5.1

Consider the samples 1-6. Use a six-sided die to obtain three different bootstrap samples and their corresponding means.

```
pop <- seq(from = 1, to = 6, by = 1)

n <- 6

s1 <- mean( sample(pop, n, replace = T) )
s2 <- mean( sample(pop, n, replace = T) )
s3 <- mean( sample(pop, n, replace = T) )
```

$\bar{x}_1^* = 4.3333333, \bar{x}_2^* = 4.3333333, \bar{x}_3^* = 3.1666667$

5.2

Consider the samples 1, 3, 4, and 6 from some distribution.

```
pop <- c(1, 3, 4, 6)

samples <- permutations(n = 4, r = 4, pop, repeats.allowed = T)
```

a.) For one random bootstrap sample, find the probability that the mean is one.

```
means <- apply(samples, 1, mean)

p <- mean( means == 1 )
```

Probability: **0.39%**

b.) For one random bootstrap sample, find the probability that the maximum is 6.

```
maxes <- apply(samples, 1, max)

p <- mean( maxes == 6 )
```

Probability: **68.36%**

c.) For one random bootstrap sample, find the probability that exactly two elements in the sample are less than 2.

```
lt2 <- apply(t(apply(samples, 1, function(x) { x < 2})), 1, sum)

p <- mean( lt2 == 2 )
```

Probability: **21.09%**

5.3

Consider the sample 1-3.

a.) List all the (ordered) bootstrap samples from this sample. How many are there?

```
samples <- permutations(n = 3, r = 3, 1:3, repeats.allowed = T)

n <- nrow(samples)
```

Samples: = 3^3 = **27**

b.) How many unordered bootstrap samples are there? For example, {1, 2, 2} and {2, 1, 2} are considered to be the same.

```
samples <- combinations(n = 3, r = 3, 1:3, repeats.allowed = T)

n <- nrow(samples)
```

Samples: = 3^3 = **10**

c.) How many ordered bootstrap samples have one occurrence of 1 and two occurences of 3?

```
samples <- permutations(n = 3, r = 3, 1:3, repeats.allowed = T)

n.ones <- apply(t(apply(samples, 1, FUN = function(x) { x == 1 })), 1, function(x) sum(x) )
n.threes <- apply(t(apply(samples, 1, FUN = function(x) { x == 3 })), 1, function(x) sum(x) )

sum((n.ones == 1 & n.threes == 2) == T)
```

```
[1] 3
```

Is this the same number of bootstrap samples that have each of 1, 2 and 3 occuring exactly once?

```
n.ones <- apply(t(apply(samples, 1, FUN = function(x) { x == 1 })), 1, function(x) sum(x) )
n.twos <- apply(t(apply(samples, 1, FUN = function(x) { x == 2 })), 1, function(x) sum(x) )
n.threes <- apply(t(apply(samples, 1, FUN = function(x) { x == 3 })), 1, function(x) sum(x) )

sum((n.ones == 1 & n.twos == 1 & n.threes == 1) == T)
```

```
[1] 6
```

No, 3 != 6.

d.) Is the probability of obtaining a bootstrap sample with one 1 and two 3's the same as the probabiliity of obtaining a bootstrap sample with each of 1, 2 and 3 occuring exactly once?

```
( sum((n.ones == 1 & n.threes == 2)) / n ) == ( sum((n.ones == 1 & n.twos == 1 &  n.threes == 1
```

```
[1] FALSE
```

No, 3% and 6% chances respectfully.