Week 8: Portfolio Optimizaton

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Preliminaries

- A portfolio consists of N financial instruments (e.g. stocks, futures, currencies) or *strategies.*
- Each instrument or strategy i has its own expected net return $\mathrm{E}(R_i) \coloneqq m_i \coloneqq$**M**, stddev of returns sd$(R_i) = \mathrm{E}((R_i - m_i)^2)^{1/2} \coloneqq s_i$.
- Important to consider: *covariance* of returns $C_{i,j} = \boldsymbol{C}$ among all instruments i, j.
  - **M** is $N \times 1$, $\boldsymbol{C}$ is $N \times N$
- Capital allocated to instrument i is F_i.
  - $\boldsymbol{F}$ is the $N \times 1$ capital allocation vector.

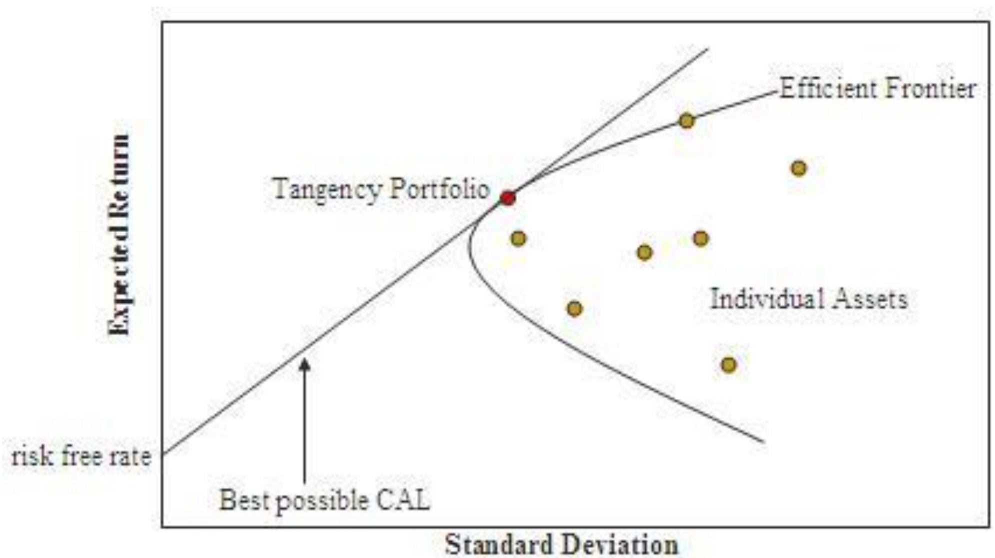
Preliminaries

- Expected return of portfolio is $\boldsymbol{F^T M}$
- Expected variance of returns for portfolio is $\boldsymbol{F^t C F}$

What is Portfolio Optimization?

- Markowitz portfolio optimization: find F that minimizes variance of return of a portfolio subject to an expected return constraint.

The Efficient Frontier (Courtesy of Wikimedia Commons)



Markowitz Portfolio Optimization?

- But what is the best *trade-off* between expected return and variance?
  - Markowitz says we should prefer the "tangency portfolio" that maximizes Sharpe ratio (i.e. expected return / volatility) … but why?
  - The reason: this portfolio has maximum compounded growth rate under the constraint of a fixed leverage!

Maximizing expected compound returns

- Recall from Week 2 exercise, expected compound growth rate=expected log return=$(m - s^2/2)$
  - m=expected net return, s=standard deviation
- For a portfolio, $m = F^T M, s^2 = F^t C F$
  - We used to solve the constrained maximization problem by finding the efficient frontier by numerical quadratic programming.
  - But analytically, maximizing $F^T M - F^t C F/2$ means taking partial derivative of this w.r.t. F_i and setting it to 0.
    - Note this is equivalent to constrained optimization by Lagrange Multipliers! (See reference on next slide.)
  - Solution: $\boldsymbol{F}^* = \boldsymbol{C}^{-1}\boldsymbol{M}$ (Kelly formula!)

Connection with Markowitz Portfolio Optimization

- It can be shown that the Kelly portfolio allocation formula results in the tangency portfolio of the efficient frontier.
  - Rf: epchan.blogspot.com/2014/08/kelly-vs-markowitz-portfolio.html
- Note that this F^* maximizes both the compound growth rate of the value of the portfolio *and* the Sharpe ratio.
- F^* also tells the what the optimal leverage is, whereas tangency portfolio doesn't.

Limitations of portfolio optimization

- Returns of stocks are notoriously difficult to predict.
  - Optimal portfolio based on historical returns may not be optimal out-of-sample.
- Covariance easier to predict than expected returns.
  - Estimates of mean do not improve with larger sample size, while estimates of covariance do.
  - See Ang, "Asset Management: A systematic Approach to Factor Investing"
- Hence minimum variance portfolio more stable than tangency portfolio.

Minimum variance portfolio

- Research also shows low volatility stocks outperform high volatility stocks out-of-sample.
  - See people.stern.nyu.edu/jwurgler/papers/faj-benchmarks.pdf
- In the last 20 years, investors have increasingly favored minimum variance over tangency portfolio.
  - E.g. Minimum volatility ETFs: SPLV (even correlations are presumed 0), ACWV, EEMV, and USMV.

Minimum variance portfolio

- Many investors distrust even covariance forecasts:
  - Allocate capital based just on inverse volatilities.
  - “Risk parity” portfolios.
  - E.g. Bridgewater Associates’ All Weather Fund
    - Blamed for 2015 Q4 financial contagion.
    - Have you read Ray Dalio’s bestseller “Principles”?
- Why should we fear volatilities?
  - Tail risks and drawdowns more scary.
  - Should minimize different risk measures e.g. VaR, ES, tail index, max drawdown, etc.

Hierarchical Risk Parity (HRP)

- Addresses shortcomings of Markowitz's portfolio, esp. poor OOS performance.
- Applies graph theory, machine learning, to build diversified portfolio based on covariance matrix.
- Invented by Marcos López de Prado who generated *multi-billion-dollar profits* for investors in Guggenheim Partners.
- papers.ssrn.com/sol3/papers.cfm?abstract_id=2708678