**Chapter 2**

## 1

You are invested in two hedge funds. The probability that hedge fund Alpha generates positive returns in a given year is 60%. The probability that hedge fund Omega generates positive returns in a given year is 70%. Assume the returns are independent.

What is the probability that both funds generate positive returns in a given year?

```
prob_up <- .6 * .7
```

**42%**

What is the probability that both funds lose money?

```
prob_down <- (1 - .6) * (1 - .7)
```

**12%**

## 2

Corporation ABC issues $100 million of bonds. The bonds are rated BBB. The probability that the rating on the bonds is upgraded within the year is 8%. The probability of a drowngrade is 4%.

What is the probability that the raiting remains unchanged?

```
prob <- (1 - 0.08 - 0.04)
```

**88%**

## 3

Stock XYZ has a 20% chance of losing more than 10% in a given month. There is also a 30% probability that XYZ gains more than 10%. What is the probability that stock XYZ either loses more than 10% or gains more than 10%?

```
prob <- (.3 + .2)
```

**50%**

## 4

There is a 30% chance that oil prices will increase over the next six months. If oil prices increase, there is a 60% chancee that the stock market will be down.

What is the probability that oil prices increase and the stock market is down over the next 6 months?

```
prob <- .3 * .6
```

**18%**

## 5

Given the following density function:

$f(x) = c(100 - x^2)$ for $-10 \leq x \leq 10$

Calculate the value of c.

$\int_{-\infty}^{\infty} f(x)dx = \int_{-10}^{10} c(100 - x^2)dx = c[100x - \frac{1}{3}x^3]_{-10}^{10}$

$c = \frac{3}{4{,}000}$

## 6

Given the following cumulative distribution function, $F(x)$, for $0 \leq x \leq 10$:

$F(x) = \frac{x}{100}(20 - x)$

Check that this is a valid CDF; that is, show that $F(0)$ and $F(1)$ = 1.

$F(x) = \frac{1}{5}x - \frac{x^2}{100}$

For 0:

$F(0) = \frac{1}{5}0 - \frac{0}{100}$

$F(0) = 0$

For 10:

$F(10) = \frac{1}{5}10 - \frac{10^2}{100}$

$F(10) = 2 - 1$

$F(10) = 1$

Calculate the probability density function, $f(x)$.

$F^{-1}(x) = \frac{1}{50}(10 - x)$

## 7

Given the probability density function, $f(x) = \frac{c}{x}$,

Where $1 \leq x \leq e$. Calculate the cumulative distribution function, $F(x)$, and solve for the constant c.

$F(x) = \int_{1}^{x} f(t)dt = \int_{1}^{x} \frac{c}{t}dt = c[ln(t)]_{1}^{x} = c \ln x$

$F(x) = ln\, x$

## 8

You own two bonds. Both bonds have a 30% probability of defaulting. Their default probabilities are statistically independent.

What is the probability that both bonds default?

```
prob <- .3 * .3
```

P[both] = **9%**

What is the probability that only one bond defaults?

```
prob <- (1 - .3) * 2*.3
```

P[one defaults] = **42%**

What is the probability that neither bond defaults?

```
prob <- (1 - .3)*(1 - .3)
```

P[neither] = **49%**

## 9

The following table gives a one-year rating transition matrix. Given a bond's rating now, the matrix gives the probability associated with the bond having a given rating in a year's time.

```
ratings <- data.table(Grade = c("A", "B", "C", "D"), A = c(.9, .1, 0, 0), B = c(.08, .8, .25, (

pretty_kable(ratings, "Bond Transition Probabilities")
```

Table 1: Bond Transition Probabilities

| Grade | A | B | C | D |
|---|---|---|---|---|
| A | 0.9 | 0.08 | 0.02 | 0.00 |
| B | 0.1 | 0.80 | 0.08 | 0.02 |
| C | 0.0 | 0.25 | 0.60 | 0.15 |
| D | 0.0 | 0.00 | 0.00 | 1.00 |

Given a B-rated bond, what is the probability that the bond default (D rating) over 1 year?

```
prob <- 0.02
```

P[default] = **2%**

What is the probability that the bond defaults over two years?

```
prob <- .8 * .02 + .08*.15 + .02
```

P[default] = **4.8%**

## 10

Your firm forecasts that there is a 50% probability that the markets will be up significantly next year, a 20% probability that the market will be down significantly next year, and a 30% probability that the market will be flat, neither up or down significantly.

You are asked to evaluate the prospects of a new portfolio manager. The manager has a long bias and is likely to perform better in an up market.

Based on past data, you believe that the probability that the manager will be up if the market is up significantly is 80%, and that the probability that the manager will be up if the market is down significantly is only 10%.

If the market is flat, the manager is just as likely to be up as to be down.

What is the unconditional probability that the manager is up next year?

$$P[X_{up}] = P[X_{up}|M_{up}] * P[M_{up}] + P[X_{up}|M_{dn}] * P[M_{dn}] * P[X_{up}|M_{flat}] * P[M_{flat}]$$

$P[X_{up}]$ = 80% * 50% + 10% * 20% + 50% * 30%

```
prob <- .8*.5 + .1*.2 + .5*.3
```

$P[X_{up}]$ = **57%**