Chapter 8

8.1

For the following data, use R to verify that the least squares regresion line is $\hat{Y} = 1.8X - 8.5$

X: 5, 8, 9, 7, 14 Y: 3, 1, 6, 7, 19

```
dat <- data.table(
   X = c(5, 8, 9, 7, 14),
   Y = c(3, 1, 6, 7, 19))

lm(Y ~ X, data = dat)
```

```

Call:
lm(formula = Y ~ X, data = dat)

Coefficients:
(Intercept)            X
     -8.478        1.823
```

Also verify that the Theil-Sen estimator, the slope is estimated to be 1.746 and the intercept is estimated to be -7.968.

```
tsreg(dat$X, dat$Y)$coef
```

```
Intercept
-5.730159  1.746032
```

8.2

Using the R function *lsfit*, compute the residuals using the data in E1,

Verify that if you square and sum the residusls, you get 46.585.

```
res <- lsfit(dat$X, dat$Y)$resid

sum(res^2)
```

```
[1] 46.58407
```

8.3

Verify that for the data in E1, if you use $\hat{Y} = 2X - 9$, the sum of the squared residuals is greater than 46.584.

```
Yhat <- 2*dat$X - 9
res <- sum( (dat$Y - Yhat)^2 )
```

```
stopifnot(res > 46.583)
res
```

```
[1] 53
```

Why would you expect a value greater than 46.584?

The x coefficent increased.

8.4

Suppose that based on $n = 25$ values, $s_x^2 = 12$ and $\sum(X_i - \bar{X})(Y_i - \bar{Y}) = 144.$

What is the slope of the least squares regression?

$$A = 144, C = (n-1)s_x^2 = 288, b_1 = A/C = 144/288 = .5$$

8.5

The following table reports breast cancer rates plus levels of solar radiation (in calories per day) for various cities in the United States. The data are stored in the file cancer_rate_dat.txt.

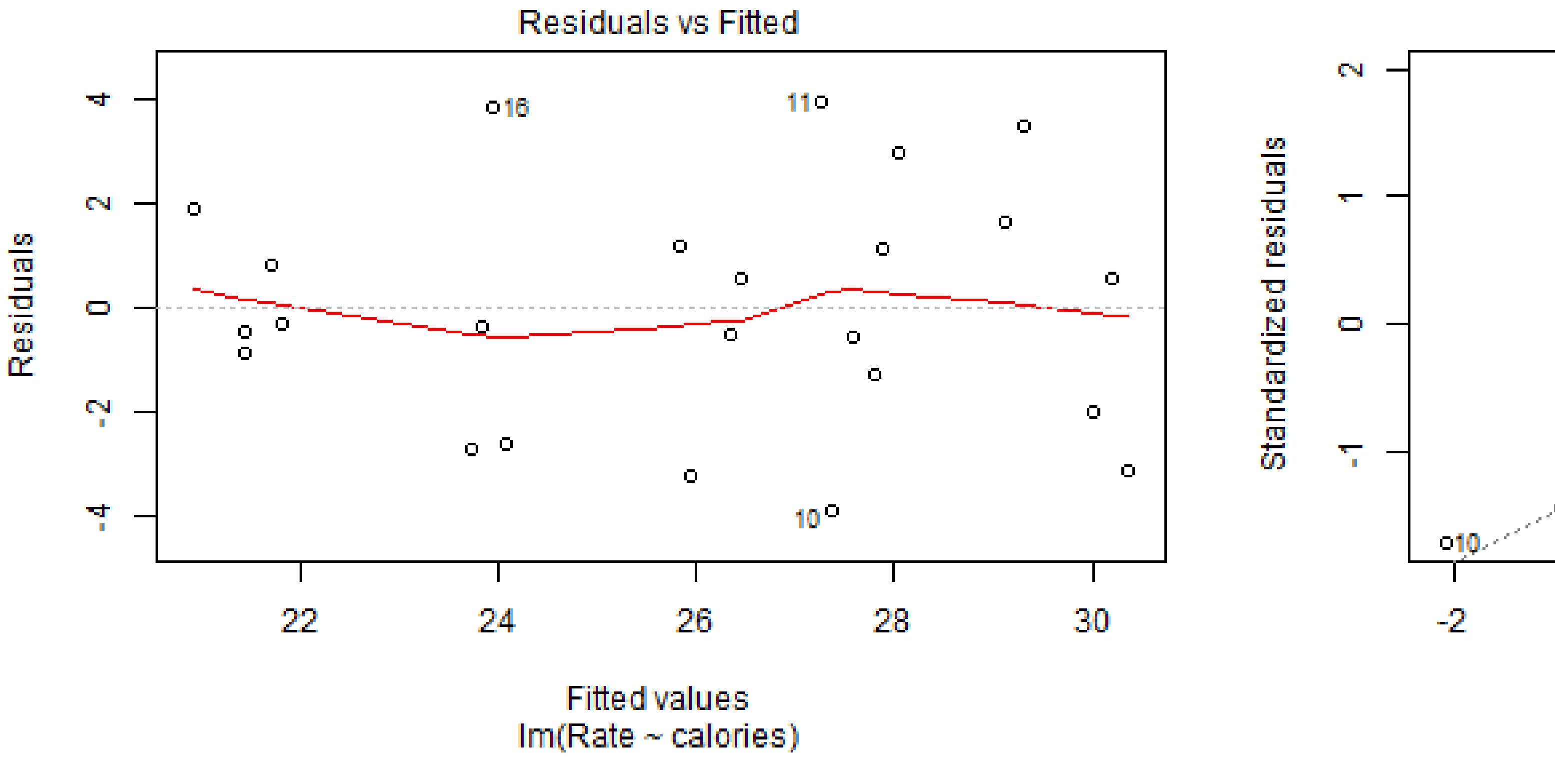
```
dat <- data.table::fread(paste0(data.dir, "cancer_rate_dat.txt"), fill = T, sep = "&")

dat
```

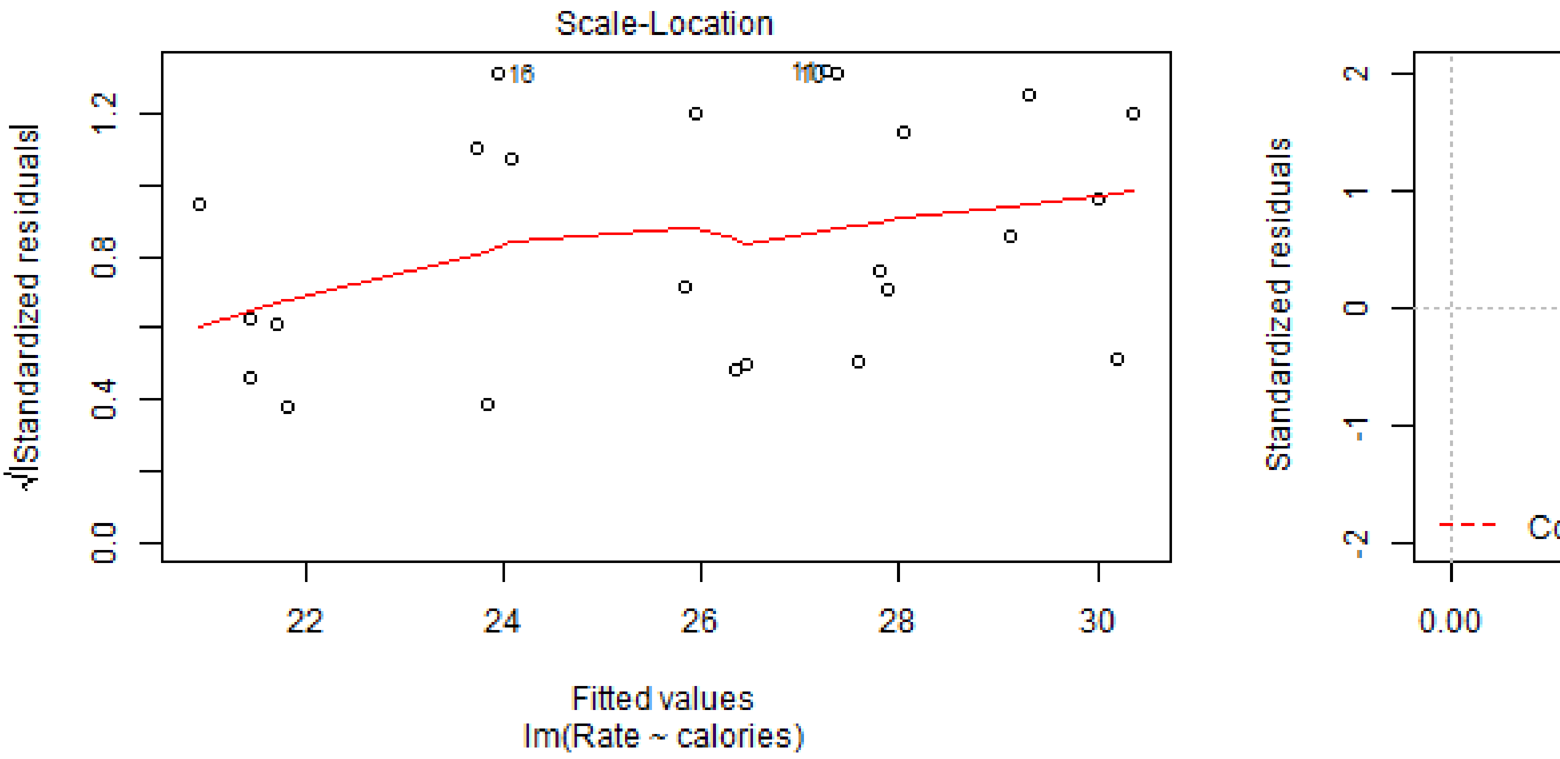
```
               City  Rate calories
 1:        New York 32.75      300
 2:         Chicago 30.75      275
 3:      Pittsburgh 28.00      280
 4:         Seattle 27.25      270
 5:          Boston 30.75      305
 6:       Cleveland 31.00      335
 7:        Columbus 29.00      340
 8:    Indianapolis 26.50      342
 9:     New Orleans 27.00      348
10:       Nashville 23.50      354
11: Washington, DC 31.20      357
12: Salt Lake City 22.70      394
13:           Omaha 27.00      380
14:       San Diego 25.80      383
15:         Atlanta 27.00      397
16:     Los Angeles 27.80      450
17:           Miami 23.50      453
18:      Fort Worth 21.50      446
19:           Tampa 21.00      456
20:     Albuquerque 22.50      513
```

```
21:       Las Vegas 21.50       510
22:        Honolulu 20.60       520
23:         El Paso 22.80       535
24:         Phoenix 21.00       520
               City  Rate calories
```

Fit a OLS regression to predict cancer rates and comment on what this suggests.

```
fit <- lm(Rate ~ calories, data = dat)

plot(fit)
```





8.6

For the following data, use R to compute the least squares regresion line for predicting GPA given SAT.

SAT: 500, 530, 590, 660, 610, 700, 570, 640 GPA: 2.3, 3.1, 2.6, 3.0, 2.4, 3.3, 2.6, 3.5

```
dat <- data.table(
   SAT = c(500, 530, 590, 660, 610, 700, 570, 640),
   GPA = c(2.3, 3.1, 2.6, 3.0, 2.4, 3.3, 2.6, 3.5)
)

fit <- lm(GPA ~ SAT, data = dat)
coef(fit)
```

```
(Intercept)         SAT
0.484615385 0.003942308
```

8.7

Compute the residuals for the data used in the previous problem and verify that sum to zero.

```
round(sum((dat$GPA - fit$fitted.values)), 5)
```

```
[1] 0
```

8.8

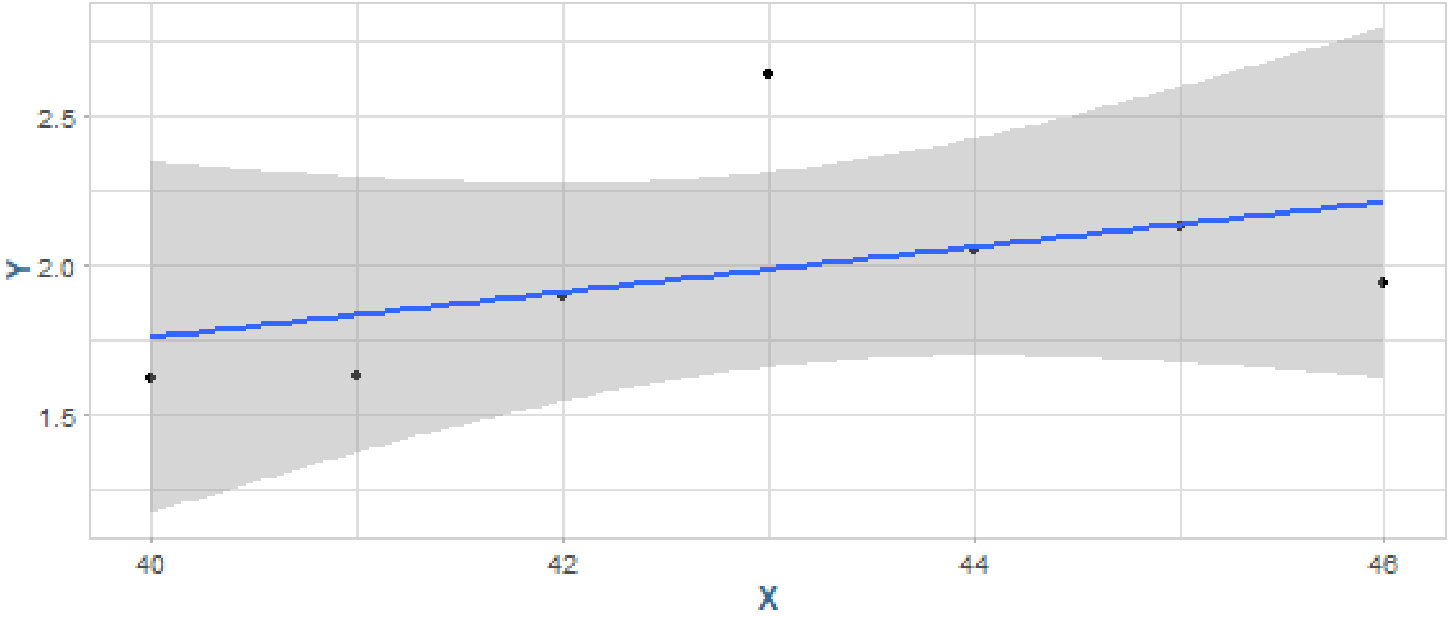
For the following data, use R to compute the least squares regression line for predicting Y from X.

X: 40, 41, 42, 43, 44, 45, 46 Y: 1.62, 1.63, 1.90, 2.64, 2.05, 2.13, 1.94

```
dat <- data.table(
   X = c(40, 41, 42, 43, 44, 45, 46),
   Y = c(1.62, 1.63, 1.90, 2.64, 2.05, 2.13, 1.94)
)

summary(fit <- lm(Y ~ X, data = dat))
```

```

Call:
lm(formula = Y ~ X, data = dat)

Residuals:
        1         2         3         4         5         6         7 
-0.141071 -0.206429 -0.011786  0.652857 -0.012500 -0.007857 -0.273214 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.25321    2.73157  -0.459    0.666
X            0.07536    0.06346   1.188    0.288

Residual standard error: 0.3358 on 5 degrees of freedom
Multiple R-squared:   0.22, Adjusted R-squared:  0.064 
F-statistic:  1.41 on 1 and 5 DF,  p-value: 0.2883
```

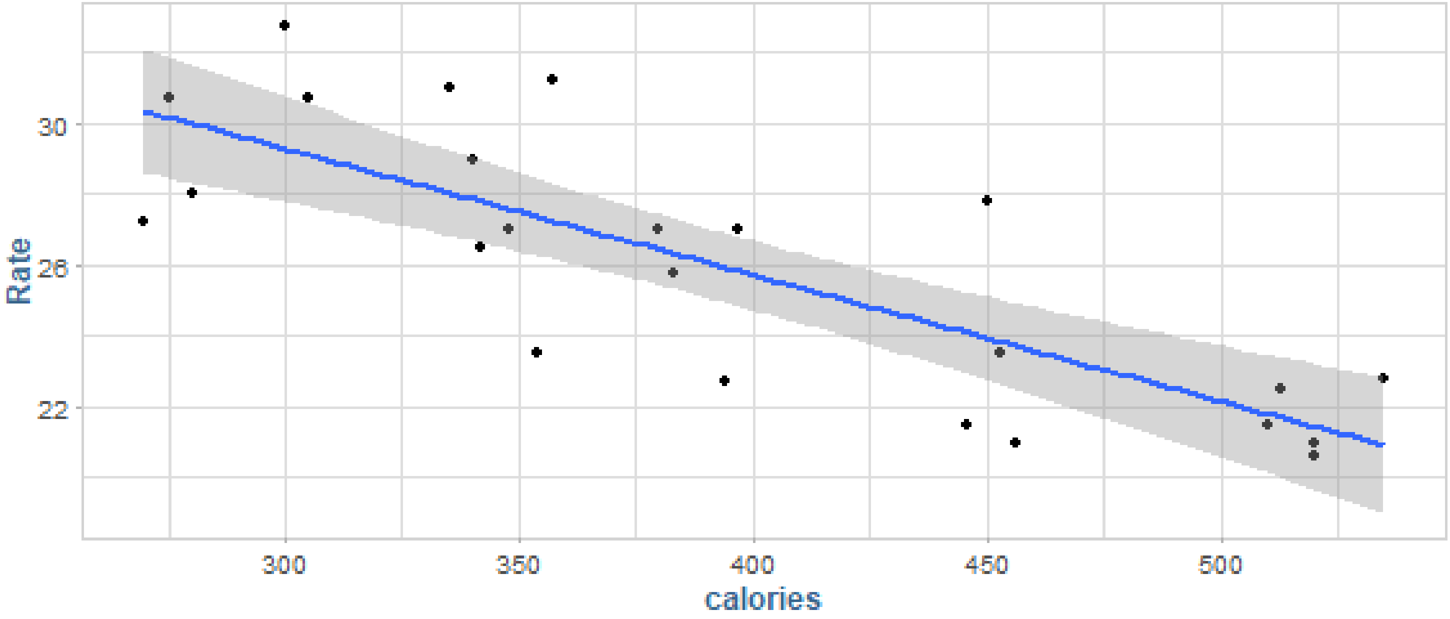
```
ggplot(dat, aes(X, Y)) +
   geom_point() +
   geom_smooth(method = "lm")
```



8.9

In exercise 5, what would be the least squares estimate of the cancer rate given a solar radiation of 600?

```
dat <- data.table::fread(paste0(data.dir, "cancer_rate_dat.txt"), fill = T, sep = "&")

ggplot(dat, aes(calories, Rate)) +
   geom_point() +
   geom_smooth(method = "lm")
```



```
fit <- lm(Rate ~ calories, data = dat)

coef(fit)
```

```
(Intercept)    calories 
39.99094634 -0.03565283 
```

```
39.99 -0.037*600
```

```
[1] 17.79
```

Why might this be unreasonable?

Because 600 is outside of the bounds of seen values (extrapolation).

8.10

8.21

8.22

8.23

8.24

8.25

8.26

8.27

8.28

8.29

8.30

8.31

8.32

8.33

8.34

8.35

8.36