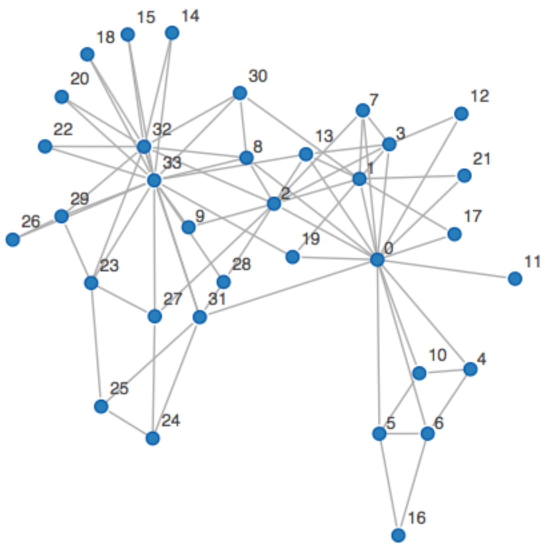
Network and Spatial Analyses Homework

March 2020

1. (2p.) List the integer numbers from 2 to 12, let these numbers be the nodes of a graph G. Draw an edge between two numbers (nodes), if one of them divides the other (e.g. 2 divides 6 , thus, there is an edge between them).
   (a) Write down the edgelist representation of the graph! (List all possible pairs, decide if there is an edge.)
   (b) Draw the graph! Label the nodes with the numbers!
   (c) How many connected components are there in this graph?
   (d) What is the diameter of the largest connected component?

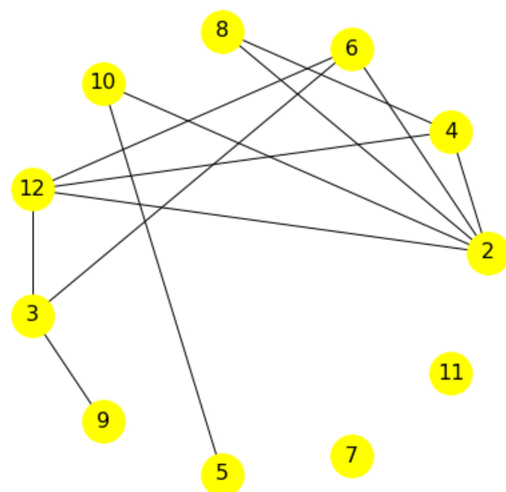
2. (2p.) The following network is Zachary's Karate Klub network, that is one of the most famous small real-world network examples. It depicts friendships within a karate klub, that later split up into two parts, with the members founding two new groups. After that, each new member belonged to one of the two new groups.



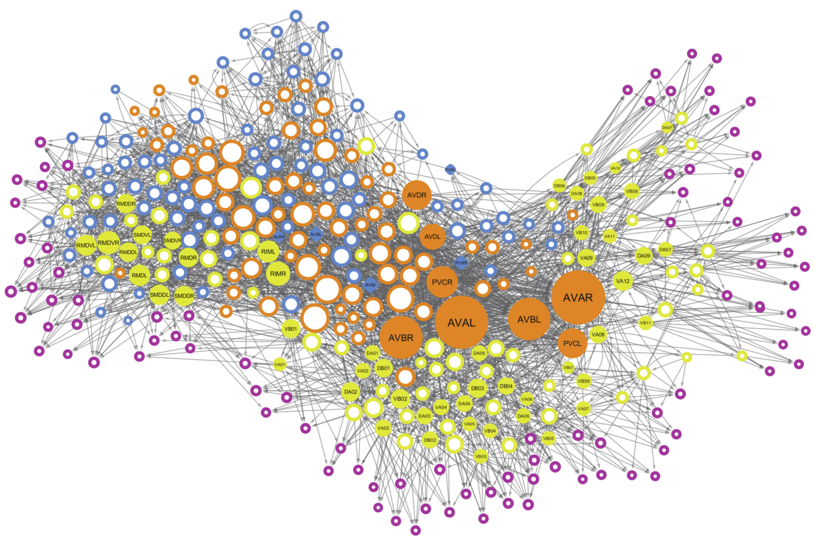
(a) Characterize the network with the learned concepts:

- (un)directed/(un)weighted;
- number of nodes/number of edges;
- average degree;
- diameter.

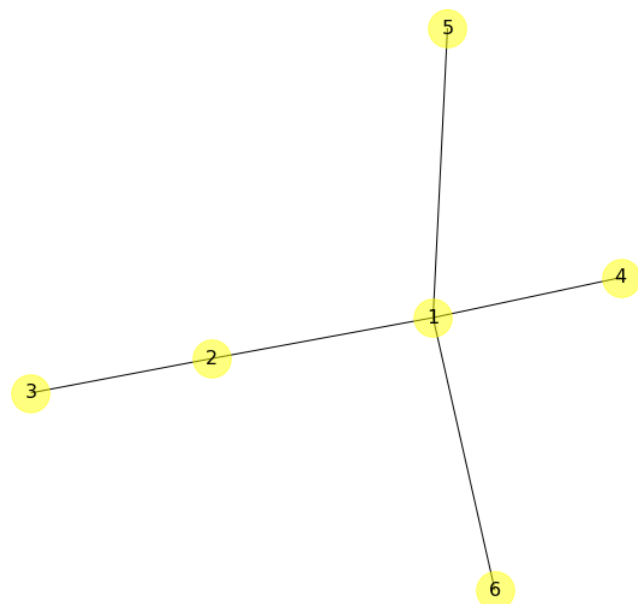
3. (4p.) Calculate the clustering coefficient for the all of the nodes and calculate the betweenness centrality for node **6** in the following network!



4. (6p.) The following figure shows the network of the neurons within the famous *C. elegans* worm, for which biologists know the exact connections between the neural cells. (We still have no such detailed model of the human brain because of the large number of nodes.) What do you think, which model could be more plausible for the emergence of this neuron network, Erdos–Rényi, Small-World or Barabási–Albert? Do you think your chosen model is a perfect explanation, or are there some features of the *C. elegans* network that you could not account for? You can have a look at some of the aggregate measures for this network at https://snap.stanford.edu/data/C-elegans-frontal.html.



5. (6p.) Here's a small graph, that has been generated in a few timesteps with a Barabási– Albert model, where each new node has been added with one single new edge to the existing graph.



If we denote the attachment probability of node i by p_i, and the degree of node i by k_i, and the total number of nodes is N at the current timestep, then:

$$p_i = \frac{k_i}{\sum_{j=1}^{N} k_j} \tag{1}$$

(a) Let's suppose we would like to add one new node with one new edge. For each old node, calculate the attachment probability!

(b) Draw an interval between 0 and 1, and divide the interval according to the previously calculated probabilities $p_1, p_2, ...p_N$. Each small interval now represents one of the nodes and the length corresponds to that node's attachment probability. Remember, the sum of the probabilities should be 1, since the new node definitely connects itself to somewhere in the existing graph. Go to the following link, that generates you a random number r between 0 and 1. Note down this number. Which of the small intervals does it fall into? Connect the new node to the corresponding old node!

(c) Repeat steps a) and b) two more times! Attention, the number of nodes, so the number of probabilities to be calculated grows by one at each timestep!

(d) Give the adjacency matrix of the final graph! Plot the degree distribution of the final graph!