Empirical exercise – The geometry of OLS

1. Aim of the exercise

Learning about the geometry of the OLS approximation.

2. Clear the memory

Clear the memory.

```
clear
```

3. Define variables as vectors in the three-dimensional Euclidean space

In a linear regression, the columns of **X** form a space in the K-dimensional Euclidean space. This space is a subspace of the N-dimensional space defined by the observations contained in a given column of **X**. Hence, there are two different spaces. The first space is the observation space, and the second is the variable space.

Consider the current exercise. The dimension of the observation space is three. Three observations define a vector or a variable. The dimension of the variable space is two as long as we are talking about the regressors.

The tail and head are the starting and ending points of a vector. The coordinates define the elements of a given vector.

```
t = [0 0 0]'; % Define the origin and let this be the the 'tail'of all vectors.
y = [3 3 4]'; % The 'head'of the y vector.
x1 = [1 2 1]';
x2 = [3 1 1]'; % N = 3, K = 2.
```

4. Projection of the dependent variable to the space spanned by the regressors and to the space orthogonal to this space.

Connecting the head of `y_hat` to the head of `y` leads to that *u_hat* is orthogonal to `y_hat`. Why?

```
N_obs = size(y,1);
I = eye(N_obs);
X = [x1 x2];
P = X*inv(X'*X)*X';
M = I-P;
y_hat = P*y; % Or, X*inv(X'*X)*X'*y = X*B_hat.
u_hat = M*y; % Or, y-y_hat.
```

5. Plot vectors in the three-dimensional Euclidean space

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```
plot3([t(1) x1(1)],[t(2) x1(2)],[t(3) x1(3)],'blue','LineWidth',1,'Marker','.','MarkerSi
% x1.
hold on
plot3([t(1) x2(1)],[t(2) x2(2)],[t(3) x2(3)],'blue','LineWidth',1,'Marker','.','MarkerSi
% x2.
hold on
plot3([t(1) y(1)],[t(2) y(2)],[t(3) y(3)],'black','LineWidth',3,'Marker','.','MarkerSize
% y.
hold on
plot3([t(1) y_hat(1)],[t(2) y_hat(2)],[t(3) y_hat(3)],'blue','LineWidth',3) % y_hat.
hold on
plot3([y_hat(1) y(1)] y(1)],[y_hat(2) y(2)],[y_hat(3) y(3)],'red ','LineWidth',3)
% u_hat. Connecting the head of y_hat vector to the head of the y vector leads
to that u_hat is orthogonal to y_hat. Why?
hold on
text(0,2,1,'S (x1,x2)','Color','blue')
hold on
text(3,2,4,'S⊥(x1,x2)','Color','red')
grid on
axis([-1 5 -1 5 -1 5])
title('Fig. 1: The geometry of OLS')
xlabel('Obs. 1 in a vector in the x-coord.')
ylabel('Obs. 2 in a vector in the y-coord.')
zlabel('Obs. 3 in a vector in the z-coord.')
legend('x1','x2','y ','y_hat = Py = XB_hat','u_hat = My','Location ','northeast')
view(-20,20)
```

Fig. 1: The geometry of OLS