Lab 6

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Markov Chain Monte Carlo

Monte Carlo basic equation

$E[h(\theta|x)] = \int \pi(\theta|x)h(\theta)d\theta \approx \frac{1}{n}\sum_{i=1}^{n} h(\theta_i)$

- $\pi(\theta|x)$ is our posterior distribution
- $h(\theta)$ is some function of the parameter values, e.g. mean, or variance
- e.g. recall that if you multiply each possible value by probability of it occurence, and then add the products, you obtain arithmetic mean
- e.g. P(1) = 0.2; P(2) = 0.3; P(3) = 0.3; P(4) = 0.2 and mean of (1, 1, 2, 2, 2, 3, 3, 3, 4, 4) are both equal to 2.5
- instead of integrating we can draw a large sample from π and use it approximate integral

Issues with standard MC

- if we know the correct form of the posterior distribution (quite rare), we can just sample from it and evaluate it
- if we don't know the correct form (usually), we can still use some sampling technique, like rejection sampling
- however, if we have multi-parameter posterior distribution (frequent), we are little bit stuck
- it is possible to do rejection sampling, but finding appropriate and efficient bounding distribution in high dimensions is very difficult

Markov Chain Monte Carlo

- a solution proposed in the field of physics (among scientist working in Manhattan Project) is Markov Chain Monte Carlo
- MCMC is used not only on Bayesian analysis, but due to its efficiency and flexibility it is one of the main Bayesian tools
- the basic idea is to draw a sample of mildly correlated values, and use it to summarize target distribution
- thus it is the same as in standard MC, but we are working with nonidependent samples
- it occurs that the problem with nonindepent samples is not so huge, given the potential gains of this alghorithm