Lab notes for *Statistics for Social Sciences II: Multivariate Techniques*

BSc in International Studies, BSc in International Studies & Political Science, Carlos III University of Madrid

Eduardo García Portugués

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Contents

Introduction

Course logistics

- **Lessons**. Lab **INF-15.S.06**.
- **Office hours**. Tuesdays from 15:00 to 18:00 at office 10.0.10 (access through 10.0.7 in *Campomanes* building). If they are incompatible with your schedule, you are welcome to send me an email to get an alternative appointment (preferable) or to drop by my office (but will remove this option if I get overwhelmed with queries…).
- **Grading**. Continuous evaluation accounts for 60% of the final grade (2 partials and a group project, each accounts for 20%). Final exam is 40%.
- **Partials**. Based on theory and practise. The first will (likely) cover Topics 1-2, and the second Topics 3-5.
- **Group project**. Students must team up in groups of 4 (± 1) and apply the statistical analyses seen to a dataset. As a rule of thumb, all students in a group will be graded evenly, so choose wisely your group! You will have time to. Specific details to be disclosed by the end of the course.

What are these notes?

These notes are a compact description of the key points and insights of the methods presented in the lessons throught the help of computer, providing at the same time an effective way of delivering statistical analysis.

These notes **are neither an exhaustive, rigorous nor comprehensive treatment** of the broad statiscal branch know as Multivariate Analysis. They are designed to be

Description of the software required

Available in all computer labs, not only **INF-15.S.06**. We will employ

- `R`, a free software environment for statistical computing and graphics. Install instructions available here.

- `Deducer`, a *free easy to use alternative to proprietary data analysis software* that *provides an intuitive graphical user interface (GUI) for `R`, encouraging non-technical users to learn and perform analyses without programming getting in their way.*

If you plan to use your personal laptop, you are responsible for the right setup of the software (follow the installation links).

0.1 How to install it in your own laptop

0.2 Why `R`? We want this very

Chapter 1

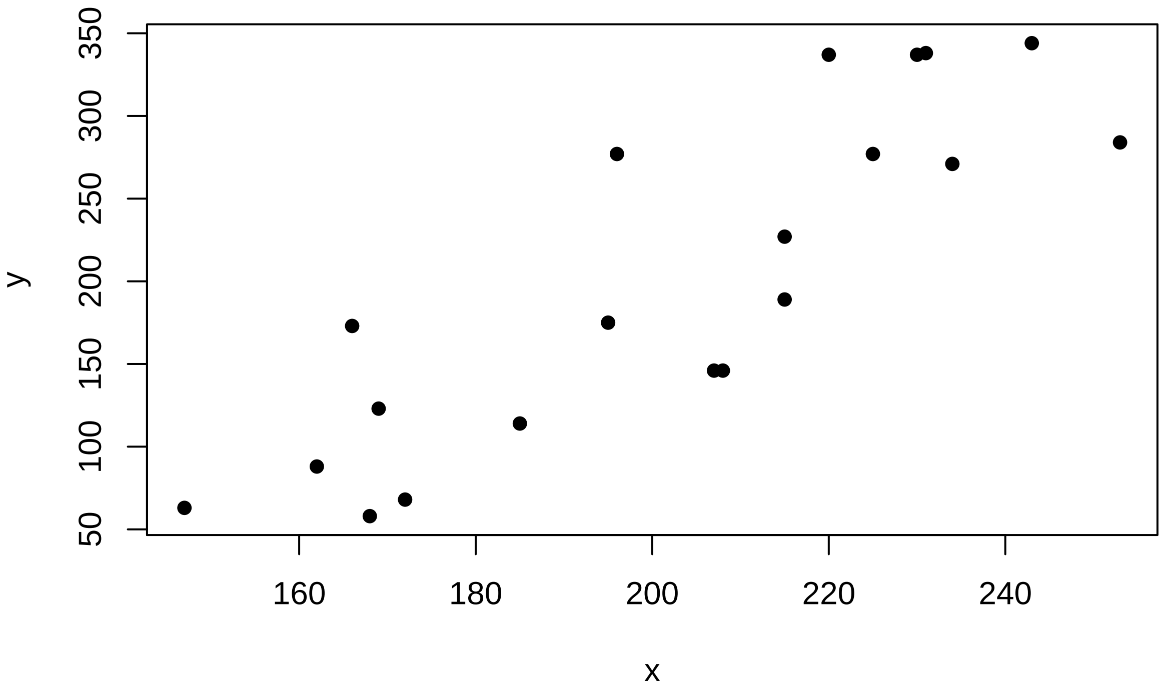
Simple linear regression

1.1 Motivation; model formulation; examples; applications; assumptions

1.1.1 Motivating example

Suppose we have a production line where we measured the time (in minutes) required to produce a number of solicited items. There are two random variables: `y` = "time required to produce an order" and `x` = "number of items in the order". We want to explain/predict `y` from `x` from a linear model.

```
# Production time
y <- c(175, 189, 344, 88, 114, 338, 271, 173, 284, 277, 337, 58, 146, 277, 123, 227, 63, 337, 146, 68)

# Number of units in the order
x <- c(195, 215, 243, 162, 185, 231, 234, 166, 253, 196, 220, 168, 207, 225, 169, 215, 147, 230, 208, 1

# Data scatterplot (pch = 16 for filled points)
plot(y ~ x, pch = 16)
```



1.1.2 The `lm` function

The linear fit in `R` is done by the `lm` function and the formula `y ~ x`, used to denote that we are interested in regressing `y` over `x`.

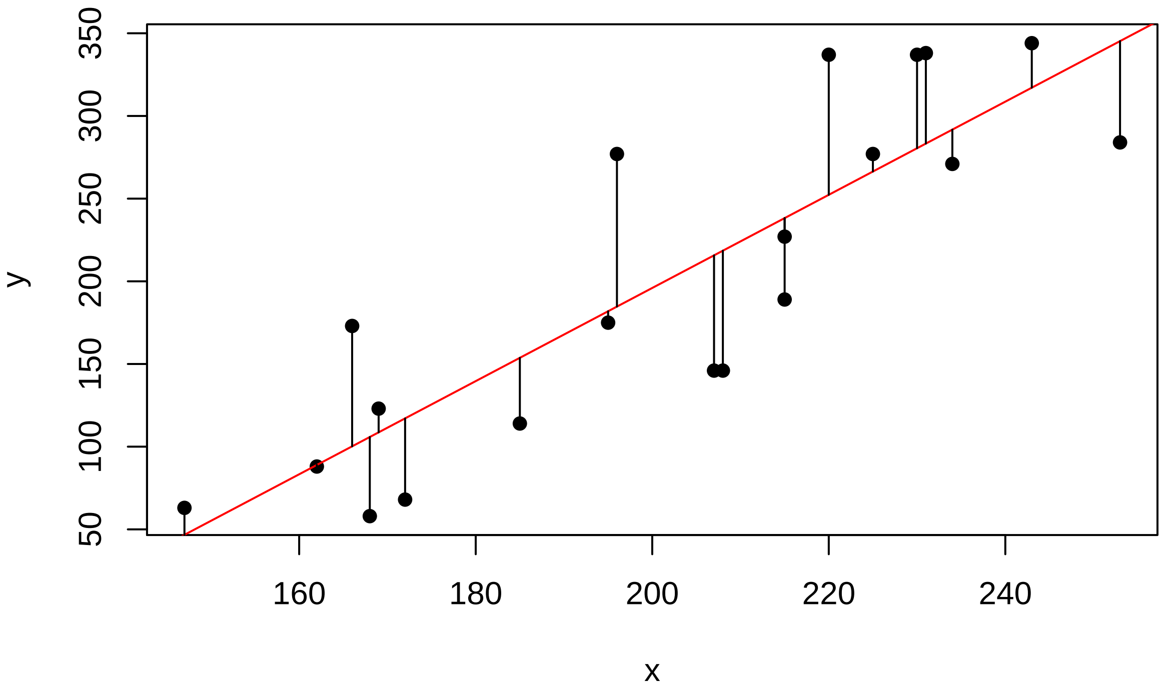
```r
# Fit a linear model y = beta0 + beta1 * x
reg <- lm(y ~ x)

# The result is a list with several objects
names(reg)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

```r
# The fitted coefficients beta0 (intercept) and beta1 (slope)
reg$coefficients
```

```
## (Intercept)           x
## -367.360643    2.816682
```

```r
# The regression line with the minimized distances
plot(y ~ x, pch = 16)
abline(reg, col = 2)
segments(x0 = x, y0 = y, x1 = x, y1 = reg$fitted.values)
```



1.1.3 The least squares estimate

We can check and visualize that `reg$coefficients` indeed contains the least squares estimates and that they minimize the Residual Sum of Squares (RSS).

```
# Create the design matrix
X <- cbind(1, x)

# The analytical solution
betahat <- solve(t(X) %*% X) %*% t(X) %*% y
betahat
```

```
##          [,1]
##   -367.360643
## x    2.816682
```

```
# Minimal RSS
sum((y - X %*% betahat)^2)
```

```
## [1] 51658.57
```

You can try to get a better RSS. Good luck! :)

1.1.4 Summary of the model

The `summary` function applied to a `lm` object gives the fitted coefficients and its significances ("Pr(>|t|)"), the R^2 ("Multiple R-squared") and the fitted error variance ("Residual standard error").

```
# Summary of the fit
summary(reg)
```

```
##
## Call:
```

```
## lm(formula = y ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -72.509 -48.159  -3.917  33.857  92.291
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -367.3606    82.4122  -4.458 0.000304 ***
## x              2.8167     0.4035   6.980 1.61e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 53.57 on 18 degrees of freedom
## Multiple R-squared:  0.7302, Adjusted R-squared:  0.7152
## F-statistic: 48.72 on 1 and 18 DF,  p-value: 1.615e-06
```

1.1.5 Prediction

Prediction of a new observation can be done via the function `predict`, which also provides conficence intervals. The `newdata` argument of `predict` needs a `data.frame`.

```
# Point in which we want a prediction for y
newx <- data.frame(x = 200)

# Prediction with 95% confidence interval
predict(reg, newdata = newx, interval = "prediction", level = 0.95)
```

```
##        fit      lwr      upr
## 1 195.9758 80.63344 311.3182
```

```
# The same prediction
reg$coefficients %*% c(1, 200)
```

```
##          [,1]
## [1,] 195.9758
```

1.1.6 Some words of caution with R^2

R^2 does not measure the correctness of a linear model but the **usefulness assuming the model is correct**.

```
# Fixed design
xf <- seq(0, 1, l = 50)

# Errors with different variance
set.seed(123456)
eps1 <- rnorm(50, sd = 0.1)
eps2 <- rnorm(50, sd = 1)

# Responses generated following a linear model
y1 <- 1 + 2 * xf + eps1
y2 <- 1 + 2 * xf + eps2

# Fits
reg1 <- lm(y1 ~ xf)
```

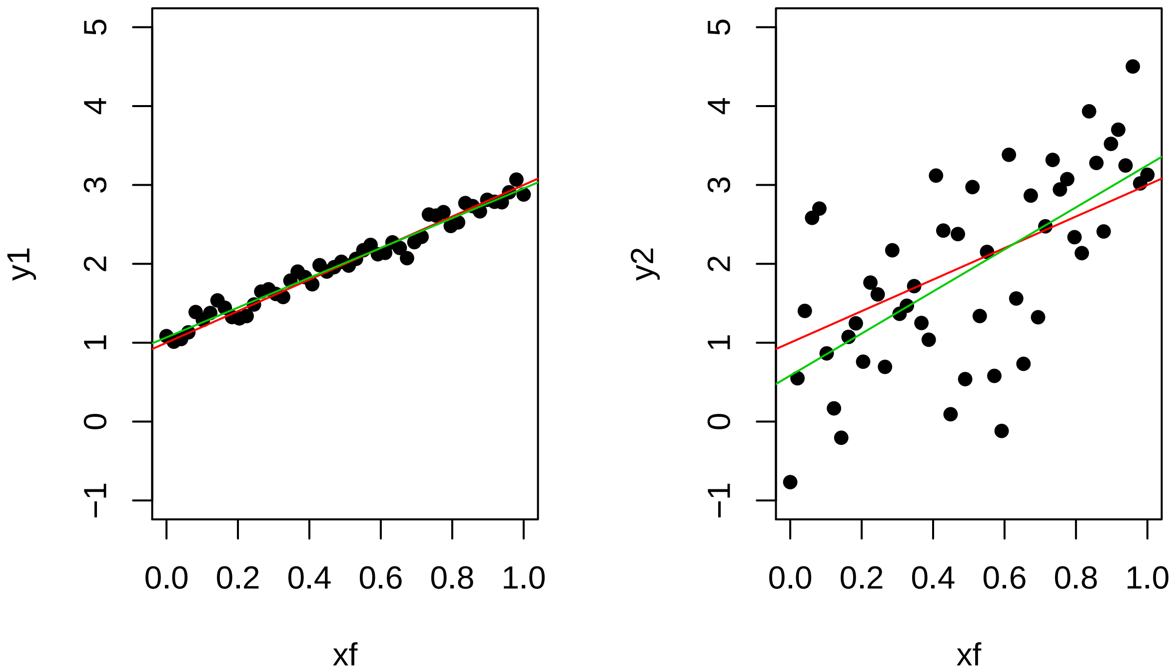
```r
reg2 <- lm(y2 ~ xf)

# R^2 depends on the
summary(reg1)
```

```
##
## Call:
## lm(formula = y1 ~ xf)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.26738 -0.08220 -0.00143  0.06930  0.19892
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.06719    0.02781   38.37   <2e-16 ***
## xf           1.88910    0.04793   39.42   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09981 on 48 degrees of freedom
## Multiple R-squared:   0.97,  Adjusted R-squared:  0.9694
## F-statistic:  1554 on 1 and 48 DF,  p-value: < 2.2e-16
```

```r
summary(reg2)
```

```
##
## Call:
## lm(formula = y2 ~ xf)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.27986 -0.59235  0.03505  0.57115  1.89983
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.5830     0.2595   2.247   0.0293 *
## xf            2.6665     0.4472   5.963 2.86e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9313 on 48 degrees of freedom
## Multiple R-squared:  0.4255, Adjusted R-squared:  0.4136
## F-statistic: 35.55 on 1 and 48 DF,  p-value: 2.858e-07
```

```r
# Plot
par(mfrow = c(1, 2))
plot(y1 ~ xf, pch = 16, ylim = c(-1, 5))
abline(a = 1, b = 2, col = 2)
abline(a = reg1$coefficients[1], b = reg1$coefficients[2], col = 3)
plot(y2 ~ xf, pch = 16, ylim = c(-1, 5))
abline(a = 1, b = 2, col = 2)
abline(a = reg2$coefficients[1], b = reg2$coefficients[2], col = 3)
```



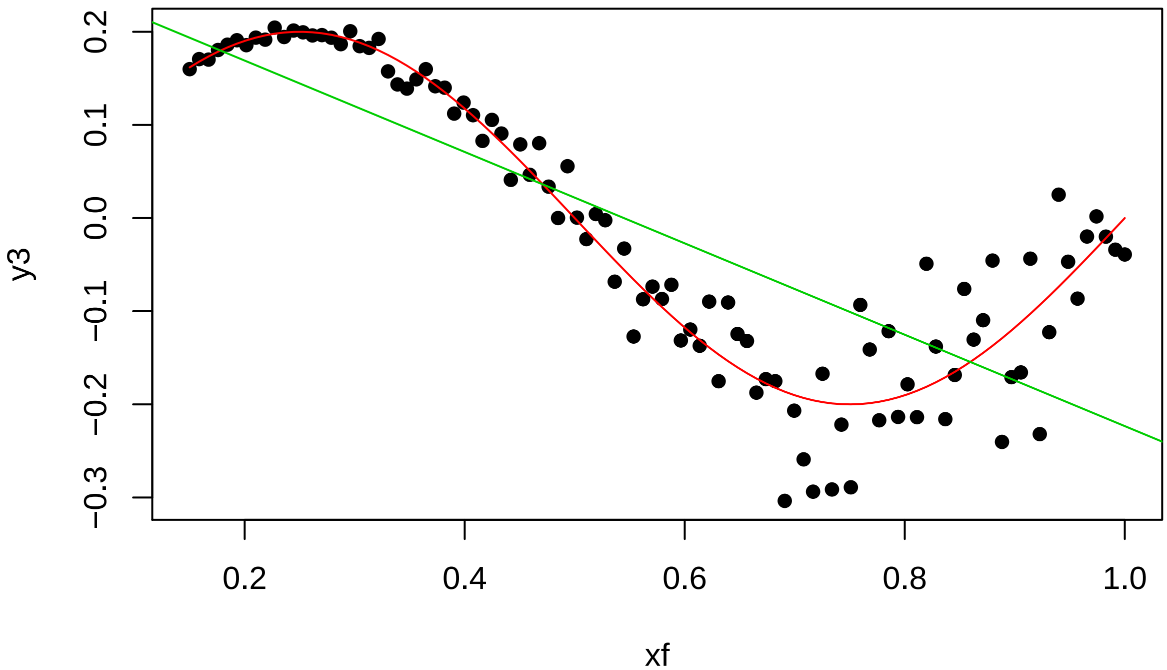
A large R^2 means *nothing* if the **assumptions of the model do not hold**.

```
# Create data that:
# 1) does NOT follow a linear model
# 2) the error is heteroskedastic
xf <- seq(0.15, 1, l = 100)
y3 <- 0.2 * sin(2 * pi * xf) + rnorm(n = 100, sd = 0.1 * xf^2)

# Great R^2!?
reg3 <- lm(y3 ~ xf)
summary(reg3)
```

```
##
## Call:
## lm(formula = y3 ~ xf)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.231885 -0.070384  0.006934  0.056374  0.218962
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.26718    0.02343   11.40   <2e-16 ***
## xf          -0.49051    0.03742  -13.11   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09274 on 98 degrees of freedom
## Multiple R-squared:  0.6368, Adjusted R-squared:  0.6331
## F-statistic: 171.8 on 1 and 98 DF,  p-value: < 2.2e-16
```

```
# But predicting is obviously problematic
plot(y3 ~ xf, pch = 16)
lines(0.2 * sin(2 * pi * xf) ~ xf, col = 2)
```

```r
abline(reg3, col = 3)
```



Simple linear regression

$$Y_i = a + bX_i + \varepsilon, \quad i = 1, \ldots, n.$$

1.2 Estimation of model parameters; least squares; inference for model parameters; forecasting.

1.3 Assessing model fit; ANOVA; model validation; model diagnostics; handling nonlinear relationships.

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter **??**. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter **??**.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```r
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 1.1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 1.1.

```r
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package (Xie, 2016) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015).

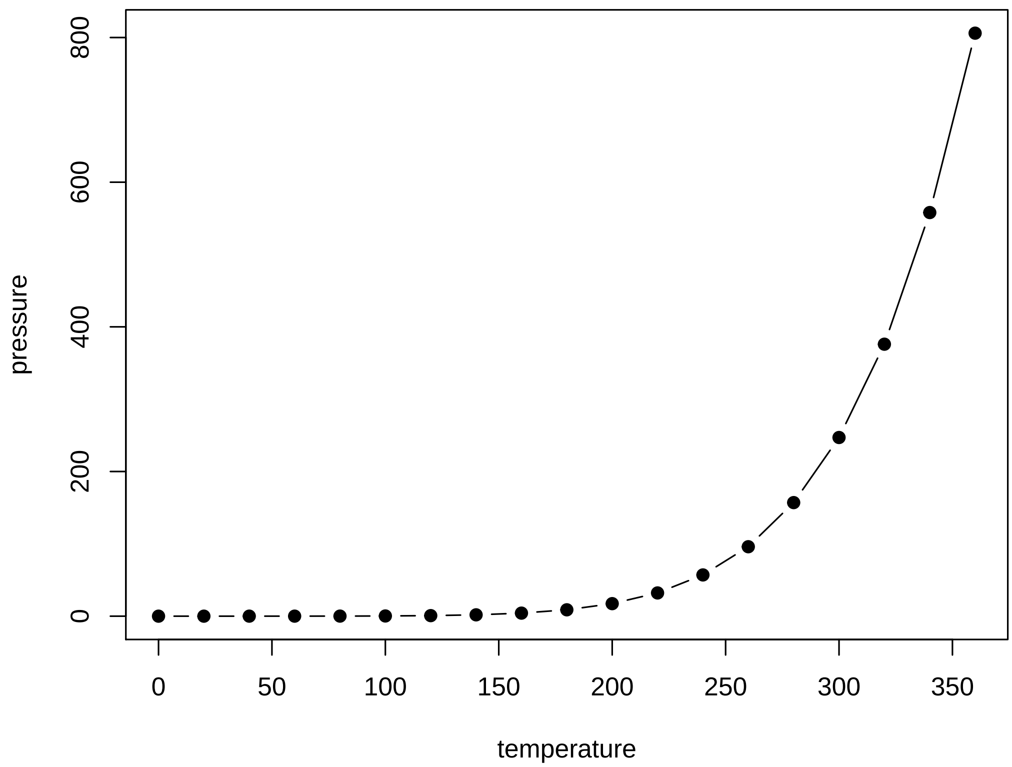


Figure 1.1: Here is a nice figure!

Table 1.1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports, e.g., a math equation $a^2 + b^2 = c^2$.

For now, you have to install the development versions of **bookdown** from Github:

```
devtools::install_github("rstudio/bookdown")
```

Remember each Rmd file contains one and only one chapter, and a chapter is defined by the first-level heading `#`.

To compile this example to PDF, you need to install XeLaTeX.

Chapter 2

Multiple linear regression

2.1 Motivation; model formulation; examples; applications; assumptions.

2.2 Estimation of model parameters; least squares; inference for model parameters; forecasting.

2.3 Assessing model fit; ANOVA; model validation.

2.4 Multicollinearity; model diagnostics.

TODO

Chapter 3

Binomial logistic regression

3.1 Motivation; model formulation; examples; applications; assumptions.

3.2 Parameter estimation; assessing model fit; significance testing; interpreting coefficients.

TODO

Chapter 4

Factor analysis and principal component analysis

4.1 Motivation; formulations; examples; applications; assumptions.

4.2 Principal components analysis; choosing the number of factors; analysis and interpretation.

4.3 Exploratory factor analysis; design, analysis and interpretation; rotation of factors.

TODO

Chapter 5

Cluster analysis

5.1 Motivation; examples; applications; hierarchical agglomerative clustering; dendrogram.

5.2 Choosing the number of clusters; assessing fit; interpretation of clusters.

TODO

Bibliography

Xie, Y. (2015). *Dynamic Documents with R and knitr*. Chapman and Hall/CRC, Boca Raton, Florida, 2nd edition. ISBN 978-1498716963.

Xie, Y. (2016). *bookdown: Authoring Books with R Markdown*. R package version 0.1.6.