

Empirical exercise – The GMM objective function and gradient of it

The code in the function file is reproduced at the end of this file. The second line defines the error vector \mathbf{e} .

Using the error vector \mathbf{e} and the instrument vector \mathbf{Z} , the third line constructs the sample moments. In particular, $\mathbf{m_bar}$ contains a set of sample moments. It contains N_mom sample moments, and hence it is a $N_mom \times 1$ vector. The first sample moment is the zero mean moment for the error term. The other sample moments result from the requirement that each of the N_mom variables in \mathbf{Z} is uncorrelated with the error \mathbf{e} . Note that we have N_par parameters to estimate in $\mathbf{B_true}$. If the number of moments is larger than the number of parameters to estimate, the model is over-identified.

Using the sample moments, the fourth line constructs the GMM criterion function. We consider a quadratic form of $\mathbf{m_bar}$. This is our objective function. The unique $\mathbf{B_hat}$ that solves this function is the GMM estimator. We ask `fminunc` to evaluate this function at $\mathbf{B_true}$. `Obj` takes a scalar value at the minimising $\mathbf{B_hat}$.

$\mathbf{G_bar}$ is the derivative of the sample moment vector with respect to parameter vector $\mathbf{B_true}$.

`GradObj` is the gradient of the objective function. The gradient is the partial derivatives of the objective function with respect to each element of $\mathbf{B_true}$. That is, the component i of the gradient is the partial derivative of the function with respect to the component i of $\mathbf{B_true}$. $\mathbf{B_true}$ has N_par elements, and therefore the gradient has N_par elements. The gradient is evaluated at $\mathbf{B_true}$ because $\mathbf{G_bar}$ is evaluated at $\mathbf{B_true}$.

Given the arguments defined above, we can now instruct MATLAB that our function is `exercisegmmfunction`, and it accepts \mathbf{y} , \mathbf{X} , \mathbf{Z} , $\mathbf{B_true}$, $\mathbf{W_hat}$, N_obs , and N_par as input arguments, and returns `Obj` and `GradObj` as output arguments.

```
function [Obj,GradObj] = exercisegmmfunction(y,X,Z,B_true,W_hat,N_obs,N_par)
e = y-exp(X*B_true);
m_bar = (1/N_obs)*Z'*e;
Obj = m_bar'*W_hat*m_bar;
G_bar = -(1/N_obs)*Z'*(X.*repmat(exp(X*B_true),[1 N_par]));
GradObj = 2*G_bar'*W_hat*m_bar;
end
```