A-AE 567 Homework Spring 2016

You will need Matlab and Simulink.

Your work must be neat and easy to read.

Clearly, identify your answers in a box.

You will loose points for poorly written work.

You must work alone.

NAME:

1 Problem

Consider the discrete time system

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} e^{-\frac{n}{50}} & 1 \\ 2 & \cos(\frac{n}{50}) \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} e^{-\frac{n}{50}} \cos(\frac{n}{50}) \\ 1 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} 1 + e^{-\frac{n}{50}} & 2 + \sin(\frac{n}{50}) \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \Big(1 + \frac{1}{2}\sin(\frac{n}{50})\Big) v(n)$$

where u and v are independent Gaussian white noise processes. The initial condition $x(0) = 0$. To generate u and v in Matlab, set

$$\text{rng(1000);} \quad u = \text{randn(1,20);}$$

$$\text{rng(2000);} \quad v = \text{randn(1,20);}$$

Let $\mathcal{M}_n = \text{span}\{y(j)\}_0^n$. Find the following

(i) $P_{\mathcal{M}_{n-1}} x_1(n)$ for n= 8,9,10.

(ii) $P_{\mathcal{M}_n} x_2(n)$ for n= 8,9,10.

(Note the indices on the state $x_1(n)$ in Part (i), and $x_2(n)$ in Part (ii).) Be careful Matlab does not have a zero index. So for example, in Matlab

$$A(0) = A\{1\} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad A(1) = A\{2\} = \begin{bmatrix} e^{-\frac{1}{50}} & 1 \\ 2 & \cos(\frac{1}{50}) \end{bmatrix} \quad \text{ect.}$$

2 Problem: The inverted pendulum problem

Part of this problem is taken from the web site:

Control tutorials for Matlab,

http://www.engin.umich.edu/group/ctm/index.html

Consider the classical inverted pendulum problem, consisting of a cart of mass m_1 with an inverted pendulum with inertia J on the cart. The linearized equations of motion in state space form are given by

$$\begin{aligned}
\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{(J+m_2l^2)\zeta}{q} & \frac{m_2^2 g l^2}{q} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m_2 l \zeta}{q} & \frac{m_2 g l (m_1+m_2)}{q} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{J+m_2l^2}{q} \\ 0 \\ \frac{m_2 l}{q} \end{bmatrix} w + \begin{bmatrix} 0 & 0 \\ \frac{1}{4} & 0 \\ 0 & 0 \\ 0 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\
y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{20} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\
q &= J(m_1+m_2) + m_1 m_2 l^2.
\end{aligned}$$

Here x is the position of the cart and φ is the angle of the inverted pendulum from the vertical position. The control force on the cart is denoted by w. Moreover, u_1, u_2, v_1 and v_2 are all independent white noise processes. Notice that u_1 is a random force on the cart and u_2 is a random torque on the pendulum. The notation is

- $m_1 = 0.5kg$ is the mass of the cart
- $m_2 = 0.2kg$ is the mass of the pendulum
- $\zeta = 0.1N/m/sec$ is the friction of the cart
- $l = 0.3m$ is the distance to the pendulum's center of mass

- $J = 0.006kgm^2$ is the inertia of the pendulum
- w is the force applied to the cart
- x is the position of the cart
- φ is the angle in radians of the pendulum from the vertical position.

Assume that all the initial conditions are zero. Design a feedback controller $w = -K\widehat{x}$ based on the steady state Kalman filter such that the cart moves not more that one meter from the origin ($|x| \leq 1$) and the angle moves no more than ± 20 degrees from the vertical position ($|\varphi| \leq 0.35$). Your state feedback gain $K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$ must satisfy $|k_j| \leq 30$ for $j = 1, 2, 3, 4$. Simulate your controller in Simulink for 30 seconds. Hand the graphs from your Simulink program for:

(i) *The position x and the estimated position of the cart $\widehat{x}$ on the same graph.*

(ii) *The velocity $\dot{x}$ and the estimated velocity of the cart $\dot{\widehat{x}}$ on the same graph.*

(iii) *The angle φ and the estimated angle of the pendulum $\widehat{\varphi}$ on the same graph.*

(iv) *The angular velocity $\dot{\varphi}$ and the estimated angular velocity of the pendulum $\dot{\widehat{\varphi}}$ on the same graph.*

(v) *Hand in your gain K.*

On the band limited white nose generators, set both the noise power and sample time equal to 0.1. The seed for u_1, u_2, v_1 and v_2 is respectively, 23341, 23342, 23343 and 23344.