Time Series Analysis

Ernest Chan, Ph.D.

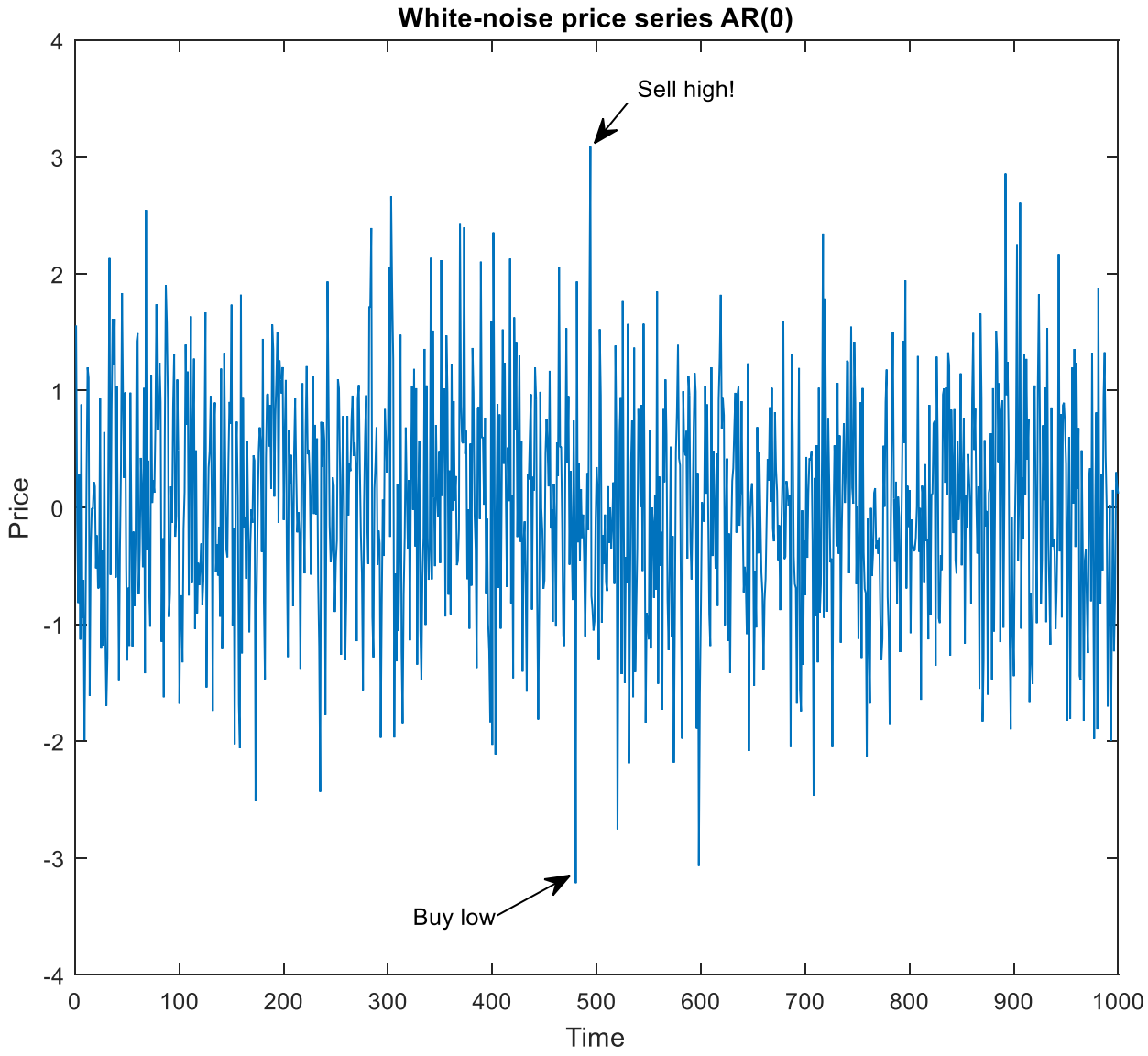
Week 5 - Predict 451

What is a time series model?

- A linear model with
  - independent variables=past prices or returns
  - dependent variable=future price or return
- AR(1): $a_t = \alpha_0 + \alpha_1 a_{t-1} + \epsilon_t$
- AR(p): $a_t = \alpha_0 + \alpha_1 a_{t-1} + \alpha_2 a_{t-2} a_{t-1} + \cdots + \alpha_p a_{t-p} + \epsilon_t$
- ϵ_t is assumed independent of a_{t-1}.
- ϵ_t has mean=0, variance=1.
- ϵ_t may have Gaussian or t-distribution.

White noise

- AR(0) is white noise.
- White noise is stationary.
  - Stationary means
    - E(price)=constant,
    - Var(prices)=constant,
    - Cov(prices, prices)=constant.
- Stationary price series is good for mean-reversion trading!


White-noise price series AR(0)
Sell high!
Buy low
Price
Time
4
3
2
1
0
-1
-2
-3
-4
0
100
200
300
400
500
600
700
800
900
1000

ARMA(p, q)

- AR(p) often requires many lags.
- Add a moving average of noise terms ϵ_t will often reduce number of lags required:

ARMA(p, q):

$$a_t = \alpha_0 + \alpha_1 a_{t-1} + \alpha_2 a_{t-2} a_{t-1} + \cdots + \alpha_p a_{t-p} + \epsilon_t + \beta_1 \epsilon_{t-1} + \cdots + \beta_q \epsilon_{t-q}$$

ARIMA(p, 1, q)

- ARIMA(p, 1, q) on log *prices*

  is identical to

  ARMA(p, q) on log *returns*.
  - Hence it is usually sufficient just to deal with ARMA(p, q) on log returns.
- We seldom (never?) need ARIMA(p, d, q) with $d > 1$.

Training and forecasting

- We train ("fit", "estimate") a model on some train data using MLE with BIC.

  m_train=auto.arima(ret[1:trainset], ic='bic').

- But to test this model, use the previously fitted model, and apply that to test data:

  m_test=Arima(ret[trainset:(trainset+testset-1)], model=m_train)

  forecasts_testset=fitted(m_test)

- Note this is 1-step-ahead forecast, not forecasting N-step-ahead.
  - I.e. we use $a_{t-1}, a_{t-2}, \cdots, a_{t-p}$ to forecast a_t, for every t.

Simulation

- ARIMA model is also useful for simulating price series.
- Simulated price series can be used for testing risk or trading models.
- See my talk at QuantCon last Saturday, or epchan.blogspot.com/2017/11/optimizing-trading-strategies-without.html