Class 2: Bayes theorem and Bayesian updating

Bayes theorem

$$Pr(A|B) = \frac{Pr(B|A) \times Pr(A)}{Pr(B)}$$

Bayes theorem

- Suppose there is a test for vampirism that correctly detects vampirism in 90% of cases, i.e. given that you test a vampire, in 90 out of 100 cases it will return positive result.
- The test has some flaws, because it can also return positive result when given to a human. Luckilly, the chance of such a result is small, 10%, i.e. given that you test a human in 10 out of 100 cases it will return positive result.
- We also know that vampires are quire rare, there is only 1 in 100 individuals.
- What is the chance that someone you tested and who obtained positive test result is a vampire?

Bayes theorem

- $Pr(positive|vampire) = .90$ and this implies $Pr(negative|vampire) = .10$
- $Pr(positive|human) = .10$ and this implies $Pr(negative|human) = .90$
- $Pr(vampire) = .01$, and this implies $Pr(human) = .99$
- $Pr(vampire|positive) = ...$

Bayes theorem

$$Pr(vampire|positive) = \frac{Pr(positive|vampire) \times Pr(vampire)}{Pr(positive)}$$

$Pr(positive) = ???$

Bayes theorem

- How frequently the test will return positive result?
- Suppose you give a test to 1000 individuals:
  - If $Pr(vampire) = .10$, then 10 will be vampires, and 990 will be humans.
  - If $Pr(positive|vampire) = .90$, then among 10 vampires, you obtain 9 positive results.
  - If $Pr(positive|human) = .10$, then among 990 humans, you obtain 99 positive resutls.
  - In sum you will obtain 108 positive results out of 1000 tests. Then $Pr(positive) = .108$

Bayes theorem

- In mathematical notation you can write the formula for faster computation.

$$Pr(p.) = [Pr(p.|v.) \times Pr(v.)] + [Pr(p.|h.) \times Pr(h.)]$$

Bayes theorem

$$Pr(v.|p.) = \frac{Pr(p.|v.) \times Pr(v.)}{[Pr(p.|v.) \times Pr(v.)] + [Pr(p.|h.) \times Pr(h.)]}$$

Bayes theorem

$$Pr(vampire|positive) = \frac{.90 \times .01}{[.90 \times .01] + [.10 \times .99]}$$

$$Pr(vampire|positive) = \frac{.90 \times .01}{.108}$$

$$Pr(vampire|positive) = \frac{.90 \times .01}{.108}$$

$$Pr(vampire|positive) = .0833$$

Your turn

Verify your computation

When does a significant p-value indicate a true effect?

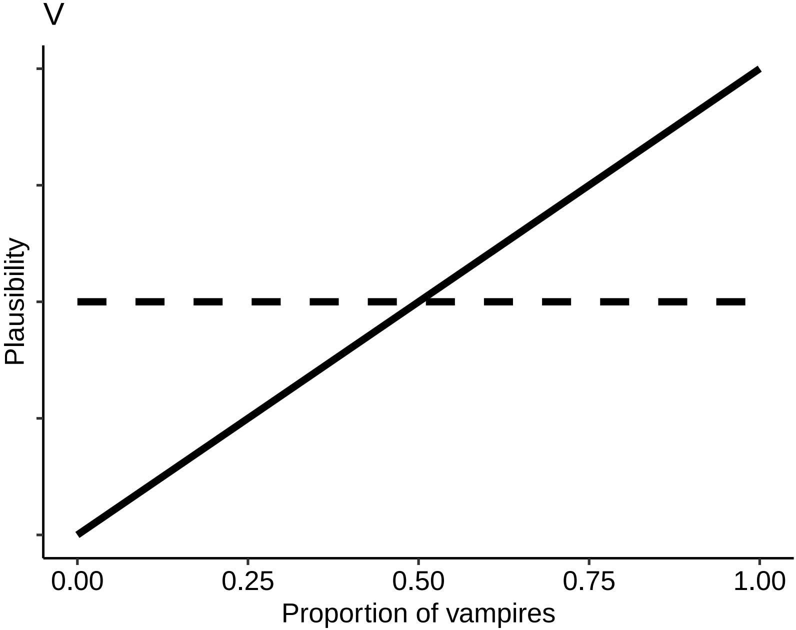
Bayes theorem

$$Posterior = \frac{Likelihood \times Prior}{AverageLikelihood}$$

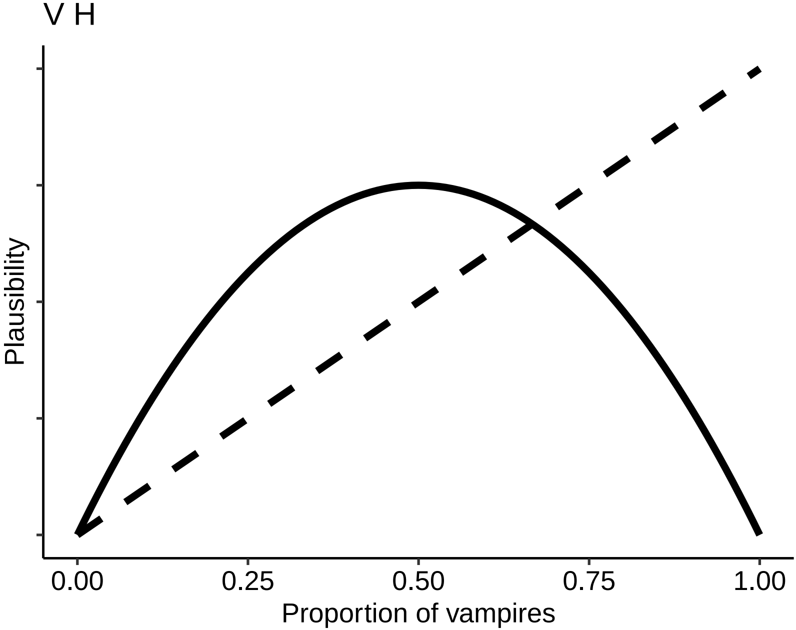
Bayesian updating

- Suppose you have build a perfect detector of vampires.
- You start examining randomly encountered people.
  - What is a random sample?
- You have obtained a sample of 10 independent records.
  - What is the definition of independent records?
- Vampire, Human, Human, Vampire, Human, Human, Human, Human, Human, Human

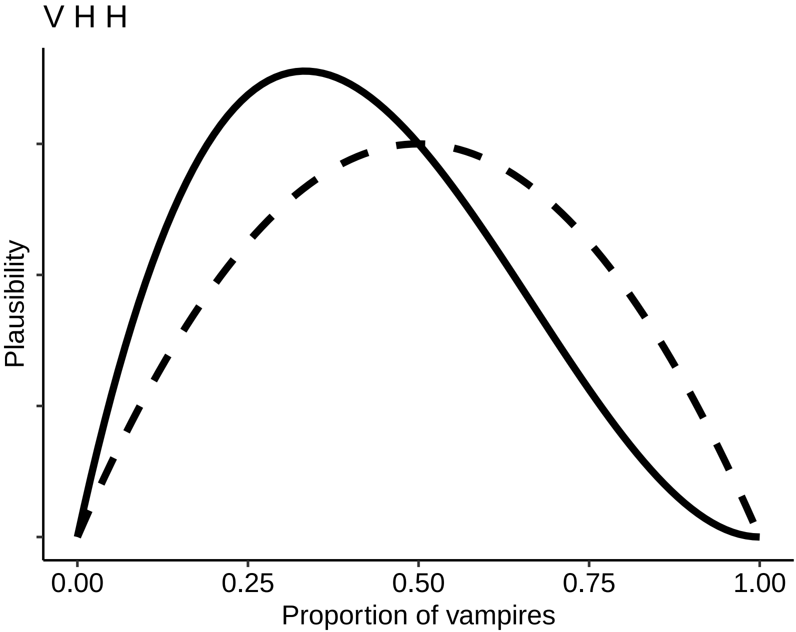
Bayesian updating



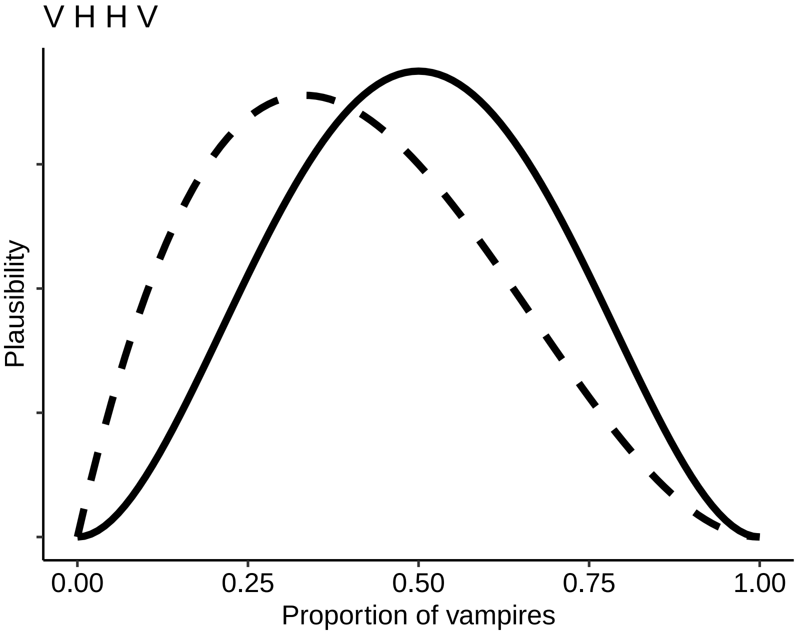
Bayesian updating



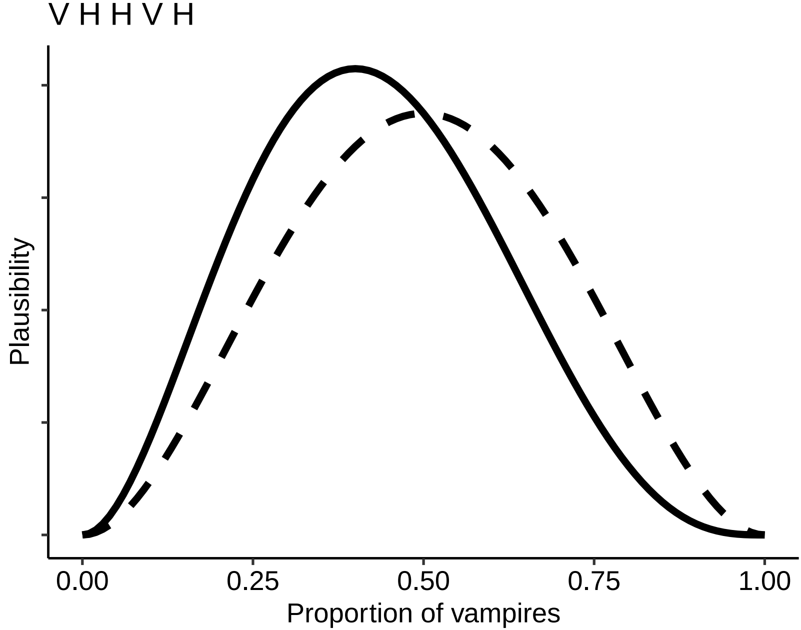
Bayesian updating



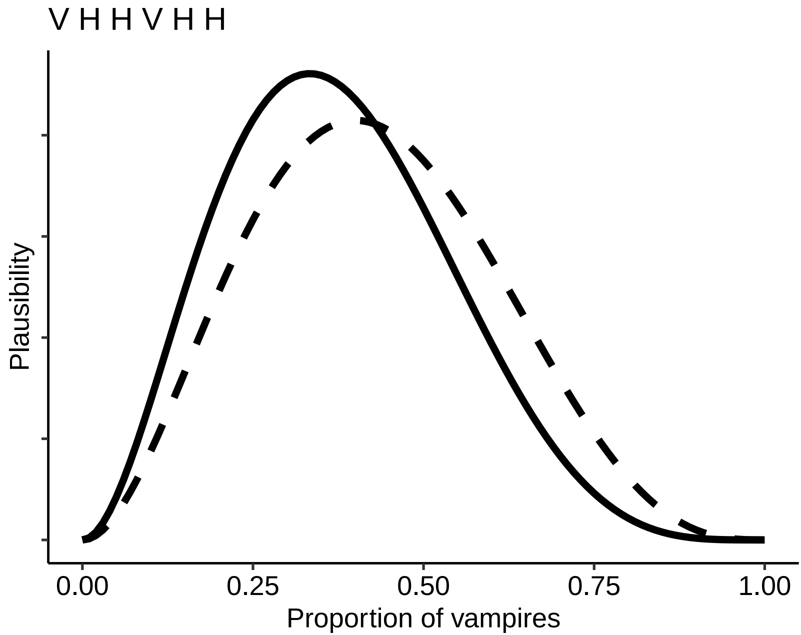
Bayesian updating



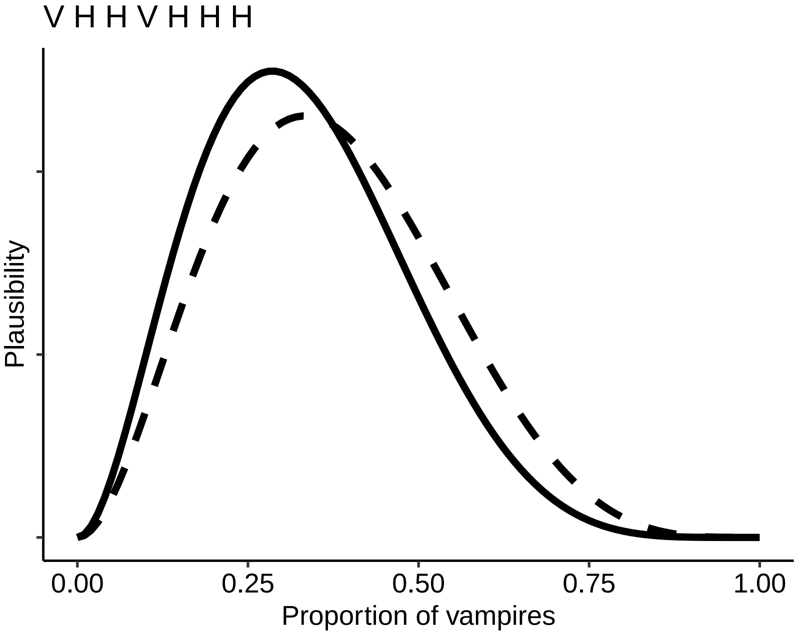
Bayesian updating



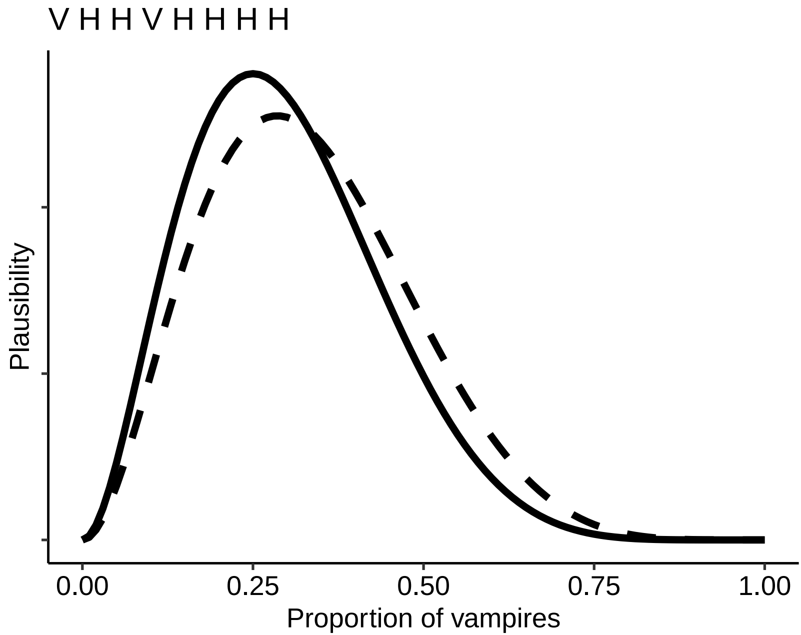
Bayesian updating



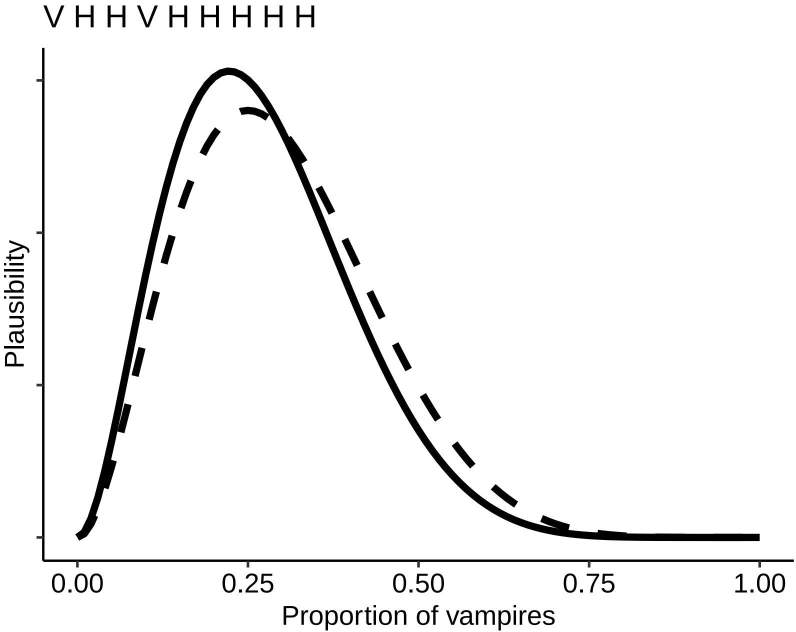
Bayesian updating



Bayesian updating



Bayesian updating



Bayesian updating