Time Series Analysis

Getting to Time Series Regression

Michael Colaresi
colaresi@msu.edu

Outline

Outline

Outline

1. Quick Review
   - Intervention Analysis
   - Temporary Shocks with Simple Interventions
   - Permanent Simple Intervention
   - Intervention Analysis: Higher Orders
2. Transfer Functions
3. Multipliers (Distributed Lag Case)
   - Mean lags
   - Median lags

Outline

Outline

What are we doing

Review

To get to regression analysis of time series, we started first with some simple intervention models so we can see the effects of various shocks on the univariate time series we are modeling.

What are we doing

Review

To get to regression analysis of time series, we started first with some simple intervention models so we can see the effects of various shocks on the univariate time series we are modeling.

Transfer Functions

Transfer functions will allow inputs that are non-binary BUT are still restricted in several ways.

Shocking

Intervention analysis is concerned with seeing the effects of various shocks on the levels of a time series. It is an effort to trace out the dynamic impacts of a change in the model on the dependent variable. There are two types of shocks to consider: temporary and permanent.

Shocking

Intervention analysis is concerned with seeing the effects of various shocks on the levels of a time series. It is an effort to trace out the dynamic impacts of a change in the model on the dependent variable. There are two types of shocks to consider: temporary and permanent.

Simple Interventions Models vs. Compound

- Interventions that change the mean or equilibrium of a series but NOT the dynamics are known as simple intervention models.
- Intervention that change both the mean or equilibrium of a series and ALSO the dynamics are known as compound intervention models

Shocking

Intervention analysis is concerned with seeing the effects of various shocks on the levels of a time series. It is an effort to trace out the dynamic impacts of a change in the model on the dependent variable. There are two types of shocks to consider: temporary and permanent.

Simple Interventions Models vs. Compound

- Interventions that change the mean or equilibrium of a series but NOT the dynamics are known as simple intervention models.
- Intervention that change both the mean or equilibrium of a series and ALSO the dynamics are known as compound intervention models

In the previous lecture, were those simple or compound?

Simple Temporary Interventions

Consider the sequence:

$$[0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0] \tag{1}$$

One type of intervention is a "temporary shock" which is specified in (1).
Plot the sequence for the following first order model below:

$$Y_t = \phi Y_{t-1} + \beta I_t \quad |\phi| < 1$$

with I_t as the intervention.

Now:

$$\begin{aligned} Y_{i+0} &= \phi Y_{i-1} + \beta I_i = \beta \\ Y_{i+1} &= \phi Y_i + \beta I_{i+1} = \phi\beta \\ Y_{i+2} &= \phi Y_{i+1} + \beta I_{i+2} = \phi^2\beta \end{aligned}$$

Rule:

$$\begin{aligned} Y_{i+n} &= \phi Y_{i+n-1} + \beta I_{i+n} \\ &= \phi\left(\phi^{n-1}\beta\right) + \beta\,(0) \\ &= \phi^n\beta \end{aligned}$$

Comment on ϕ :
If $\phi \to 1$ spike has a lasting effect.
If $\phi \to 0$ spike has a rapid decay.

The sum of the changes in Y is equal to

$$\sum_{i=0}^{\infty} \phi^i \beta = \frac{\beta}{(1-\phi)}$$

Computing the effect of a temporary intervention: Consider the following temporary intervention model:

$$Y_t = 0.8Y_{t-1} + 0.2I_t$$

Then,

n	$\phi^n\beta$	Y_{t+n}
0	$0.8^0(0.2)$	0.20
1	$0.8^1(0.2)$	0.16
2	$0.8^2(0.2)$	0.13
3	$0.8^3(0.2)$	0.11
4	$0.8^4(0.2)$	0.08
5	$0.8^5(0.2)$	0.07

Permanent Simple Intervention

Consider a different sequence:

$$[0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ \ldots\ 1] \qquad (2)$$

Here the intervention is a "permanent shock." Sequence (1) can be obtained by differencing this sequence.

Plot the sequence for the following first order model below:

$$Y_t = \phi Y_{t-1} + \beta I_t \quad |\phi| < 1$$

where I_t is the intervention.
Now:

$$\begin{aligned}
Y_i &= \phi Y_{i-1} + \beta(1) = \beta \\
Y_{i+1} &= \phi Y_i + \beta(1) = \phi\beta + \beta \\
Y_{i+2} &= \phi Y_{i+1} + \beta(1) = \phi(\phi\beta) + \phi\beta + \beta = \phi^2\beta + \phi\beta + \beta \\
Y_{i+3} &= \phi Y_{i+2} + \beta(1) = \phi\left(\phi^2\beta + \phi\beta + \beta\right) + \beta = \phi^3\beta + \phi^2\beta + \phi\beta +
\end{aligned}$$

Rule:

$$Y_{i+n} = \left(1 + \phi + \ldots + \phi^j\right)\beta$$

This gives the "permanent shock" after j periods.

The total long-run effect is:

$$\sum_{i=0}^{N} \phi^i \beta_i = \frac{\beta}{(1-\phi)}$$

Computing the effect of a permanent intervention: Consider the following permanent intervention model:

$$Y_t = 0.8Y_{t-1} + 0.2I_t$$

Then,

n	$\phi^n\beta$	$\sum_{j=0}^{N} \phi^j\beta$
0	(0.2)	0.20
1	$0.8^1(0.2)$	0.36
2	$0.8^2(0.2)$	0.49
3	$0.8^3(0.2)$	0.59
4	$0.8^4(0.2)$	0.67
5	$0.8^5(0.2)$	0.73

The total effect is found to be:

$$\frac{0.2}{(1-0.8)} = 1$$

Note that as $\phi \to 1$, a change in levels takes a longer period of time to approach the total effect.

A Higher Order

Consider the following second order model:

$$Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \beta I_{t-1} + \epsilon_t \tag{3}$$

where I_{t-1} is the permanent shock.

Now, calculate the “total effect”:

$$\begin{aligned}
\left(1-\phi_1 B-\phi_1 B^2\right) Y_t &= \alpha+\beta I_{t-1}+\epsilon_t \\
Y_t &= \frac{\alpha}{(1-\phi_1 B-\phi_1 B^2)} \qquad (4)\\
&\quad +\frac{\beta I_{t-1}}{(1-\phi_1 B-\phi_1 B^2)}+\frac{\epsilon_t}{(1-\phi_1 B-\phi_1 B^2)} \qquad (5)
\end{aligned}$$

Consider an alternative second order model:

$$Y_t = \gamma + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \beta_1 I_t + \beta_2 I_{t-1} + u_t \tag{6}$$

where I_t and I_{t-1} are permanent shocks.
Again, calculate the "total effect":

$$\begin{aligned}\left(1 - \phi_1 B - \phi_1 B^2\right) Y_t &= \gamma + \beta_1 I_t + \beta_2 I_{t-1} + u_t \\ Y_t &= \frac{\gamma}{(1 - \phi_1 B - \phi_1 B^2)} \quad (7) \\ &\quad + \frac{\beta_1 I_t + \beta_2 I_{t-1}}{(1 - \phi_1 B - \phi_1 B^2)} + \frac{u_t}{(1 - \phi_1 B - \phi_1 B^2)} \quad (8)\end{aligned}$$

Let's focus on the "intervention" in (4) and (7). If (3) had the following results:

$$Y_t = 0.68 + 1.35Y_{t-1} - 0.47Y_{t-2} - 0.13I_{t-1} + \epsilon_t$$

then the intervention term in (4) equals:

$$\frac{-0.13}{1 - 1.35 + 0.47} = -1.0833$$

Now if (6) has the results:

$$Y_t = 0.68 + 1.35Y_{t-1} - 0.47Y_{t-2} + 0.5I_t + 0.42I_{t-1} + u_t$$

The intervention term(s) in (7) equal:

$$\frac{0.50 + 0.42}{1 - 1.35 + 0.47} = 7.6667.$$

Practical Advice

1. Since MAs represent temporary shocks, many people model noise when doing intervention models as ARs.
2. Initially model noise before and after intervention in two different estimations to determine if dynamics change with the intervention. If so, then a compound transfer function model is necessary.

Transfer Functions

Definition

A methodology for finding a parsimonious and estimable parameterization of an infinite order lag model.

$$
\begin{aligned}
Y_t &= \Sigma_{i=0}^{\infty} v_i * X_{t-i} + N_t \\
&= (\Sigma_{i=0}^{\infty} v_i * B^i) * Xt + N_t \\
&= v(B) * X_t + N_t
\end{aligned}
$$

$v(B)$ is the transfer functions and the individual v_i's are known as the impulse responses.

Steps to Estimation

1. Prewhitening I: create a filter for he input series, so that series is white noise.
2. Prewhitening II: Apply that same filter to the OUTPUT series.
3. Compute the cross-correlation between the series: this will be proportional to the dynamics of the relationship between the series.
4. Pick the lag representation of X_{t-i} that summarizes the cross-correlations and estimate that model. This can be done with the impulse response function.
5. Check residuals for white noise, and re-estimate with a better noise model
6. Check correlation between white noise residuals and input

Multipliers (on the DL)

Consider the following distributed lag model

$$Y_t = \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} \tag{9}$$

This is a distributed lag model. We can define the following concepts for such a model

- The distributed lag **Total Multiplier** is:

$$\sum_{i=0}^{n} \beta_i \tag{10}$$

- An increase in the level of X by one unit increases (decreases) Y by the sum of $\beta's$ over the period 0 to n.

Impact and Interim Multipliers

- The distributed lag **Impact Multiplier** is:

$$\beta_0$$

  This is the "initial effect."

- The distributed lag **Interim Multiplier** is:

$$\frac{\sum_{i=0}^{m} \beta_i}{\sum_{i=0}^{n} \beta_i} \tag{11}$$

  where $m < n$
  This is the percentage of adjustment of time period "i".

Example

Consider the model:

$$Y_t = 0.2X_{t-2} + 0.4X_{t-3} + 0.1X_{t-4}$$

- Then, the **total multiplier** is $0.2 + 0.4 + 0.1 = .7$.
- The **impact multiplier** does not exist, since there is no β_0.
- The **interim multiplier** is

$$\begin{aligned} m &= 2 \rightarrow \frac{0.2}{0.7} = .57143 \\ m &= 3 \rightarrow \frac{0.2 + 0.4}{0.7} = .85714 \end{aligned}$$

Blind Example

Consider the model:

$$Y_t = .9X_{t-1} + 0.3X_{t-2} + 0.2X_{t-3} + 0.1X_{t-4}$$

- Total multiplier?

Blind Example

Consider the model:

$$Y_t = .9X_{t-1} + 0.3X_{t-2} + 0.2X_{t-3} + 0.1X_{t-4}$$

- Total multiplier?
- Impact Multiplier?

Blind Example

Consider the model:

$$Y_t = .9X_{t-1} + 0.3X_{t-2} + 0.2X_{t-3} + 0.1X_{t-4}$$

- Total multiplier?
- Impact Multiplier?
- Interim Multiplier? $1 > m > n$

Mean lags

$$Mean\ Lag = \frac{\sum_{i=0}^{n} i\beta_i}{\sum_{i=0}^{n} \beta_i} \tag{12}$$

"What is the average lag for the total effect to take place?" (Assume non-negative coefficients.) Based on above, The mean lag is computed as

$$\frac{0.2\,(2) + 0.4\,(3) + 0.1\,(4)}{0.7} = 2.\,8571$$

Median lags

$$(i_{\beta<0.5}) + \sum_{i=0}^{m} \beta^m$$

"When did 50% of the effect occur?" Assume non-negative coefficients. And:

$$\sum_{i=0}^{m} \beta^m = \frac{0.5 - \beta^*_B}{\beta^*_{B+1} - \beta^*_B}$$

where:

$$\begin{aligned} \beta^*_B &= \text{largest interim multiplier less than 0.5} \\ \beta^*_{B+1} &= \text{first interim multiplier greater than 0.5} \end{aligned}$$

Median Lag example

$$\left.\begin{array}{rcl} \beta^*_{0,0} & = & 0 \\ \beta^*_{0,1} & = & 0 \\ \beta^*_{0,2} & = & 0.29 \\ \beta^*_{0,3} & = & 0.86 \\ \beta^*_{0,4} & = & 0.14 \end{array}\right\}$$

$$2 + \frac{0.5 - 0.29}{0.86 - 0.29} = 2.3684$$

Koyck and Dirty

In the distributed lag case we can write:
$\beta_j = \lambda^j \beta$ and
$Y_t = \beta \sum_{j=0}^{\infty} \lambda^j X_{t-j}$ Now,

$$\begin{aligned} \beta &= \text{“}\textit{impact multiplier}\text{”} \\ \sum \beta_j &= \beta \sum \lambda^j = \frac{\beta}{1-\lambda} = \text{“}\textit{total multiplier}\text{”} \end{aligned}$$

Nestle Koyck

We need to do something to rid ourselves of the infinite lags of X:

$$Y_t = \beta\left(X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \lambda^3 X_{t-3} + \ldots\right). \tag{13}$$

Lag this equation back one period and multiply by λ:

$$\beta\left(\lambda X_{t-1} + \lambda^2 X_{t-2} + \lambda^3 X_{t-3} + \ldots\right) \tag{14}$$

Subtract (14) from (13) and we get:

$$\begin{aligned} Y_t - \lambda Y_{t-1} &= \beta X_t \\ \Rightarrow \quad Y_t &= \lambda Y_{t-1} + \beta X_t \end{aligned}$$

This is the **Koyck transformation**.

Koyck-tinued

We are now prepared to determine how long a specific change took place. After 1 period, let's say $(1-\lambda)$ of the change occurs; therefore, λ is the change remaining. Continuing:

$$\begin{aligned}
2nd\ period &: (1-\lambda)+\lambda(1-\lambda)=\left(1-\lambda^2\right) \\
3rd\ period &: (1-\lambda)+\lambda(1-\lambda)+\lambda^2(1-\lambda)=\left(1-\lambda^3\right) \\
4th\ period &: (1-\lambda)+\lambda(1-\lambda)+\lambda^2(1-\lambda)+\lambda^3(1-\lambda)=\left(1-\lambda^4\right)
\end{aligned}$$

Koyck or Pepsi

Let $1 - \lambda^n =$ the amount of change after n periods.
$P = 1 - \lambda^n$
where λ is the coefficient of the lagged dependent variable.

$$\begin{aligned} \lambda^n + P &= 1 \\ n \ln \lambda &= \ln(1-P) \\ n &= \frac{\ln(1-P)}{\ln \lambda} \end{aligned}$$

The Koyck and the Dead

With this information now determine the dynamics of the following equation:
$Y_t = \alpha + \beta \sum_{j=0}^{\infty} \lambda^j X_{t-j}$ Now, take a Koyck transformation:
$Y_t = \alpha (1 - \lambda) + \lambda Y_{t-1} + \beta X_t$ Using this transformation, when will 90% of the change occur?
To calculate the result, let $P = 0.90$ and assume that $\lambda = 0.5$ (the coefficient of the lagged dependent variable):
$n = \frac{\ln(.1)}{\ln(0.5)} = 3.3219$ "*shorter adjustment*"
Now assume that $\lambda = 0.9$:
$n = \frac{\ln(0.1)}{\ln(0.9)} = 21.854$ "*long adjustment*"

Nested L-R Tests: The End of Puns

ARIMA Models

Restricted Model	Unrestricted Model	Nested?
$(1,0,0)$	$(2,0,0)$	Yes
$(1,1,0)$	$(2,1,0)$	Yes
$(0,1,1)$	$(0,1,2)$	Yes
$(0,0,1)$	$(0,0,2)$	Yes
$(1,1,0)$	$(0,1,1)$	No
$(2,1,0)$	$(0,1,2)$	No

What about $(3,1,2)$ and $(4,1,4)$

Hypothesis Testing

Issue: There are many hypothesis testing concerns that go beyond the standard t-statistic. In particular, we will concern ourselves with

1. Model order
2. Restrictions on structural change.

LR tests

Model Order: Tests for model order tend to be of the likelihood ratio variety (LR). LR's can be computed as:

$$\begin{aligned} LR &= -2\left([\ln \mathcal{L}_R] - [\ln \mathcal{L}_U]\right) \sim \chi^2_k \\ \ln \mathcal{L}_R &= \text{log-likelihood of the ``\textbf{restricted} model''} \\ \ln \mathcal{L}_U &= \text{log-likelihood of the ``\textbf{unrestricted} model''} \end{aligned}$$

Note that this test may look quite different in different applications because likelihoods will be different. For example, for linear regression, the likelihood ratio tests is

$$LR = T\left(\ln|\Sigma_R| - \ln|\Sigma_U|\right) \sim \chi^2_k$$

where T is the sample size, Σ_R and Σ_U are the sample covariance matrices from the respective models.

AIC

A test that is useful for cases in which the null hypothesis is unknown, which is likely the case when testing model order, is the **Akaike Information Criterion (AIC):**

$$AIC = \ln\left(s^2\right) + 2\left(\frac{k}{T}\right)$$

where s^2 is the sample error variance, k is the number of parameters and T is the total number of cases. The model that minimizes the AIC is in theory the best specified model. There are other similar tests that differ from the AIC in the amount they punish having additional parameter estimates in the model.

Chow Test

Often we need to test whether intercepts/slopes are constant for the sample period. Tests for this question are called *Chow* tests:

$$\frac{\frac{RSS_R - RSS_U}{K_R}}{\frac{RSS_U}{(T-K)}} \sim F\left(K_R, T-K\right),$$

where,

- T = sample size
- RSS_R = residual sun of squares (restricted model)
- RSS_U = residual sum of squares (unrestricted model)
- K_R = number of restrictions
- K = number of regressors (including the intercept) in the unrestricted regression.

Chow Test Uses

Tests for structural change center on 3 specific cases:

1. Changes in the slope and intercept.
2. Changes in the slope.
3. Changes in the intercept.

Intercept and Slope Change

Restrict analysis to two periods: $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ *v.* (α^*, β^*).

Step 1: Estimate restricted regression for all data points,

$$Y_t^* = \alpha^* + \beta_r^* X_t^* + u_t^*$$

and calculate RSS_R.

Step 2: Estimate the unrestricted regression for data subsets:

$$\begin{aligned} Y_{1t} &= \alpha_1 + \beta_1 X_{1t} + u_{1t} &\rightarrow \text{ calculate } RSS_1 \\ \textit{and } Y_{2t} &= \alpha_2 + \beta_2 X_{2t} + u_{2t} &\rightarrow \text{ calculate } RSS_2. \end{aligned}$$

$\Rightarrow RSS_1 + RSS_2 = RSS_U$

Now construct the tests,

$$\begin{aligned} H_0 &: \alpha_1 = \alpha_2; \ \beta_1 = \beta_2 \\ H_A &: \alpha_1 \neq \alpha_2 \textit{ and/or } \beta_1 \neq \beta_2 \end{aligned}$$

Step 3: Calculate Chow-test where $RSS_U = RSS_1 + RSS_2$.

Slope Change Only

Same as in previous section, but specification is different for restricted model for all data points:

$$Y_t^* = \alpha + \gamma * dummy + \beta_R X_t^* + u_t^* \quad \rightarrow \text{ calculate } RSS_R$$

where the dummy variable is zero for one period of data and one for he other period of data.
Same unrestricted models.
Chow test also the same.

$$\begin{aligned} H_0 &: \beta_1 = \beta_2 \\ H_A &: \beta_1 \neq \beta_2 \end{aligned}$$

Intercept Change

Step 1: Estimate restricted regression for all data points,

$$Y_t^* = \alpha^* + \beta_r^* X_t^* + u_t^* \;\rightarrow\; \text{calculate } RSS_R.$$

Step 2: Estimate the unrestricted regression for data subsets:

$$Y_t^* = \alpha + \gamma * dummy + \beta_R X_t^* + u_t^* \;\rightarrow\; \text{calculate } RSS_U$$

Now construct the tests,

$$\begin{aligned} H_0 &: \alpha = \alpha^* \\ H_A &: \alpha \neq \alpha^* \end{aligned}$$

Step 3: Calculate Chow-test.

Econometric Issue	Diagnostic Test	Source
Exogeneity	Recursive Estimation	Dufour (1982)
1) Weak	Break Point Tests	Chow (1960)
2) Super	Conditional Model Specification	Engle and Hendry (1987)
Dynamic Specification	Cointegration (Error Correction)	Engle and Granger (1987)
Specification	RESET	Ramsey (1969)
	White-test	White (1980)
	Schwartz Criterion	Schwartz (1978)
	Akaike Criterion	Akaike (1969)
Sample Size Inference	Normality	Jarque and Bera (1980)
Collinearity	R^2 / Linear Transformation	Johnston, Henry and Morgan (1988)
Residual Autocorrelation		
1) 1st-order	Durbin-Watson	Durbin and Watson (1950, 1951)
2) Q'th-order	Residual Correlation (Q'th-order)	Box and Pierce (1970); Godfrey (1978)
3) ARCH	ARCH (Q'th-order)	Engle (1982)
4) Restrictions	COMFAC	Hendry and Mizon (1978); Sargan (1980)
Foreasting	Break Point	Hendry (1989); Chow (1960)
Encompassing	F-test	Mizon and Richard (1986)
	Cox-test	Cox (1961)
	Sargan-test	Sargan (1980)