

The Bayesian Way

utility function for plotting a posterior

```
plot_posterior <- function(bsim) {
  res <- data.table(mu = bsim@coef, sigma = bsim@sigma)
  colnames(res) <- c("mu", "sigma")

  pmain <- ggplot(res, aes(mu, sigma)) +
    geom_point(alpha = .65, col = "darkgrey") +
    labs(x = "", y = "") +
    stat_density_2d(aes(fill = stat(level), alpha = ..level..), geom = "polygon")

  xbox <- axis_canvas(pmain, axis = "x") +
    geom_histogram(data = res, aes(mu, y = ..density.., fill = ..count..), size = .2, bins = 5) +
    geom_density(data = res, aes(mu), col = "darkgrey", alpha = .85) +
    theme(legend.position = "none")

  ybox <- axis_canvas(pmain, axis = "y", coord_flip = T) +
    geom_histogram(data = res, aes(sigma, y = ..density.., fill = ..count..), size = .2, bins = 5) +
    geom_density(data = res, aes(sigma), col = "darkgrey", alpha = .85) +
    coord_flip() +
    theme(legend.position = "none")

  suppressWarnings({
    p1 <- insert_xaxis_grob(pmain, xbox, grid::unit(.2, "null"), position = "top")
    p2 <- insert_yaxis_grob(p1, ybox, grid::unit(.2, "null"), position = "right")
  })

  ggdraw(p2)
}
```

Bayes Therom

$$P(\theta|y) = \frac{P(\theta)P(y|\theta)}{P(y)}$$

Estimating the Mean

The model of the data is:

$$y \sim \text{Norm}(\theta, \sigma^2)$$

Given the data has three measurements,

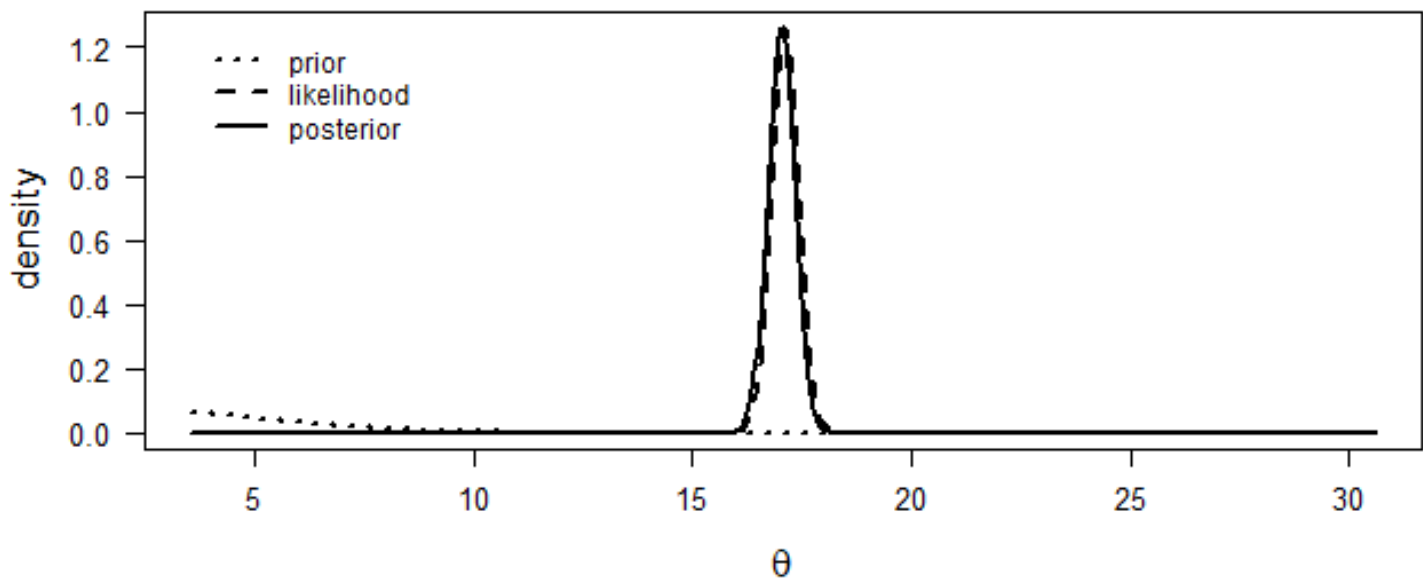
$$y_1 = 27.1, y_2 = 14.6, y_3 = 14.6$$

$$\sigma^2 = 20$$

```
n <- 200
```

```
x <- rnorm(n, mean = 17, sd = .5)
```

```
triplot.normal.knownvariance(n=n, theta.data = x, variance.known = 20, prior.theta = 0, prior.var = 100)
```



```
$posterior.mean
```

```
[1] 17.02758
```

```
$posterior.variance
```

```
[1] 0.0994152
```

\$x

```
[1] 16.614582 14.479351 16.422536 21.988484 12.608296 10.478518 31.382553
[8] 21.466955 18.405122 27.784221 16.817070 22.736172 15.688076 8.030159
[15] 18.599014 13.621072 16.805453 19.316468 18.487361 24.258960 14.489658
[22] 21.637876 13.346508 12.151978 11.430892 19.144929 19.070124 18.717060
[29] 11.558430 22.125091 13.327593 18.962811 17.037585 8.163602 18.667601
[36] 21.795684 14.079756 14.220209 12.318212 19.976698 8.520700 14.269608
[43] 17.860962 19.390677 15.647639 14.348594 13.424696 17.382181 13.274943
[50] 19.750342 7.725293 15.062817 17.176315 20.608786 20.553384 10.729883
[57] 16.127392 24.553823 18.148820 17.231256 19.193319 20.472602 15.485832
[64] 11.379506 16.236498 25.189300 16.371926 21.210208 9.483422 16.198414
[71] 20.795147 24.573382 21.549207 12.852323 12.081636 14.506917 14.486189
[78] 21.711543 22.385677 20.796538 22.359110 24.655461 20.493558 9.528454
[85] 14.997922 23.742668 13.425632 12.589178 20.210691 19.309679 17.967088
[92] 15.572175 23.739530 13.041454 11.165332 18.255672 11.005990 16.522002
[99] 11.579503 15.640548 18.328670 14.067745 15.292591 18.681861 10.576683
[106] 19.768761 13.150513 21.792521 26.819666 8.691833 12.605308 21.267534
[113] 11.388947 19.387487 22.286676 16.863562 19.876737 15.445097 22.138465
[120] 22.045010 18.361515 21.478582 9.753181 18.233680 21.577144 19.509812
[127] 18.301589 6.769061 15.857400 10.156756 16.915561 10.145906 17.913488
[134] 23.431495 15.725773 9.712470 21.697568 13.792234 22.171706 12.204138
[141] 17.253901 21.683838 24.729813 18.776416 14.663541 14.449474 12.395364
[148] 19.973472 21.563859 22.596038 19.782161 17.625738 19.502452 17.757982
[155] 22.594471 12.605141 16.780826 9.273247 18.429260 14.892441 11.547474
[162] 15.022207 17.535743 25.025851 20.522398 14.257858 14.259835 7.032736
[169] 22.681483 17.568467 11.425206 22.149547 14.458458 15.576967 17.772075
[176] 18.743409 19.436544 21.655831 11.147784 16.484315 19.015891 22.719301
[183] 22.102055 20.906203 18.875554 18.626792 20.887796 16.048650 22.910583
[190] 10.129344 17.448811 9.875615 17.406378 20.254181 19.087851 14.266831
[197] 13.199224 21.130891 18.292482 13.783221
```

Estimating Mean and Variance using Simulating

```
# Simulate hypothetical body height measurements

true.mean <- 165
true.sd <- 10
y <- round(rnorm(10, mean = true.mean, sd = true.sd))

mod <- lm(y ~ 1)
mod
```

Call:

```
lm(formula = y ~ 1)
```

```
Coefficients:  
(Intercept)  
      165.5
```

```
summary(mod)$sigma
```

```
[1] 10.45891
```

$y \sim \text{Norm}(\hat{\theta} = 166.3, \sigma^2 = 13.3)$

```
nsim <- 5000
```

```
bsim <- sim(mod, n.sim=nsim)
```

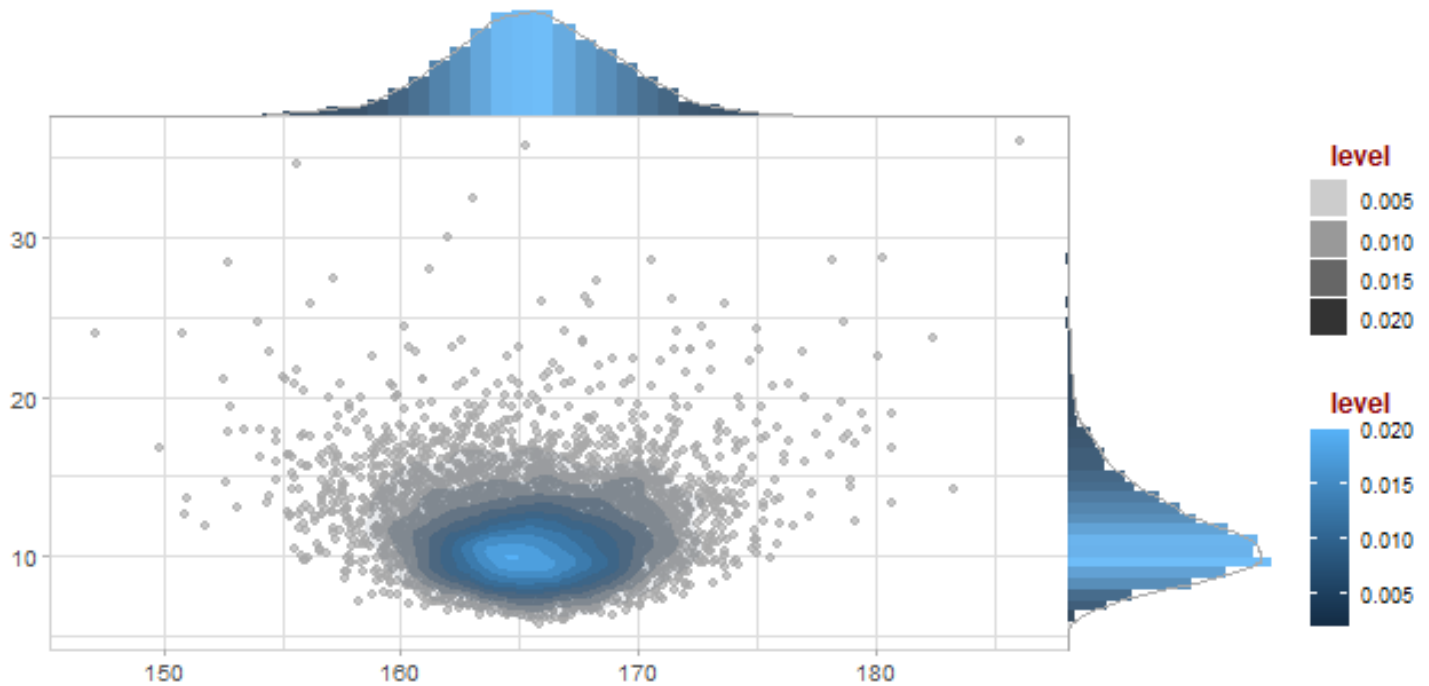
```
str(bsim)
```

```
Formal class 'sim' [package "arm"] with 2 slots  
..@ coef : num [1:5000, 1] 165 165 163 167 161 ...  
.. ..- attr(*, "dimnames")=List of 2  
.. .. ..$ : NULL  
.. .. ..$ : chr "(Intercept)"  
..@ sigma: num [1:5000] 11.16 9.47 13.67 9.48 14.94 ...
```

```
str(bsim, max.level = 2)
```

```
Formal class 'sim' [package "arm"] with 2 slots  
..@ coef : num [1:5000, 1] 165 165 163 167 161 ...  
.. ..- attr(*, "dimnames")=List of 2  
..@ sigma: num [1:5000] 11.16 9.47 13.67 9.48 14.94 ...
```

```
plot_posterior(bsim)
```



```
quantile(bsim@coef, prob = c(.025, 0.5, 0.975))
```

```
      2.5%      50%      97.5%
157.8031 165.4623 173.0343
```

```
quantile(bsim@sigma, prob = c(0.025, 0.5, 0.975))
```

```
      2.5%      50%      97.5%
 7.092488 10.931606 18.815159
```

```
HPDinterval(as.mcmc(bsim@coef))
```

```
      lower      upper
(Intercept) 158.352 173.3526
attr("Probability")
[1] 0.95
```

```
sum(bsim@coef > 160) / nsim
```

```
[1] 0.9362
```

```
mean(bsim@coef > 160)
```

```
[1] 0.9362
```

```
cvsim <- bsim@sigma / bsim@coef
```

```
quantile(cvsim, prob = c(0.025, 0.5, 0.975))
```

```
      2.5%      50%      97.5%
```

0.04283702 0.06618611 0.11460771

The Frequentist Way