Homework 1. Stat 202a. Due Thu, Oct 16, 10:30am. **Late hws will not be accepted!**

You must work on the homework INDEPENDENTLY! Collaborating on this homework will be considered cheating. Submit your homework via the CLCC website https://ccle.ucla.edu/course/view/14F-STATS202A-1 .Your homework solution should be a single PDF document. The first pages should be your *output* from the problems below. This preferably means you typing out your answers and explaining them briefly in a clear, readable way, rather than simply cutting and pasting your R output. After that, on subsequent pages, include all your *code* for these problems. Your functions pi2 and pi3, for example, should be included in the code section.

1. Assessing estimates of the 90th percentile of 100 iid uniform(0,1) random variables.

The *R* function *quantile()* implements a somewhat complex interpolation method in order to estimate a particular quantile, such as the 90th percentile. We will compare the estimate in *quantile()* with simpler estimates.

a) Write a function that takes as input a vector of length 100 and outputs the 90th of the 100 values sorted from smallest to largest. Note that the input vector might not be sorted.

b) Write a function to find the 91st of the sorted vector of 100 values.

c) Write a function that outputs the average of the 90th and 91st of the sorted vector of 100 values.

d) For each of your functions in parts a-c, as well as the function *quantile(x,0.9)*, do the following:

(i) Generate 100 iid uniform (0,1) random variables, and calculate your estimate of the 90th percentile.

(ii) Repeat step (i) 100,000 times.

(iii) Plot the sample mean of the first *m* of your estimates, as a function of *m.*

e) Report the ultimate sample mean of your 100,000 estimates, for each of the four estimates. In 1-2 sentences, indicate which of the 4 estimates appears to be the best, and why.

2. Approximating π.

a) Write a function called *pi2(n)* that approximates π as a function of n, using the approximation $\pi = \lim_{n \to \infty} \sqrt{[6 \sum_{k=1}^{n} k^{-2}]}$. Evaluate *pi2*($10^j$) for $j = 0,1,2,\ldots,6$.

b) Write a function *pi3(n)* that approximates π as a function of n, by simulating random points in the square with vertices (-1,-1), (-1,1), (1,1), and (1,-1), seeing what fraction of them are in the unit circle [the circle with radius 1 centered at the origin], and then converting this fraction into an estimate of π. Evaluate *pi3*(10^j) for $j = 0,1,2,\ldots,6$. For j=6, plot your simulated points, using different plotting symbols for simulated points inside and outside the unit circle. There is no need for you to plot the unit circle also.