$$i = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1} \qquad P_i = i(i'i)^{-1}i' \qquad y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}_{n\times 1}$$

$$P_i\, y = \underset{i}{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}} \underset{(}{\Bigg[} \underset{i'}{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}} \underset{i}{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}} \underset{)}{\Bigg]}^{-1} \underset{i'}{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}} \begin{bmatrix} y_1 \\ \\ y_n \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \cdot \underbrace{\left(\sum_{t=1}^{n} 1^2 \right)^{-1}}_{\frac{1}{n}} \cdot \left(\sum_{t=1}^{n} y_t \right)$$

$$= \begin{bmatrix} \bar{y} \\ \vdots \\ \bar{y} \end{bmatrix}$$

$$M_i y = (I - P_i)\, y = \begin{bmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{bmatrix} \qquad \Leftarrow M_i + P_i = I$$