Problem Background

The following code block computes the MLE for a bivariate t-distribution fit to CRSP returns data.

```
data(CRSPday, package = "Ecdat")

Y = CRSPday[, c(5,7)]

loglik = function(par) {

  mu = par[1:2]

  A = matrix( c(par[3], par[4], 0, par[5] ),
              nrow=2, byrow=T )

  scale_matrix = t(A) %*% A

  df = par[6]

  -sum(log(dmt(Y,mean=mu,S=scale_matrix,df=df)))
}

A = chol(cov(Y))

start = as.vector( c(apply(Y,2,mean), A[1,1], A[1,2], A[2,2], 4) )

fit_mvt = optim( start,
                 loglik,
                 method="L-BFGS-B",
                 lower=c(-0.02,-0.02,-0.1,-0.1,-0.1,2),
                 upper=c(+0.02,+0.02,+0.1,+0.1,+0.1,15),
                 hessian=T )

params = fit_mvt$par
params.disp <- round(params, 7)
```

Let $\hat{\theta} = (\mu_1, \mu_2, A_{1,1}, A_{1,2}, A_{2,2}, \upsilon)$, where μ_j is the mean of the $ijth$ variable, $A_1, A_2, and A_3$ are the nonzero element of A, and υ is the degrees of freedom parameter.

Problem 1

***What does the code A = chol(cov(Y)) do*?**

cov(Y) computes the covariate matrix for *IBM* and *CRSP* returns in the CSPday dataset (from the Ecdat package).

The covariate matrix is then passed into **chol**, which computes the "Square Root" of the cov matrix using the Cholesky factorization method.

The resulting 2x2 matrix (since we have 2 stocks in this data set) is then stored in **A**.

Problem 2

Find $\hat{\theta}_{ML}$, the MLE of $\hat{\theta}$.

Table 1: θ_{ML} parameter estimates

μ_1	μ_2	A_1	A_2	A_3	υ
0.0003789	0.0008317	0.0126907	0.0026859	0.0051011	4.261839

Problem 3

Find the MLE of the covariance matrix of the returns.

```
A = matrix( c(params[3], params[4], 0, params[5] ),
            nrow=2, byrow=T )

df <- params[6]
M.cov <- df / ( df - 2 ) * crossprod(A)

kable(format(round(M.cov, 7), scientific = F), "latex", caption = "Covariate Matrix", booktabs
  kableExtra::kable_styling(latex_options = "hold_position")
```

Table 2: Covariate Matrix

0.0003035	0.0000642
0.0000642	0.0000626

Problem 4

Find the MLE of p, the correlation between the two returns ($Y_1 and Y_2$).

```
p <- M.cov[1,2] / sqrt(M.cov[1,1]*M.cov[2,2])

M.cor <- matrix(c(1, p, p, 1), nrow = 2)

kable(format(round(M.cor, 7), scientific = F), caption = "MLE p, Correlation Matrix") %>%
  kableExtra::kable_styling(latex_options = "hold_position")
```

Table 3: MLE p, Correlation Matrix

1.0000000	0.4659025
0.4659025	1.0000000