Time Series Analysis

Equilibrium, Estimation, and Intervention Models

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Outline

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Equilibrium of a Series: The Point of Return

- Take this: $y_t = .2y_{t-1} + \epsilon_t$
  - Shock it one unit ($\epsilon_0 = 1, \epsilon_i = 0 \ \forall \ i \neq 0$). Where does it return back too?
- Take this, now: $y_t = .9 + .1y_{t-1} + \epsilon_t$
  - Do the same thing: Simulate a one unit shock.
- What about, $y_t = \epsilon_t - .3\epsilon_{t-1}$
- and, $y_t = .9 + \epsilon_t - .3\epsilon_{t-1}$

What does this tell us?

Equilibrium: AR(1)

- For AR(1), $y_t = \alpha + \phi y_{t-1} + \epsilon_t$, Equil. is $\frac{\alpha}{1-\phi}$

Solution: set $y_0 = y_1$. For ex. $y_t = .9 + .1y_{t-1} + \epsilon_t$

$$\begin{aligned}
y_0 &= .9 + .1y_0 + \epsilon_t && (1)\\
y_0 - .1y_0 &= .9 + \epsilon_t && (2)\\
y_0(1 - .1) &= .9 + \epsilon_t && (3)\\
y_0 &= \frac{.9}{1 - .1} + \frac{\epsilon_t}{1 - .1} && (4)\\
E(y_0) &= \frac{.9}{1 - .1} = 1 && (5)
\end{aligned}$$

- For $AR(p) = \frac{\alpha}{1-\sum_{i=1}^{p} \phi_i}$
- For all MA, much easier.

ARIMA Estimation: Nuts and Bolts

- Many ARIMA(p,d,q) models are essentially linear transformations of the observed data.
  - AR(p) where we drop the first p observations, looks like an OLS regression.
  - Coefficients on the lagged dep. variables are ϕ's
  - OLS has "fine" properties given white noise residuals (more later).
- ARIMA models with non-zero q are NOT linear transformations of the observed data.
  - Take MA(1): $y_t = \epsilon_t - \theta\epsilon_{t-1}$ there is nothing to regress y_t on. Both θ and ϵ_t are unobserved

What do we do? Maximize the Likelihood!

ML is 5 Minutes

First: Take an MLE class. No substitute.

- ML is nothing more than efficient trial and error (with apologies to Fisher)
- Steps:
    - You need a function to maximize, which will be the log likelihood of the equation of interest.
    - Find an algorithm that smart people have thought up for literally guessing parameter values in sequence. Begin plugging in parameter values and calculating the respective likelihood that these values "generated" the data.

Do I have any time left?

- Since we will return to this later, important to understand what is going on conceptually:
  - We have data y, and want to know the parameter values, call them θ
  - Best would be: $p(\theta|y)$, but we can not get that from frequentist statistics (from Bayes rule, would need a prior, which would be a no no– but we will break this rule later)
  - Fisher's contribution was to define $\mathcal{L}(\theta|y) = p(y|\theta)$.

MLE for MA(1)

- MA(1): $y_t = \epsilon_t - \theta\epsilon_{t-1}$
- $\epsilon_t \sim N(0, \sigma^2)$
- ASSUME pre-history is zero for now (Conditional ML)
- Problem: Find θ such that $\mathcal{L}(\theta|y_t)$ is a maximum.

$$Ln\mathcal{L}(\theta) = -\frac{T}{2}log(2\Pi) - \frac{T}{2}log(\sigma^2) - \sum_{t=1}^{T}\frac{\epsilon_t^2}{2\sigma^2} \tag{6}$$

NOTE: We still need ϵ_t

MLE for MA(1) Cont.

- If we “knew” both θ and ϵ_0, it would be easy get everything we needed
  - $\epsilon_t = y_t + \theta\epsilon_{t-1}$
  - Beginning at time 1: $\epsilon_1 = y_1 + \epsilon_0$
  - plug that in for time 2: $\epsilon_2 = y_2 + \epsilon_1$
  - and so on. We can recursively solve all the way to $t = T$.

SOOOOOO, assuming ϵ_0 is the key.

MLE for MA(1) Cont.

- If we assume $\epsilon_0 = 0$, and then maximize the likelihood based on that, we have CONDITIONAL MAXIMUM LIKELIHOOD
  - The assumption might not be quite right, but its effect will die out as we continue our recursions (STATIONARITY)
- A more useful approach would be to impute (backcast) ϵ_t from the MLE(θ). This is UNCONDITIONAL MAXIMUM LIKELIHOOD.
  - $y_t = \epsilon_t - \theta\epsilon_{t-1}$, so $\epsilon_{t-1} = \frac{y_t - \epsilon_t}{-\theta}$
  - Need to use an estimate of ϵ_{T+1},0, but that is not so bad (dies off by the time we reach $t = 0$).
  - In practice, we can either recursively backcast or use the conditional MLE to get us started.

Algorithms to generate guesses

- Newton Approximation: Excellent if you are close to the truth, prone to local maxima.
- Newton-Raphson: Linear approximation of derivate. Only as good as the approximation.
- Berndt, Hall, Hall and Hausman: Only works for certain types of problems, but strong global properties when it applies (ARIMA c).
- BFGS (Broyden, Fletcher, Goldfarb, Shanno): Good global properties, but a bit slow if you are already close (Newton-Raphson can be faster if in neighborhood).

Can use multiple. Stata switches between BHHH and BFGS by default.

Dynamic regression with a dummy variable

- Typical regression without dynamics.
  $y_t = \alpha + \omega_1 I_t + \epsilon_t$
- Two types of dummy variables are of interest
  - Permanent: $I_{Pt} = \left\{ \begin{array}{cc} 0 & t < t_i \\ 1 & t \geq t_i \end{array} \right\}$
  - Temporary: $I_{Tt} = \left\{ \begin{array}{cc} 0 & t \neq t_i \\ 1 & t = t_i \end{array} \right\}$
- Note: $I_{Tt} = \Delta I_{Pt}$

A Change is a Change. Right?

- What do they mean?
  What do they imply about the series?
  What is the theoretical implications of choosing one or the other?
  - Permanent change intervention?

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  What is the theoretical implications of choosing one or the other?
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- Think about how these answers change when you add dynamics.

Intervention Analysis

Box and Tiao 1975

"Given a known intervention, is there evidence that change in the series of the kind expected actually occurred, and, if so what can be said of the nature and magnitude of the change?"

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Impact Assessment

A test of the null hypothesis that a postulated event caused a change in the social processes measured as a time-series.

Examples

- impact of air pollution laws (Box and Tiao 1975)
- impact of political realignments (Lewis-Beck 1979)
- impact of "rally events" (Clarke, et al 2001)
- *etc*

A Dynamic Dummy

- The full impact of an event can be written as:

$$Y_t = f(I_t) + N_t \tag{7}$$

  where,
  I_t represents the intervention. N_t represents the noise component of the ARIMA model (ar, ma and white noise).

- Because the model is linear,

$$Y_t^* = Y_t - N_t = f(I_t) \tag{8}$$

Abrupt and Permanent Intervention Effect

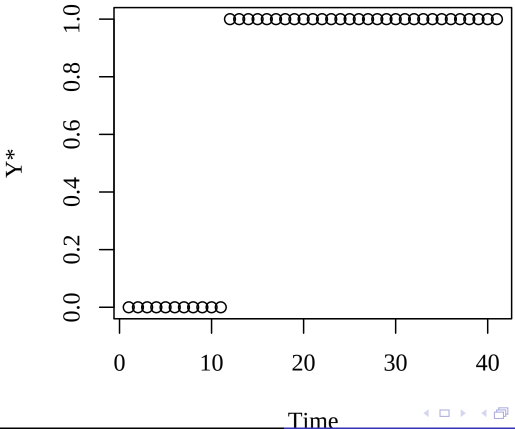
$$Y_t - N_t = Y_t^* = \omega_0 I_t$$

where

$$I_t = \left\{ \begin{array}{ll} 0 & t < t_i \\ 1 & t \geq t_i \end{array} \right\}$$

- We have defined $f(I_t)$ as $\omega_0 I_t$, with constant ω_0
- What does this mean we have hypothesized about Y_t^*?

Step To It



Gradual Permanent Intervention Effect

$$Y_t^* = f(I_t) = \frac{\omega_0}{1 - \delta B} I_t$$

where

$$I_t = \left\{ \begin{array}{ll} 0 & t < t_i \\ 1 & t \geq t_i \end{array} \right\}$$

We can rewrite the equation for Y_t^* as:

$$\begin{aligned} Y_t^* &= \frac{\omega_0}{1 - \delta B} I_t \\ (1 - \delta B) Y_t^* &= \omega_0 I_t \\ Y_t^* - \delta Y_{t-1}^* &= \omega_0 I_t \\ Y_t^* &= \delta Y_{t-1}^* + \omega_0 I_t \end{aligned}$$

Gradual Permanent Intervention Cont.

What are the implied recursions for Y_t^*?

- Pre-event

Gradual Permanent Intervention Cont.

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What are the implied recursions for Y_t^*?

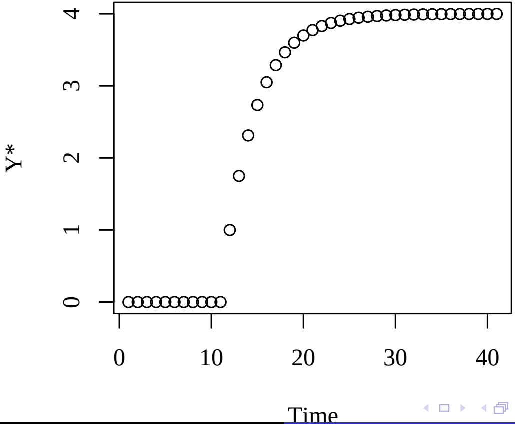
- Pre-event
- During event
- Aftermath

Gradual Permanent Intervention Cont.

What are the implied recursions for Y_t^*?

- Pre-event
- During event
- Aftermath
- So, what would you expect to see with $\omega_0 = 1$ and $\delta = .75$?

I think I can, I think I can,



Abrupt and Temporary Effect

Now lets look at at a temporary pulse intervention:

$\Delta I_t \leftarrow$Differencing a Permanent Intervention

$\ldots,\ (0-0),\ (0-0),\ (1-0),\ (1-1),\ (1-1),\ \ldots$

$\ldots,\ 0,\ 0,\ 1,\ 0,\ 0,\ \ldots$

This yields:

$$(1-B)I_t = \left\{ \begin{array}{ll} 0 & t < t_i \\ 1 & t = t_i \\ 0 & t > t_i \end{array} \right\}$$

Without Dynamics:

$$Y_t^* = \omega_0(1 - B)I_t$$

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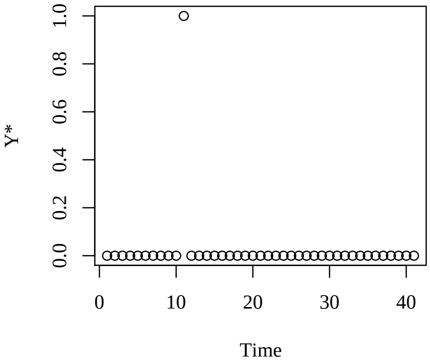

Abupt Temporary with Dynamics

The first order transfer function becomes:
$Y_t^* = \frac{\omega_0}{1-\delta B}(1-B)I_t$
Which gives us:

$$\begin{aligned}(1-\delta B)Y_t^* &= \omega_0(1-B)I_t \\ Y_t^* &= \delta Y_{t-1}^* + \omega_0(1-B)I_t\end{aligned}$$

Abrupt and Temporary Continued

What are the simulated predictions then?

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- What is the interpretation of $\omega_0 = 1$ and $\delta = .75$?

Abrupt and Temporary Continued

What are the simulated predictions then?

- What is the interpretation of $\omega_0 = 1$ and $\delta = .75$?
- What happens with $\delta = 1$

Shoots and Ladders