509-HW7

Q1

1. The daily adjusted closing price data for Amazon, Apple, and NASDAQ over the past year are in Data folder on Canvas.

(a) Utilizing a regression analysis, determine the α, β of both Amazon and Apple relative to the NASDAQ index, and quantify what portion of the variance each of these two assets is market variance. Assume that the risk-free rate of return as 2.5% – or return per trading day is $.025/250 = 0.001$

From the summary below, we can notice that $\alpha_{AAPL} = 0.0003006$, $\beta_{AAPL} = 1.1819814$.63.63% cariance of AAPL is market variance. $\alpha_{AMZN} = 0.0003848$, $\beta_{AMZN} = 1.5683034$.77.12% cariance of AMZN is market variance.

```r
AAPL <- read.csv("AAPL-Mar21_2018_Mar20_2019.csv")
AMZN <- read.csv("AMZN-Mar21_2018_Mar20_2019.csv")
NASDAQ <- read.csv("NASDAQ-Mar21_2018_Mar20_2019.csv")
show(c(nrow(AAPL), nrow(AMZN), nrow(NASDAQ)))
```

```
## [1] 251 251 251
```

```r
AAPL_ret <- diff(AAPL$Adj.Close) / AAPL$Adj.Close[1:nrow(AAPL)-1]
AMZN_ret <- diff(AMZN$Adj.Close) / AMZN$Adj.Close[1:nrow(AMZN)-1]
NASDAQ_ret <- diff(NASDAQ$Adj.Close) / NASDAQ$Adj.Close[1:nrow(NASDAQ)-1]
rf <- 0.025/250

ex_r_aapl <- AAPL_ret - rf
ex_r_amzn <- AMZN_ret - rf
ex_r_nasdaq <- NASDAQ_ret - rf
var_nasdaq <- var(NASDAQ_ret)
var_aapl <- var(AAPL_ret)
var_amzn <- var(AMZN_ret)
fitAAPL <- lm(ex_r_aapl ~ ex_r_nasdaq)
summary(fitAAPL)
```

```
##
## Call:
## lm(formula = ex_r_aapl ~ ex_r_nasdaq)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.063994 -0.005351 -0.000042  0.005366  0.053158
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0003006  0.0007364   0.408    0.683
## ex_r_nasdaq 1.1819814  0.0567465  20.829   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01164 on 248 degrees of freedom
## Multiple R-squared:  0.6363, Adjusted R-squared:  0.6348
## F-statistic: 433.9 on 1 and 248 DF,  p-value: < 2.2e-16
```

```
fitAMZN <- lm(ex_r_amzn ~ ex_r_nasdaq)
summary(fitAMZN)
```

```
##
## Call:
## lm(formula = ex_r_amzn ~ ex_r_nasdaq)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.050298 -0.005480 -0.000138  0.005896  0.035434
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0003848  0.0007038   0.547    0.585
## ex_r_nasdaq 1.5683034  0.0542374  28.916   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01113 on 248 degrees of freedom
## Multiple R-squared:  0.7712, Adjusted R-squared:  0.7703
## F-statistic: 836.1 on 1 and 248 DF,  p-value: < 2.2e-16
```

```
Prop_AAPL <- fitAAPL$coef[2]^2 * var_nasdaq / var_aapl
Prop_AMZN <- fitAMZN$coef[2]^2 * var_nasdaq / var_amzn
show(c(Prop_AAPL, Prop_AMZN))
```

```
## ex_r_nasdaq ex_r_nasdaq
##   0.6362858   0.7712403
```

(b) Assuming independent regression errors and assuming the regression models in (a), compute the covariance between Amazon and Apple returns. Derive the empirical covariance between Apple and Amazon, and compare to this regression-derived covariance.

From below, covariance between them is 0.0003048114, while based on rgression, the covariance is 0.0003133438, which is a little larger.

```
cov(AAPL_ret, AMZN_ret)
```

```
## [1] 0.0003048114
```

```
fitAAPL$coef[2] * fitAMZN$coef[2] * var_nasdaq
```

```
##  ex_r_nasdaq
## 0.0003133438
```

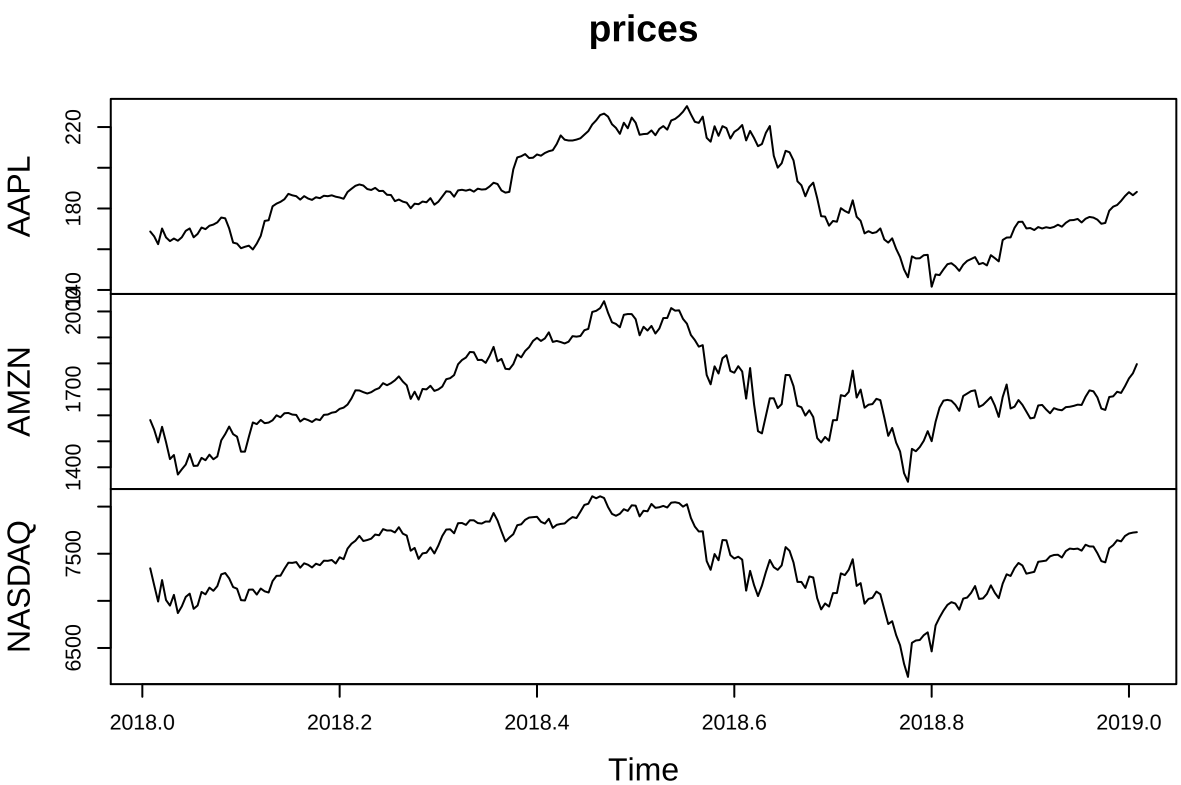
Q2

2. Consider the time series from problem 1.

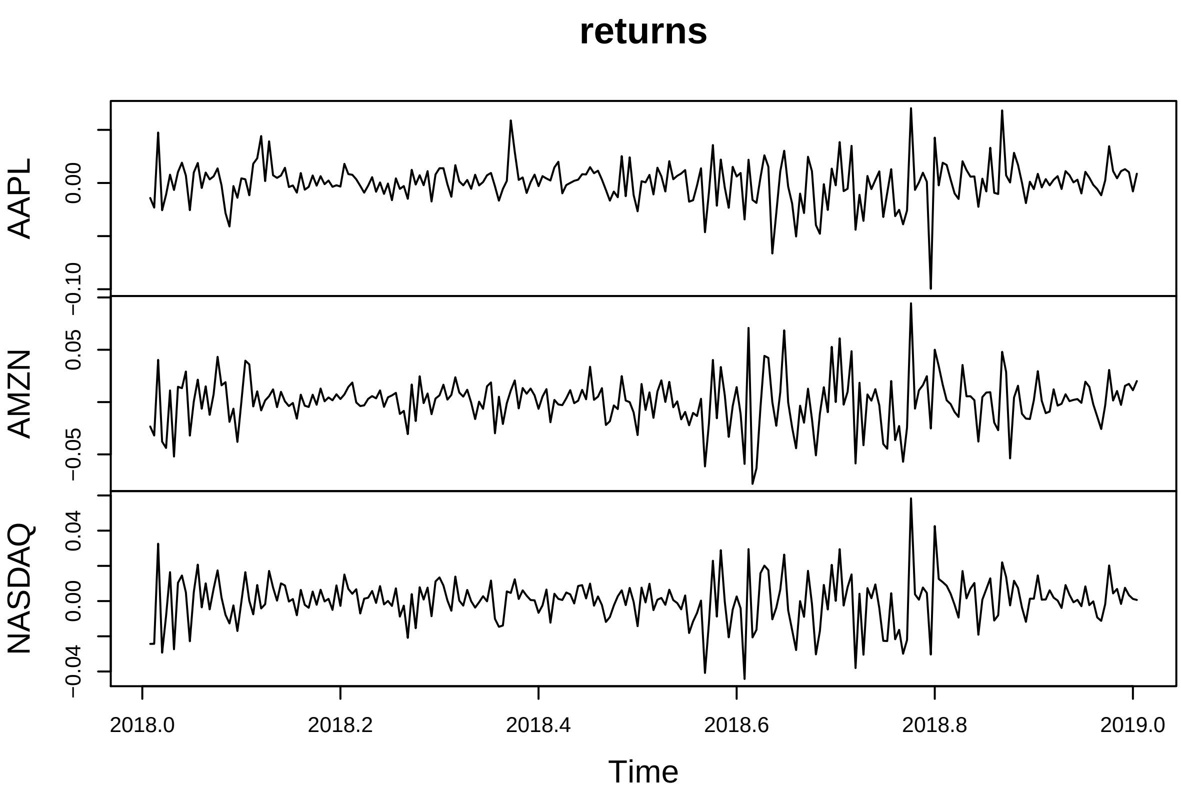
(a) Put the data into a R-time series object, and show time-series plots of the adjusted closing prices and the returns for each of assets Amazon, Apple, and NASDAQ. Discuss what the plots show for each of these assets.

The price and return plots looks similar, meaning stock prices of both AAPL and AMZN seems to highly related to the NASDAQ, and so do their returns.

```
prices <- ts(cbind(AAPL=AAPL$Adj.Close, AMZN=AMZN$Close, NASDAQ=NASDAQ$Adj.Close), start=c(2018, 3), fr
plot(prices)
```




```
returns <- ts(cbind(AAPL=AAPL_ret,AMZN=AMZN_ret, NASDAQ=NASDAQ_ret), start=c(2018, 3),frequency = 250)
plot(returns)
```



(b) Compute/plot the auto-correlation functions for the NASDAQ, Apple, and Amazon ad- justed closing prices over the past year. Discuss the plot and what it indicates about whether the closing prices are stationary.

From the ACF plts, the sticks are all outside of the confidence inverval, and it is also confirmed by the result of Box-Ljung test that p-value is small, meaning the closing prices are not stationary.

```r
par(mfrow=c(2, 2))
acf(as.vector(AAPL$Adj.Close), lag=20, main='NASDAQ.adj')
acf(as.vector(AMZN$Adj.Close), lag=20, main='NASDAQ.adj')
acf(as.vector(NASDAQ$Adj.Close), lag=20, main='NASDAQ.adj')

Box.test(AAPL$Adj.Close, lag=20, type='Ljung')
```

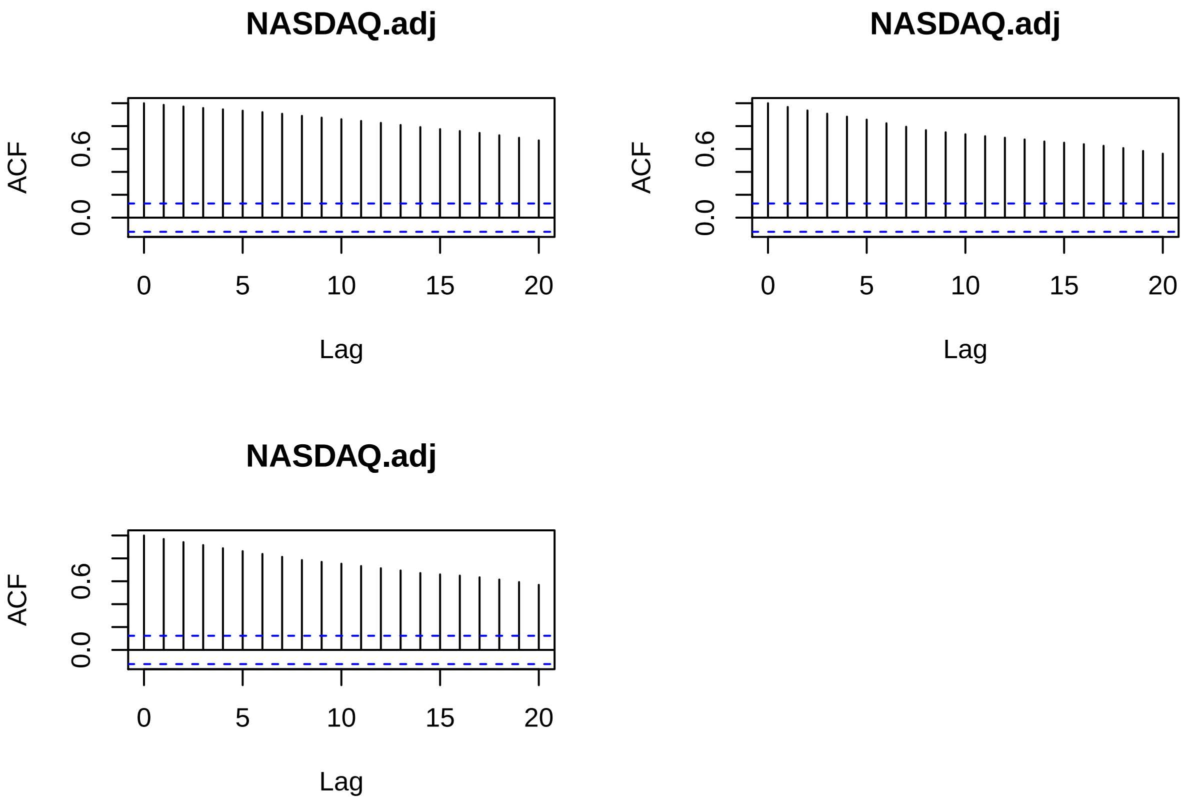
```
##
##  Box-Ljung test
##
## data:  AAPL$Adj.Close
## X-squared = 3795, df = 20, p-value < 2.2e-16
```

```r
Box.test(AMZN$Adj.Close, lag=20, type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  AMZN$Adj.Close
## X-squared = 2963.5, df = 20, p-value < 2.2e-16
```

```r
Box.test(NASDAQ$Adj.Close, lag=20, type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  NASDAQ$Adj.Close
## X-squared = 3053.5, df = 20, p-value < 2.2e-16
```



(c) Compute/plot the auto-correlation functions for the NASDAQ, Apple, and Amazon re- turns over the past year. Discuss the plot and what it indicates about whether the returns are stationary.

From the ACF plts, the sticks is outside of the confidence inverval of lag 18 for AAPL, 8 for AMAZON, and 7, 14 for NASDAQ. Check the result of Box-Ljung test, both AAPL and AMAZON have large p-value (0.4929, 0.2899), so we can reject the null that they are not stationary. As for NASDAQ, given the plot, and p-value = 0.08811, we might say NASDAQ is also stantioanry but not as significant as the other two stocks.

Interesting things is that by adjusting returns to absolute value, all of them are not stationary, which is indicated by both plots and quite small p-value of Box-Ljung test.

```r
par(mfrow=c(2, 2))
acf(as.vector(AAPL_ret), lag=20, main='NASDAQ.ret')
acf(as.vector(AMZN_ret), lag=20, main='NASDAQ.ret')
acf(as.vector(NASDAQ_ret), lag=20, main='NASDAQ.ret')

Box.test(AAPL_ret, lag=20, type='Ljung')
```

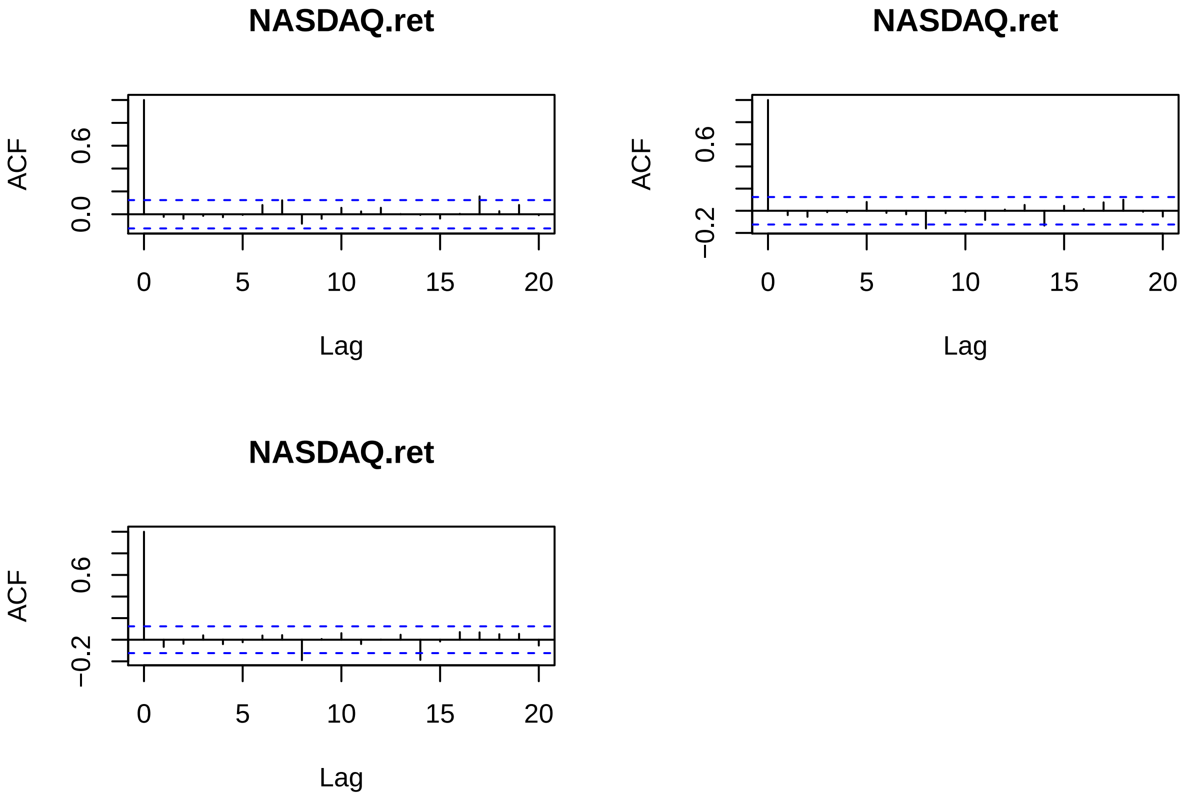
```
##
##  Box-Ljung test
##
## data:  AAPL_ret
## X-squared = 19.448, df = 20, p-value = 0.4929
```

```r
Box.test(AMZN_ret, lag=20, type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  AMZN_ret
## X-squared = 22.977, df = 20, p-value = 0.2899
```

```r
Box.test(NASDAQ_ret, lag=20, type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  NASDAQ_ret
## X-squared = 28.982, df = 20, p-value = 0.08811
```




```r
par(mfrow=c(2, 2))
acf(as.vector(abs(AAPL_ret)), lag=20, main='NASDAQ.ret')
acf(as.vector(abs(AMZN_ret)), lag=20, main='NASDAQ.ret')
acf(as.vector(abs(NASDAQ_ret)), lag=20, main='NASDAQ.ret')


Box.test(abs(AAPL_ret), lag=20, type='Ljung')
```

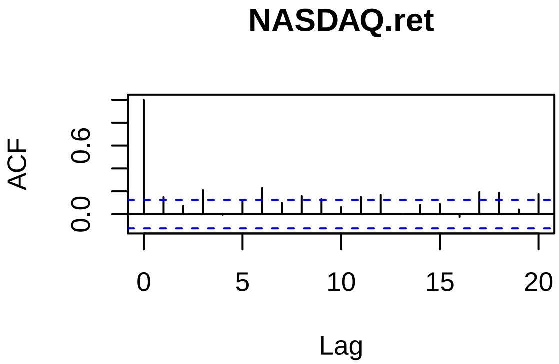
```
##
##  Box-Ljung test
##
## data:  abs(AAPL_ret)
## X-squared = 96.218, df = 20, p-value = 5.948e-12
```

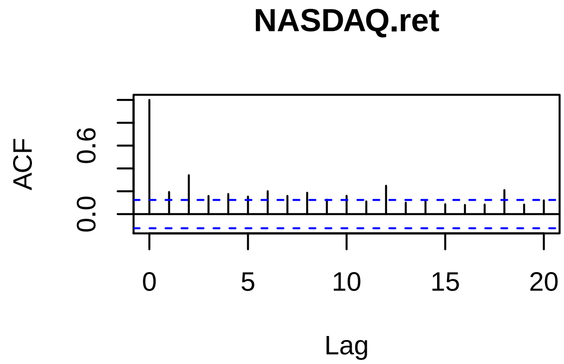
```r
Box.test(abs(AMZN_ret), lag=20, type='Ljung')
```

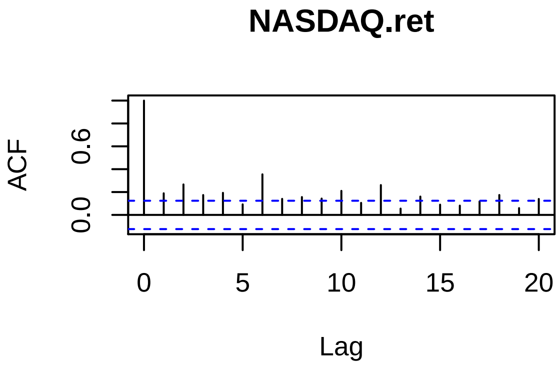
```
##
##  Box-Ljung test
##
## data:  abs(AMZN_ret)
## X-squared = 145.59, df = 20, p-value < 2.2e-16
```

```r
Box.test(abs(NASDAQ_ret), lag=20, type='Ljung')
```

```
##
##  Box-Ljung test
##
## data:  abs(NASDAQ_ret)
## X-squared = 158.63, df = 20, p-value < 2.2e-16
```







Q3

3. Suppose U,V are 2 independent uniform Uniform(0,2π) random variables and suppose Y1,Y2,… and Z1,Z2,… are 2 stationary process with means μ_Z, μ_Y and autocovariances γ_Z, γ_Y, which are independent of each other and from U, V .Also, Y_n is ergodic and has a mean mu_Y= 0.

(a) Show that Xn = Yn + Zn is a stationary process and derive its mean, auto-covariance, and auto-correlation functions.

We know that $Y_n,\ and\ Z_n$ are indep and stationary, we have $E(X_n) = E(Y_n + Z_n) = \mu_Z + \mu_Y = \mu_Z$;

$Var(X_n) = Var(Y_n + Z_n) = \gamma_Y(0) + \gamma_Z(0)$ which is a constant.

Auto-covariance $\gamma_X(t-s) = Cov(X_t, X_s) = Cov(Y_t+Z_t, Y_s+Z_s)$ by independence, it equals to $Cov(Y_t, Y_s)+Cov(Z_t, Z_s) = \gamma_Z(t-s) + \gamma_Y(t-s)$. So $\gamma_X(h) = \gamma_Y(h) + \gamma_Z(h)$

auto-correlation $\rho_X(t-s) = Corr(X_t, X_s) = Corr(X_t, X_s)/\sqrt{Var(X_t) * Var(X_s)} = (\gamma_Z(t-s) + \gamma_Y(t-s))/(\gamma_Y(0) + \gamma_Z(0))$ So $\rho_X(h) = (\gamma_Y(h) + \gamma_Z(h))/(\gamma_Y(0) + \gamma_Z(0))$ By definition, as the mean and covariance function have the same form, we can know that Xn = Yn + Zn is a stationary.

(b) Show that $Xn' = Yn * Zn$ is stationary process and derive its mean and auto-covariance functions. By independetn, $E(X_n) = E(Y_n * Z_n) = E(Y_n) * E(Z_n) = \mu_Z * \mu_Y = 0$; $Var(X_n) = Var(Y_n * Z_n) = \gamma_Y(0) * \gamma_Z(0)$ which is a constant.

Auto-covariance $\gamma_X(t-s) = Cov(X_t, X_s) = E(X_t X_s) - E(X_t)E(X_s) = E(Y_t * Y_s) * E(Z_t * Z_s) - 0$ and it equals to $E(Y_t * Y_s) * E(Z_t * Z_s) = (E(Y_t) * E(Y_s) + Cov(Y_t, Y_s))(E(Z_t) * E(Z_s) + Cov(Z_t, Z_s)) = \gamma_y(t-s) * (\mu_z^2 + \gamma_z(t-s))$ By definition, we can know that Xn = Yn * Zn is a stationary.

(c) Consider the time series of $W_n = cos(\frac{n\pi}{8})Y_n + sin(\frac{n\pi}{8})V$ Derive the mean and auto-covariance function of Wn and determine if it is stationary.

$E(W_n) = E(cos(\frac{n\pi}{8})Y_n + sin(\frac{n\pi}{8})V) = cos(\frac{n\pi}{8}) * E(Y_N) + sin(\frac{n\pi}{8}) * E(V) = \mu_Y * cos(\frac{n\pi}{8}) + sin(\frac{n\pi}{8}) * \pi = sin(\frac{n\pi}{8}) * \pi$

auto-covariance $\gamma_W n(t-s) = (cos(\frac{t\pi}{8})cos(\frac{s\pi}{8}) * Cov(Y_t, Y_s) + sin(\frac{n\pi}{8})sin(\frac{s\pi}{8}) * Cov(V, V)$ and plug in $Cov(V, V) = Var(V) = \pi^2/3$, then auto-covariance has different form, as it is not a function of n-m, so it is not stationary.

(d) Consider the time series of $W_n = cos(\frac{n\pi}{8} + U)Y_n + sin(\frac{n\pi}{8} + U)V$ and show that it is stationary, but not ergodic.

First, $E(cos(\frac{n\pi}{8} + U)) = \int_0^{2\pi} cos(\frac{n\pi}{8} + u) * 1/(2\pi)du = 1/(2\pi) * [sin(\frac{n\pi}{8} + 2\pi) - sin(\frac{n\pi}{8})] = 0$, so does it for $E(sin(\frac{n\pi}{8} + U))$ or becuase sin is symetric with respect to axis x.; Therefore, E(W_n)=0

Consider $Cov(cos(\frac{n\pi}{8} + U)) = E(cos^2(\frac{n\pi}{8} + U)) - 0 = 1/2 * E(cos(\frac{n\pi}{4} + 2U) + 1)$. With $E(cos(\frac{n\pi}{4} + 2U)) = \int_0^{2\pi} cos(\frac{n\pi}{4} + 2u) * 1/(2\pi)du = 0$ for same reasons above, then $Cov(cos(\frac{n\pi}{8} + U)) = 1/2 + 0 = 1/2$, and also happens for $Cov(sin(\frac{n\pi}{8} + U)) = 1/2$. Therefore, we know $sin(\frac{n\pi}{8} + U)$ and $cos(\frac{n\pi}{8} + U)$ are stationary. Then with 3(b) and above, $cos(\frac{n\pi}{8} + U) * Y$ and $sin(\frac{n\pi}{8} + U) * U$ should be stationary. Then sum of them W is stationary.

We have y_h is ergodic, so when h appraoches infinity, the cov(y_h)=0. But $cos(\frac{n\pi}{8} + U) = 1/2$ no matter n for which value.Therefore, $Cov(W_n) = E(Wn * Wn) + 0$ cannot be zero for infinity h. Then, it's not ergodic.