Chapter 2

2.1

Compute the mean $\bar{x}$ and median, m ,of the six numbers: 3, 5, 8, 15, 20, 21, 24, then apply the natural log to the data.

```r
x <- c(3, 5, 8, 15, 20, 21, 24)
xt <- log(x)
```

Does $\bar{x}$ = $\tilde{x}$?

```r
log(mean(x)) == mean(xt)
```

```
[1] FALSE
```

Does m = $\tilde{m}$?

```r
log(median(x)) == median(xt)
```

```
[1] TRUE
```

2.2

Compute the median $\bar{x}$ and median of the eight numbers: 1, 2, 4, 5, 6, 8, 11, 15.

Let $f(x) = \sqrt{x}$

Apply the transformation, then compute the mean, $\tilde{x}$ and median, m, of the transformed data.

```r
x <- c(1, 2, 4, 5, 6, 8, 11, 15)
xt <- sqrt(x)
```

- Is $f(\bar{x}) = \tilde{x}$?

```r
sqrt(mean(x)) == mean(xt)
```

```
[1] FALSE
```

- Is $f(m) = \tilde{m}$?

```
sqrt(median(x)) == median(xt)
```

```
[1] FALSE
```

2.4

Import the flights data.

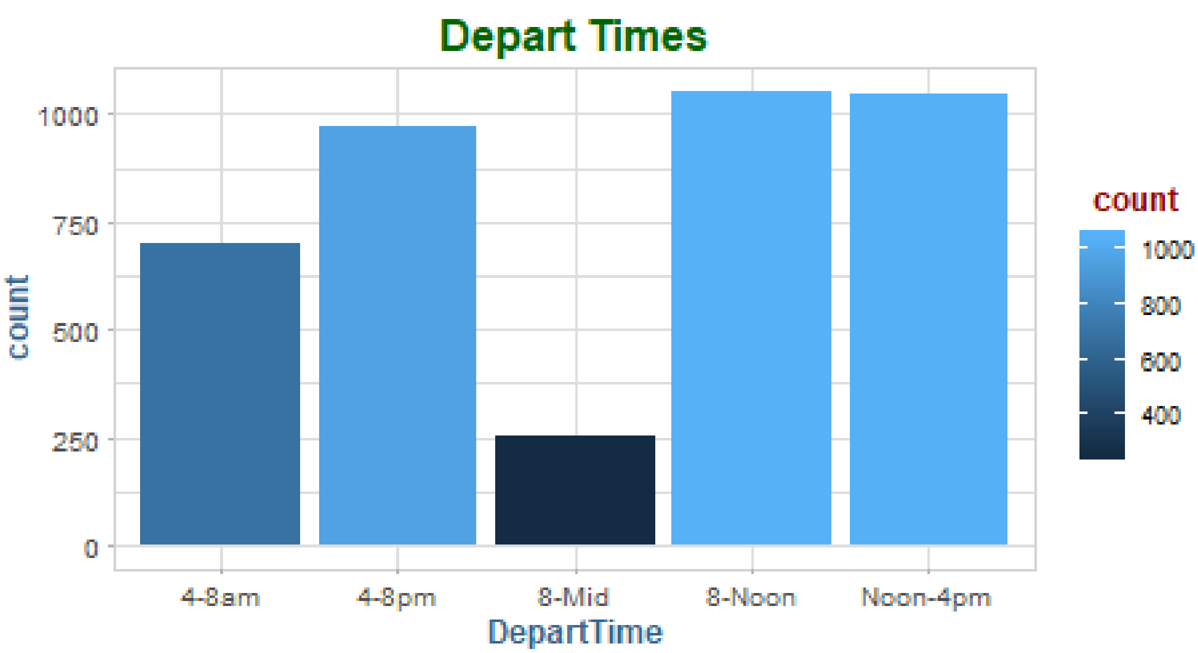
```
flights <- data.table(read.csv(paste0(data.dir, "FlightDelays.csv"),
                  header = T))
```

a.) Create a table and bar chart of the departure times *(DepartTimes)*

```
table(flights$DepartTime)
```

```
  4-8am   4-8pm   8-Mid  8-Noon Noon-4pm
    699     972     257    1053     1048
```

```
ggplot(flights, aes(DepartTime)) +
   geom_bar(aes(fill = ..count..)) +
   labs(title = "Depart Times")
```



b.) Create a contingency table of the variables *Day* and *Delay30*.

```
delay <- table(flights$Day, flights$Delayed30)

pretty_kable(delay, "Flight Delays")
```

Table 1: Flight Delays

	No	Yes
Fri	493	144
Mon	569	61
Sat	406	47
Sun	507	44
Thu	434	132
Tue	535	93
Wed	488	76

Show the proportions of delayed flights, by day:

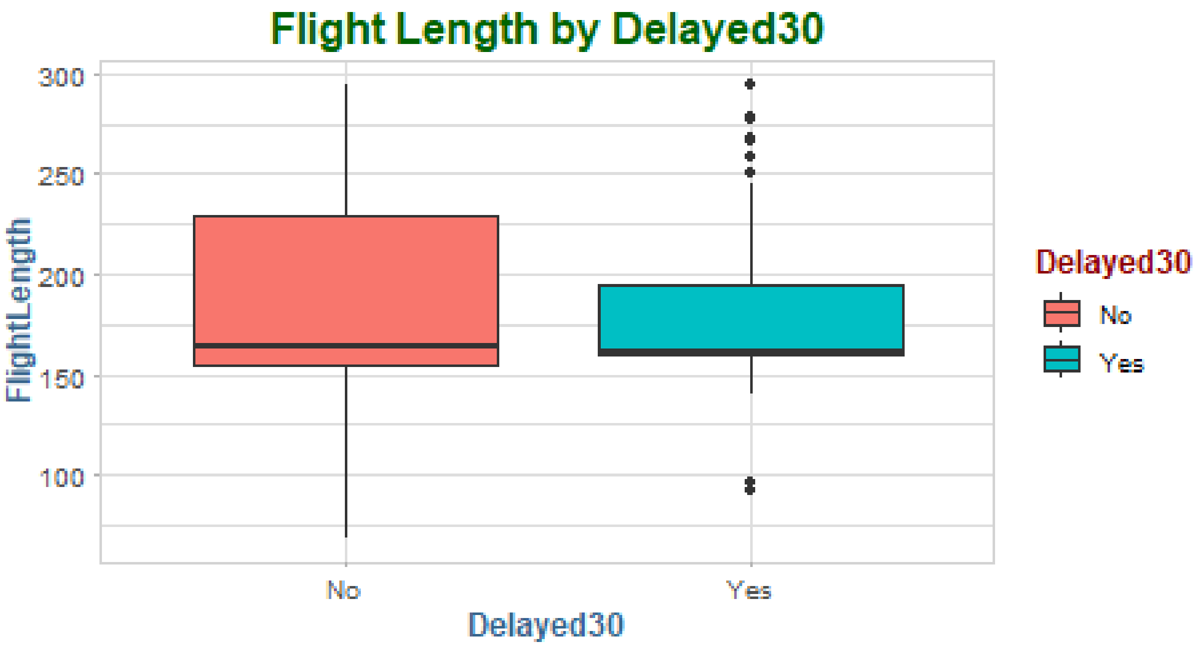
```
pretty_kable(round(prop.table(delay), 4) * 100, "Flight Delays Proportions")
```

Table 2: Flight Delays Proportions

	No	Yes
Fri	12.24	3.57
Mon	14.12	1.51
Sat	10.08	1.17
Sun	12.58	1.09
Thu	10.77	3.28
Tue	13.28	2.31
Wed	12.11	1.89

c.) Create side-by-side boxplots of the lengths of flight times, grouped by wether or not the flight was delayed at least 30 minutes:

```
ggplot(flights) +
  geom_boxplot(aes(Delayed30, FlightLength, fill = Delayed30)) +
  labs(title = "Flight Length by Delayed30")
```



d.) Do you think there is a relationship between the length of the flight and whether or not the departure time is delayed by at least 30 minutes?

The average flight time is the same, however, the flights that are delayed 30 minutes or more seem to be shorter overall.

2.5

Import the General Social Survey data.

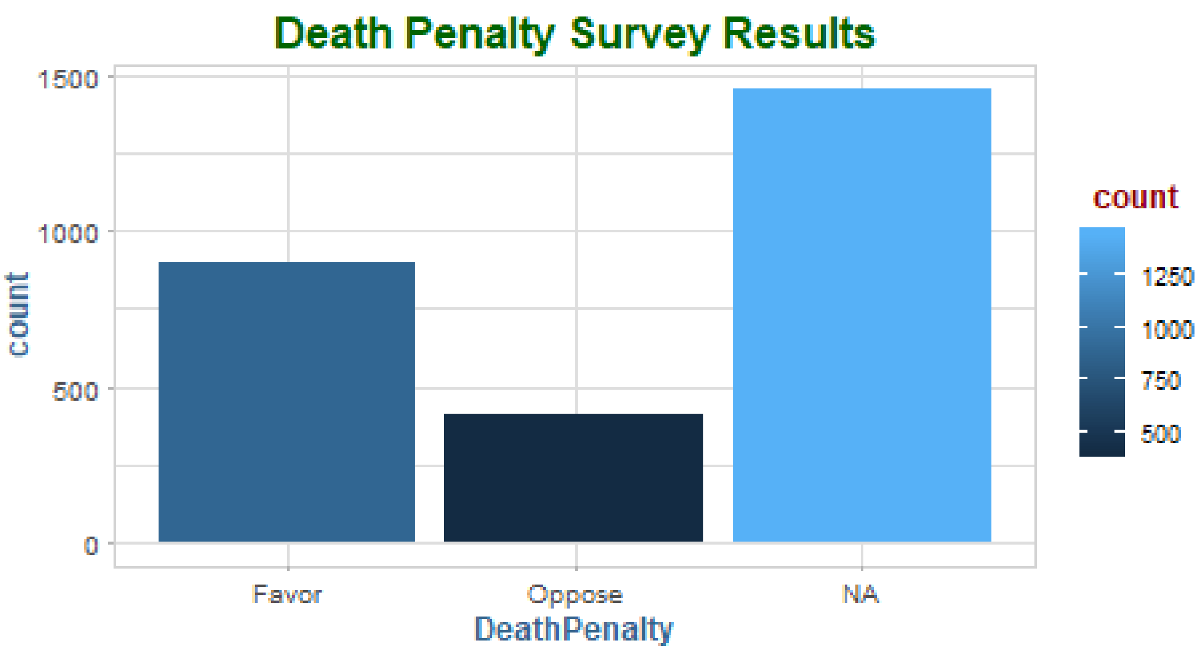
```
gss <- data.table(read.csv(paste0(data.dir, "GSS2002.csv"),
                 header = T))
```

a.) Create a table and a bar chart of the response to the question about the death penalty.

```
table(gss$DeathPenalty)
```

```

 Favor Oppose
   899    409
```

```r
ggplot(gss, aes(DeathPenalty)) +
  geom_bar(aes(fill = ..count..)) +
  labs(title = "Death Penalty Survey Results")
```



b.) Use the *table* command and the summary command in R on the gun ownership variable. What additional information does the summary command give that the table does not?

```r
table(gss$OwnGun)
```

```
    No Refused     Yes
   605       9     310
```

```r
summary(gss$OwnGun)
```

```
    No Refused     Yes    NA's
   605       9     310    1841
```

The summary tells us how many people didn't respond at all to the question.

c.) Create a contingency table displaying the relationship between opinions about the death penalty to that about gun ownership.

```r
with(gss, {
  table(OwnGun, DeathPenalty)
})
```

```
         DeathPenalty
OwnGun    Favor Oppose
  No        375    199
  Refused     7      2
  Yes       243     59
```

d.) What proportion of gun owners favor the death penalty? Does it appear to be different from the proportion among those who do not own guns?

```
round(prop.table(with(gss, {
   table(OwnGun, DeathPenalty)
})) * 100, 2)
```

```
         DeathPenalty
OwnGun    Favor Oppose
  No      42.37  22.49
  Refused  0.79   0.23
  Yes     27.46   6.67
```

It does seem that gun owners are overwhelmingly in favor of the death penalty compared to those who do not own a gun.

2.6

Import the data from the recidivism case study in section 1.4.

Import the General Social Survey data.

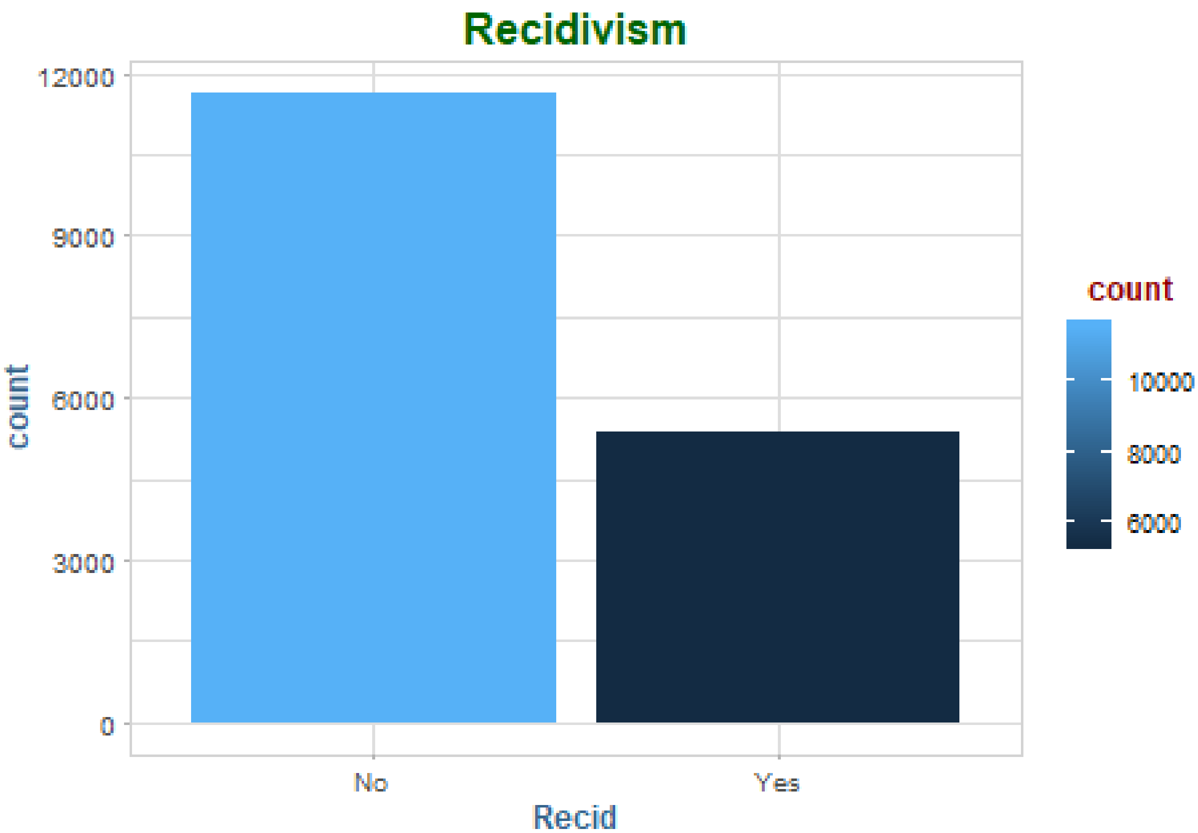
```
recid <- data.table(read.csv(paste0(data.dir, "Recidivism.csv"),
                 header = T))
```

a.) Create a table and bar chart of the **Recid** variable.

```
table(recid$Recid)
```

```
   No   Yes
11636  5386
```

```
ggplot(recid, aes(Recid)) +
   geom_bar(aes(fill = ..count..)) +
   labs(title = "Recidivism")
```



b.) Create a contingency table summarizing the relationship between recidivism (**Recid**) by age (**Age25**).

```
with(recid, {
   table(Recid, Age25)
})
```

```
     Age25
Recid Over 25 Under 25
  No      9679     1954
  Yes     4263     1123
```

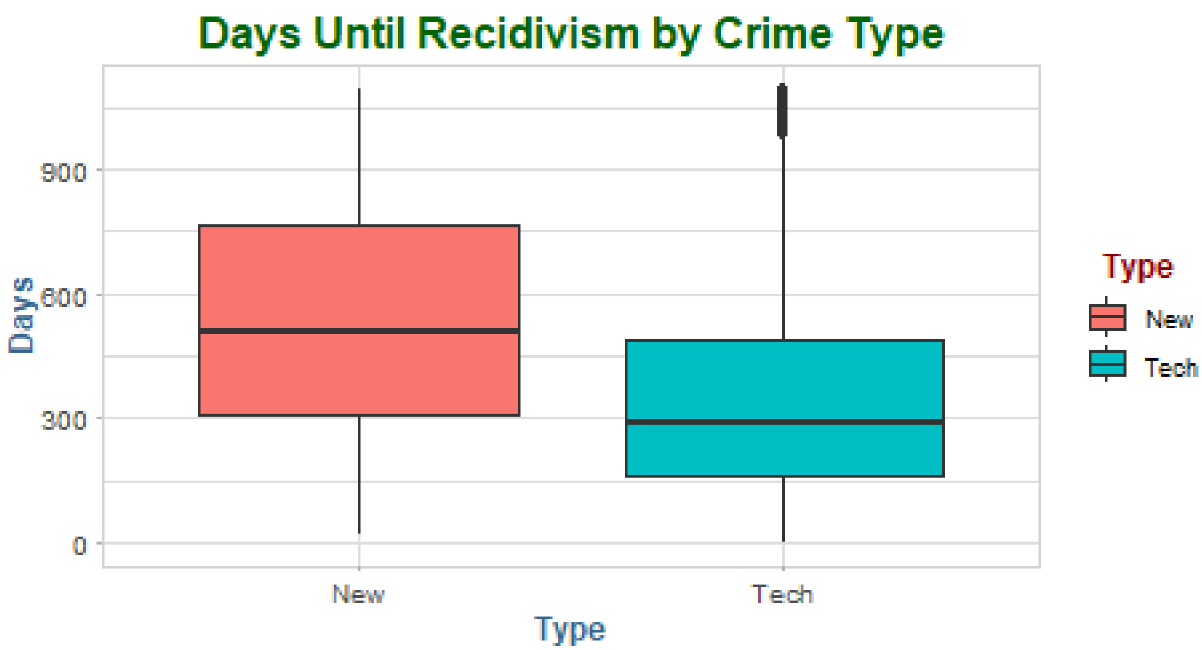
Of those over 25 years of age, what proportion were sent back to prision?

```
round(prop.table(with(recid, {
   table(Recid, Age25)
})) * 100, 2)
```

```
     Age25
Recid Over 25 Under 25
  No     56.87    11.48
  Yes    25.05     6.60
```

c.) Create side-by-side boxplots of the number of days to recidivism grouped by type of violation, and give three comparative statements about the distributions.

```
ggplot(recid[!is.na(Days)], aes(Type, Days)) +
   geom_boxplot(aes(fill = Type)) +
   labs(title = "Days Until Recidivism by Crime Type")
```



- 1.) Technical violations seem to happen quicker after initial release than new crimes.
- 2.) There are more outliers present in the technical violations, with a cluster of them occuring after 900 days.
- 3.) The variance is days until recidivism is larger for new crimes. The chances of a technical violation occuring after 450 days drop dramatically.

```
recid[!is.na(Days), .(Variance = comma(var(Days))), by = Type]
```

```
   Type  Variance
1: Tech 63,200.01
2:  New 76,898.53
```

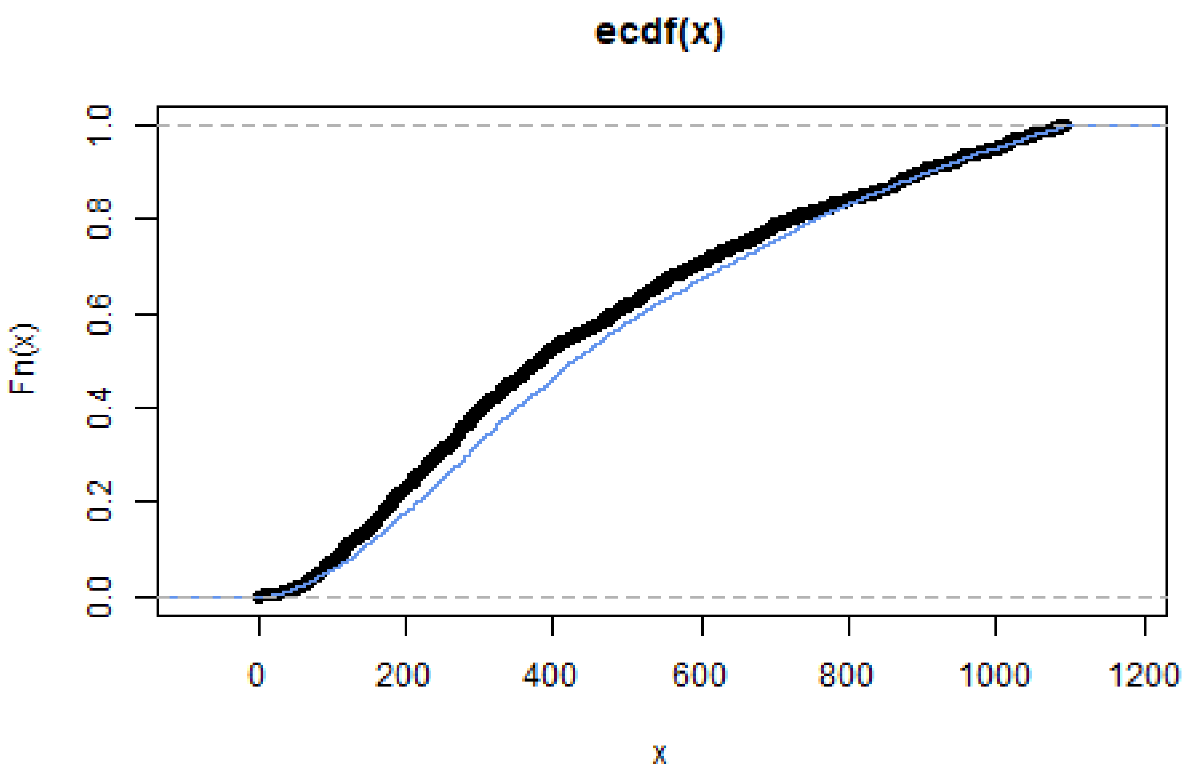
d.) Use the quantile command to obtain the quartiles of the number of days to recidivism. Since there are missing values (**NA**) for those released offeneders who had not recidivated, you will need to add the argument **na.rm = T** to the **quartile** command to exclude those observations.

```
quantile(recid$Days, na.rm = T)
```

```
  0%  25%  50%  75% 100% 
   0  241  418  687 1095
```

e.) Create ecdf's of days to recidivism for those under 25 years of age and those 25 years of age or older.

```
with(recid, {
   recid$U25 <- Age25 == "Under 25"

   plot.ecdf(recid[U25 == T]$Days)
   plot.ecdf(recid[U25 == F]$Days, col="cornflowerblue", add = T)
})
```



Approximately what proportion in each age group were sent back to prision 400 days after release?

Of those who were sent back to prision,

```
recid[!is.na(Days),
      .(Days, O400 = Days >= 400), by = Age25][,
                                             .(Proportion = sum(O400) / .N), by = Age25]
```

```
      Age25 Proportion
1: Under 25  0.4746215
2:  Over 25  0.5378841
```

went back after being release 400 days or more.

2.7

Import the data from the black spruce case study in 1.10.

```
spruce <- data.table(read.csv(paste0(data.dir, "Spruce.csv"),
                 header = T))
```

a.) Compute the numeric summaries for the hight changes of the seedlings.

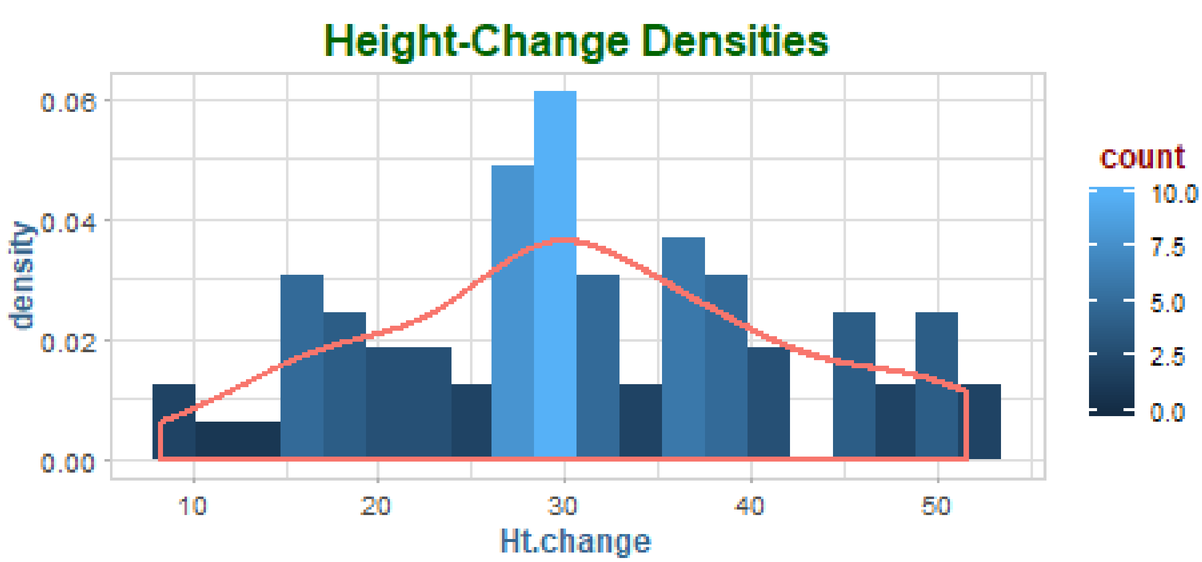
```
summary(spruce$Ht.change)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   8.30   23.20   30.10   30.93   38.17   51.50
```

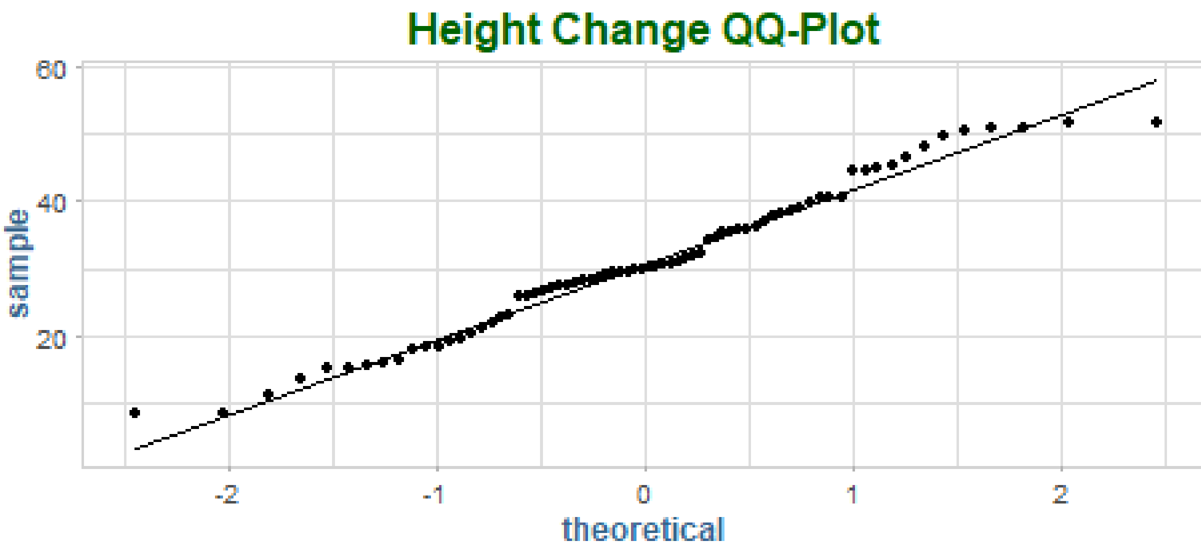
b.) Create a histogram and normal quantile plot for the height changes of the seedlings.

Is the distribution approximately normal?

```
ggplot(spruce) +
   geom_histogram(aes(x = Ht.change, y = ..density.., fill = ..count..), bins = 20) +
   geom_density(aes(x = Ht.change, y = ..density.., col = "darkred"), lwd = 1) +
   guides(col = "none") +
   labs(title = "Height-Change Densities")
```



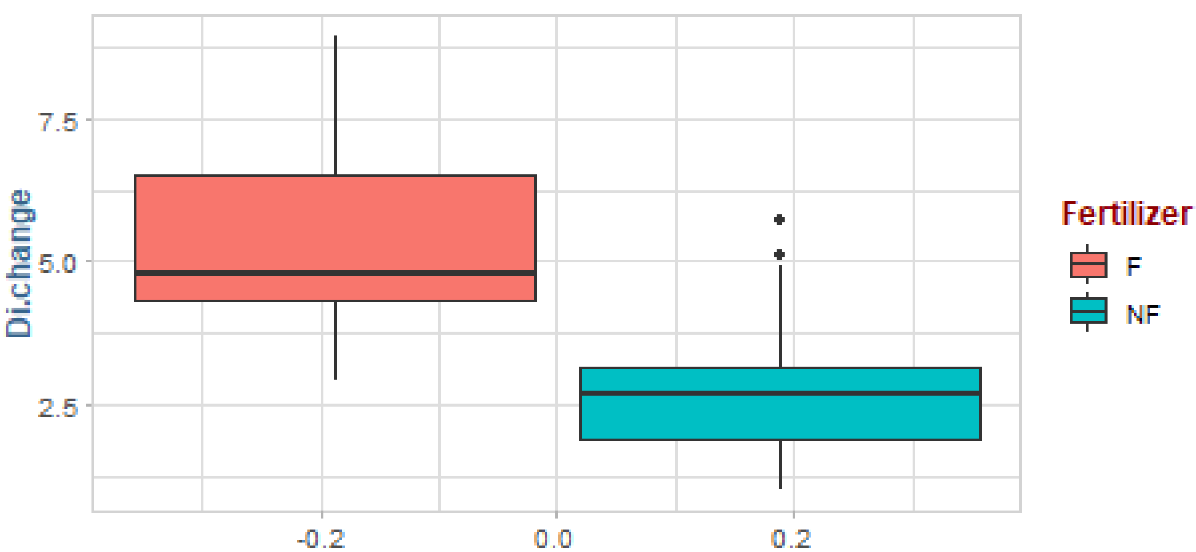
```
ggplot(spruce) +
  geom_qq(aes(sample = Ht.change)) +
  geom_qq_line(aes(sample = Ht.change)) +
  labs(title = "Height Change QQ-Plot")
```



For the height change variable, we see an uneven density plot, qq-plot with outliers in the tails. The data appers to be "approximately" normal.

c.) Create a boxplot to compare the distribution of the change in diameters of the seedlings (**Di.change**), grouped by wheather or not they were in fertilized plots.

```
ggplot(spruce) +
  geom_boxplot(aes( y = Di.change, fill = Fertilizer ))
```



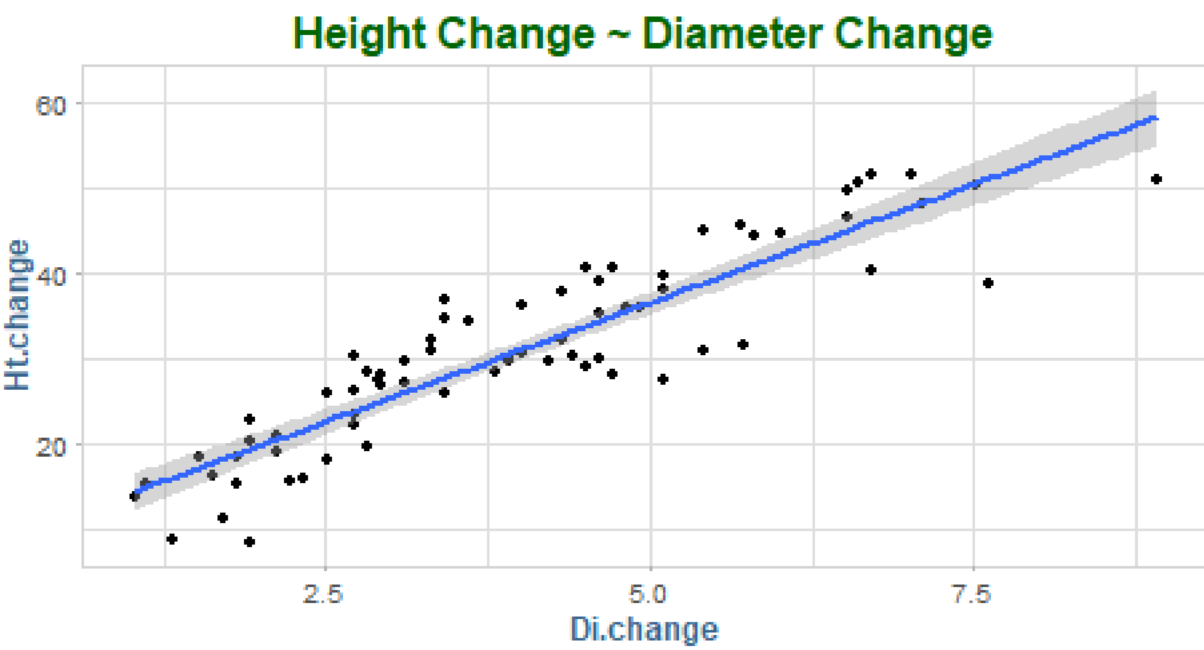
d.) Use the **tappy** command to find the numeric summaries of the diameter changes for the two levels of fertilization.

```
tapply(spruce$Di.change, spruce$Fertilizer, summary)
```

```
$F
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  2.913   4.318   4.763   5.274   6.518   8.919

$NF
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.019   1.915   2.712   2.718   3.165   5.713
```

e.) Create a scatter plot of the hight change against the diameter changes, and describe the relationship.

```
ggplot(spruce, aes(Di.change, Ht.change)) +
   geom_point() +
   geom_smooth(method = "lm") +
   labs(title = "Height Change ~ Diameter Change")
```



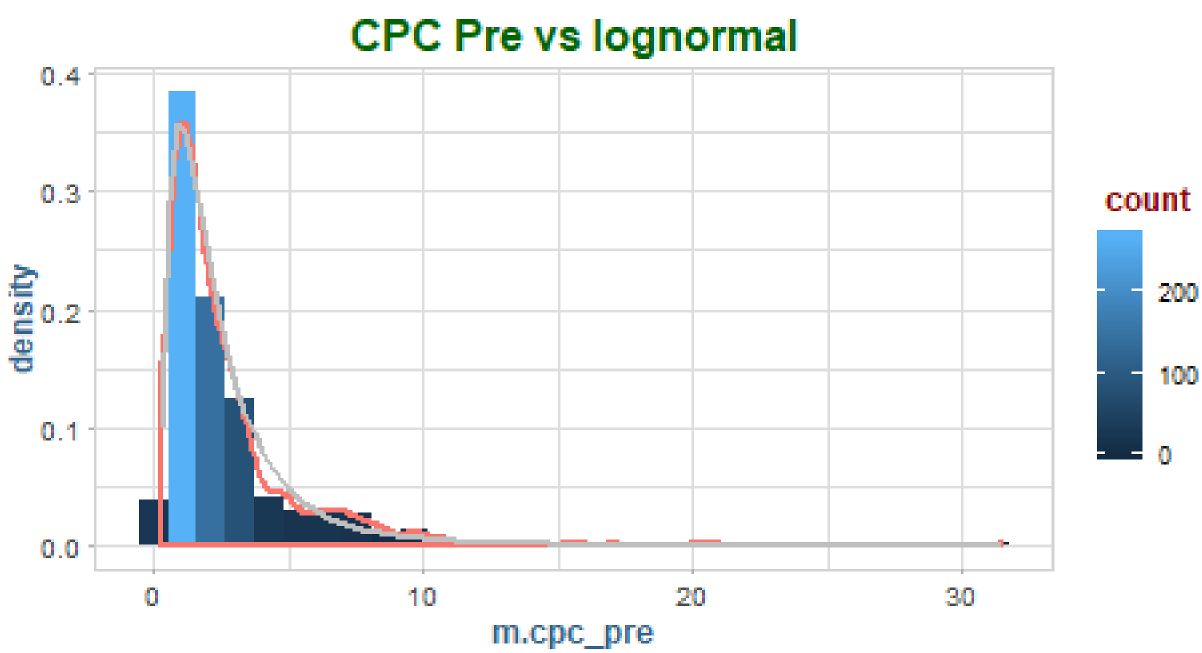
2.8

Import the mobile ads data from section 1.12.

```
mobile <- data.table(read.csv(paste0(data.dir, "MobileAds.csv"),
                  header = T))
```

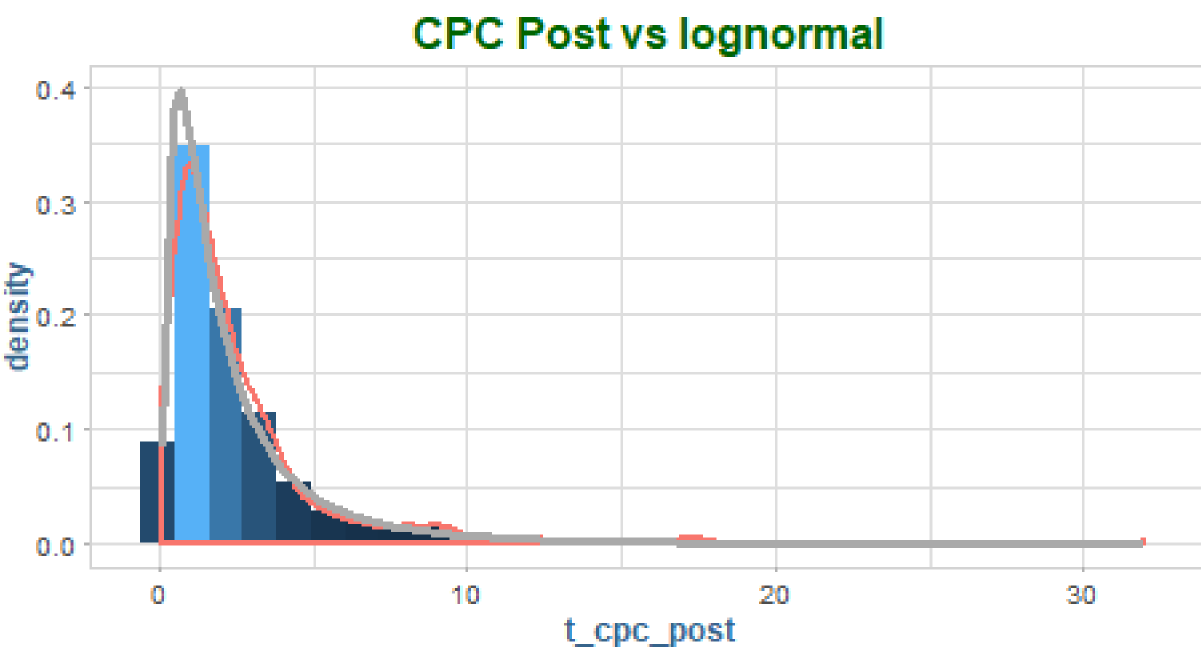
a.) Create histograms of the variables **m.cpc_pre** and **m.cpc_post**, and describe their distributions.

```
fit <- fitdistr(mobile$m.cpc_pre, "lognormal")

ggplot(mobile, aes(m.cpc_pre)) +
   geom_histogram(aes(y = ..density.., fill = ..count..), bins = 30) +
   geom_density(aes(y = ..density.., col = "darkred"), lwd = 1) +
   stat_function(fun = dlnorm, size = 1, color = 'gray',
                     args = list(mean = fit$estimate[1], sd = fit$estimate[2])) +
   guides(col = "none") +
   labs(title = "CPC Pre vs lognormal")
```




```
pct_zero <- sum(mobile$m.cpc_post == 0) / nrow(mobile) # 7% of values zero

# we will replace the zeros with half the min value here to fit a lognormal.

t_cpc_post <- mobile$d.cpc_post
rep_val <- min( t_cpc_post[t_cpc_post > 0] ) / 2
```

```
t_cpc_post[t_cpc_post == 0] <- rep_val

fit <- fitdistr(t_cpc_post, "lognormal")

ggplot(mobile, aes(t_cpc_post)) +
   geom_histogram(aes(y = ..density.., fill = ..count..), bins = 30) +
   geom_density(aes(y = ..density.., col = "darkred"), lwd = 1) +
   stat_function(fun = dlnorm, size = 1.2, color = "darkgrey",
                 args = list(mean = fit$estimate[1], sd = fit$estimate[2])) +
   guides(col = "none") +
   labs(title = "CPC Post vs lognormal") +
   theme(legend.position = "none")
```



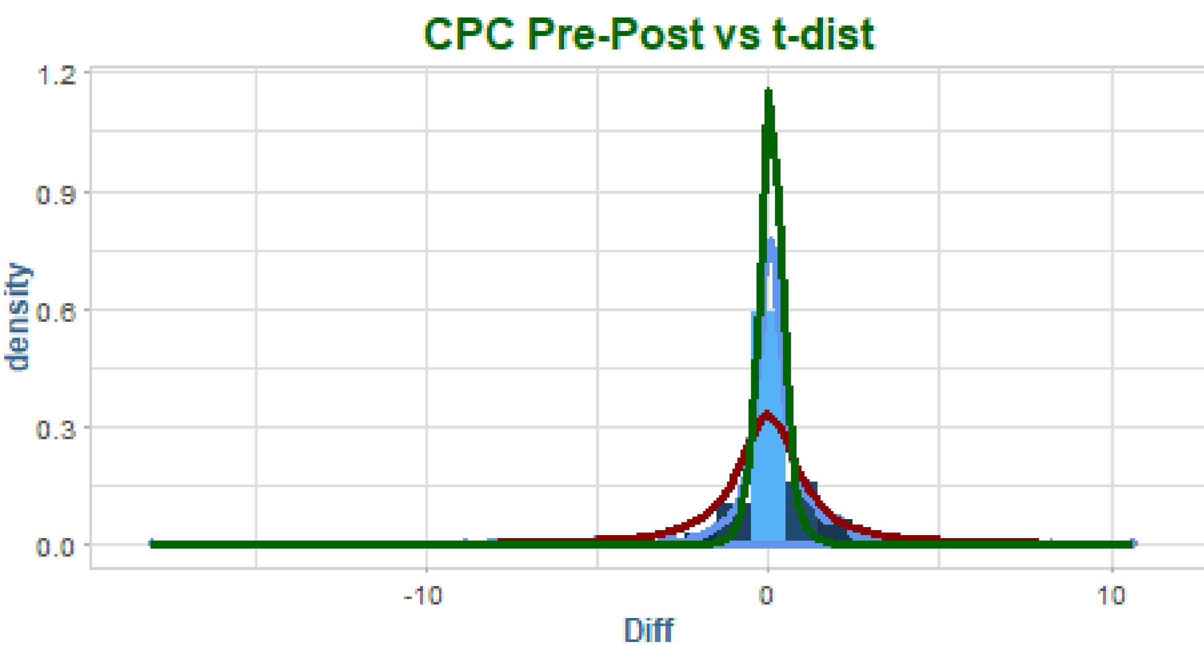
The distribution of cpc pre and post seem to follow an approximately lognormal distribution.

b.) Compute the difference between these two variables, create a histogram, and describe the distribution.

```
diff <- data.table(Diff = with(mobile, { m.cpc_pre - m.cpc_post}))[, Index := .I ]

suppressWarnings({
   fit <- fitdistr(diff$Diff, "t")
})

ggplot(diff, aes(Diff)) +
   geom_histogram(aes(y = ..density.., fill = ..count..))+
   geom_density(aes(y = ..density..), col = "cornflowerblue", lwd = 1.2) +
   stat_function(fun = dt, size = 1.2, color = "darkred",
```

```
                      args = list(df = fit$estimate[3])) +
    stat_function(fun = dstd, size = 1.2, color = "darkgreen",
                      args = list(mean = fit$estimate[1], sd = fit$estimate[2])) +
    guides(col = "none") +
    labs(title = "CPC Pre-Post vs t-dist") +
    theme(legend.position = "none")
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

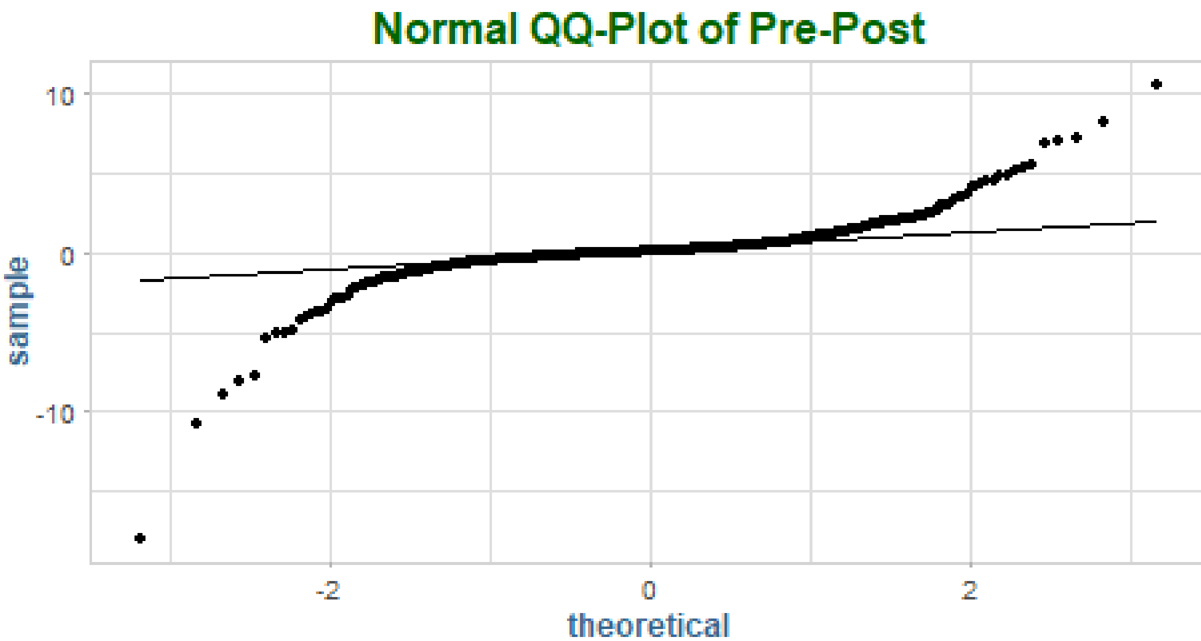


We fit the delta data to a t-distribution, estimating the parameters with **fitdist**. The distribution is way too tail heavy for a normal, and the standard t-distribution has tails that are too light, while the normal t-distribution has tails that are too heavy.

We also note that the true tails in the distribution are polynomial, so getting a close fit would be a bit challenging.

d.) Create a normal quantile plot of the difference. Does it appear to be normally distributed?

```
ggplot(diff, aes(sample = Diff)) +
    geom_qq() +
    geom_qq_line() +
    labs(title = "Normal QQ-Plot of Pre-Post")
```



This data definitely does not fit a normal distribution. As we saw with the density plots, there are numerious problems with fitting the tails of the data. A simple normal estimate is a terrible fit, while even parametric approaches have problems due to the polynomial tails.

2.9

Let $x_1 < x_2 < ... < x_n$ and $y_1 < y_2 < ... < y_n$ be two sets of data with means $\bar{x}, \bar{y}$ and means m_x, m_y, respectively.

Let $w_i = x_i + y_i$ for $i = 1, 2, \ldots, n$.

a.) Prove or give a counterexample: $\bar{x} + \bar{y}$ is the mean of $w_1, w_2, ..., w_n$.

$$\bar{w} = \bar{x} + \bar{y} = \frac{1}{n}\sum_{i=1}^{n} x_n + \frac{1}{n}\sum_{i=1}^{n} y_n$$

$$\ldots = \frac{1}{n}\sum_{i=1}^{n}[x_n + y_n]$$

b.) Prove or give a counterexample: $m_x + m_y$ is the median of $w_1, w_2, ..., w_n$.

$$m_x = \frac{x_{[(\#x+1)\div 2]} + x_{[(\#x+1)\div 2]}}{2}$$

$$m_y = \frac{y_{[(\#y+1)\div 2]} + y_{[(\#y+1)\div 2]}}{2}$$

$$m_x + m_y = \frac{x_{[(\#x+1)\div 2]} + x_{[(\#x+1)\div 2]}}{2} + \frac{y_{[(\#y+1)\div 2]} + y_{[(\#y+1)\div 2]}}{2}$$

$$\ldots = \frac{1}{2}(x_{[(\#x+1)\div 2]} + x_{[(\#x+1)\div 2]} + y_{[(\#y+1)\div 2]} + y_{[(\#y+1)\div 2]})$$

2.10

Find the median **m** and first and third quartiles for the random variable X having:

a.) The exponential distribution with $pdf\, f(x) = \lambda e^{-\lambda x}$

m = $\frac{ln(2)}{\lambda}$ =

$q_p = -\frac{ln(1-p)}{\lambda}$, where q in .25, .5, .75.

b.) The Pareto distribution with parameter $\alpha > 0$ with $pdf\, f(x) = \frac{\alpha}{\chi^{\alpha+1}}$

2.11

Let the random variable X have a Cauchy distribution with $pdf\, f(x) = \frac{1}{\pi(1+x(x-\theta)^2))}$ for $-\infty < x < \infty$.

a.) Show that the mean of X does not exist.

b.) More generally, will $\mathbb{E}[X^k]$ exist? (k = 1, 2, 3, …).

c.) Show that θ is the median of the distribution.