Confidence intervals

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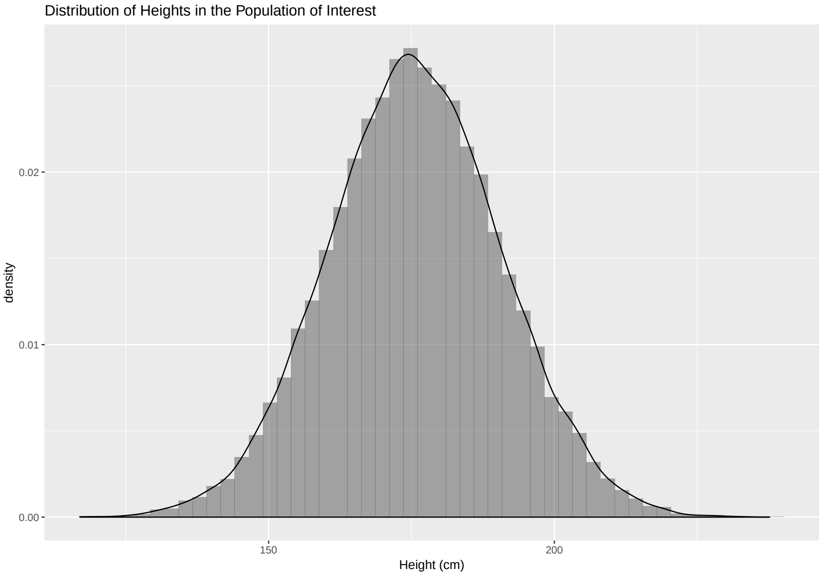
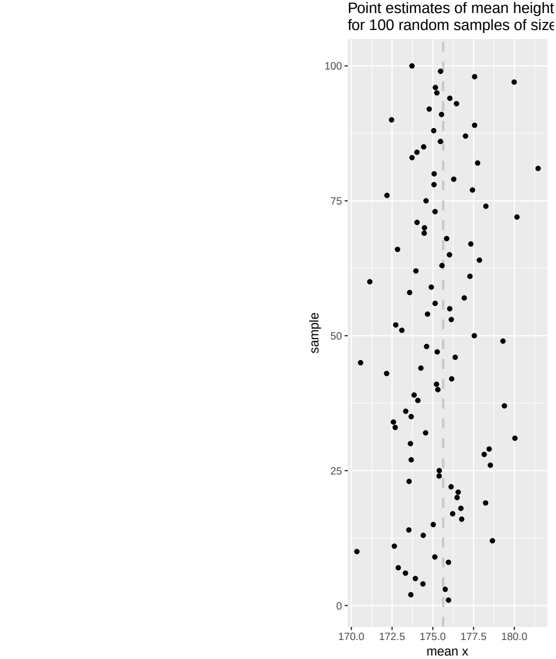
Population of interest

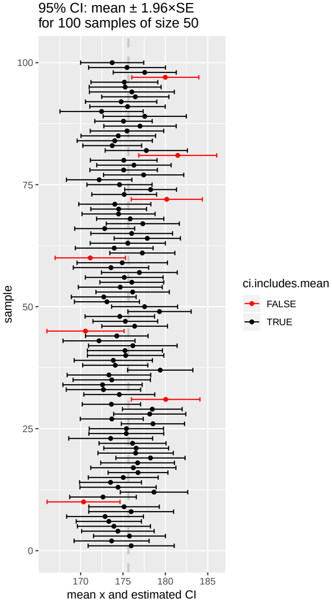


The population distribution of heights is $N(175.7, 15.19)$

Point estimates of the mean for random samples of size 50



95% Confidence intervals for the mean for random samples of size 50



Interpretting confidence intervals

From NIST page on confidence intervals:

> *As a technical note, a 95 % confidence interval does not mean that there is a 95 % probability that the interval contains the true mean. The interval computed from a given sample either contains the true mean or it does not. Instead,* ***the level of confidence is associated with the method of calculating the interval*** *. . . That is, for a 95% confidence interval, if many samples are collected and the confidence interval computed, in the long run about 95% of these intervals would contain the true mean.*

Confidence intervals: general formulation

We define the $(100 \times \beta)\%$ confidence interval for the statistic ϕ as the interval:

$$CI_\beta = \phi_n \pm (z \times SE_{\phi,n})$$

Where:

- ϕ_n is the statistic of interest in a random sample of size n
- $SE_{\phi,n}$ is the standard error of the statistic ϕ (via simulation or analytical solution)

And the value of z is chosen so that:

- across many different random samples of size n, the true value of the ϕ in the population of interest would fall within the interval approximately $(100 \times \beta)\%$ of the time