Writing Bayesian Hierarchical Models

Models for Socio-Environmental Data

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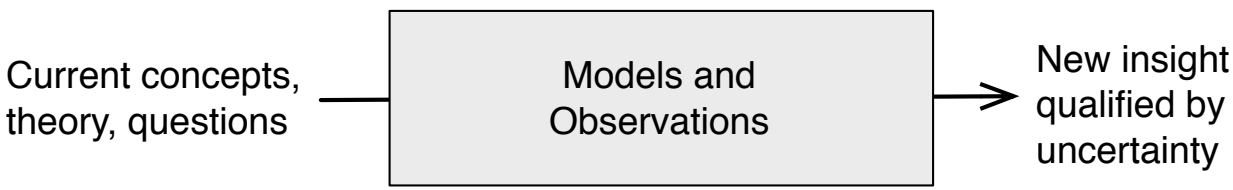
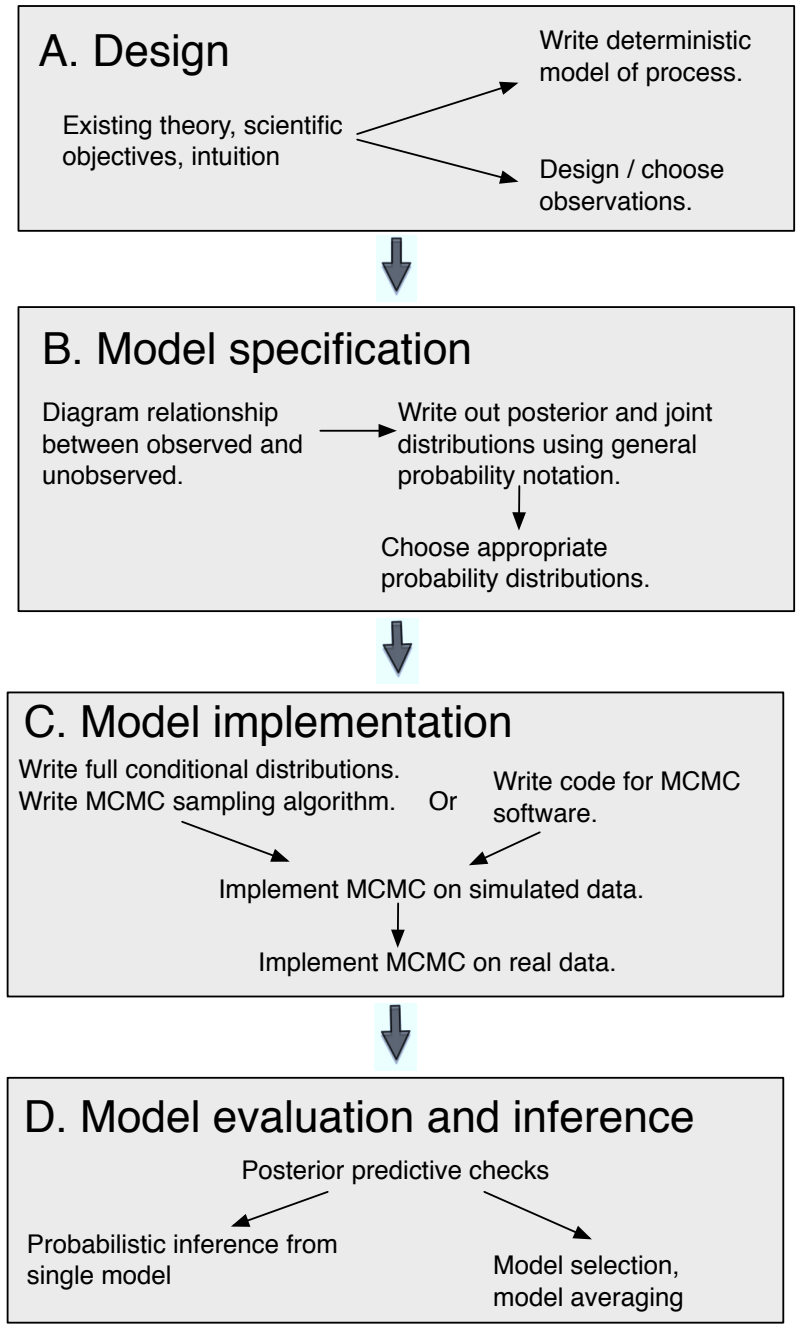
What is this course about?



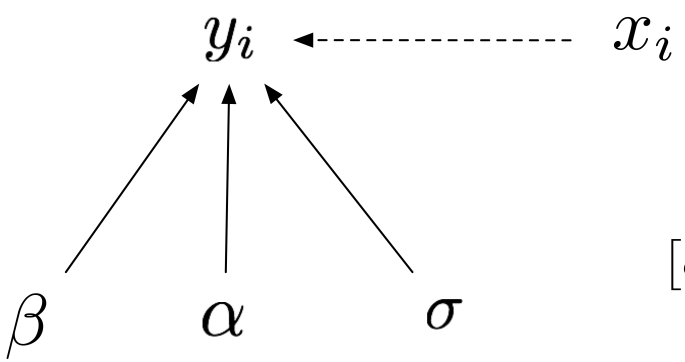
All modeling problems have idiosyncrasies

- Different types of data
- Different deterministic models
- Sampling error in the predictors or responses
- Calibration error for predictors or responses
- Prior knowledge of parameters
- Missing data
- Multiple scales of data (group level effects)
- Prediction and forecasting
- Spatial or temporal dependence
- Derived quantities

The Bayesian method



Cross cutting theme



$$\mu_i = \frac{m x_i^a}{h^a + x_i^a}$$

$$[a, h, m, \sigma^2 \mid y] \propto \prod_{i=1}^{n} [y_i | \mu_i, \sigma^2][a][h][m][\sigma^2]$$

```
model{

      for(i in 1:length(y)){

            mu[i] <- (m*x[i]^a)/(h^a+x[i]^a)
            y[i] ~ dgamma(mu[i]^2/sigma^2,mu[i]/sigma^2)

      }

a ~ dnorm(0,.0001)
m ~ dgamma(.01,.01)
h ~ dgamma(.01,.01)
sigma ~ dunif(0,5)
}
```

Things to watch for today

- Sampling error in x's and y's
- Calibration error in y's
- Derived quantities
- Group level effects
- Treatment effects

All of these will appear in the exercises.

Things to watch for today

Partitioning uncertainty

- Process variance
- Sampling variance
- Calibration variance (aka observation variance)
- Group level variance

Steps in writing Bayesian models

1. Write your deterministic model. Be careful about support.
2. Draw Bayesian network (DAG) describing relationships between observed and unobserved quantities.
3. Use the Bayesian network to write proportionality between posterior and joint distributions using bracket notation $[\,|\,]$.
   - 3.1 Posterior distribution: $[\text{unobserved quantities}\,|\,\text{data}]$
   - 3.2 Joint distribution
     - 3.2.1 All nodes in Bayesian network at the heads of arrows (children) must be on the left hand side of a conditioning symbol.
     - 3.2.2 All nodes in Bayesian network at the tails of arrows (parents) must be on the right hand side of a conditioning symbol $|$.
     - 3.2.3 All nodes at the end of an arrow with no arrow coming into them must be expressed unconditionally, i.e., they must have numeric arguments.
4. Assign specific PDF or PMF to each of the brackets.
5. Choose numeric values for parameters of prior distributions. Do this sensibly! Do not default to vague priors.