Chapter 4

4.1

If the possible values for x are 0, 1, 2, 3, 4, 5, and the corresponding values for $P(x)$ are 0.2, 0.2, 0.15, 0.3, 0.35, 0.2, 0.1, respectively, does $P(x)$ qualify as a probability function?

```
x <- c(0, 1, 2, 3, 4, 5)
p <- c(0.2, 0.2, 0.15, 0.3, 0.35, 0.2, 0.1)

sum(p)
```

```
[1] 1.5
```

No. P > 1.

4.2

If the possible values for x are 2, 3, 4, and the corresponding values for $P(x)$ are 0.2, -0.1, 0.9, respectively, does $P(x)$ qualify as a probability function?

No. $0 \leq x \leq 1$

4.3

If the possible values for x are 1, 2, 3, 4, and the corresponding values for $P(x)$ are 0.1, 0.15, 0.5, 0.25, respectively, does $P(x)$ qualify as a probability function?

```
x <- c(1, 2, 3, 4)
p <- c(0.1, 0.15, 0.5, 0.25)

sum(p)
```

```
[1] 1
```

Yes, this is a valid probability function.

4.4

If the possible values for x are 2, 3, 4, 5, and the corresponding values for $P(x)$ are 0.2, 0.3, 0.4, 0.1, respectively, what is the probability of observing a value less than or equal to 3.4?

```r
x <- c(2, 3, 4, 5)
p <- c(0.2, 0.3, 0.4, 0.1)

stopifnot( sum(p) == 1 )

prob <- sum( p[ x <= 3.4 ] )
```

Probability: **50%**

4.5

For the previous distribution, what is the probability of observing a 1?

Zero.

4.6

For the previous distribution, what is the probability of observing a value greater than 3?

```r
prob <- sum( p[ x > 3 ])
```

Probability: **50%**

4.7

For the previous distribution, what is the probability of observing a value greater than or equal to 3?

```r
prob <- sum( p[ x >= 3 ])
```

Probability: **80%**

4.8

If the probability of observing a value less than or equal to 6 is 0.3, what is the probability of observing a value greater than 6?

```r
prob <- 1 - .3
```

Probability: **70%**

4.9

For the probability function:

$x\ :\ 0, 1$

$P(x)\ : 0.7, 0.3$

Verify that the mean and variance are 0.3 and 0.21, respectively.

```r
x <- c(0, 1)
p <- c(0.7, 0.3)

mu <- sum( x * p )
variance <- sum( (x - mu)^2 * p)
```

$\mu = 0.3, \sigma^2 = 0.21$

What is the probability of getting a value less than the mean?

50%

4.10

Imagine that an auto manufacturer wants to evaluate how potential customers will rate handling for a new car being considered for production. Also, suppose that if all potential customers were to rate handling on a four-point scale, 1 being poor and 4 being excellent, the corresponding probabilities associated with these ratings would be:

$P(1) = 0.2, P(2) = 0.4, P(3) = 0.3, P(4) = 0.1$

Determine the population mean, variance and standard deviation.

```r
x <- 1:4
p <- c(0.2, 0.4, 0.3, 0.1)

stopifnot(sum(p) == 1)

mu <- sum(x * p)
variance <- sum( (x - mu)^2 * p)
stdDev <- sqrt(variance)
```

$\mu = 2.3, \sigma^2 = 0.81, \sigma = 0.9$

4.11

If the possible values for x are 1, 2, 3, 4, 5, with probabilities 0.2, 0.1, 0.1, 0.5, 0.1, respectfully, what are the population mean, variance, and standard deviation?

```
x <- 1:5
p <- c(0.2, 0.1, 0.1, 0.5, 0.1)

mu <- sum( x * p )
variance <- sum( (x - mu)^2 * p )
sigma <- sqrt(variance)
```

$\mu = 3.2$, $\sigma^2 = 1.76$, $\sigma = 0.9$

4.12

In the previous exercise, determine the probability of getting a value within one standard deviation of the mean.

That is, $\mu - \sigma \leq x \leq \mu + \sigma$

```
vals <- mu + c(-1, 1)*sigma
round(vals, 4)
```

```
[1] 1.8734 4.5266
```

```
sum( p[ x >= vals[1] & x <= vals[2] ] )
```

```
[1] 0.7
```

4.13

If the possible values for x are 1, 2, 3, with probabilities 0.2, 0.6, and 0.2, respectively, what is the mean and standard deviation?

```
x <- 1:3
p <- c(0.2, 0.6, 0.2)

mu <- sum( x * p )
variance <- sum( (x - mu)^2 * p)
stdDev <- sqrt(variance)
```

$\mu = 2$, $\sigma^2 = 0.4$, $\sigma = 0.6324555$

4.14

In the previous excersize, suppose the possible values for x are now 0, 2, 4 with the same probabilities as before.

Will the standard deviation increase, decrease or stay the same?

Increase.

```
x <- c(0, 2, 4)
p <- c(0.2, 0.6, 0.2)

mu <- sum( x * p )
variance <- sum( (x - mu)^2 * p)
stdDev <- sqrt(variance)
```

$\mu = 2, \sigma^2 = 1.6, \sigma = 1.2649111$

4.15

For the probability function:

$x : 1, 2, 3, 4, 5$ $P(x) : 0.15, 0.2, 0.3, 0.2, 0.15$

Determine the mean, the variance, and the probability that a value is less than the mean.

```
x <- 1:5
p <- c(0.15, 0.2, 0.3, 0.2, 0.15)

mu <- sum( x * p)
variance <- sum( ( x - mu)^2 * p )
stdDev <- sqrt(variance)

sum( p[x < mu] )
```

```
[1] 0.35
```

$\mu = 3, \sigma^2 = 1.6, \sigma = 1.2649111$

4.16

For the probability function:

$x : 1, 2, 3, 4, 5$ $P(x) : 0.1, 0.25, 0.3, 0.25, 0.1$

Would you expect the variance to be larger or smaller than the previous pdf?

Larger.

4.17

For the probability function:

$x : 1, 2, 3, 4, 5$ $P(x) : 0.2, 0.2, 0.2, 0.2, 0.2$

Would you expect the variance to be larger or smaller than the previous pdf?

Smaller.

4.18

For the following probabilities:

Income			
Age	High	Medium	Low
< 30	0.030	0.180	0.090
30-50	0.052	0.312	0.156
Over 50	0.018	0.108	0.054

a.) The probability that someone is under 30.

.03 + 0.18 + 0.09 = .30

b.) The probability that someone has a high income given that they are under 30.

.03 / .3 = .01

c.) The probability of someone having a low income given that they are under 30.

0.09 / .3 = 0.3

d.) The probability of a medium income given that they are over 50.

0.018 + 0.108 + 0.054 = .18

.108 / .18 = .6

4.19

For the previous data, are income and age independent?

Yes.

4.20

Attitude		
Member	1	0
Yes	757	496
No	1,071	1,074

```r
d <- matrix(c(757, 496, 1071, 1074), nrow = 2, byrow = T)

prop.table(data.table(d))
```

```
         V1        V2
1 0.2227781 0.1459682
2 0.3151854 0.3160683
```

a.) Probability of boy choosing "yes".

.4

b.) P(yes|1)

.22

c.) P(1|yes)

.41

d.) is yes independent of attitude?

No, the probabilities are disproportionate

4.21

Let Y be the cost of a home and let X be a measure of the crime rate. If the variance of the cost of a home changes with X, does this mean that the cost of a home and the crime rate are dependent?

Yes, this can only happen when the conditional probabilites change when told X.

4.22

If the probability of Y < 6 is .4 given that X = 2, and if the probability of Y < 6 is .3 given that X = 4, does this mean that X and Y are dependent?

Yes.

4.23

If the range of possible Y values varies with X, does this mean that X and Y are dependent?

Absolutely.

4.24

For a binomial with n = 10 and p = .4, determine:

a.) $P(0)$

```r
dbinom(0, size = 10, prob = .4)
```

```
[1] 0.006046618
```

b.) $P(X \leq 3)$

```r
pbinom(3, size = 10, prob = .4)
```

```
[1] 0.3822806
```

c.) $P(X < 3)$

```r
pbinom(2, size = 10, prob = .4)
```

```
[1] 0.1672898
```

d.) $P(X > 4)$

```r
1 - pbinom(4, size = 10, prob = .4)
```

```
[1] 0.3668967
```

e.) $P(2 \leq X \leq 5)$

```r
pbinom(5, size = 10, prob = .4) - pbinom(1, size = 10, prob = .4)
```

```
[1] 0.787404
```

4.25

For a binomial with n = 15 and p = 0.3, determine.

a.) $P(0)$

```r
dbinom(x = 0, prob = .3, size = 15)
```

```
[1] 0.004747562
```

b.) $P(X \leq 3)$

```r
pbinom( q = 3, prob = .3, size = 15)
```

```
[1] 0.2968679
```

c.) $P(X < 3)$

```r
pbinom(2, size = 15, prob = .3)
```

```
[1] 0.1268277
```

d.) $P(X > 4)$

```r
pbinom(4, size = 15, prob = .3, lower.tail = F)
```

```
[1] 0.4845089
```

e.) $P(2 \leq X \leq 5)$

```r
pbinom(5, size = 15, prob = .3) - pbinom(1, size = 15, prob = .3)
```

```
[1] 0.6863538
```

4.26

For a binomial with n = 15, p = 0.6 determine the probability of exactly 10 successes.

```r
dbinom(10, size = 15, prob = .6)
```

```
[1] 0.1859378
```

4.27

For a binomial with n = 7 and p = 0.35, what is the probability of exactly 2 successes?

```
dbinom(2, size = 7, p = .35)
```

```
[1] 0.2984848
```

4.28

For a binomial with n = 18 and p = 0.6, determine the mean, variance of X, the total number of successes.

```
n <- 18
p <- 0.6
q <- 1 - p

mu <- n * p
variance <- mu * q
```

$\mu = 10.8, \sigma^2 = 4.32$

4.29

For a binomial with n = 22 and p = .2, determine the mean and variance of X, the total number of successes.

```
n <- 22
p <- .2
q <- 1 - p

mu <- n * p
variance <- mu * q
```

$\mu = 4.4, \sigma^2 = 3.52$

4.30

For a binomial with n = 20 and p = .7, determine the mean and variance of $\hat{p}$, the proportion of observed success.

```
n <- 20
p <- .7
q <- 1 - p

mu <- n * p
variance <- mu * q
```

4.31

For a binomial with n = 30 and p = 0.3, determine the mean and variance of $\hat{p}$.

```
n <- 30
p <- .3
q <- 1 - p

phat <- p / n
variance <- p*q / n
```

$\hat{p} = 0.01, \sigma^2 = 0.007$

4.32

For a binomial with n = 10 and p = 0.8, determine:

```
n <- 10
p <- 0.8
q <- 1 - p

variance <- p*q / n
```

a.) the probability that $\hat{p}$ is less than or equal to 0.7.

b.) the probability that $\hat{p}$ is greater than or equal to 0.8.

c.) the probability that $\hat{p}$ is exactly equal to 0.8.

4.33

A coin is rigged so that when it is flipped, the probability of a head is 0.7. If the coin is flipped three times, which is the more likely outcome, exactly three heads or two heads and a tail?

```
dbinom(3, 3, .7) # 3 heads
```

```
[1] 0.343
```

```
dbinom(2, 3, .7) # 2 heads 1 tail
```

```
[1] 0.441
```

Two heads, 1 tail.

4.34

Imagine that the probability of heads when flipping a coin is given by the binomial probability function with p = 0.5.

If you flip the coin nine times and get nine heads, what is the probability of a head on the 10th flip?

```
# independent events.
dbinom(1, 1, .5)
```

```
[1] 0.5
```

4.35

The Department of Agriculture of the United States reports that 75% of all people who invest in the futures market lose money. Based on the binomial probability function, with n = 5, determine:

a.) the probability that all 5 lose money.

$P(x) = 0$

```
dbinom(5, size = 5, prob = .75)
```

```
[1] 0.2373047
```

b.) the probability that all five make money.

```
dbinom(5, size = 5, prob = .25)
```

```
[1] 0.0009765625
```

c.) the probability that at least two lose money.

```
pbinom(q = 3, size = 5, prob = .25)
```

```
[1] 0.984375
```

4.36

If for a binomial distribution p = 0.4 and n = 25, determine:

```
n <- 25
p <- .4
q <- 1 - p
```

a.) $P(X < 11)$

```
pbinom(10, size = n, prob = p)
```

```
[1] 0.585775
```

b.) $P(X \leq 11)$

```
pbinom(11, size = n, prob = p)
```

```
[1] 0.7322822
```

c.) $P(X > 9)$

```
pbinom(9, size = n, prob = p, lower.tail = F)
```

```
[1] 0.575383
```

d.) $P(X \geq 9)$

```
pbinom(8, size = n, prob = p, lower.tail = F)
```

```
[1] 0.7264685
```

4.37

In the previous problem, determine the mean of X, the variance of X, the mean of $\hat{p}$, and the variance of $\hat{p}$.

```
mu <- n * p
variance <- mu * q

phat <- p
v <- p*q/n
```

$\mu = 10$, $\sigma^2 = 6$, $\hat{p} = 0.4$, $\sigma^2 = 0.0096$

4.38

Given that Z has a standard normal distribution, determine:

a.) $P(Z \geq 1.5)$

```
pnorm(1.5, lower.tail = F)
```

```
[1] 0.0668072
```

b.) $P(Z \leq -2.5)$

```
pnorm(-2.5)
```

```
[1] 0.006209665
```

c.) $P(Z < -2.5)$

```
pnorm(-2.5)
```

```
[1] 0.006209665
```

d.) $P(-1 \leq Z \leq 1)$

```
# P(Z > -1) - P(Z > 1)
pnorm(-1, lower.tail = F) - pnorm(1, lower.tail = F)
```

```
[1] 0.6826895
```

```
# 1 - 2*tail_area
1 - 2 * pnorm(1, lower.tail = F)
```

```
[1] 0.6826895
```

4.39

If Z has a standard normal distribution, determine:

a.) $P(Z \leq 0.5)$

```r
pnorm(0.5)
```

```
[1] 0.6914625
```

b.) $P(Z > -1.25)$

```r
pnorm(-1.25, lower.tail = F)
```

```
[1] 0.8943502
```

c.) $P(-1.2 < Z < 1.2)$

```r
1 - 2 *pnorm(1.2, lower.tail = F)
```

```
[1] 0.7698607
```

d.) $P(-1.8 \leq Z \leq 1.8)$

```r
pnorm(1.8, lower.tail = T) - pnorm(-1.8)
```

```
[1] 0.9281394
```

4.40

4.41

4.42

4.43

4.44

4.45

4.46

4.47

4.48

4.49

4.50

4.51

4.52

4.53

4.54

4.55

4.56

4.57

4.58

4.59

4.60

4.61

4.62