the information contained in the prior distribution.



An informative prior influences the posterior distribution. A vague prior exerts minimal influence.

Influence of data and prior information

$$\text{beta}\left(\phi|y\right)=\frac{\text{binomial}\left(y|\phi,n\right)\text{beta}\left(\phi|\alpha_{prior},\beta_{prior}\right)}{\left[y\right]}$$

$$\alpha_{posterior}=\alpha_{prior}+y$$
$$\beta_{posterior}=\beta_{prior}+n-y$$

Influence of data and prior information

$$\text{gamma}\left(\lambda|\mathbf{y}\right) = \frac{\prod_{i=1}^{4}\text{Poisson}\left(y_i|\lambda\right)\text{gamma}\left(\lambda|\alpha_{prior},\beta_{prior}\right)}{[\mathbf{y}]}$$

$\alpha_{posterior} = \alpha_{prior} + \sum_{i=1}^{4} y_i$
$\beta_{posterior} = \beta_{prior} + n$

A vague prior is a distribution with a range of uncertainty that is clearly wider than the range of reasonable values for the parameter (Gelman and Hill 2007:347).

Also called: diffuse, flat, automatic, nonsubjective, locally uniform, objective, and, incorrectly, "non-informative."

Vague priors are *provisional* in two ways:

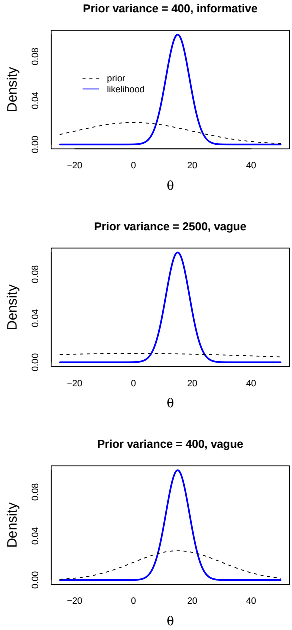
1. Operationally provisional: We try one. Does the output make sense? Are the posteriors sensitive to changes in parameters? Are there values in the posterior that are simply unreasonable? We may need to try another type of prior.
2. Strategically provisional: We use vague priors until we can get informative ones, which we prefer to use.

Scaling

Vague priors need to be scaled properly.

Suppose you specify a prior on a parameter, $\theta \sim \text{normal}(\mu = 0, \sigma^2 = 1000)$. Will this prior influence the posterior distribution?

Scaling vague priors

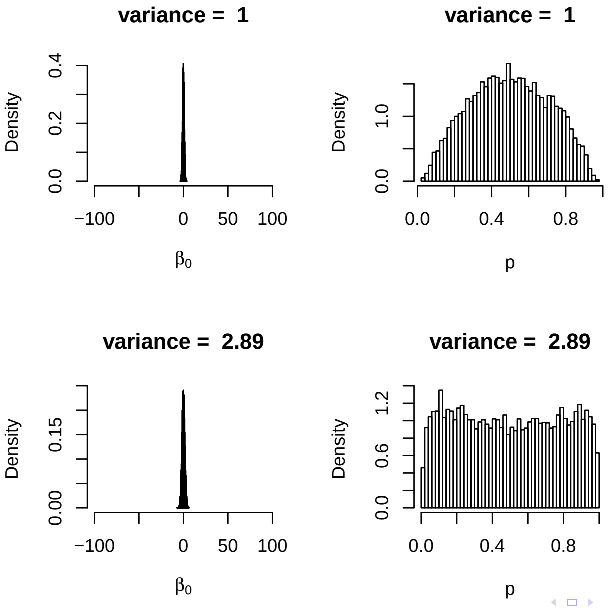


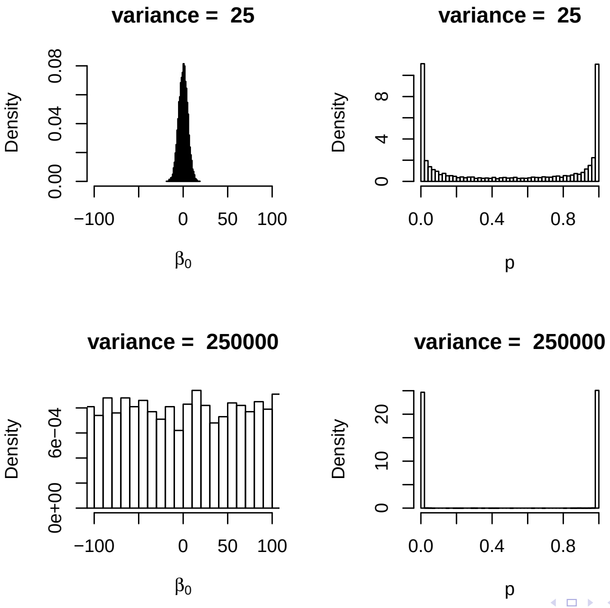
Problems with excessively vague priors

- Computational: failure to converge, slicer errors, failure to calculate log density, etc.
- Cause pathological behavior in posterior distribution, i.e, values are included that are unreasonable.
- Sensitivity: changing the parameters of "vague" priors meaningfully changes the posterior.
- Non-linear functions of parameters with vague priors have informative priors.

"Priors" on nonlinear functions of parameters

$$
\begin{aligned}
p_i = g(\boldsymbol{\beta}, x_i) &= \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \\
[\boldsymbol{\beta} | \mathbf{y}] &\propto \prod_{i=1}^{n} \text{Bernoulli}(y_i | g(\boldsymbol{\beta}, x_i)) \times \\
& \quad \text{normal}(\beta_0 | 0, 10000)\text{normal}(\beta_1 | 0, 10000)
\end{aligned}
$$


variance = 1
Density
0.0
0.2
0.4
−100
0
50
100
β0
variance = 1
Density
0.0
1.0
0.0
0.4
0.8
p
variance = 2.89
Density
0.00
0.15
−100
0
50
100
β0
variance = 2.89
Density
0.0
0.6
1.2
0.0
0.4
0.8
p


variance = 25
Density
0.00
0.04
0.08
−100
0
50
100
β_0
variance = 25
Density
0
4
8
0.0
0.4
0.8
p
variance = 250000
Density
0e+00
6e−04
−100
0
50
100
β_0
variance = 250000
Density
0
10
20
0.0
0.4
0.8
p

Islands data

Vague priors for probability of occupancy[1]

$$\beta_0 \sim \text{normal}(0, 2.7) \tag{1}$$
$$\beta_1 \sim \text{normal}(0, 2.7) \tag{2}$$

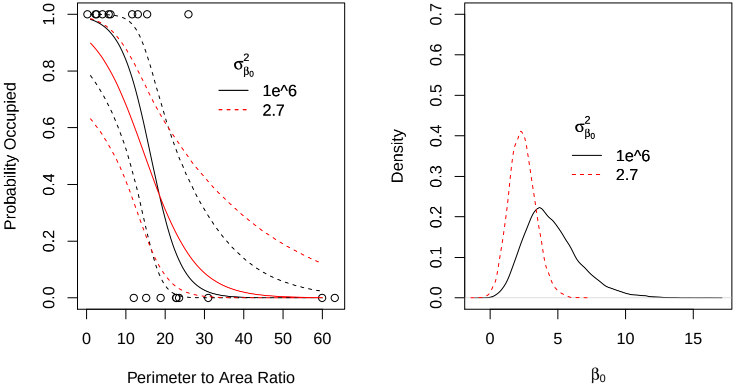
Vague priors for parameters[2]

$$\beta_0 \sim \text{normal}(0, \text{1e+6}) \tag{3}$$
$$\beta_1 \sim \text{normal}(0, \text{1e+6}) \tag{4}$$

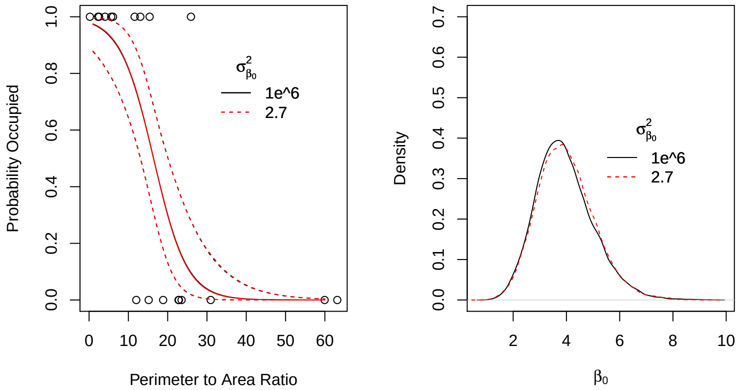
[1] Remember these are variances. JAGS needs precisions, $\tau = .37$

[2] Remember these are variances. JAGS needs precisions, τ = 1e-6

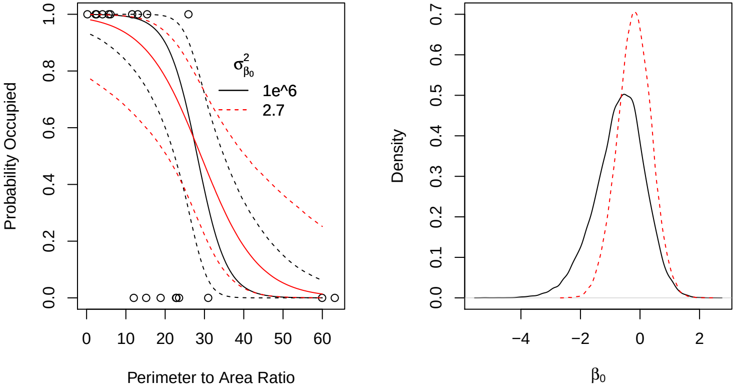
Islands data



Islands data x 3



Standardize the original data

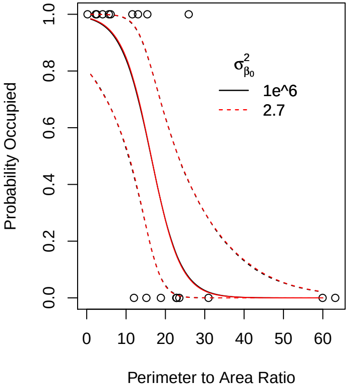


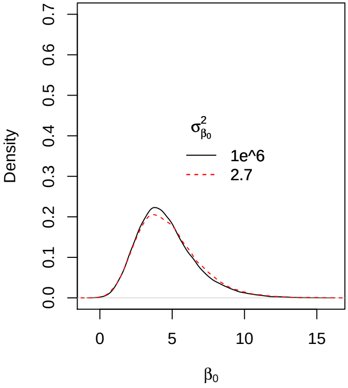
Slightly more informed priors with original data

$$\begin{aligned}\beta_0 &\sim \text{normal}(3, \sigma^2_{\beta_0})\\ \beta_1 &\sim \text{normal}(-1, \sigma^2_{\beta_1})\end{aligned}$$

We center β_0 on 3 using the reasoning that large islands are almost certainly (p=.95 at PA = 0) occupied. Choosing a negative value for the slope make sense because we *know* the probability of occupancy goes down as islands get smaller.

Weakly informative priors on parameters





Guidance

- Know that priors that are vague for parameters can influence non-linear functions of parameters.
- Explore sensitivity of all non-linear models to priors.
- Always use informative priors when you can.
- Always standardize data for non-linear models.
- Set variance $\approxeq$2.7 for normal priors on parameters in inverse logit models (precision $\approxeq .37$). Set means at "reasonable" values if possible.