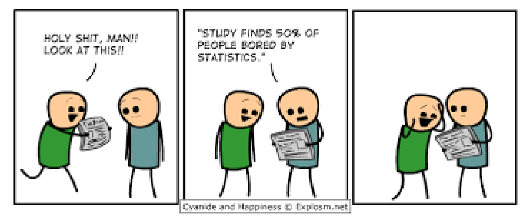
Applied Linear Modeling

October 22, 2019

All the things for today

- Type I & II error warm-up
- Mini-lecture on the theory behind the logistic model
- Logistic regression basketball workshop
  - *R package to download: odds.n.ends*
- Slides and workshop packet on GitHub



What is logistic regression

- A statistical model used to predict or explain a **binary** outcome variable
- For example:
  - *What predicts whether or not someone uses the library?*
  - *What predicts whether or not someone is a smoker?*
  - *What predicts whether or not someone owns a gun?*

The statistical form of the logistic model

- Because the outcome variable is binary, the linear regression model would not work (it requires a continuous outcome!)
  - *Linear model:* $y = b_0 + b_1 x_1 + b_2 x_2 \ldots.$
- The linear regression model can be transformed using a *logit transformation* based on the logistic function in order to model binary outcomes
- The logistic function is officially defined:

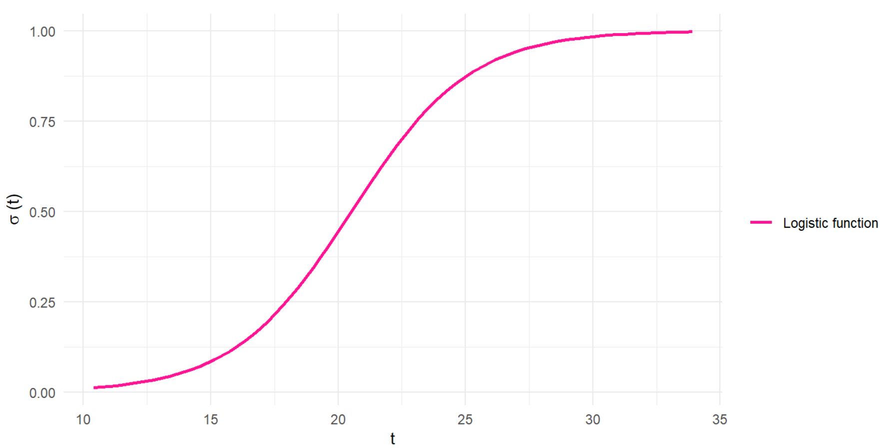
$$\sigma(t) = \frac{e^t}{e^{t+1}}$$

- Simplified to:

$$\sigma(t) = \frac{1}{1+e^{-t}}$$

The logistic function

- When you graph the logistic function, it has a sigmoid shape that stretches from -∞ to ∞ on the x-axis and from 0 to 1 on the y-axis
- The function $\sigma(t) = \frac{1}{1+e^{-t}}$ can take any value of t along the x-axis and give the corresponding value of $\sigma(t)$ that is between 0 and 1 on the y-axis



From the logistic function to the logistic model

- Logistic function to logistic model
  - *In the case of logistic regression, the value of t will be the right-hand side of the regression model, which looks something like* $b_0 + b_1x_1 + \ldots$
  - *The left hand side of the logistic regression model is the probability of y or* $p(y)$
- Start with the logistic function

$$\sigma(t) = \frac{1}{1+e^{-t}}$$

- Substitute in the regression notation of x and y and slope and intercept to make the logistic model

$$p(y) = \frac{1}{1+e^{-(b_0+b_1x_1+b_2x_2\ldots)}}$$

Reading the logistic model

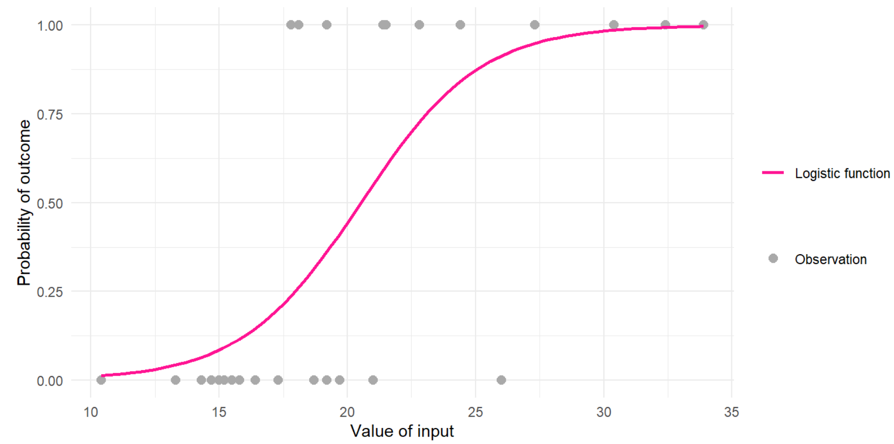
- Logistic model

$$p(y) = \frac{1}{1 + e^{-(b_0 + b_1 x_1 + b_2 x_2)\ldots}}$$

- Where:
  - *y is the binary outcome variable*
  - *p(y) is the probability of the outcome*
  - b_0 *is the y-intercept*
  - x_1, x_2, *etc are predictors of the outcome*
  - b_1, b_2, *etc are the slopes/coefficients for* x_1, x_2, *etc.*

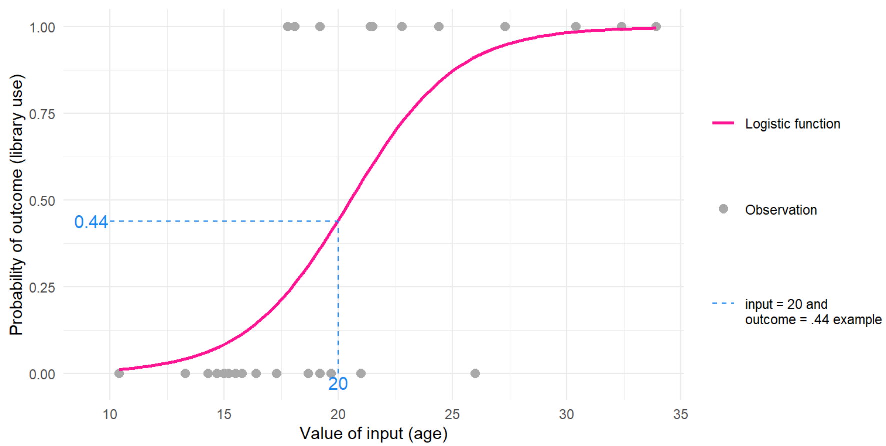
The logistic function with data

- The data points represent library users (y = 1) and non-users (y = 0)
- The x-axis represents age in years
- The equation for the curve shown would be something like $p(y) = \frac{1}{1+e^{-(b_0+b_1*age)}}$



Example of probability of y for a value of x

- What is the probability of library use for a 20-year old?
  - *Starting at 20 on the x-axis, trace a straight line up to the logistic function curve and from there look to the y-axis for a value*
    - This logistic curve might represent the regression equation $p(y) = \frac{1}{1+e^{-(.15-.02*age)}}$
  - *For the logistic model represented by this graph, the model would predict a probability of y around .44 or 44%*



Interpreting the probability

- If this were a model predicting library use from age, it would it predict a 44% probability of library use for a 20-year-old [p(y) = .44]
- Since 44% is lower than a 50% probability of the value of y, the model is predicting that the 20-year-old does not have the outcome
- So, if the outcome is library use, the logistic model would predict this 20-year-old was not a library user

Let’s play logistic regression basketball!