Regression,2.4

Afsaneh Mohammadnejad

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Suppose we have these assumptions for a stright-line regression through the origion:

i)Y is related to x by simple linear regression model $Y_i = \beta x_i + e_i$ (i=1,...,n)(i.e,E(Y| X= x_i)=β x_i

ii) The errors e_1,e_2,...,e_n are independent.

iii)The errors have a common variance σ^2

iv)The errors are normally distributed with mean 0 and variance σ^2

Since the regression model is conditional in X we can assume that the values of predictors x_1,x_2,...,x_n are known fixed constants.

Our task is to show the following :

a)The least square estimate of β is: $\hat{\beta}$= $\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$

We know RSS=$\sum_{i=1}^n e_i^2$=$\sum_{i=1}^n (y_i - \hat{y_i})^2$=$\sum_{i=1}^n (y_i - \beta x_i)^2$ in order to

minimize the residual sum of square(RSS) we can derivate it with respect to β so,

$\frac{\partial RSS}{\partial \beta}$=$-2\sum_{i=1}^n (y_i - \hat{\beta x_i})x_i$=0

$\sum_{i=1}^n (y_i x_i - \hat{\beta x_i^2})$=0

Therfore:

$\hat{\beta}$= $\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$

b)Under the above assumption I want to show $E(\hat{\beta}|X = x_i)$=β

From the previous exercise I know $\hat{\beta}$= $\sum_{i=1}^n x_i y_i / \sum_{i=1}^n x_i^2$,so by replacing it in

the formula:

$E(\sum_{i=1}^n x_i y_i / \sum_{i=1}^n x_i^2 | X = x_i)$=$\frac{1}{\sum_{i=1}^n x_i^2}$.$\sum_{i=1}^n$ $x_i.E(Y_i|X = x_i) = \frac{1}{\sum_{i=1}^n x_i^2}. \sum_{i=1}^n x_i(\beta x_i) = \frac{1}{\sum_{i=1}^n x_i^2}$

$.\beta. \sum_{i=1}^n x_i^2 = \beta$

C)I want to show that $var(\hat{\beta}|X = x_i)$=$\frac{\sigma^2}{\sum_{i=1}^n x_i^2}$

Again like the previous exercise we know

$var(\sum_{i=1}^n x_i y_i / \sum_{i=1}^n x_i^2 | X = x_i)$=$\frac{1}{(\sum_{i=1}^n x_i^2)^2}$.$var(\sum_{i=1}^n x_i \ y_i | X = x_i)$

=$\frac{\sum_{i=1}^n x_i^2}{(\sum_{i=1}^n x_i^2)^2}$.$var(y_i|X = x_i)$=$\frac{\sigma^2}{\sum_{i=1}^n x_i^2}$

d) I want to indicate that $\hat{\beta}|X \sim N(\beta, \frac{\sigma^2}{\sum_{i=1}^n x_i^2})$

Under the assumption iv we know the errors are normaly distributed and $Y_i = \beta x_i + e_i$,$Y_i|X$ is normally distributed. since $\hat{\beta}$|X is a linear combination of y_i's ,so$\hat{\beta}$|X is normally distributed.