Lab session week 18 : Exercises Chapter 8

Exercise 6

- Estimate $\int_0^1 \exp(x^2)dx$

```r
# generate the first 100 data values:
U <- runif(100)
X <- exp(U^2)
sm <- mean(X) # sample mean
sv <- sum((X-sm)^2)/99 # sample variance
```

Exercise 6

```
# generate values until S/sqrt(k) < d:
k <- 0

while(sqrt(sv)/sqrt(k) >= .01 ){
  U <- runif(1)
  X <- append(X, exp(U^2))
  sm <- mean(X)
  sv <- sum((X-sm)^2)/(99+k)
  k <- k+1
}

sm # result
```

```
## [1] 1.47295
```

Exercise 12

```
x <- c(102, 112, 131, 107, 114, 95, 133, 145, 139,
       117, 93, 111, 124, 122, 136, 141, 119, 122,
       151, 143)
X <- 0
S2 <- 0
X[1] <- x[1]
for(i in 2:20){
  X[i] <- X[i-1] + (x[i]-X[i-1])/(i)
  S2[i] <- (1 - 1/(i-1))*S2[i-1] + i*(X[i]-X[i-1])^2
}
S = sqrt(S2)

2.58*S[20]/sqrt(20)
```

```
## [1] 9.692004
```

Exercise 14

```
set.seed(1)
n=2
M=sapply(1:1000,function(x)
  sample(c(1,3),2,replace=TRUE))

samplemean=apply(M,2,function(x) mean(x))
samplevariance=c()

for(i in 1:100){
  samplevariance=append(samplevariance,
      sum(sapply(M[,i], function(x)
        (x-samplemean[i])^2))/(n-1))
  }

var(samplevariance)
```

```
## [1] 1.008485
```

Recursion

Recursion is used when a function need to call for itself. For instance the factorial function

```
factorialrecursive = function(i){
if (i == 0){
  value = 1
}else{
  tmp <- factorialrecursive(i-1)
  value = i*tmp
}
return(value) }
```

Exercises

Define respectively the mean estimator as

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

1. Choose any distribution with mean known and generate a N-sample. With the recursion method, write a program that returns $\bar{X}_n$ according to $n \in \{1, \cdots, N\}$. Use the following relation

$$\bar{X}_n = \bar{X}_{n-1} + \frac{X_n - \bar{X}_{n-1}}{n}, \qquad \bar{X}_0 = 0.$$

Exercises

2. Write a program that returns the mean and variance bootstrap estimates for the median estimators with the uniform distribution. If n is the sample size, compare your results with the true mean and variance functions, respectively

$$mean(n) = \frac{\lceil n/2 \rceil}{n+1}, \qquad var(n) = \frac{\lceil n/2 \rceil (n - \lceil n/2 \rceil + 1)}{(n+1)^2 (n+2)}.$$