https://dicook.github.io/Statistical_Thinking/tutorials/lab11/lab11help.pdf

Remember SETU surverys!

Question One - Monte Carlo Approximations

Your tasks:

1. Sample coordinates from the unit square.
2. Decide if that point is inside the unit circle.
3. Count the number of times the point is inside the circle for $n \in \{100, 1000, 10000, 100000\}$.
4. Calculate your approximation for π.

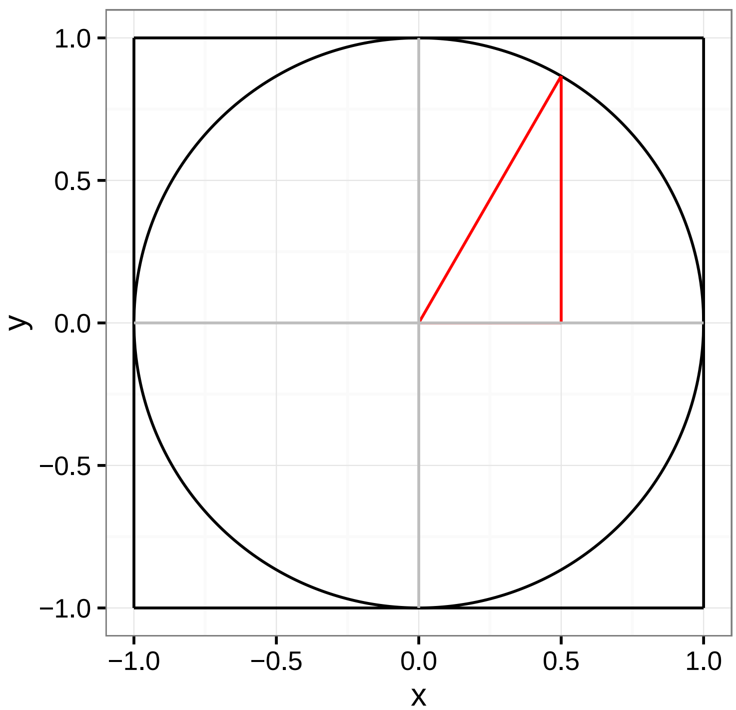

Figure 1: If we have even odds of picking any point in the square, the proportion of points in the circle should be the ratio of the two areas.

Question Two - Accept/Reject sampling

Part A

1. Sample $x^{(i)}$ from the proposal distribution $g(x)$.
2. Calculate $\frac{f(x^{(i)})}{cg(x^{(i)})}$.
3. Sample u from $\mathcal{U}(0, 1)$.
4. Decide to accept or reject $x^{(i)}$ depending on if $u < \frac{f(x^{(i)})}{cg(x^{(i)})}$.

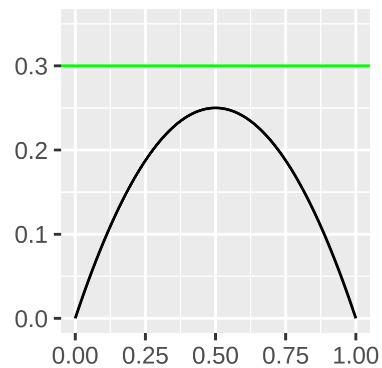

Figure 2: Let c = 0.3. If we sample a point x from a uniform distribution, it is accepted as something that could be from f(x) with probability f(x)/c, the ratio of the height of the two lines at x. Points where f(x) is high are more likely to be accepted, so the accepted sample resembles f(x).

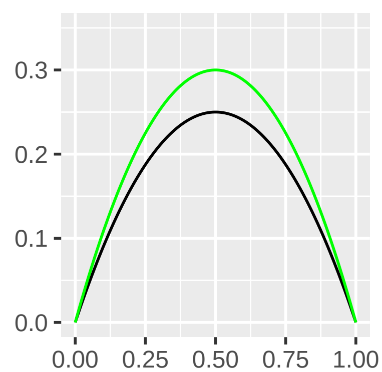

Figure 3: g(x) is now a beta(2, 2) distribution (with c = 0.2). The acceptance ratio is higher close to 0 and 1, but now most of our draws will be close to the middle, so the final sample will still resemble f(x).

Part B

1. For each c run the function 1000 times. Try using replicate(1000, function).
2. Count the number of rejections.
3. Calculate $f(x)$ for a sequence of x between 0 and 1.
4. Plot the density of the accepted draws against a line plot of $(x, f(x))$.

Question Three - Metropolis Hastings MCMC

1. Pick an arbitary starting value that could come from a Cauchy, which is the same as a Students' t with one degree of freedom. You can get stuck near the starting value for a while but eventually the Markov chain will converge to the posterior so the starting value doesn't really matter.
2. Generate a candidate draw y from the proposal distribution.
3. Calculate $r(x, y)$ where x is the previous draw.
4. Decide to keep the new draw or repeat x.
5. Calculate the rejection rate.
6. Plot (1:n, draws) as a line plot.
7. Plot the draws as a density plot against a Cauchy density.

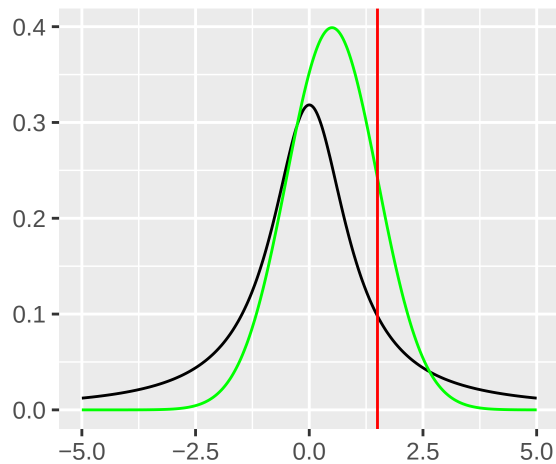

Figure 4: I start with $x^{(0)} = 0.5$. Then I sample $y = 1.5$ from a $\mathcal{N}(x^{(0)}, 1)$ distribution. Then I calculate $r(x = 0.5, y = 1.5) = 0.38$ and randomly accept $x^{(1)} = y$ with probability 0.38 or repeat $x^{(1)} = x^{(0)}$ with probability $1 - 0.38$. Values y that are more likely to come from a Cauchy distribution than my current draw $x^{(0)}$ are more likely to be accepted.

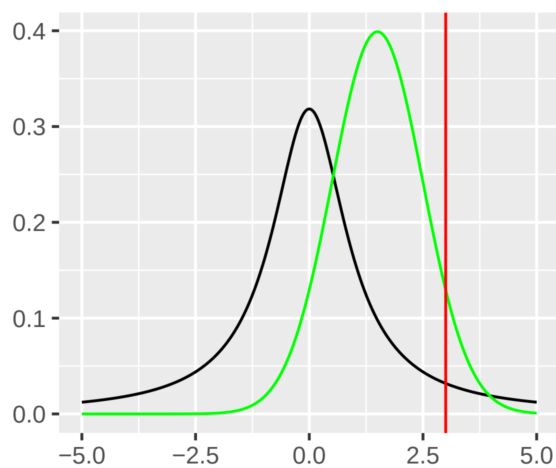

Figure 5: After accepting $x^{(1)} = 1.5$ I sample $y = 3$ from a $\mathcal{N}(x^{(1)}, 1)$ distribution. Now my acceptance probability is 0.325 and it's pretty likely I'll reject y and set $x^{(2)} = x^{(1)}$. I might end up repeating this value until I draw a candidate value y that is more likely to come from a Cauchy distribution than my current 1.5.