Chapter 2

2.1

Suppose,

$X_1 = 1, X_2 = 3, X_3 = 0, X_4 = -2, X_5 = 4, X_6 = -1, X_7 = 5, X_8 = 2, X_9 = 10.$

```
x <- c(1, 3, 0, -2, 4, -1, 5, 2, 10)
```

Find:

a.) $\sum X_i$

```
sum(x)
```

```
[1] 22
```

b.) $\sum_{i=3}^{5} X_i$

```
sum( x[3:5] )
```

```
[1] 2
```

c.) $\sum_{i=1}^{4} X_i^3$

```
sum(x[1:4]^3)
```

```
[1] 20
```

d.) $(\sum X_i)^2$

```
(sum(x))^2
```

```
[1] 484
```

e.) $\sum 3$

```
3 * length(x)
```

```
[1] 27
```

f.) $\sum (X_i - 7)$

```
sum(x - 7)
```

```
[1] -41
```

g.) $3\sum_{i=1}^{5} X_i - \sum_{i=6}^{9} X_i$

```
3 * sum( x[1:5] ) - sum( x[6:9] )
```

```
[1] 2
```

h.) $\sum 10X$

```
sum( 10 * x)
```

```
[1] 220
```

i.) $\sum_{i=2}^{6} iX_i$

```
i <- 2:6
sum( i * x[i]  )
```

```
[1] 12
```

j.) $\sum 6$

```
6 * length(x)
```

```
[1] 54
```

2.2

Express the following in summation notation.

a.) $X_1 + \frac{X_2}{2} + \frac{X_3}{3} + \frac{X_4}{4}$

$\ldots = \frac{X_1}{1} + \frac{X_2}{2} + \frac{X_3}{3} + \frac{X_4}{4}$

$\ldots = \sum{}^4{}_{i=1} \frac{X_i}{i}$

b.) $U_1 + U_2^2 + U_3^3 + U_4^4$

$\ldots = U_1^1 + U_2^2 + U_3^3 + U_4^4$

$\ldots = \sum_{i=1}^{4} U_i^i$

c.) $(Y_1 + Y_2 + Y_3)^4$

$(\sum_{i=1}^{3} Y_i)^4$

2.3

Show by numerical example that $\sum X_i^2$ is not necessarily equal to $(\sum X_i{}^2)$.

```
x <- 1:10

sum(x^2)
```

```
[1] 385
```

```
(sum(x))^2
```

```
[1] 3025
```

2.4

Find the mean and median of the following sets of numbers.

a.) -1, 0, 3, 0, 2, -5.

```
x <- c(-1, 0, 3, 0, 2, -5)

mean(x)
```

```
[1] -0.1666667
```

```
median(x)
```

```
[1] 0
```

b.) 2, 2, 3, 10, 100, 1,000

```
x <- c(2, 2, 3, 10, 100, 1000)

mean(x)
```

```
[1] 186.1667
```

```
median(x)
```

```
[1] 6.5
```

2.5

The final exam scores for 15 students are: 73, 74, 92, 98, 100, 72, 74, 85, 76, 94, 89, 73, 76, 99.

Compute the mean, 20% trimmed mean, and median using R.

```
scores <- c(73, 74, 92, 98, 100, 72, 74, 85, 76, 94, 89, 73, 76, 99)

mean(scores)
```

```
[1] 83.92857
```

```
mean(scores, trim = .2)
```

```
[1] 83.1
```

```
median(scores)
```

```
[1] 80.5
```

2.6

The average of 23 numbers is 14.7. What is the sum of these numbers?

```
23 * 14.7
```

```
[1] 338.1
```

2.7

Consider the 10 values: 3, 6, 8, 12, 23, 26, 37, 42, 49, 63.

The mean is $\bar{X} = 26.9$

```
x <- c(3, 6, 8, 12, 23, 26, 37, 42, 49, 63)
mean(x)
```

```
[1] 26.9
```

a.) What is the value of the mean if the largest value, 63, is increased to 100?

```r
x <- c(3, 6, 8, 12, 23, 26, 37, 42, 49, 100)
mean(x)
```

```
[1] 30.6
```

b.) What is the mean if 633 is increased to 1,000?

```r
x <- c(3, 6, 8, 12, 23, 26, 37, 42, 49, 1000)
mean(x)
```

```
[1] 120.6
```

c.) What do these results illustrate about the mean?

The mean is very sensitive to outliers.

2.8

Repeat the previous exercise, only compute the median instead.

```r
x <- c(3, 6, 8, 12, 23, 26, 37, 42, 49, 100)
median(x)
```

```
[1] 24.5
```

b.) What is the mean if 633 is increased to 1,000?

```r
x <- c(3, 6, 8, 12, 23, 26, 37, 42, 49, 1000)
median(x)
```

```
[1] 24.5
```

2.9

In general, how many values must be altered to make the sample mean arbitrarily large?

One.

2.10

What is the minimum number of values that must be altered to make the 20% trimmed mean and sample median arbitrarily large?

mean = $g = (.2N), g + 1$

median = ~$.5N$