The Bayesian Way

```
# utility function for plotting a posterior

plot_posterior <- function(bsim) {
  res <- data.table(mu = bsim@coef, sigma = bsim@sigma)
  colnames(res) <- c("mu", "sigma")

  pmain <- ggplot(res, aes(mu, sigma)) +
    geom_point(alpha = .65, col = "darkgrey") +
    labs(x = "", y = "") +
    stat_density_2d(aes(fill = stat(level), alpha = ..level..), geom = "polygon")

  xbox <- axis_canvas(pmain, axis = "x") +
    geom_histogram(data = res, aes(mu, y = ..density.., fill = ..count..), size = .2, bins = 5
    geom_density(data = res, aes(mu), col = "darkgrey", alpha = .85) +
    theme(legend.position = "none")

  ybox <- axis_canvas(pmain, axis = "y", coord_flip = T) +
    geom_histogram(data = res, aes(sigma, y = ..density.., fill = ..count..), size = .2, bins
    geom_density(data = res, aes(sigma), col = "darkgrey", alpha = .85) +
    coord_flip() +
    theme(legend.position = "none")

  suppressWarnings({
    p1 <- insert_xaxis_grob(pmain, xbox, grid::unit(.2, "null"), position = "top")
    p2 <- insert_yaxis_grob(p1, ybox, grid::unit(.2, "null"), position = "right")
  })

  ggdraw(p2)
}
```

Bayes Therom

$P(\theta|y) = \frac{P(\theta)P(y|\theta)}{P(y)}$

Estimating the Mean

The model of the data is:

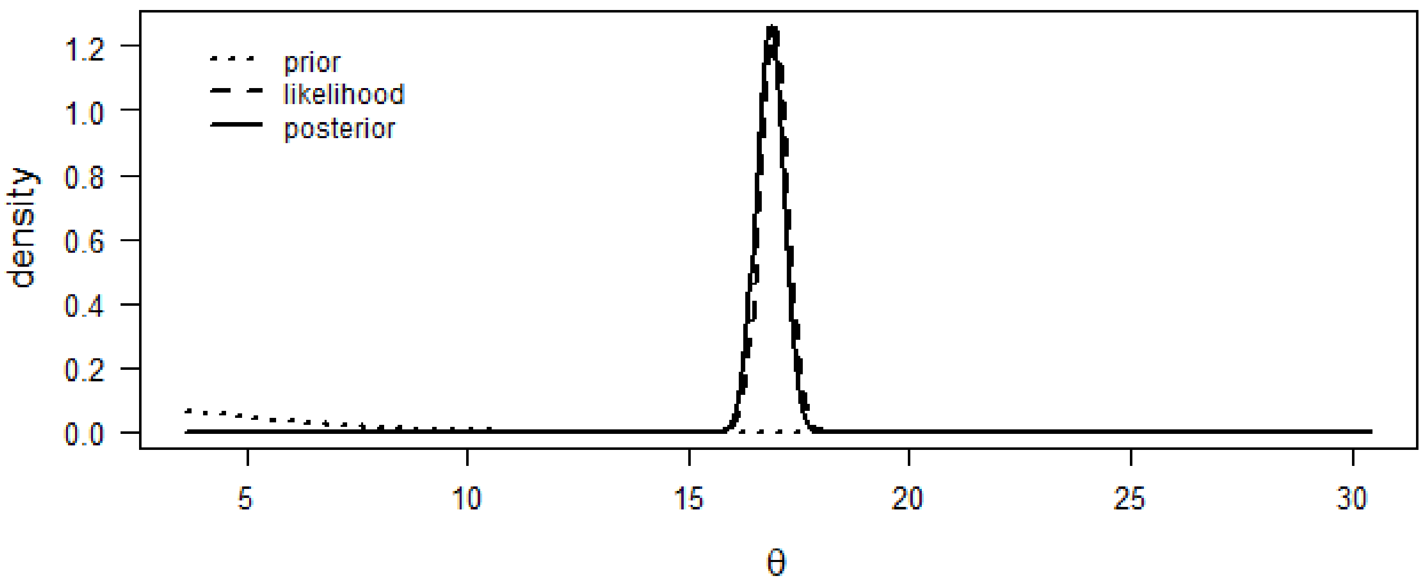
$y \sim Norm(\theta, \sigma^2)$

Given the data has three measurements,

$y_1 = 27.1, y_2 = 14.6, y_3 = 14.6$

$\sigma^2 = 20$

```
n <- 200

x <- rnorm(n, mean = 17, sd = .5)
triplot.normal.knownvariance(n=n, theta.data = x, variance.known = 20, prior.theta = 0, prior.v
```




```
$posterior.mean
[1] 16.83473

$posterior.variance
[1] 0.0994152
```

```
$x
  [1] 17.258372 16.831426 17.647787 14.464943 21.693290 14.073198 14.142938
  [8] 26.488045 14.220806 24.429206 19.941599 18.731802 16.561183 22.466308
 [15] 16.965709 16.639374 24.257867 19.791178 15.354534 15.682744 19.156921
 [22] 22.958277 10.122966 18.241888 18.188589 22.161871 16.448033 16.114960
 [29] 23.117066 12.098331  9.833202 17.194074 11.852056 15.168891 20.301470
 [36] 24.013888 13.998450 19.674567 22.913687 17.827256  9.045690 13.293926
 [43] 15.437994 14.446031 15.345662 22.055755 26.656163 19.377686 14.877305
 [50] 12.141283 19.877618 19.965275 18.238791 20.684483 22.156126 24.636169
 [57]  7.134514 18.487195 14.968464 14.241076 18.881224 10.795125  9.703999
 [64] 14.057318 15.724782 15.246420 26.998129 12.352180 15.692438 15.499390
 [71] 25.056989 17.802133 16.022754 14.384516 16.667176 16.159587 16.457771
 [78]  8.567717 12.144057 12.637865 18.022268 16.193526 12.899744 21.108026
 [85] 23.954511 20.422121 25.809136 14.151558 12.546424 14.598275 16.469266
 [92] 18.503788 17.323361 20.143510 18.065488 13.221952 23.332854 19.921262
 [99] 13.878530 15.524281 11.816388 17.853027  8.961025 20.220152 22.051239
[106] 15.815864 16.294106  6.697379 19.343343 21.000153 11.054982 14.087158
[113] 20.791379 16.532916  9.191497 18.899369 22.190406 16.500571 18.325354
[120] 17.854719 20.983257 15.710101 20.521102 12.908412 28.651680 22.086939
[127] 23.642810 21.925471 13.073041 18.638566 14.177240 13.286624 18.015564
[134] 14.685223 22.501519 12.552534 25.432241 16.450641 10.295349  9.459107
[141] 18.225397  9.459095  9.481256 10.537706 18.237554 23.812443 11.243861
[148] 25.670347 15.765362 12.490997 17.277952 21.047589 18.012128 13.838623
[155] 20.154620 16.428595 19.439234 14.654432 22.650653 19.479447 11.930002
[162] 14.930436 11.004486 12.023318 16.828710 17.968702 13.810529 17.950133
[169] 16.541024 13.169008 22.958646 14.016201  8.005813 20.021008 24.896082
[176] 10.564477 18.728342 23.124659 15.321027 13.681014 18.115127 18.198922
[183] 10.720993  9.162371 18.077181 12.210352 14.921375 13.660769 11.583069
[190] 18.175927 20.526024 20.507284 17.561385 12.357303 16.048825 15.095235
[197] 17.834052 18.774147 17.406850 20.918414
```

Estimating Mean and Variance using Simulating

```r
# Simulate hypothetical body height measurements

true.mean <- 165
true.sd <- 10
y <- round(rnorm(10, mean = true.mean, sd = true.sd))

mod <- lm(y ~ 1)
mod
```

```
Call:
```

```
lm(formula = y ~ 1)

Coefficients:
(Intercept)
        164
```

```r
summary(mod)$sigma
```

```
[1] 7.45356
```

$y \sim Norm(\hat{\theta} = 166.3, \sigma^2 = 13.3)$

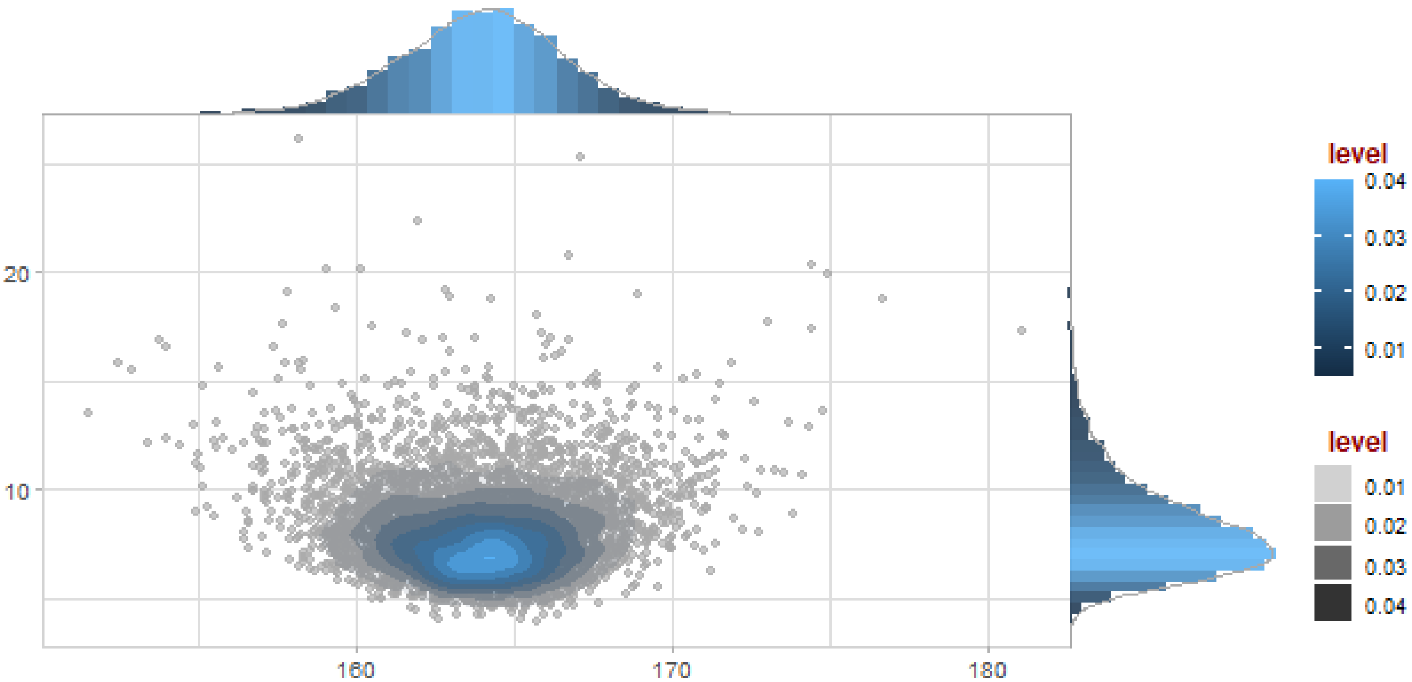
```r
nsim <- 5000
bsim <- sim(mod, n.sim=nsim)

str(bsim)
```

```
Formal class 'sim' [package "arm"] with 2 slots
  ..@ coef : num [1:5000, 1] 168 158 170 162 164 ...
  .. ..- attr(*, "dimnames")=List of 2
  .. .. ..$ : NULL
  .. .. ..$ : chr "(Intercept)"
  ..@ sigma: num [1:5000] 7.35 11.12 13.62 6.94 7.85 ...
```

```r
str(bsim, max.level = 2)
```

```
Formal class 'sim' [package "arm"] with 2 slots
  ..@ coef : num [1:5000, 1] 168 158 170 162 164 ...
  .. ..- attr(*, "dimnames")=List of 2
  ..@ sigma: num [1:5000] 7.35 11.12 13.62 6.94 7.85 ...
```

```r
plot_posterior(bsim)
```




```r
quantile(bsim@coef, prob = c(.025, 0.5, 0.975))
```

```
    2.5%      50%    97.5% 
158.5635 164.0392 169.2480 
```

```r
quantile(bsim@sigma, prob = c(0.025, 0.5, 0.975))
```

```
     2.5%       50%     97.5% 
 5.156631  7.769071 13.624715 
```

```r
HPDinterval(as.mcmc(bsim@coef))
```

```
               lower    upper
(Intercept) 158.5454 169.2123
attr(,"Probability")
[1] 0.95
```

```r
sum(bsim@coef > 160) / nsim
```

```
[1] 0.9316
```

```r
mean(bsim@coef > 160)
```

```
[1] 0.9316
```

```r
cvsim <- bsim@sigma / bsim@coef
quantile(cvsim, prob = c(0.025, 0.5, 0.975))
```

```
      2.5%         50%       97.5%
```

```
0.03143427 0.04739495 0.08268764
```

The Frequentist Way