Bayesian Models in Psychology

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October 10, 2017

Lab 2 | Binomial Models

Bayesian updating

Bayes theorem

$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}$

Bayesian updating

$P(\theta|\mathcal{D}) = \frac{P(\mathcal{D}|\theta) \times P(\theta)}{P(\mathcal{D})}$

- $\mathcal{D}$ - data, i.e. results of an experiment or observation, fixed
- θ - parameters responsible for generating data, random

Example 1

- When tossing a coin we have some beliefs about the frequencies of heads and tails - θ. If we believe that the coin is fair, we expect that $P(heads) = P(tails) = 1/2$. When actually tossing a coin (suppose 10 times) we are obtaining some proportion of heads and tails, say 6 to 4 - $\mathcal{D}$.

Example 2

- Before conducting a study on sleeping disorders (SD), we may have some beliefs about the frequency of SD. Say we believe that it is present among 25% percent of undergraduate students - θ. After conducting the study on N = 100 undergraduates, we observe SD among 40 of undergraduates - $\mathcal{D}$

Example 3

- It is well known that IQ has a mean, $\mu = 100$ and standard deviation, $\sigma = 15$ - we can jointly name these two values as θ. We can expect these values in a random sample. However, after conducting a sample of school students we observe mean, m = 97, and standard deviation, SD = 20 - $\mathcal{D}$

Bayesian updating

$P(\theta|\mathcal{D}) = \frac{P(\mathcal{D}|\theta) \times P(\theta)}{P(\mathcal{D})}$

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Bayesian updating

$P(\theta|\mathcal{D}) \propto P(\mathcal{D}|\theta) \times P(\theta)$

posterior = likelihood $\times$ prior

Each new observation changes our beliefs about parameters responsible for generating data.

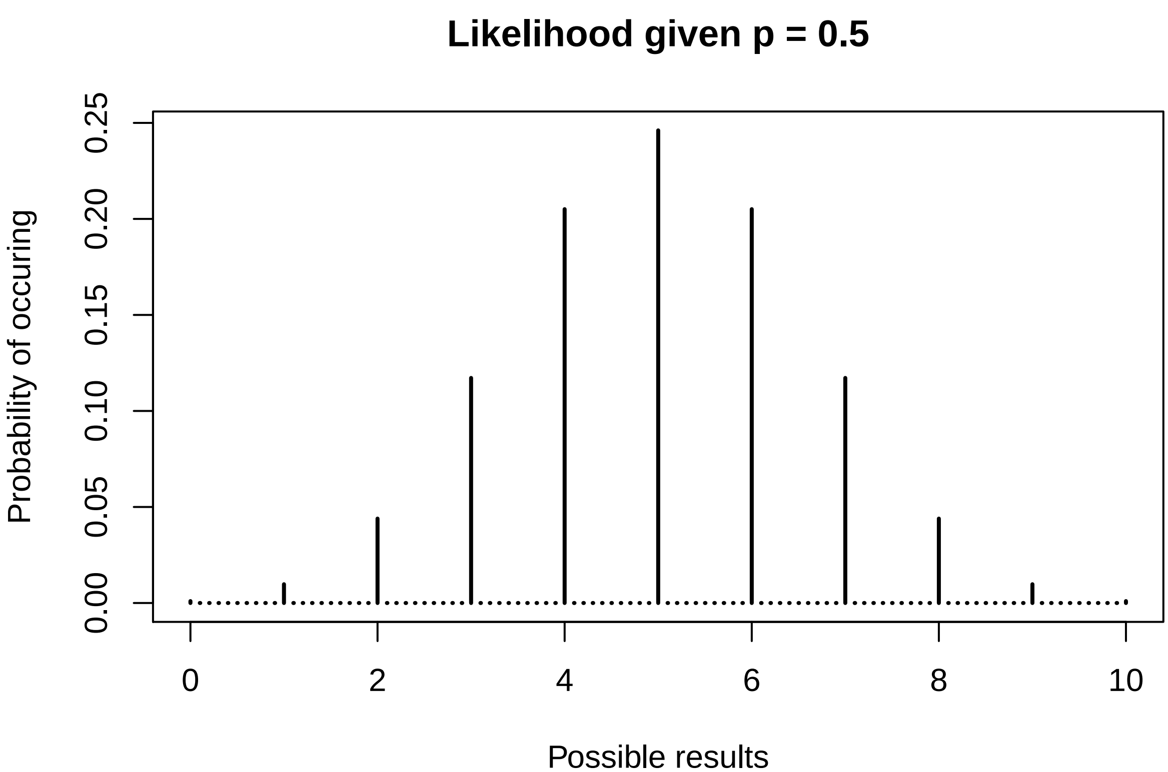
Prior $\times$ likelihood = posterior

- Prior - beliefs about about a state of the world **before** conducting a study
- Posterior - beliefs about a state of the world **after** conducting a study
- Likelihood - description of how certain quantities/qualities manifest in obervations; expected results given a certain value of parameter(s)

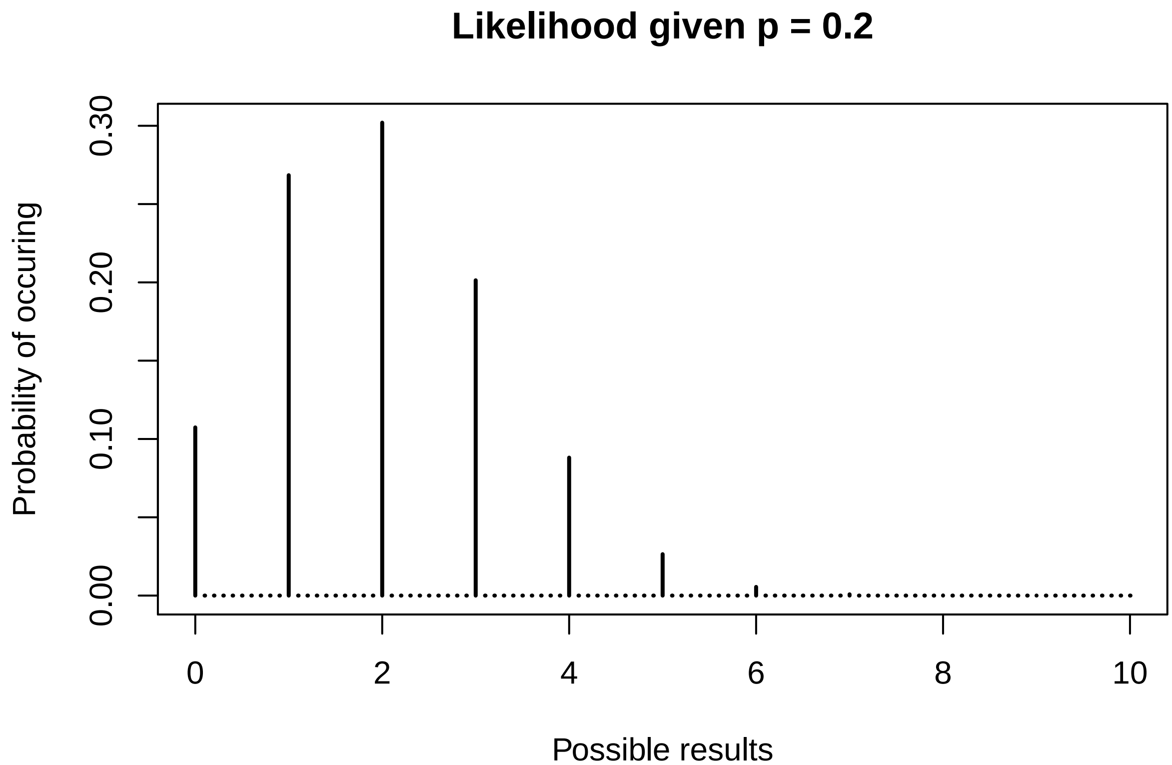
Prior - degree of belief

- Prior - degree of belief in parameter values
- Parameters values rarely take form of dichotomous or polytomous states, e.g. yes or no
- Parameters can also take discrete form, e.g. 4, 5, 6
- And frequently continuous form, e.g. 2.3242, 6.6875, 5.6789
- To assign degree of beliefs to different parameter values we use some distribution (e.g. Normal, Student t, F, χ^2)

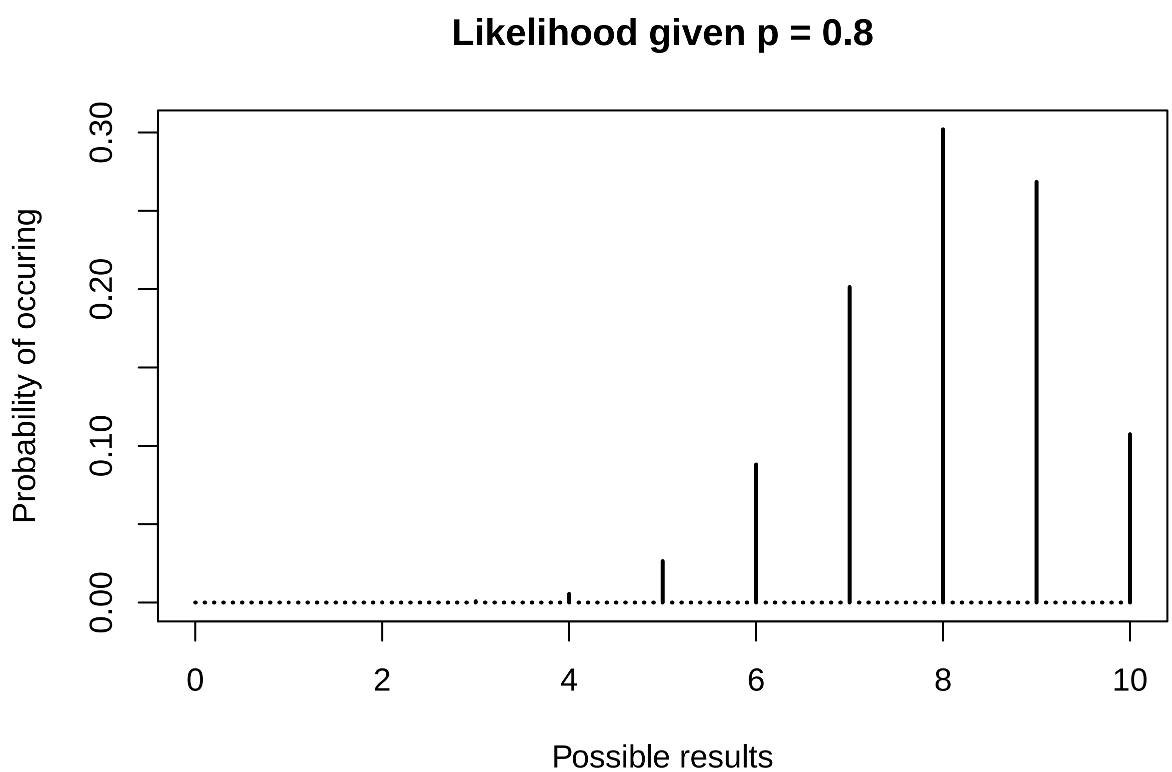
Binomial likelihood



Binomial likelihood



Binomial likelihood



Binomial likelihood

$L(p|S,F) \propto p^S \times (1-p)^F, 0 < p < 1$

- p - probability (parameter)
- S - number of successes or positive observations (data)
- F - number of failures or negative observations (data)

Working with beta distribution

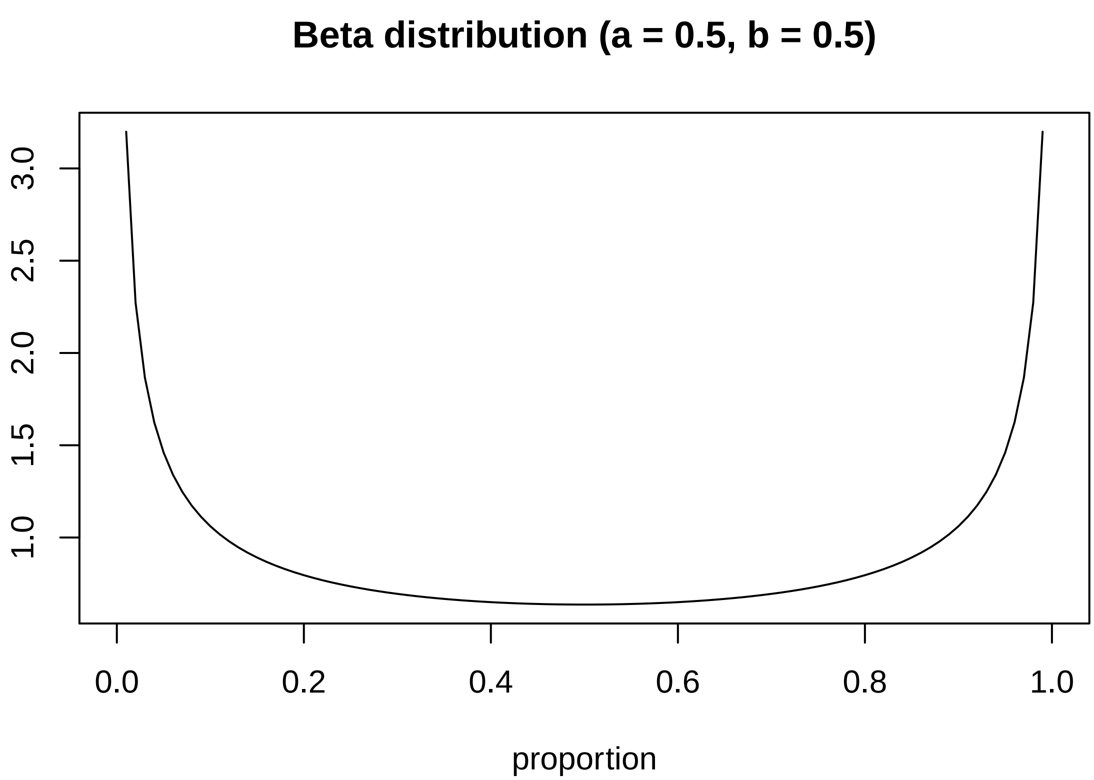
- Assigning degree of beliefs to parameter values can be very tedious, and with continuous parameters almost impossible
- We usually describe uncertainty associated with parameters by using some existing well known distribution
- When working with parameters reflecting proportion (i.e. taking values from 0 to 1), we usually use **beta distribution**

Working with beta distribution

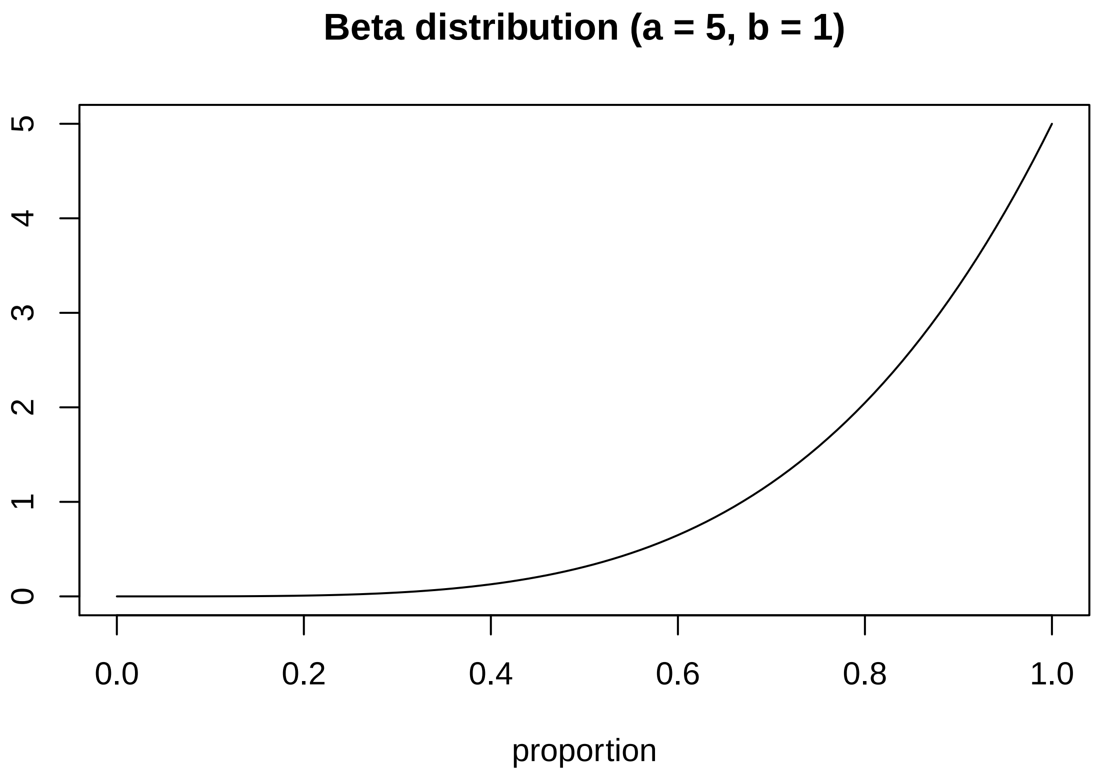
$\pi(p) \propto p^{a-1} \times (1-p)^{b-1}, 0 < p < 1$

- p - probability (parameter)
- a and b - hyperparameters reflecting the users prior beliefs about p
- mean of beta distribution is m = a/(a + b)
- variance of beta distribution is v = m(1-m)/(a+b+1)

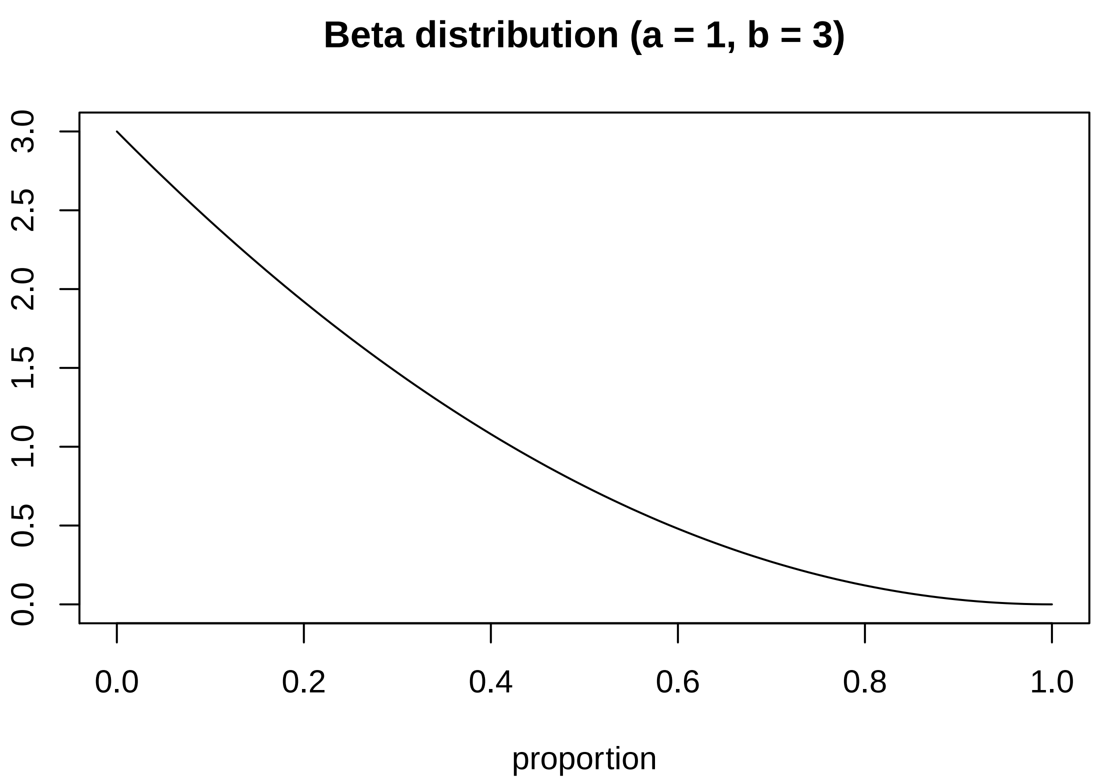
Working with beta distribution



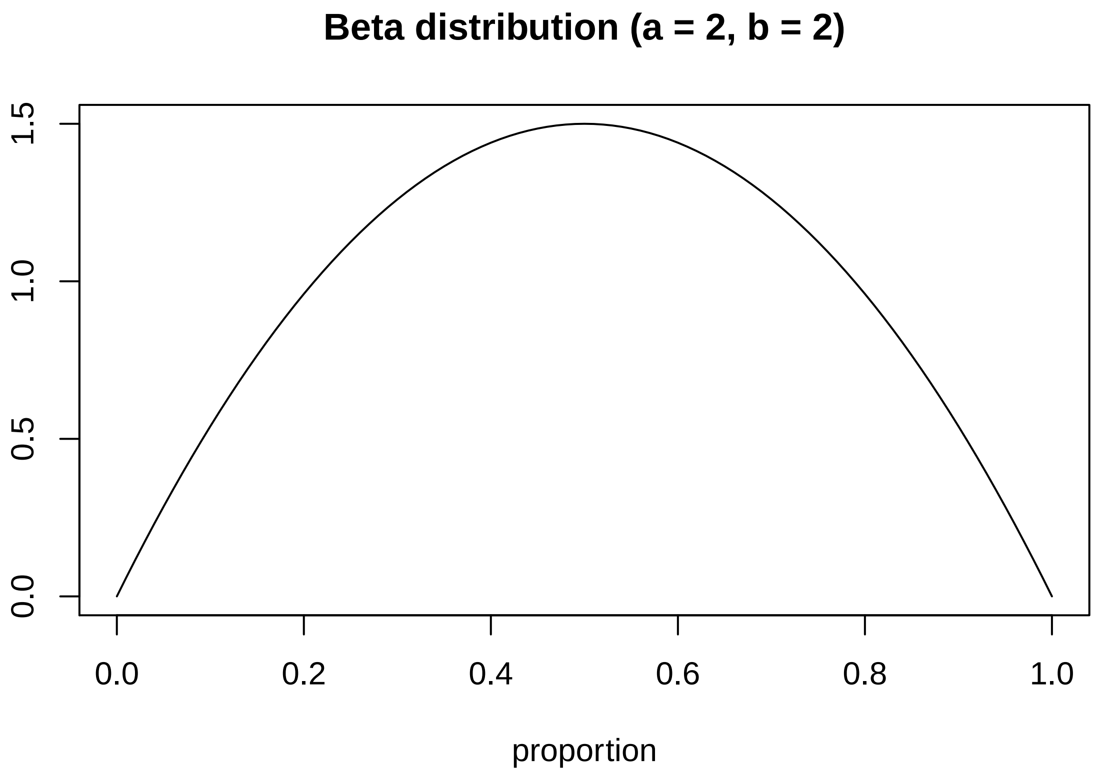
Working with beta distribution



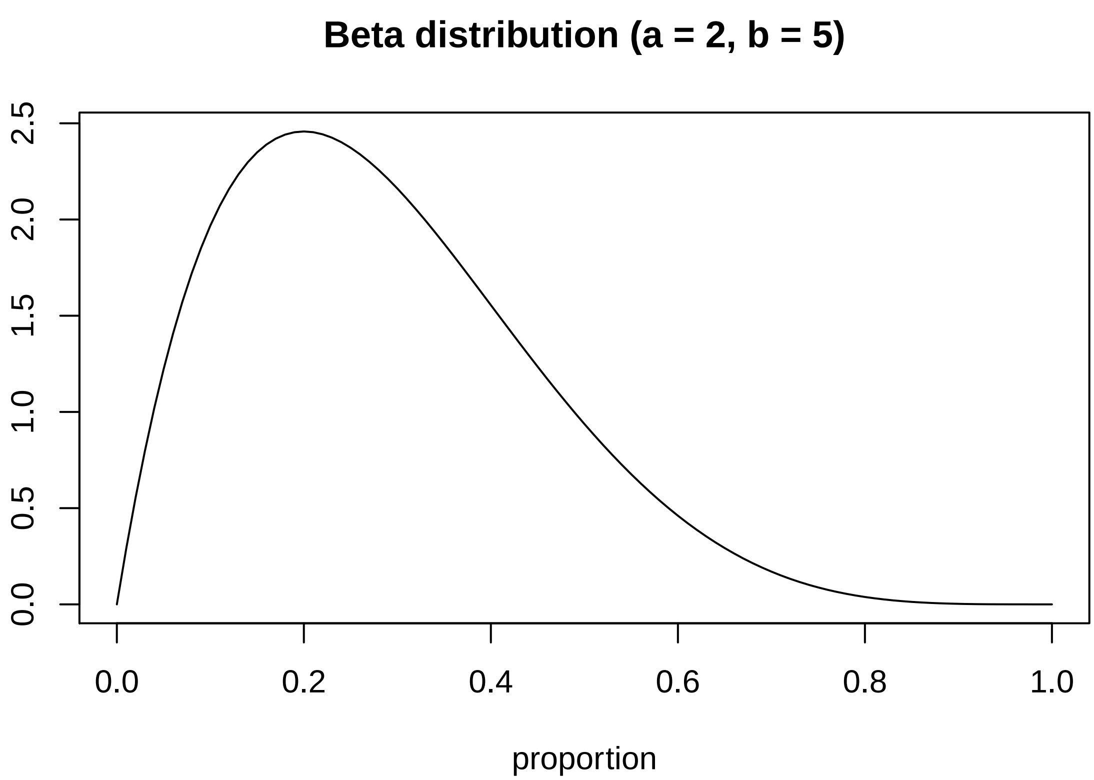
Working with beta distribution



Working with beta distribution



Working with beta distribution



Beta-binomial model

- Note how similar are formulas for binomial and beta distributions (up to a constant)

Binomial likelihood

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Beta prior

$\pi(p) \propto p^{a-1} \times (1-p)^{b-1}, 0 < p < 1$

Beta-binomial model

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Beta posterior

$L(p) \times \pi(p) = p^{(a+S)-1} \times (1-p)^{(b+F)-1}$

Beta binomial model