Lab 9

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December 5th 2017

Bayes Factor

Posterior = likelihood x prior...

$P(\theta|\mathcal{D}) = \frac{P(\mathcal{D}|\theta) \times P(\theta)}{P(\mathcal{D})}$

... given our model is true

$P(\theta|\mathcal{D}, \mathcal{M}) = \frac{P(\mathcal{D}|\theta, \mathcal{M}) \times P(\theta|\mathcal{M})}{P(\mathcal{D}|\mathcal{M})}$

where $\mathcal{M}$ = model used to describe data. We implicitly assume that our model is true, and thus usually don't include $\mathcal{M}$ in equations.

Bayesian model comparison

Different models result in different posterior distributions

$P(\theta_1|\mathcal{D}, \Updownarrow\infty) = \frac{P(\mathcal{D}|\theta_1, \Updownarrow\infty) \times P(\theta_1|\Updownarrow\infty)}{P(\mathcal{D}|\Updownarrow\infty)}$

versus

$P(\theta_2|\mathcal{D}, \Updownarrow\in) = \frac{P(\mathcal{D}|\theta_2, \Updownarrow\in) \times P(\theta_2|\Updownarrow\in)}{P(\mathcal{D}|\Updownarrow\in)}$

Posterior for a model

We can use Bayes theorem to compute posterior for the model - probability that the model is true given the data.

$P(\Updownarrow\infty|\mathcal{D}) = \frac{P(\mathcal{D}|\Updownarrow\infty) \times P(\Updownarrow\infty)}{P(\mathcal{D})}$

and

$P(\Updownarrow\in|\mathcal{D}) = \frac{P(\mathcal{D}|\Updownarrow\in) \times P(\Updownarrow\in)}{P(\mathcal{D})}$

Explicit comparision

With 2 models we can explicitly compare them by using odds of posterior distributions.

$\frac{P(\Updownarrow\infty|\mathcal{D})}{P(\Updownarrow\in|\mathcal{D})} = \frac{P(\mathcal{D}|\Updownarrow\infty)}{P(\mathcal{D}|\Updownarrow\in)} \times \frac{P(\Updownarrow\infty)}{P(\Updownarrow\in)}$

posterior odds = ???? x prior odds

Bayes Factor

$\frac{P(\Updownarrow\infty|\mathcal{D})}{P(\Updownarrow\in|\mathcal{D})} = Bayes\ Factor \times \frac{P(\Updownarrow\infty)}{P(\Updownarrow\in)}$

Bayes Factor denotes how much our belief in model 1 in comparison to model 2 increases, by seeing the data. It can be described how much more we favor one model over another.

Interpreting Bayes Factor

Provided by Jeffreys:

value	strength of evidence for model 1 over model 2
less than 1	negative (supports model 2)
1 to 3.16	barely worth mentioning
3.16 to 10	substantial
10 to 31.62	strong
31.62 to 100	very strong
more than 100	decisive

Interpreting Bayes Factor

Provided by Kass and Raftery:

value	strength of evidence for model 1 over model 2
less than 1	negative (supports model 2)
1 to 3	not worth more than a bare mention
3 to 20	positive
20 to 150	strong
more than 150	very strong

Calculating Bayes Factor

Computing Bayes Factor is as simple as dividing denominators of Bayes theorem from both models.

$Bayes\ Factor = \frac{\int P(\mathcal{D}|\theta,\Updownarrow\infty)\times P(\theta|\Updownarrow\infty)}{\int P(\mathcal{D}|\theta,\Updownarrow\in)\times P(\theta|\Updownarrow\in)}$

Obviously, this is just a joke! Dividing two unknown integrals can be very hard - worse yet MCMC is not very helpful in this case. However there are some approaches to approximate Bayes Factor.

Pros and cons of Bayes Factor

Pros:

- it is quite intuitive and easy to interpret
- we can measure a strength of evidence for null hypothesis

Cons:

- we have to be very carefull with priors we use - it is SUPERSENSITIVE to our prior assumptions

- there are no noninformative prior distributions