# Lab 13

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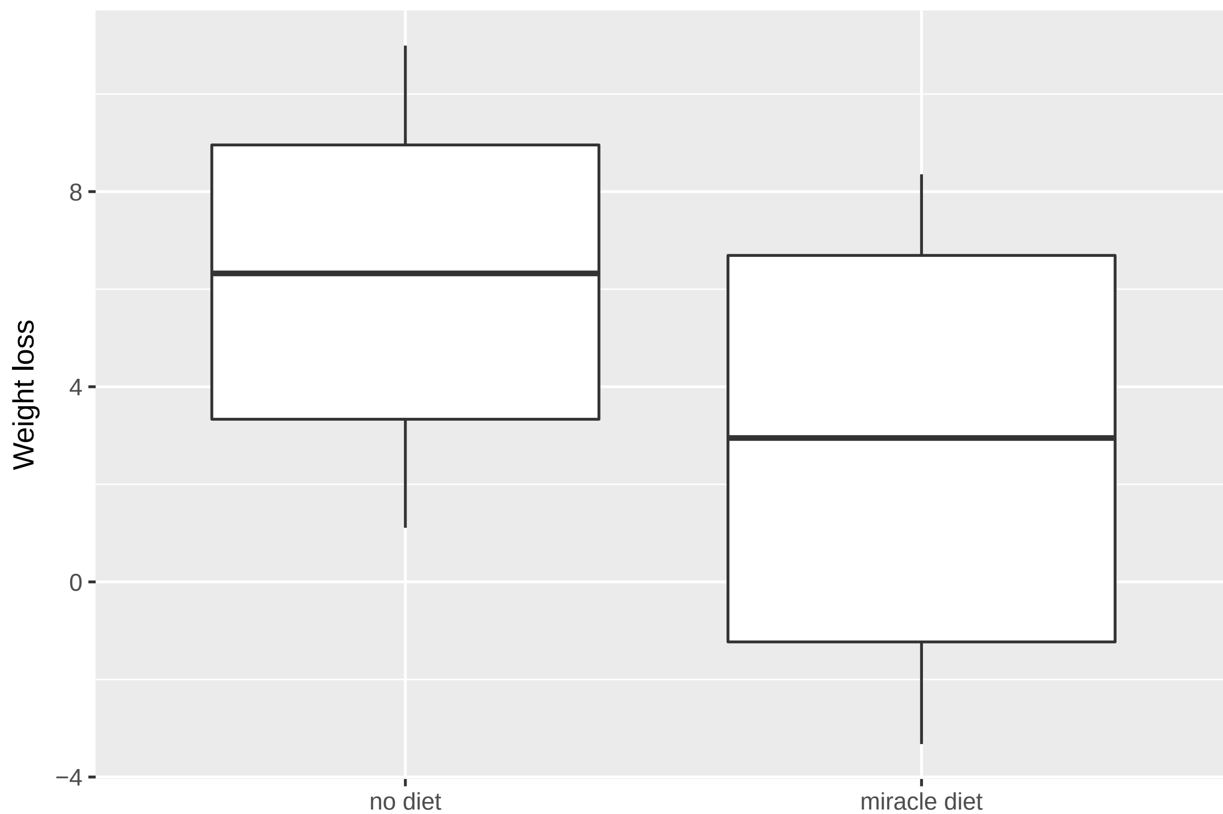
## Bayesian Analysis of Factorial Desings

### Why factorial designs?

- Say we are interested in how some miracle diet affects weight loss.
- We are conducting a simple study and compare no diet to miracle diet.

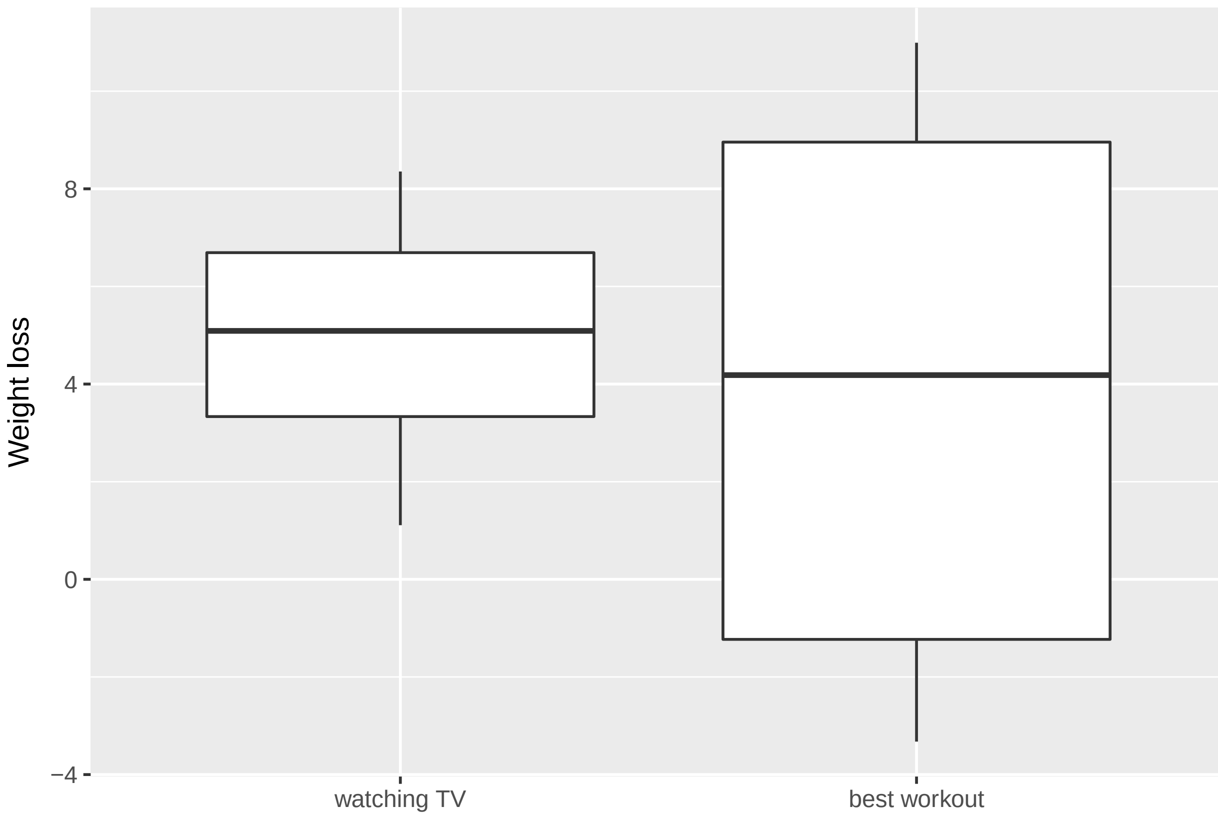
### Why factorial designs?




### Why factorial designs?

- Ahh, the miracle diet... What is really important is a proper workout.
- We should conduct another experiment, should we?

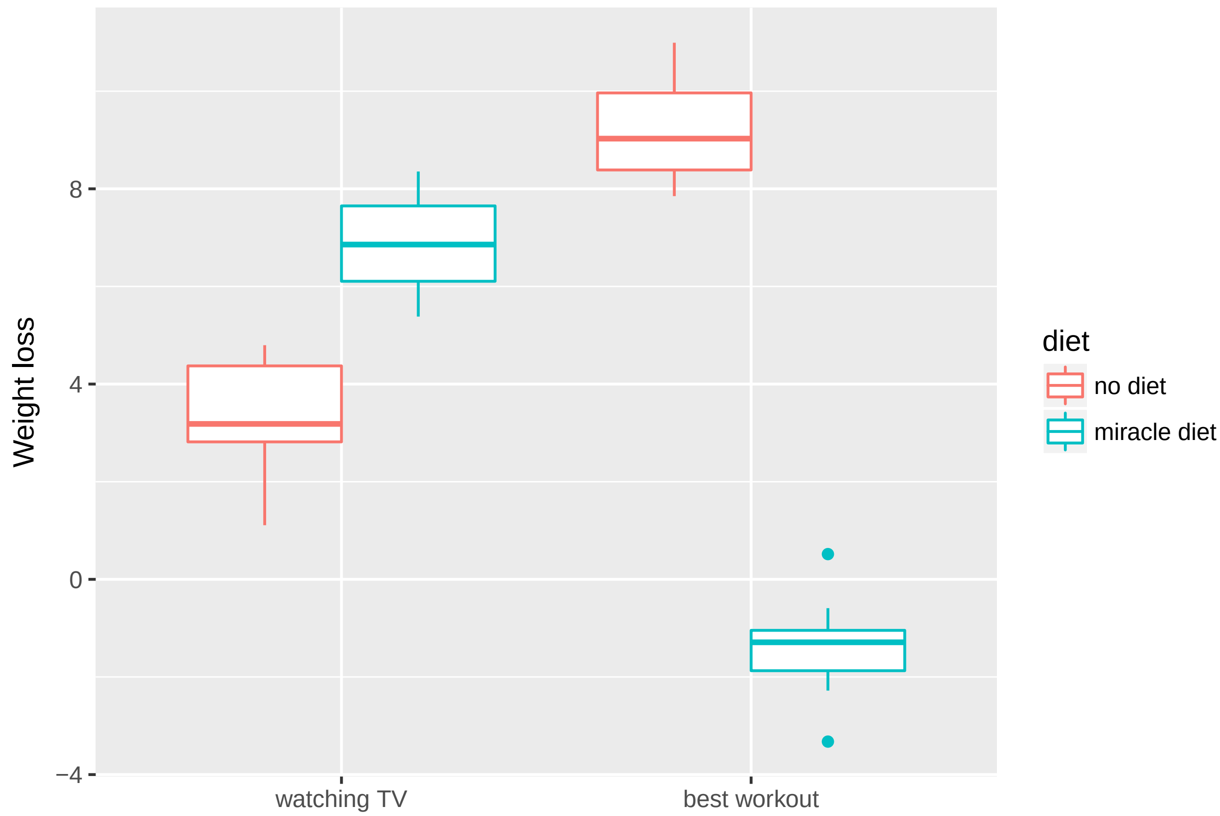
## Why factorial designs?




## Why factorial designs?

- If we know in advance important $n$ factors responsible for changes in DV, we should rather conduct $n$ factorial experiment.
- E.g. with diet and workout.

## Why factorial designs?




## Model for factorial designs

$y_{ijk} = \alpha + \beta_{1j} + \beta_{2k} + \beta_{1\times 2jk} + \epsilon_{ijk}$

- $\alpha$ - grand mean
- $\beta_{1j}$ - effect of factor 1 for level indicated by subscript $j$
- $\beta_{2k}$ - effect of factor 2 for level indicated by subscript $k$
- $\beta_{1\times 2jk}$ - interactive effect of factors 1 and 2 for their levels indicated by subscripts $j$ and $k$
- $\epsilon_{ijk}$ - individual level variation for a subject $i$ in groups $j$ of factor 1 and $k$ of factor 2
- we assume that $E(\epsilon_{ijk})$ and $Var(\epsilon_{ijk}) = \sigma^2$, ie. variance does not depend on group membership

## Understanding the model

- Say $\alpha = 4.5$

| | no diet | diet |
|---|---|---|
| watching TV | 4.5 | 4.5 |
| workout | 4.5 | 4.5 |

## Understanding the model

- Say $\beta_{1j}$ refers to diet, and is equal to a vector of length equal to the number of levels of diet: (-2, 2)

| | no diet | diet |
|---|---|---|
| watching TV | 2.5 | 6.5 |
| workout | 2.5 | 6.5 |

## Understanding the model

- Say $\beta_{2j}$ refers to workout, and is equal to a vector of length equal to the number of levels of workout: (-3, 3)

| | no diet | diet |
|---|---|---|
| watching TV | 1.5 | 1.5 |
| workout | 7.5 | 7.5 |

## Understanding the model

- The additive effect of $\beta_{1j}$ and $\beta_{2j}$ is just a combination of previous operations

| | no diet | diet |
|---|---|---|
| watching TV | -0.5 | 3.5 |
| workout | 5.5 | 9.5 |

## Understanding the model

- What's left are interaction terms, which are specific to each cell (ie. each combination of factors)
- Say we have this interaction effects:

| | no diet | diet |
|---|---|---|
| watching TV | -1 | 1 |
| workout | 1 | -1 |

## Understanding the model

- When we add respective cells we will obtain estimated means

| | no diet | diet |
|---|---|---|
| watching TV | -1.5 | 4.5 |
| workout | 6.5 | 8.5 |

## Bayesian factorial model

- The implementation follows the guidelines of Kruschke
- We assume $y_{ijk} \sim \mathcal{N}(\mu_{jk}, \sigma_y^2)$
- $\sigma_y^2 \sim \mathcal{U}(low, high)$ where low and high indicate values respective to the scale of the DV
- $\mu_{jk} = \alpha + \beta_{1j} + \beta_{2k} + \beta_{1\times 2jk}$
- $\alpha \sim \mathcal{N}(M, S^2)$ where M and S are mean and multiple of observed DV variance
- $\beta_{xj} \sim \mathcal{N}(0.0, \sigma_x^2)$, are deflections related to $x$ (where $x$ is factor 1, 2, or interaction)
- $\sigma_x^2 \sim \mathcal{G}(Shape, Rate)$, are variances/standard deviations of deflections related to $x$ (where $x$ is factor 1, 2, or interaction), and shape and rate are calculated w.r.t. distribution of DV