

The Effect of Taxation on Labor Supply: Evaluating the Gary Negative Income Tax Experiment

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A model of labor supply is formulated which takes explicit account of nonlinearities in the budget set which arise because the net, after-tax wage depends on hours worked. These nonlinearities may lead to a convex budget set due to the effect of progressive marginal tax rates, or they may lead to a nonconvex budget set due to the effect of government transfer programs such as AFDC or a negative income tax. The nonlinearities affect both the marginal wage and the “virtual” nonlabor income which the individual faces. The model is estimated on a sample of prime-age males from the Gary negative income tax experiment.

The economic theory of consumer choice derives consumer demands under the assumption of a constant price which is independent of quantity demanded by the consumer. Empirical estimation of consumer-demand functions then depends on a functional relationship between the price of a commodity and the amount of the quantity which is consumed. Many actual situations do not conform to these classical assumptions. Totally within the private sector, nonlinear prices may arise in industries with substantial fixed costs. Here average price and marginal price are not equal so the choice of the “correct” price to put in the demand function is not straightforward. Electricity prices are the most important example of this situation. The existence of government income tax and income

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transfer programs, however, is the primary source of nonlinear consumer prices. Net, after-tax wage rates almost always depend on the number of hours of work supplied. For instance, workers facing a progressive income tax have net wage rates that decline as gross earnings rise. Other important examples are AFDC programs and social security payments for individuals between 65 and 72 years of age. Here income transfers are accompanied by high marginal tax rates. These programs not only cause prices to be nonlinear but also cause budget sets to be nonconvex, which further complicates the theory and estimation of labor supply.

Most econometric studies of labor supply assume that the most preferred level of work effort for an individual depends on a single wage rate that is independent of the chosen level of hours of work. Thus the endogeneity of the net wage rate is ignored. When it has been considered, only reduced-form estimates have been computed. These estimates have the disadvantage that they depend on the particular sample information used and cannot be used to evaluate the expected effect of policy changes. In this paper we propose an alternative method of estimating labor-supply functions in the presence of nonlinear net wages. The technique follows from a structural model of individual labor-supply choice when the net wage depends on hours of work supplied. Thus individual choice depends on all net wages which comprise the budget set so that policy changes can be evaluated using the parameter estimates. The model we estimate has one other important difference from usual models of labor supply. We allow for a distribution of preferences in the population for the labor-leisure choice. This broadening of the traditional model seems called for by the observed data in which otherwise-identical individuals have widely differing labor-supply choices. Our findings confirm this observation since a very skewed distribution of preferences is observed.

The model developed here is used to evaluate the effects of a negative income tax (NIT) experiment in Gary, Indiana. In this experiment, income transfer payments were made to families on the basis of an income support level which depended on family size and a tax rate of either 40 percent or 60 percent. Beyond a given number of hours worked, an individual's earnings were taxed at the usual federal and state tax rates. A complicated budget constraint resulted which consisted of linear segments connected by kink points. We estimate the unknown parameters of the uncompensated labor-supply function together with the associated indirect utility function to evaluate the income and substitution effects of the NIT. An important question considered is, How large are the appropriate income and wage elasticities? We further consider the likely pattern of response to a NIT in the population. The estimates found here can be compared with labor-supply response estimates from other NIT experiments in New Jersey, Seattle, and Denver. Our findings imply that a NIT has only a very small effect on a substantial proportion of the population but that a significant

number of individuals' labor-supply decisions may be affected quite substantially.

The first section of the paper discusses the problem of nonlinear net wages. It shows how a progressive income tax leads to a convex budget set, while government transfer programs like a NIT lead to nonconvex budget sets in which certain choices of hours worked can never be optimum. Thus the importance of knowing the form of both the labor-supply function and the associated indifference-curve map is emphasized. In Section II we use the modern theory of duality to derive the indifference-curve map through the indirect utility function. This utility function is derived from an ordinary specification of labor supply. Restrictions from the theory of consumer demand are derived so that the parameter estimates will not lead to violation of the theory of individual choice. Next we consider utility maximization and consumer choice in the presence of nonlinear net wages. We calculate the preferred point along a budget line at which individual choice jointly determines both hours worked and net marginal wage. To complete the specification of the model, we propose a stochastic specification which allows for both individual variation in preferences and the more traditional deviation between preferred hours of work and actual observed hours of work. Section IV discusses the operation of the Gary NIT experiment and the actual calculation of individual budget sets. Since some individuals acted as controls, we also consider budget sets of individuals facing only a progressive income tax. Potential data problems are also discussed. In Section V we present the results of maximum-likelihood estimation of our structural model. While an important income-elasticity response is found, no associated uncompensated wage-elasticity response seems to be present. Finally, we discuss the policy implications of the results and indicate possible future research.

I. Labor Supply with Nonlinear Net Wages

The economic theory of labor supply is a straightforward application of utility maximization. Individuals face a given market-determined wage along with prices of other consumption goods. Workers are assumed to choose the desired amount of hours of work which corresponds to the most preferred point on their budget sets.¹ In the familiar two-good diagram of hours supplied and expenditure on other goods, the slope of the budget set is the normalized wage $w = \tilde{w}/p$ and the intercept $y = \tilde{y}/p$ is normalized nonlabor income, where \tilde{w} and \tilde{y} are the market wage and nonlabor

¹ The theory of labor supply is sometimes stated as a theory of leisure demand given full income (see Becker 1965). However, we will treat hours supplied as the variable of interest, using a minus sign for hours worked in the utility function to maintain the usual monotonicity conventions. Also, in the econometric estimation, we will account for the fact that individuals may not be able to work their desired amount.

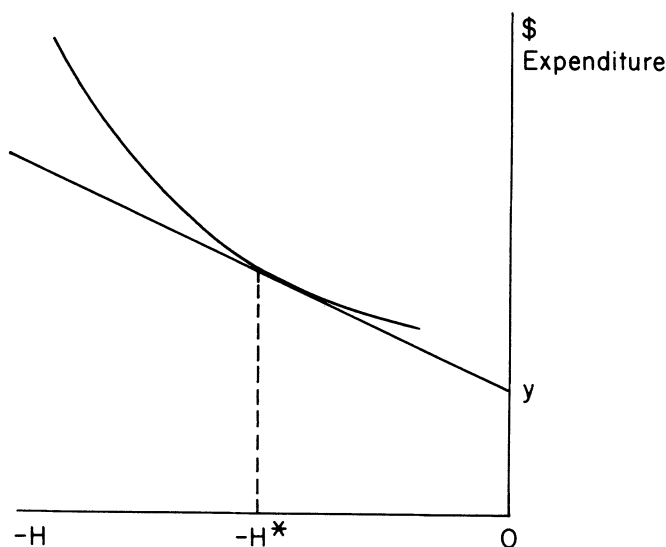


FIG. 1

income, respectively, and we use the price of the consumption good as the numeraire. In figure 1, $-H^*$ is the point which corresponds to the most preferred point created by the tangency of the indifference curve to the budget set. However, an important shortcoming of this analysis is the failure to incorporate the fact that the individual faces a nonconstant net wage, $w(1 - t)$, where t is the marginal tax rate. If t were a constant independent of labor supplied, and y were also independent of $-H$, then the budget line would simply be rotated counterclockwise. The previous analysis would be correct using the correctly measured net wage.

However, proportional tax systems are used only at the state and local tax level so that the analysis must account for nonlinear budget sets created by progressivity in tax formulas. Thus the budget set is piecewise linear, with kinks at points where income rises sufficiently to put the individual into the next higher tax bracket. The effect of a progressive tax system is to create a quasi-convex budget set like the one shown in figure 2.

In this highly simplified version of a progressive tax system, $-H_1$ and $-H_2$ correspond to the kink points induced by the tax system and $-H^*$ is the preferred amount of labor supply. This convex nonlinearity creates problems for the theory and estimation of labor supply. Theoretically, the usual comparative statics results must take account of the kink points and how their location depends on the gross wage. For estimation the problems are especially severe. Typical labor-supply specifications have the form

$$H = g(w, y, z, \epsilon), \quad (1)$$

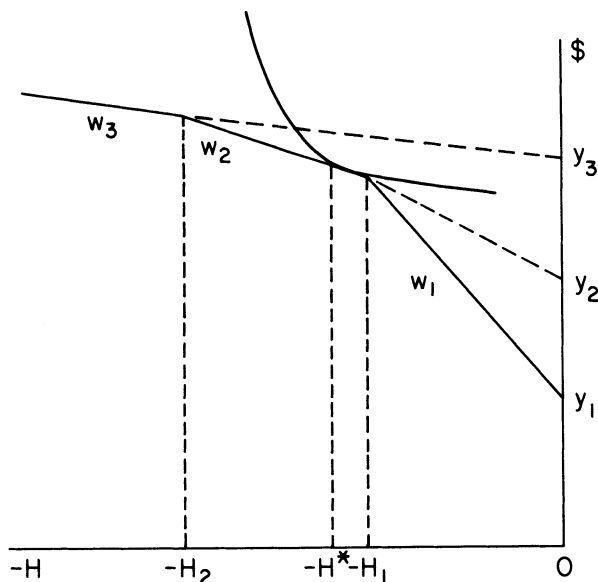


FIG. 2

where z is a vector of individual characteristics and ε is a stochastic term. Within the context of this type of labor-supply function, it is not obvious which net wage should be included as the variable explaining labor supply. Nor is it straightforward to decide which level of nonlabor income to specify. For instance, if the net wage corresponding to the second segment of the budget set of figure 2, w_2 , were chosen, we might want to use "virtual income," y_2 , corresponding to the intercept which equals nonlabor income of the budget set that the individual faces at the margin.²

Hall (1973) noted that a worker can be considered to be facing a linear budget constraint that is tangent to his actual budget set at the observed level of hours of work. For example, the individual facing the budget set drawn in figure 2 and observed to be working H^* hours can be considered to be facing a single wage rate, w_2 , and a single level of virtual income, y_2 . While this procedure is an important advance over using the gross wage, it cannot yield unbiased single-equation estimates because of the presence of the stochastic term ε in equation (1). Since both the net wage and virtual income are functions of hours worked, H , they will be correlated with ε , inducing a simultaneous-equations or errors-in-variables problem into the estimation procedure. Hausman and Wise (1976), Hurd

² Certainly the gross wage should not be used if substantial divergence exists between the gross wage and the net wage, since an upward-biased errors-in-variables problem will result.

(1976), and Rosen (1976) have each proposed different forms of instrumental variable procedures to account for the simultaneity problem. However, all these approaches are essentially “reduced-form” approaches to the extent that predictions for alternative budget sets cannot be made on the basis of the sample estimates.³

This problem of a nonlinear budget set would be solved if we had sufficient knowledge about the form of the utility function. Suppose we choose a particular form of the utility function and then adopt an additive stochastic specification of the labor-supply function. The labor-supply function can be written

$$H = h(w_1, w_2, w_3, y_1, y_2, y_3, z) + \varepsilon, \quad (2)$$

where $h(\cdot)$ is the consumer's supply function for labor, derived from maximizing the individual utility function.⁴ Discrepancies from utility maximization would be represented by ε , but since all the right-hand-side variables can be treated as exogenous, the unknown parameters of the utility function could be estimated. No problems of “which” net wage or “which” level of nonlabor income to use would arise since they are fully taken into account in the utility maximization.

When government programs beyond the progressive income tax are considered, the situation becomes even more complex since the resulting budget set may be nonconvex. Consider the operation of the NIT program that we study empirically in this paper. An income transfer, T , is calculated on the basis of a family's income guarantee, the NIT marginal tax rate, and family income. At the break-even point, \tilde{H} , in figure 3, the income transfer is completely taxed away due to high family earnings, and the earner returns to the federal and state income tax schedules. At this point the net wage rate rises, creating a nonlinearity in the budget set. Not only are four wage rates encountered, but an additional problem arises. Since the break-even point represents a nonconvex kink point, there exists an interval along the budget line in the vicinity of break even that may never contain a global maximum if, as is generally assumed, indifference curves are convex.⁵ Furthermore, the exact size and location of this interval

³ Another possible approach is to use an ordered probit model to estimate the probability that an individual's preferred point is on a particular linear segment and then to estimate hours worked as a random variable, which is censored at the kink points. Again, this approach is reduced form since the probit model would change with different budget sets.

⁴ Note that for nearly all utility functions $h(\cdot)$ will be a complicated function of net wages and nonlabor income. In fact, it will probably not exist in closed form for nonlinear budget constraints. However, it can be calculated easily by numerical techniques on a computer. The actual procedure used will be discussed in Sec. III. Heckman (1974a) proposed an alternative procedure in which net wages are estimated as a function of hours worked. If the form of the budget set is known, this relationship would be exact so that a random variable, hours worked, and an exact nonlinear transformation of it appear on both sides of the equation. Also, nonconvex budget sets cannot be treated with this approach.

⁵ None of the instrumental variable procedures takes account of this feature of the budget set introduced by the nonconvexity.

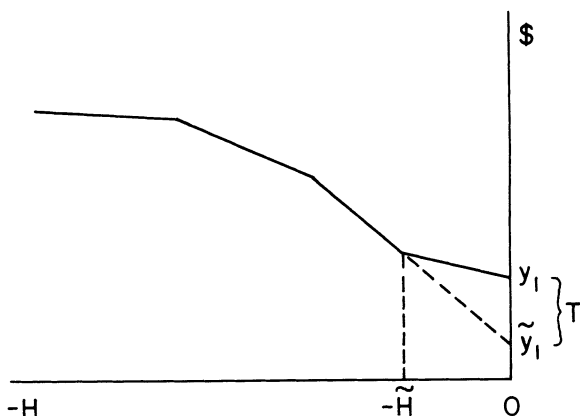


FIG. 3

depends on the specification and unknown parameters of the underlying utility function. Observed hours of work may sometimes fall in this interval if there are errors in optimization or institutional rigidities, but the implied restrictions on globally optimal hours should be taken into account in the estimation of labor supply.

The case of a nonconvex budget set emphasizes the importance of knowledge about the underlying utility function. This type of nonconvexity is encountered not only in the negative income tax but also in other earnings-related taxes or subsidies—for example, child-care payments, social security payments for individuals from 62 to 72 years old, AFDC, and food-stamp subsidies. Knowledge about the form of the utility function would permit estimation within the context of equation (2), although determination of the utility-maximizing point would be more complicated than in the previous case due to the nonconvexity of the budget set.

In this section we have specified the complications in the theory of labor supply which arise when individuals face nonlinear budget sets due to government tax and subsidy programs where the marginal tax rate is determined by earnings. The main problem for econometric purposes is the multiplicity of net wage rates which the individual faces in deciding on his labor supply. We have emphasized how knowledge of the utility function, up to its unknown parameters, would help to solve the problem. Yet all our empirical knowledge arises from observing the uncompensated labor-supply function of equation (1), since utility is never observed. In the next section we show how knowledge of the uncompensated labor-supply function can be used to derive knowledge of the utility function for purposes of econometric estimation.

II. Derivation of the Indirect Utility Function

Nonconvexity of the budget set requires specification of a parametric form for the indifference curves to determine the set of points which cannot correspond to utility-maximizing behavior. In the case of both convex and nonconvex budget sets, a complete model of consumer behavior requires knowledge of the indifference curve to determine the appropriate prices that the consumer faces. Two possible approaches to the problem are apparent. The direct approach is to specify a form of the direct or indirect utility function and then to derive the consumer-demand equations. For example, a Cobb-Douglas utility function could be specified leading to a leisure-demand (labor-supply) equation which can be estimated. This approach, taken by Burtless (1976), places strong restrictions on the labor-supply elasticities.⁶ Less restrictive utility specifications such as the second-order flexible-form utility functions outlined by Diewert (1974) might also be used. However, the flexible-form specifications lead to complicated labor-supply equations which would be extremely difficult to estimate given a nonlinear budget set.

A second approach, which we will use here, arises from the theory of duality. In the context of consumer-demand theory, Roy (1947) did pioneering research, including the derivation of the identity relating consumer demand to the indirect utility function.⁷ Define the consumers utility-maximization problem as maximizing a utility function $u(x)$ where $x = (x_1, \dots, x_N)$, an N -dimensional utility function, subject to the budget constraint $p \cdot x \leq y$, where $p = (p_1, \dots, p_N)$ is the vector of prices and y is the individual's income. Then the indirect utility function relates the maximum utility the consumer can attain, as a function of the exogenous variables p and y .⁸ The function is determined by the solution to the utility-maximization problem

$$v(p, y) \equiv \max_x [u(x) : p \cdot x \leq y]. \quad (3)$$

Because utility is an unobserved variable, estimation can take place only by observing consumer demand. Here Roy's Identity simplifies matters since it relates consumer demand to the indirect utility function by the formula

$$x_i \equiv - \frac{\partial v(p, y)}{\partial p_i} \bigg/ \frac{\partial v(p, y)}{\partial y}, \quad i = 1, \dots, N. \quad (4)$$

⁶ Since the first draft of this paper was completed, we have found Wales and Woodland (1977) using a CES utility function. However, they do not permit variation in preferences in the population as Burtless does.

⁷ Hotelling, Wold, Samuelson, and Houthakker all made important contributions to the use of duality in consumer-demand theory. Other contributions and a review of the theory are found in Diewert (1974).

⁸ For the present section we are assuming linear prices and a straight-line budget set.

We propose to use Roy's Identity "in reverse." That is, considerable empirical knowledge has been built up about labor supply together with the functional forms useful in estimating it. In fact, all our knowledge about the specific form of utility functions, both direct and indirect, must arise from observations of consumer demand. Thus our approach is to integrate Roy's Identity to derive the form of the indirect utility function rather than to specify the utility function a priori. The resulting indirect utility function will be consistent with consumer theory since it is derived using only the assumptions of utility maximization and consistent with the data to the extent that the specified labor-supply function is supported by observed behavior.⁹

Integration of equation (4) over all N goods raises the integrability problem that the function obtained must satisfy restrictions on the Slutsky matrix: rank $N - 1$, symmetry, and negative semidefiniteness. This integration would be difficult and also fruitless since we do not have observations over consumer demand for all goods. Instead, we simplify to a two-good case where the goods are labor supply and expenditure on all other goods. This two-good approach has been (implicitly) taken in almost all studies of labor supply. In order to aggregate $N - 1$ goods, justifications may be offered. A possible justification arises from assuming homogeneous weak separability of preferences between labor supply and the other $N - 1$ goods. This approach, due to Leontief, seems a reasonable assumption since labor supply differs so much from other consumption goods. It also permits the two-good approach to be applied to cross-section data where individuals do not face approximately the same prices.

Given a two-good model, the integrability problem dissolves. One diagonal element of the 2×2 Slutsky matrix determines the other three elements, since all expenditures not made on labor supply (leisure demand) must go to the remaining good. Thus the only requirements imposed by the theory of consumer demand are that the compensated labor-supply derivative with respect to the wage be greater than or equal to zero and that the indirect utility function be monotone nondecreasing in the wage and in nonlabor income.¹⁰ Once we have estimated the unknown parameters in the indirect utility function we have obtained all the observable information possible about the consumer's indifference map. This information will, however, be sufficient to estimate the labor-supply effects of other government tax and subsidy programs.

To obtain the indirect utility function we need to specify a model of labor supply. We use the constant elasticity specification because it has

⁹ Rosen (1974) emphasizes the importance of using the observed consumer behavior to achieve a proper specification of the utility function.

¹⁰ The compensated derivative is positive, not negative, since work is supplied while leisure is demanded. These requirements correspond to the quasi-convexity and monotonicity properties of indirect utility functions. For further details see Diewert (1974, pp. 120–33).

been successfully used in other labor-supply investigations (Hausman and Wise 1976, Lillard 1977) and leads to a convenient indirect utility function. The labor-supply function is

$$h = kw^\alpha y^\beta, \quad (5)$$

where h is hours worked over the appropriate period, k is a constant determined by individual characteristics, w is the net wage, and y is nonlabor income.¹¹ Using Roy's Identity of equation (5) with the insertion of a negative sign since labor supply is a "bad,"

$$-kw^\alpha y^\beta = - \frac{\partial v(w, y)}{\partial w} \bigg/ \frac{\partial v(w, y)}{\partial y}. \quad (6)$$

To derive the indirect utility function we use the implicit function theorem

$$kw^\alpha dw = -y^{-\beta} dy. \quad (7)$$

Then integrating both sides using the separability of the differential equation where c is the constant of integration,¹²

$$k \frac{w^{1+\alpha}}{1+\alpha} = - \frac{y^{1-\beta}}{1-\beta} + c. \quad (8)$$

We choose the constant of integration c as our cardinal measure of utility, and rearranging terms leads to the indirect utility function¹³

$$c = v(w, y) = k \frac{w^{1+\alpha}}{1+\alpha} + \frac{y^{1-\beta}}{1-\beta}. \quad (9)$$

Equation (10) is thus the indirect utility function that corresponds to the constant elasticity labor-supply function. The monotonicity properties are satisfied if utility is nondecreasing in w and y .¹⁴ To derive the Slutsky matrix restriction on the indirect utility function, we use the Slutsky equation

$$\frac{\partial h}{\partial w} = s_{ww} + h \frac{\partial h}{\partial y}, \quad (10)$$

¹¹ Wage and income are both divided by the consumer-goods price deflator so we take the composite price of other goods as numeraire.

¹² Separability of the labor-supply function in the wage and nonlabor income is the crucial simplification which permits this approach. A more general specification is $h = kr(w)s(y)$. Note that the integration is only done locally over the range of the observed data so that boundary conditions can be ignored.

¹³ In principle the direct utility function may be derived from the indirect utility function by a constrained minimization problem. For the constant elasticity specification a closed form does not exist. However, nowhere is the direct utility function needed, since labor supply and the effects of alternative tax and subsidy programs can be calculated solely from the indirect utility function and the expenditure function.

¹⁴ These properties are satisfied globally with the limiting case of $\alpha = -1.0$ and $\beta = +1.0$, corresponding to the Cobb-Douglas specification.

where s_{ww} is the compensated wage derivative. Upon taking derivatives and simplifying,

$$s_{ww} = \frac{1}{h} \left(\frac{\alpha}{w} - \frac{\beta h}{y} \right). \quad (11)$$

Then the Slutsky restriction $s_{ww} > 0$ implies $\alpha > \beta h w / y$. Taking the expected case of $\beta < 0$,

$$\frac{k w^{1+\alpha}}{y^{1-\beta}} > \frac{\alpha}{\beta}. \quad (12)$$

This quasi-convexity condition is automatically satisfied if $\alpha \geq 0$, as we would expect for a sample of low-income males since $k > 0$. If $\alpha < 0$, then we would need to check the observations in the sample to make certain that inequality (12) holds.

In this section we have derived the indirect utility function corresponding to an ordinary model of labor supply using Roy's Identity. The sign restrictions to insure monotonicity and quasi convexity of the indirect utility function have also been derived. However, these derivations were based on the classical assumptions of linear prices and a straight-line budget constraint. Yet almost all government income tax and income transfer programs create nonlinear prices, leading to convex or nonconvex budget sets. In the next section we demonstrate how to calculate labor supply in the more complicated case using the derived indirect utility function.

III. Nonlinear Budget Sets and Stochastic Specification

A consumer is assumed to select his most preferred level of hours, H^* , on the basis of a set of preference parameters $\theta = (k, \alpha, \beta)$. Given values of these parameters, a worker facing a convex budget set of the type pictured in figure 2 or a nonconvex budget set of the type in figure 3 will choose a certain level of labor supply, and this choice may be calculated by using the indirect utility function. Note that direct use of the uncompensated labor-supply function is impossible due to the multiplicity of net wages faced by the consumer. In the case of a nonconvex budget set we must also ascertain the range of the interval around the break-even point that can never contain a utility-maximizing choice. Once the utility-maximizing problem is solved for a single individual, we proceed to a statistical specification that permits differences among individuals to be reflected in differences in the parameters of their indirect utility functions. Given the significant variation observed in hours worked for observationally equivalent individuals, it seems inappropriate to simply add an additive stochastic

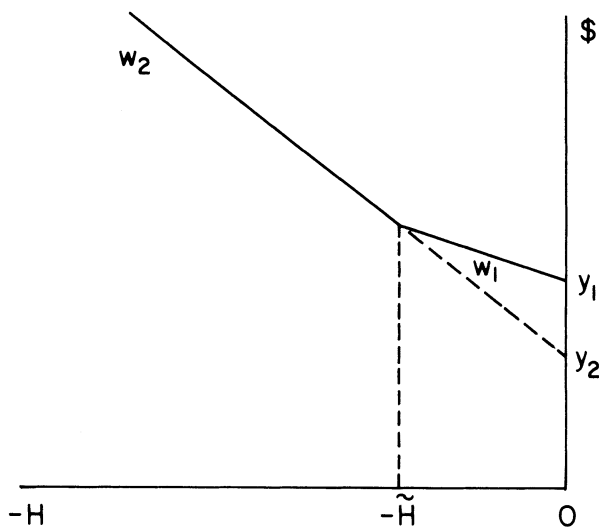


FIG. 4

disturbance to the estimated labor-supply function. Allowing for a distribution of preferences for work in the population is therefore an important component of our model of labor supply.

To demonstrate how preferred hours of work, H^* , are derived given an indirect utility function with parameters θ when an individual is faced with a nonlinear budget set, consider figure 4. The first budget segment is described by the slope, representing net wage w_1 , and the intercept y_1 , representing nonlabor income at zero hours of work. Similarly, the second budget segment is described by net wage w_2 and by virtual nonwage income y_2 . For a given set of preference parameters the maximum indirect utility corresponding to the net wage w_1 and nonlabor income y_1 is calculated as $v_1(w_1, y_1)$. The preferred hours of work, $h_1(w_1, y_1)$, may be greater than the break-even point \tilde{H} , and thus the apparent utility maximum point may lie below the second budget segment in the interior of the budget set. Likewise, we calculate the maximum indirect utility, $v_2(w_2, y_2)$, and preferred hours of work, $h_2(w_2, y_2)$, corresponding to the net wage w_2 and virtual income y_2 . Then the maximum maximum, $v^*(w_1, w_2, y_1, y_2)$, which equals the greater of v_1 or v_2 , determines the global maximum of hours of work, $H^*(w_1, w_2, y_1, y_2)$. Since the individual is better off on the boundary than he is in the interior for any number of hours worked, H^* will always lie along the boundary of the budget set. Figure 2 is treated in a similar manner. We restrict attention to the first two budget segments. Preferred hours, h_1 and h_2 , are calculated in the manner described above. Note that h_1 and h_2 cannot both be feasible. If either h_1 or h_2 is feasible,

the feasible choice is optimal, since it has greater utility than the most preferred attainable point along the other budget segment, which is the kink point, H_1 . Alternatively, if neither h_1 nor h_2 is feasible, the preferred amount of labor supply corresponds to the kink point H_1 . Extension to an indefinite number of budget segments follows by making similar comparisons of maximized utility corresponding to the net wage and virtual income of the different segments of the budget set.

Since the goal of empirical work is to estimate the unknown parameters of the uncompensated labor-supply function, we now specify a stochastic theory of labor-supply variation in a cross-section of individuals. Indexing individuals by i , we expect random differences to occur between observed hours supplied, H_i , and preferred hours of work, H_i^* . This random variation may be the result of measurement error, but a more important source of randomness arises because of unexpected variations in hours worked. Unexpected temporary layoffs, involuntary overtime, or short time due to cyclical downturns all provide potential reasons actual hours may diverge from "normal" hours associated with a given job. These variations in hours are unanticipated by the individual and cause his actual hours H_i to differ from his preferred hours H_i^* . As an empirical matter these sources of randomness turn out to explain only a very small fraction of the total variation of hours worked compared with variation in preferences among the population which we now specify.

Another form of randomness in the data occurs because of individual variation in tastes. Two individuals with the same personal characteristics who face the same budget sets may prefer to work substantially different amounts. From a policy standpoint these individual differences are very important in determining the response to alterations in the budget set induced by government programs. In estimating the unknown parameters k , α , and β , all may be specified to be functions of measurable and unmeasurable individual differences. However, this very general specification leads to an intractable estimation problem. In our empirical work a number of specifications were attempted using an instrumental variable estimator for the uncompensated labor-supply function. This experimentation suggested that k_i may best be treated as a function of measurable individual differences, while α_i and β_i are apparently independent of differences in measured personal characteristics. The actual specification used in this paper is presented, however, with the caveat that significant additional research is needed.

The constant term in the labor-supply function of equation (6) is therefore specified to be a function of measured individual differences. Since we have assumed that there will be random variation in uncompensated labor supply due to random divergences between actual hours H_i and preferred hours H_i^* , it will be convenient to subsume this random disturbance in our specification of the constant term. Thus,

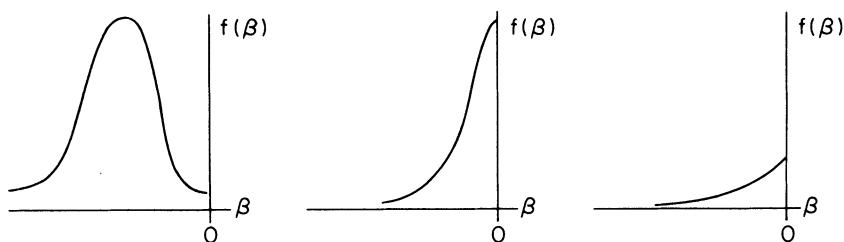


FIG. 5

$$k_i = \exp(\zeta_i \delta + \varepsilon_{2i}), \quad (13)$$

where ζ_i is a vector of individual characteristics and ε_{2i} is assumed to be distributed $N(0, \sigma_2^2)$. The two other individual parameters α_i and β_i may both be expected to vary in the population. Technically both distributions are identified so that, given ideal data, the parameters of both distributions can be estimated. As a practical matter, estimation currently seems limited to allowing one of the two parameters to vary. Thus we decided to permit either α_i or β_i , but not both, to vary in the population. Empirical results led us to specify $\alpha_i = \bar{\alpha}$ a constant and to specify β_i as a random variable in the population.

The integrability conditions discussed in Section II impose restrictions on the possible distribution of the β_i . The wage elasticity $\bar{\alpha}$ is expected to be nonnegative in a sample of low-income workers. In addition, there is a strong expectation that the income elasticity is nonpositive, which implies that the integrability inequality of equation (12) is satisfied globally. A convenient distribution which imposes the negativity restriction on β_i is the truncated normal with the truncation point at zero. As figure 5 shows, a wide variety of shapes of probability densities can be accommodated with this specification. The individual parameter β_i can then be written as $\beta_i = \mu_\beta + \varepsilon_{1i}$ where $\varepsilon_{1i} \sim TN(0, \sigma_1^2)$ with a truncation point from above of $-\mu_\beta$. We assume that ε_{1i} and ε_{2i} are independent sources of random variation. Given this stochastic specification, the unknown parameters of the model are $\theta = (\delta, \bar{\alpha}, \bar{\beta}, \sigma_1^2, \sigma_2^2)$, where $\bar{\beta}$ is the mean of the truncated distribution. We now use this stochastic specification to derive the likelihood function for a sample of observations.

The analysis is confined to budget constraints with only two linear segments, although generalization to more segments is straightforward. A control observation faces a convex budget set while an experimental one faces a nonconvex budget set of the type drawn in figure 4. The probability of the point actually observed, H_i , depends on the unknown parameters δ and α and the densities for β_i and ε_{2i} . Neglecting ε_{2i} momentarily, let us calculate the probability that a particular point, H , is the global maximum. For large negative values of β_i the individual will have a global maximum

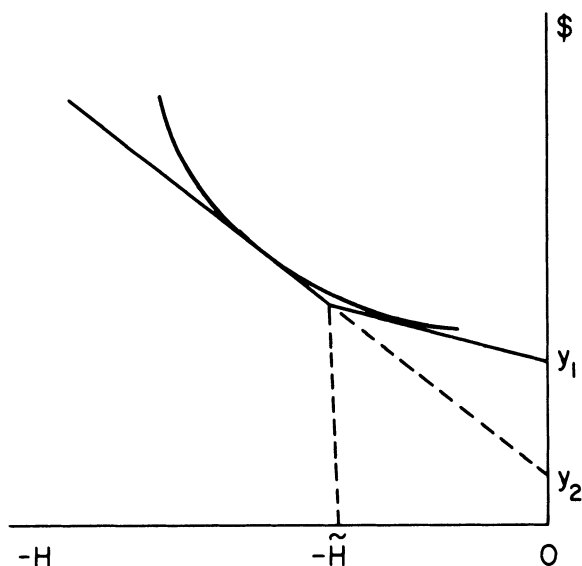


FIG. 6

on the first segment; H^* , the global maximum, will be less than the break-even point, \tilde{H} , and the net wage on the margin will be w_1 with associated virtual income y_1 . As β_i increases toward zero the global maximum point moves along the first segment toward \tilde{H} , until a critical β_i is reached at which the individual is indifferent between a solution on the first segment and a solution on the second. This critical β_i , say β_i^* , depends on the underlying utility function and on the unknown parameters in that function. Using equation (10), β_i^* is calculated quite easily by solving the following equation:

$$\frac{e^{Z_i \delta} w_{1i}^{1+\bar{\alpha}}}{1+\bar{\alpha}} + \frac{y_{1i}^{1-\beta_i^*}}{1-\beta_i^*} = \frac{e^{Z_i \delta} w_{2i}}{1+\bar{\alpha}} + \frac{y_{2i}^{1-\beta_i^*}}{1-\beta_i^*}. \quad (14)$$

For every experimental observation, this equation must be solved for β_i^* each time the parameters change; however, the solution may be cheaply obtained on a computer. The indifference curve corresponding to β_i^* is drawn in figure 6, which illustrates that the break-even point \tilde{H} cannot correspond to a utility maximizing point. As β_i rises from β_i^* to its limiting value, the global maximum moves upward along the second segment. Each value of β_i between $-\infty$ and zero, therefore, has an associated global maximum level of hours; the probability that a particular level of hours is a global maximum is the same as the probability of the associated income elasticity, β_i . We may now extend the analysis to observed hours of work by noting the relationship between observed hours, H_i , and desired hours of work effort, H_i^* ,

$$\log H_i = \log H_i^* + \varepsilon_{2i}. \quad (15)$$

For any particular H_i , there are an infinite number of combinations of H_i^* and ε_{2i} that satisfy (15). By successively determining, for every possible H_i^* , the probability that the global maximum is H_i^* and the stochastic term ε_{2i} equals the difference between the logs of H_i^* and H_i , we can ascertain the probability that actual hours, H_i , will be observed. Letting $f(\beta)$ be the truncated normal density with associated distribution $F(\beta)$ and $\phi(\cdot)$ and $\Phi(\cdot)$ the standard normal density and distribution, respectively, the probability of observing H_i is

$$PNC_i = \int_{-\infty}^{\beta_i^*} \frac{1}{\sigma_2} \phi\left(\frac{\log H_i - \log H_{1i}^*}{\sigma_2}\right) f(\beta) d\beta \\ + \int_{\beta_i^*}^0 \frac{1}{\sigma_2} \phi\left(\frac{\log H_i - \log H_{2i}^*}{\sigma_2}\right) f(\beta) d\beta, \quad (16)$$

where $\log H_{ij}^* = \zeta_i \delta + \bar{\alpha} \log w_{ij} + \beta \log y_{ij}$ and j is an index of the budget segment. Evaluation of these integrals is equivalent to evaluation of a normal distribution $\Phi(z)$ and is thus inexpensive to perform on a computer.¹⁵ The truncated density for β poses no problem since it is a normal density divided by a standard normal distribution which remains constant across all observations.

The case of a convex budget set is slightly different because there exists a range for β , say BL_i to BU_i , for which the utility maximizing point is at the kink point \tilde{H} in figure 7. If the individual's β_i is considerably less than zero, the global maximum H_i^* will be on the first segment with associated net wage w_1 and nonlabor income y_1 . If his income elasticity is quite near zero, the utility maximum will lie along the second segment corresponding to net wage w_2 and virtual and nonlabor income y_2 . The range of β_i that places the utility maximum at the kink point is easily computed from the uncompensated labor-supply function. The lower point of the range, BL_i , is the greatest β_i on the first budget segment that leads to a utility maximum at the kink point,

$$BL_i = \frac{\log \tilde{H}_i - \zeta_i \delta - \bar{\alpha} \log w_{1i}}{\log y_{1i}}. \quad (17)$$

Correspondingly, the upper point of the range, BU_i , is the smallest β_i consistent with a global maximum on the second segment, and therefore

$$BU_i = \frac{\log \tilde{H}_i - \zeta_i \delta - \bar{\alpha} \log w_{2i}}{\log y_{2i}}. \quad (18)$$

All β_i 's that lie between BL_i and BU_i thus lead to a utility maximum at the kink point, \tilde{H}_i . The probability that the observed level of hours H_i corresponds to a utility maximum at the kink point is

¹⁵ See the Appendix for derivation of the evaluation procedure for the integrals.

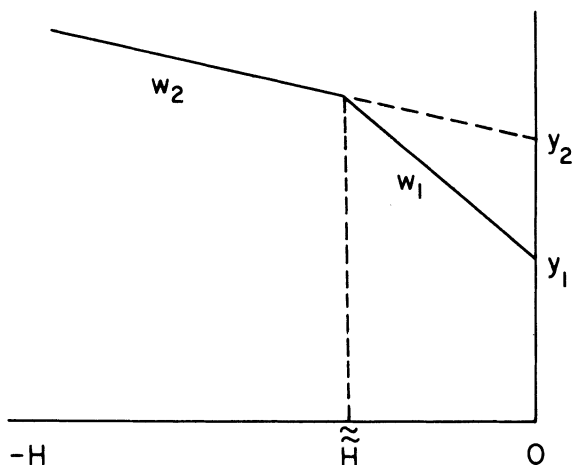


FIG. 7

$$\begin{aligned}
 pr(\log H_i^* + \log \varepsilon_{2i} = \log H_i \mid \log H_i^* = \log \tilde{H}_i) &= pr(\log H_i^* = \log \tilde{H}_i) \\
 &= \frac{1}{\sigma_2} \phi \left(\frac{\log H_i - \log \tilde{H}_i}{\sigma_2} \right) \int_{BL_i}^{BU_i} f(\beta) d\beta.
 \end{aligned} \quad (19)$$

For observed hours of work H_i corresponding to β_i 's outside the range BL_i to BU_i , the probabilities are similar to those calculated in equation (16). Thus for the case of a convex budget set the probability of observing actual hours worked H_i is

$$\begin{aligned}
 PC_i &= \int_{-\infty}^{BL_i} \frac{1}{\sigma_2} \phi \left(\frac{\log H_i - \log H_{1i}^*}{\sigma_2} \right) f(\beta) d\beta \\
 &+ \frac{1}{\sigma_2} \phi \left(\frac{\log H_i - \log \tilde{H}_i}{\sigma_2} \right) [F(BU_i) - F(BL_i)] \\
 &+ \int_{BU_i}^0 \frac{1}{\sigma_2} \phi \left(\frac{\log H_i - \log H_{2i}^*}{\sigma_2} \right) f(\beta) d\beta.
 \end{aligned} \quad (20)$$

Given our stochastic specification of the model, we are able to specify the probability of observing actual hours worked as a function of the unknown parameter values. The natural method of estimation is then maximum-likelihood estimation, in which the unknown parameter values are chosen so as to maximize the probability of observing the sample. Our method can be extended, in principle, to cover the case of an arbitrarily large number of budget segments per individual, although this extension will not be undertaken here. In the next section we apply our methodology to evaluate the results of the Gary income maintenance experiment.

IV. Calculation of Budget Sets, Data, and Sample Considerations

We use the labor-supply model to estimate a structural model of labor supply for adult married males who participated in the Gary income maintenance experiment. The experiment, which took place from 1971 to 1974, had as participants residents of low-income neighborhoods in Gary, Indiana. All participants were black. The families were not chosen at random for the experiment, a problem which we will discuss later in this section. Participants in the Gary experiment were randomly assigned to one of four NIT plans or to control status. (Control families received no benefits except a small payment for their continued participation.) Each of the NIT plans can be described in terms of two parameters: the constant marginal tax rate and the basic support level. In two of the plans, wage and nonwage income was subject to a 40 percent tax rate; in the remaining two, income was taxed at a 60 percent rate. Two of the Gary NIT plans offered basic income supports, scaled according to family size, that were equal to slightly more than the poverty level. The other two plans offered basic supports, also scaled to family size, that were one-quarter less. All federal, state, and FICA income tax liabilities were fully reimbursed for income up to the break-even point \tilde{H} . Earned income above the break-even point was taxed according to the federal, state, and FICA tax tables.

Thus for individuals eligible for NIT payments, the intercept y_1 in figure 4 equals the NIT income guarantee plus net (after tax) nonwage income. The slope of the first budget segment, w_1 , is determined by the worker's gross wage rate times one minus the experimental NIT tax rate. For the second segment of the NIT budget line, the virtual income intercept, y_2 , and net wage, w_2 , are calculated in the same manner as the second segment of a control individual's budget set, the calculation of which we now describe.

Control families are assumed to face a budget line with only two linear segments. This assumption results in a substantial simplification in budget lines, since the federal income tax schedule has a large number of kinks at lower levels of taxable income. However, low-income families face only one important kink in this schedule, one which occurs at the point where family exemptions and deductions are equal to countable family income. At that point the federal tax rate rises from 0 to 14 percent. Thereafter, the tax rate changes in relatively small steps. In calculating the budget lines for control individuals, we assume that the marginal tax rate along the first segment equals 5.85 percent for FICA plus the 2 percent Indiana state income tax rate. The second budget segment is calculated on the assumption that workers face an additional 18 percent marginal tax rate because of the federal income tax. The kink point \tilde{H} is calculated by assuming that workers took standard income exemptions and used the low-income tax deduction available in 1973. Nonwage income is assumed

to be nontaxable. The break-even point for individuals eligible for NIT payments \tilde{H} always exceeds the income tax kink point \tilde{H} so that two segments are sufficient to characterize their budget sets.

Data on workers' hours, wages, nonwage income, and personal characteristics were taken from the first, fourth, and seventh of the periodic interviews administered to participants in the experiment during the period of NIT payments. To be included in the sample, workers must have responded to at least two of the three interviews.¹⁶ Since we are interested in long-run labor-supply response, the measure of labor supply is an average of working hours in the three representative weeks during the experiment.¹⁷ Individuals not observed to be working are omitted from the current sample. This feature of the labor-supply model may be modified by specifying and estimating a separate behavioral equation for an individual's participation in the labor force as Heckman (1974*b*) and Hanoch (1976) have done for women. This extension is not undertaken here since the sample consists of prime-age males who are heads of households. The proportion of the sample that does not participate in the labor force in this sample is quite small and, presumably, for a sample of average yearly hours worked the number of nonparticipants would decline even further. Nevertheless, we intend to add the behavioral equation for labor-force participation in future work on average yearly labor supply when such data become available.

A problem referred to in passing is that initial sample selection was not random but was based on current earnings. Thus the possibility of substantial bias exists since hours worked is one component of earnings.¹⁸ Sample truncation did not occur in Gary, but families whose earnings are above 2.4 times the poverty limit were undersampled by a factor of three. Note that the cutoff line was substantially higher in Gary than in New Jersey, so that even if total truncation had occurred the effect on the conditional mean would be less.¹⁹ In estimating the effects of the Gary NIT we used the consistent weighted estimator proposed by Hausman and

¹⁶ Since approximately 35 percent of the individuals dropped out of the experiment with attrition of controls 10 percent higher than attrition for experimentals, a problem of attrition bias might occur. However, Hausman and Wise (1979), in a study of possible attrition bias on this sample, concluded that while it is serious for analysis of variance models, it does not pose a problem for structural models which control for individual characteristics and experimental-design parameters.

¹⁷ A more long-run measure of labor supply is desirable, but such data are not available at the present time.

¹⁸ In fact, in the New Jersey Negative Income Tax Experiment where sample truncation occurred at 1.5 times the poverty limit, Hausman and Wise (1976, 1977*b*) found that the ratio of estimated coefficients rose by a factor as high as 200 percent when sample truncation was accounted for.

¹⁹ Hausman and Wise (1977*c*) propose and estimate a maximum-likelihood estimator which accounts for the nonrandom sample selection. However, they find little indication of bias in an earnings equation.

Wise (1979). Since the results differed only slightly from the nonweighted estimates, in discussing our results we will present only the nonweighted estimates. Apparently the combination of the high cutoff line of 2.4 times the poverty limit and the presence of an undersampled group above the cutoff led to little or no truncation bias.

In this section we have discussed computation of the nonlinear budget set for each individual, pointing out how marginal tax rates are computed. We then discussed the sample used from the Gary income maintenance experiment as well as possible biases resulting from attrition bias, non-labor-force participation, and truncation bias. In the next section we present the specification of the individual intercept k_i in the uncompensated labor-supply function as a function of individual characteristics. We then present and discuss our results, concluding with policy implications and ideas for future research.

V. Results

Both the uncompensated labor-supply function, equation (6), and the associated indirect utility function, equation (10), include the unknown parameters k , α , and β . It will be recalled that α and β are assumed to be independent of individual characteristics, while k is specified to depend on these characteristics through $k_i = \exp(Z_i\delta + \varepsilon_{2i})$. A wide variety of personal characteristics may affect tastes for work; the following were chosen by reference to earlier research on the NIT experiments: (1) constant—equal to one; (2) education—a dummy variable is used for individuals whose educational attainment is less than 9 years; (3) number of adults—number of persons aged 16 or more residing in the household; (4) poor health—a dummy variable is used if the individual reported his health to be “poor in relation to others” and zero otherwise; and (5) age—a variable equal to the age of the respondent minus 45 years was used if this age exceeded 45 years, otherwise the age variable was set to zero. The other unknown structural parameters of the model are the wage elasticity $\bar{\alpha}$ and parameters in the distribution of the income elasticity $f(\beta)$.

The sample consists of 380 individuals assumed to be independent observations. Once the budget lines have been determined for each individual, the unknown parameters can be estimated by the method of maximum likelihood. The log likelihood equals the sum of the logs of the probabilities of actual hours worked by the NIT-eligible individuals, $\log PNC_i$, from equation (18), and the logs of the probabilities of actual hours worked by the control individuals, $\log PC_i$, from equation (20). Thus the log-likelihood function has the form

$$L = \sum_{i=1}^{N_1} \log PNC_i + \sum_{i=1}^{N_2} \log PC_i, \quad (21)$$

TABLE 1
ESTIMATES OF LABOR SUPPLY AND INDIRECT UTILITY FUNCTION

Variable	Parameter Estimates
Constant	3.75043 (.02555)
Primary education01078 (.00558)
Adults (N)03300 (.01272)
Poor health	-.02224 (.00438)
Age	-.00869 (.01347)
Wage elasticity, $\bar{\alpha}$00003 (.01632)
Mean income elasticity, $\bar{\beta}$	-.04768 (.00465)
Variance of β distribution, σ_1^206751 (.00399)
Variance of ε_{2i} , σ_2^200135 (.00022)

NOTE.—Observations (N) = 380; log of the likelihood function = - 196.27. Asymptotic standard errors in parentheses.

where the number of experimentals N_1 and controls N_2 equals 247 and 133, respectively. Approximately 65 percent of the sample was eligible for the NIT. Because of technical reasons discussed in the Appendix, the likelihood function was maximized using the gradient method of Berndt et al. (1974) as well as the no-derivative conjugant gradient method of Powell (1964). Both techniques converged to the same maximum of the likelihood function. A variety of starting points converged to the same optimum, leading us to conclude that we have found the global maximum.

Results are presented in table 1. All the elements of k_i believed to affect tastes for work have the expected effects. Poor health reduced expected labor supply by 2.25 percent while a 60-year-old is expected to work 12 percent less due to his age, other things being equal. Increased family size, however, is related to higher levels of expected work effort, which leads to the conclusion that endogeneity of nonwage income is probably not a serious problem. Moreover, the effect of relatively low levels of educational attainment is in the expected direction, under the assumption that more educated workers have a wider variety of activities to pursue in their non-work time. The estimates of these parameters are relatively precise except for the effect of increased age; all except the coefficient of age are significantly different from zero at the 5 percent level.

The parameter estimates most important to the design of a negative income tax are the ones that measure work response to the level of the income guarantee and to the marginal tax rate. Our first finding is the lack of a perceptible effect on labor supply of variations in the NIT tax rate. The estimated wage elasticity is .00003 and, even at a range of two

standard errors, is less than .04 in magnitude.²⁰ This estimate is well below the Hausman and Wise (1976) estimate for white males in the New Jersey NIT. This finding is consistent, however, with the Hausman and Wise (1977*c*) findings for the same Gary sample. Using a reduced-form earnings specification, they found the labor response to the level of the income guarantee to be much more important than the response to the marginal tax rate. While no direct effect of different marginal tax rates is found here, an indirect effect is present through the effect of taxes on a family's nonwage income. Consider two individuals with identical gross wage rates and nonwage income who are offered identical NIT income guarantees but have different NIT tax rates. They will have substantially different budget sets because the tax rate affects the locus of the break-even point \bar{H} in figure 4. The individual with the lower tax rate is more likely to work less than the break-even level of hours and is therefore likely to respond to a higher level of nonwage income than the individual with the higher NIT tax rate. Nonetheless, the finding of essentially zero wage elasticity leads to the conclusion that the wide variation in after-tax wage rates had little effect on labor supply among the black males in the Gary NIT experiment.

However, the estimate of the income elasticity was found to be quite significantly different from zero. The average income elasticity in the sample is estimated at $-.04768$. The wage elasticity and income elasticity estimates are not too dissimilar from a model estimated using instrumental variables. There, with no truncation of β , the estimate of $\bar{\beta}$ is $-.036$ with an asymptotic standard error of .022, while the wage elasticity $\bar{\alpha}$ is estimated to be .018 with an asymptotic standard error of .042. Neither coefficient estimate differs significantly from the maximum-likelihood estimates. To assess the effect of the income guarantee under the NIT, consider a family of four with one other adult present, with the worker in good health, under 45 years of age, and with a ninth-grade education (all near the mean of the sample). Using our estimates, the worker's expected preferred hours of work at a pretax wage of \$3.50 per hour with the (convex) income tax budget set are 39.943 hours per week.²¹ For comparison with a NIT plan, we assign an income guarantee of \$3,500 and a marginal tax rate of 60 percent. Preferred hours of work fall to 36.985, a change of 2.958 hours per week or 7.69 percent.²² Weekly income under the NIT rises to \$119.07 from after-tax income of \$115.54 without the NIT. Thus a significant

²⁰ Note that the integrability condition of eq. (12) is satisfied in the sample. When the distribution $f(\beta)$ was not truncated at zero, the estimate of $\bar{\alpha}$ was .0305, although the estimate was not very precise.

²¹ It is important to take the expectation with respect to $f(\beta)$ rather than at the mean $\bar{\beta}$ due to the skewness of the $f(\beta)$ distribution. Thus we calculate $EH^* = \int H^*(\beta)f(\beta)d\beta$.

²² This estimate is approximately equal to earlier estimates for the total response in the Gary sample by Kehrer et al. (1976) and by Hausman and Wise (1977*b*) who found an average response of 7.97 percent among NIT individuals.

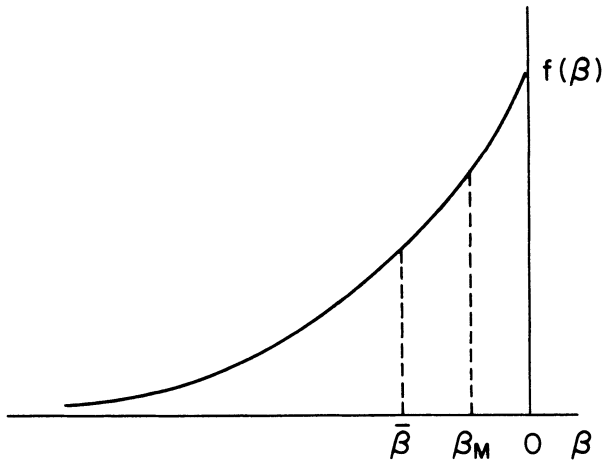


FIG. 8

work response to introduction of a NIT is found to exist, although its magnitude is not especially large. The effects of other NIT plans may be estimated in a similar manner, averaging over different family characteristics to find the average population response. In table 2 we present the expected response in the population for the average characteristics in \mathcal{Z} to the different plans in the Gary experiment. Also included is the probability for each budget set that the individual will be "on the NIT"; that is, he is below the break-even point \tilde{H} and is receiving benefits.

Since a distribution of income elasticities was estimated for the population, it is interesting to consider the estimated density $f(\beta)$. As figure 8 shows, the truncated normal distribution consists of the extreme left tail of a regular normal distribution. Thus while the mean $\bar{\beta} = -0.04768$, the median $\beta_M = -0.03331$, which means a substantial proportion of the population had a very low income elasticity. In fact, about 20 percent of the population is estimated to have an income elasticity of between 0 and 1 percent. Since the variance of ε_{2i} is very small compared with the variance of the β distribution, we can conclude that most of the observed variation in response to the NIT experiment results from differences in individual preferences rather than from random difference between the utility maximum and observed hours of work. Given this conclusion, we can interpret the estimate of the β distribution as suggesting that a small proportion of the Gary sample is substantially more responsive to the presence of an income guarantee in making their labor-supply decision than is the rest of the population. This pattern of response indicates that most individuals will vary their labor supply very little in response to the introduction of NIT plans similar to those used in the Gary experiment. A few individuals, however, will react with large reductions in labor supply. From estimates in the

TABLE 2
LOCATION OF 15 BUDGET LINES IN GARY EXPERIMENT

Financial Plan and Gross Wage/Hr (\$)	w_1 (\$)	w_2 (\$)	y_1 (\$)	y_2 (\$)	Hours at Kink Point	Expected Hours	Change from Control (%)	95% Confidence Range of Expected Hours	Probability of below Break-even Point
Control:									
2.25	2.07	1.67	2.72	27.82	43.16	43.55	...	36.8, 45.38	...
4.25	3.92	3.15	2.72	27.82	22.85	40.37	...	36.8, 45.38	...
6.25	5.76	4.63	2.72	27.82	27.82	40.34	...	36.8, 45.38	...
40% tax/low guarantee:									
2.25	1.35	1.67	78.63	27.82	159.59	38.68	-11.8	34.15, 44.95	1.0
4.25	2.53	3.15	78.63	27.82	81.77	38.68	-4.3	34.15, 44.95	1.0
6.25	3.75	4.63	78.63	27.82	57.45	38.68	-4.2	34.15, 44.95	1.0
60% tax/low guarantee:									
2.25	0.90	1.67	78.09	27.82	65.42	38.69	-11.8	34.17, 44.96	1.0
4.25	1.70	3.15	78.09	27.82	34.63	39.62	-1.9	34.16, 43.48	.21
6.25	2.50	4.63	78.09	27.82	23.55	40.23	-.3	34.16, 45.38	.02
40% tax/high guarantee:									
2.25	1.35	1.67	102.63	27.82	234.97	38.27	-12.9	33.50, 44.85	1.0
4.25	2.53	3.15	102.63	27.82	120.39	38.27	-5.3	33.50, 44.85	1.0
6.25	3.75	4.63	102.63	27.82	84.59	38.27	-5.3	33.50, 44.85	1.0
60% tax/high guarantee:									
2.2590	1.67	102.09	27.82	96.66	38.29	-12.9	33.50, 44.84	1.0
4.25	1.70	3.15	102.09	27.82	51.17	38.29	-5.3	33.50, 44.84	1.0
6.25	2.50	4.63	102.09	27.82	34.86	39.38	-2.4	33.50, 45.38	.23

Gary sample, this responsiveness seems to be the result of increases in non-wage income rather than increases in the marginal tax rate on earned income. A possible explanation of this result is that some individuals take an increased amount of time in between jobs if they have an income guarantee. They do not search and find jobs with higher wages since the wage distribution remains virtually identical for control individuals and NIT individuals. Thus the income effect is much more important than the uncompensated wage effect in determining the response to introduction of a NIT.

VI. Conclusion

Given estimates of the unknown parameters in the uncompensated labor-supply function of equation (6) and the associated indirect utility function of equation (10), we could, in principle, do an applied welfare-economics analysis in designing a negative income tax to maximize various welfare measures subject to a budget constraint. However, since our estimate of α is zero, the derivative of the indirect utility function with respect to changes in the marginal tax rate is simply a constant proportional to the tax change since no direct labor-supply response is expected. Nor is the estimated-income response very high for most individuals. Thus we might more simply conclude that within the range of guarantees and marginal tax rates considered in the Gary NIT experiment the combination of a high guarantee and a high tax rate would lead to the fulfillment of one goal of a negative income tax, which is to provide a basic level of income support at the poverty line without at the same time causing payments to be made to families with relatively high levels of earnings or causing a substantial reduction in population labor supply.

Considerable future research is desirable for estimating the effect on labor supply of government programs which create nonconvexities in the budget set. These programs may induce large distortions on individual economic activity, and the size of this effect is an important consideration in evaluating such programs. The type of model developed here can be extended to cover a wide variety of such situations.

Appendix

Evaluation of the log-likelihood function requires evaluation of the integrals in equation (16) for nonconvex budget sets and equation (20) for convex budget sets. Two types of integrals are present. The more complicated integral has the form

$$\int_{-\infty}^{\beta_i^*} \frac{1}{\sigma_2} \phi \left(\frac{\log H_i - \zeta_i \delta - \alpha \log w_{1i} - \beta \log y_{1i}}{\sigma_2} \right) f(\beta) d\beta. \quad (\text{A1})$$

The truncated normal density has the form

$$f(\beta) = \frac{\phi\left(\frac{\beta - \mu_\beta}{\sigma_\beta}\right)}{\sigma_\beta \left[1 - \Phi\left(\frac{\mu_\beta}{\sigma_\beta}\right)\right]}, \quad (\text{A2})$$

where μ_β and σ_β are the parameters of the corresponding untruncated distribution. The standard normal density in the numerator can be combined with the other normal density in the integral. To combine the two densities, note that without truncation $\log H_i$ is distributed normally with mean $Z_i \delta + \alpha \log w_{1i} + \mu_\beta \log y_{1i}$ and variance $\sigma_\beta^2 (\log y_{1i})^2 + \sigma_2^2$. Now considering the joint distribution of β and $\log H_i$, we write it as

$$f(\beta, \log H_i) = f(\beta | \log H_i) f(\log H_i) \quad (\text{A3})$$

where $f(\cdot)$ stands for the appropriate density. The conditional density $f(\beta | \log H_i)$ (without truncation) is distributed normally with conditional mean $\mu_\beta + (\sigma_\beta^2 \log y_{1i}) / [\sigma_\beta^2 (\log y_{1i})^2 + \sigma_2^2] (\log H_i - Z_i \delta - \alpha \log w_{1i} - \mu_\beta \log y_{1i})$ and conditional variance $\sigma_\beta^2 \sigma_2^2 / [\sigma_\beta^2 (\log y_{1i})^2 + \sigma_2^2]$. Using equations (A3) and (A2) to simplify the integral of equation (A1) and evaluating it yields

$$\begin{aligned} & \frac{1}{[1 - \Phi(\mu_\beta / \sigma_\beta)]} \int_{-\infty}^{\beta_i^*} f(\varepsilon_i | \beta) f(\beta) d\beta \\ &= \frac{1}{[1 - \Phi(\mu_\beta / \sigma_\beta)] \sqrt{\sigma_\beta^2 \tilde{y}^2 + \sigma_2^2}} \phi \left[\frac{(\log H_i - Z_i \delta - \alpha \tilde{w} - \mu_\beta \tilde{y})}{\sqrt{\sigma_\beta^2 \tilde{y}^2 + \sigma_2^2}} \right] \\ & \times \Phi \left[\frac{(\beta_i^* - \mu_\beta) \sqrt{\sigma_\beta^2 \tilde{y}^2 + \sigma_2^2}}{\sigma_\beta \sigma_2} - \frac{\sigma_\beta \tilde{y} (\log H_i - Z_i \delta - \alpha \tilde{w} - \mu_\beta \tilde{y})}{\sigma_2 \sqrt{\sigma_\beta^2 \tilde{y}^2 + \sigma_2^2}} \right], \end{aligned} \quad (\text{A4})$$

where $\tilde{w} = \log w_{1i}$ and $\tilde{y} = \log y_{1i}$. The somewhat formidable expression on the right-hand side of equation (A4) is quite simple to evaluate, requiring evaluation of one normal density and two normal distributions where the distribution in the denominator remains constant across observations. The only other type of integral appears as the middle term in the convex budget set probability of equation (20). It is easily evaluated as

$$\begin{aligned} & \frac{1}{\sigma_2} \phi \left(\frac{\log H_i - \log \tilde{H}_i}{\sigma_2} \right) [F(BU_i) - F(BL_i)] \\ &= \frac{1}{\sigma_2 [1 - \Phi(\mu_\beta / \sigma_\beta)]} \phi \left(\frac{\log H_i - \log \tilde{H}_i}{\sigma_2} \right) \left[\Phi \left(\frac{BU_i - \mu_\beta}{\sigma_\beta} \right) - \Phi \left(\frac{BL_i - \mu_\beta}{\sigma_\beta} \right) \right]. \end{aligned} \quad (\text{A5})$$

Two techniques were used to maximize the likelihood function. Convergence was obtained using the modified scoring method proposed by Berndt et al. (1974). Only first derivatives are required for this algorithm. However, as a check to make certain that the global maximum was achieved, the no-derivative conjugate gradient routine of Powell (1964) was also used to verify the parameter estimates. The reason for this caution is that, while the log-likelihood function of equation (21) is everywhere differentiable in the parameters, the derivatives are not everywhere continuous because of the kink point. While proofs of the usual large sample properties of maximum likelihood were not attempted in this nonregular case, consistency of the estimates would follow from the usual type of proof. However, proof of asymptotic normality of the estimates is complicated by the lack of con-

tinuous derivatives, and the reported asymptotic standard errors should be interpreted with this problem in mind. Starting values for the maximum-likelihood programs were estimated using an instrumental variable technique to predict the net wage and nonlabor income at the sample mean of 35 hours of work.

One last econometric note concerns the question of whether the gross wage should be treated as endogenous. Many studies in the past have treated it as endogenous, but the reasons advanced for the usual specification of a triangular system of a wage equation excluding hours in addition to the hours equation are not present here since we use the appropriate net wages. Previous studies used only one net wage, and since the level of utility-maximizing labor supply is observed with error the single net wage rate is also observed with an error that is correlated with the error between actual and preferred hours. Thus a simultaneous-equations problem existed. Here since all appropriate net wages are observed and used in the labor-supply specification the main cause of the simultaneous-equations problem will not occur. Nevertheless, in our preliminary investigations with the Gary data we did specify and estimate such a system making the gross (market) wage a function of personal characteristics. The results of the joint estimation were similar to those obtained when the gross wage was taken to be exogenous, and a specification error test of Hausman (1978) failed to reject the null hypothesis that the market wage could be treated as exogenous and measured without significant error. Thus simultaneous-equations estimation results are not presented in the paper.

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