Missing Data

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Your **CAMPUS**



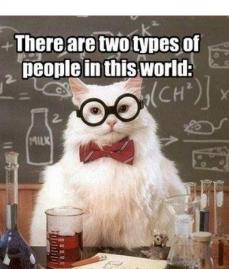
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Today's Lecture

- Types of missing data
- Ways to describe missing data
- Multiple imputation



Those who can extrapolate from incomplete data

Best practices

Hard to argue with an approach that does the following:

- quantify the completeness of covariate data
- present and discuss patterns of or reasons for missing data
- provide details about your approach for handling missing data in the analysis

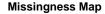
Proposed guidelines for reporting missing covariate data (Burton and Altman 2004)

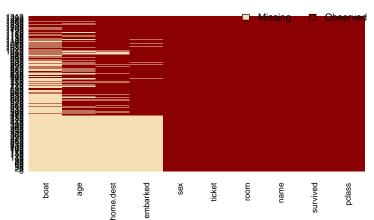
Quantifying missing data

```
library(Hmisc)
getHdata(titanic)
colnames(titanic)
## [1] "pclass" "survived" "name"
                          "age"
                                 "embarked"
 [6] "home.dest" "room"
##
                  "ticket"
                          "boat"
                                 "sex"
na.pattern(titanic)
## pattern
##
     279
            315
##
      51
            95
                         41
                                      478
```

Quantifying missing data

library(Amelia)
missmap(titanic)





Quantifying missing data

What percentage of each variable's observations are missing?

```
nrow(titanic)
## [1] 1313
colSums(is.na(titanic))
##
      pclass survived
                             name
                                         age
                                              embarked home.dest
                                                                       room
           0
                      0
                                0
                                         680
                                                   492
                                                              559
##
##
      ticket
                   boat
                              sex
##
                   966
                                0
```

Formal Missing Data Classifications

Missing Completely at Random (MCAR)

■ No data, observed or unobserved, are related to missingness.

Missing at Random (MAR)

No unobserved data are related to missingness, but missingness may depend on observed data.

Missing Not at Random (MNAR) or unignorable missingness

Missingness relationship cannot be simplified: it depends on unobserved data!

What kind of missingness did the titanic dataset have?

Missing Completely at Random (MCAR)

■ No data, observed or unobserved, are related to missingness.

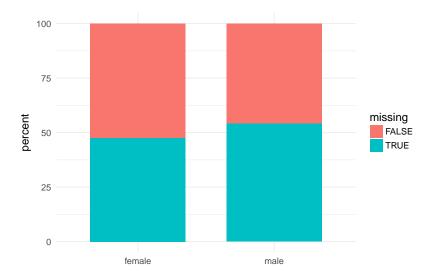
Missing at Random (MAR)

No unobserved data are related to missingness, but missingness may depend on observed data.

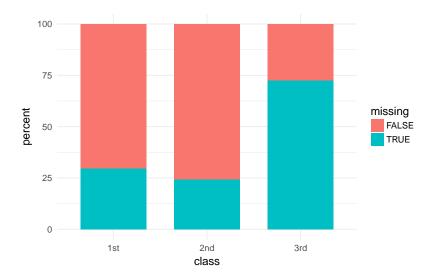
Missing Not at Random (MNAR) or unignorable missingness

Missingness relationship cannot be simplified: it depends on unobserved data!

What kind of missingness did the titanic dataset have?



What kind of missingness did the titanic dataset have?



Example code used to create the last graphic

Harder than it should be, it felt like... Code adapted from this page.

```
t3 <- titanic %>%
  group_by(pclass, age_mis) %>%
  summarise(count=n()) %>%
  mutate(perc=count/sum(count))

ggplot(t3, aes(x = pclass, y = perc*100, fill = age_mis)) +
  geom_bar(stat="identity", width = 0.7) +
  labs(x = "class", y = "percent", fill = "missing") +
  theme_minimal(base_size = 14)
```

Testing for the different types of data

Tests about the type of data you have

- MAR vs. MNAR: Not a definitive test here. Best option is to use your domain-specific knowledge about the data.
- MCAR vs. MAR: Little's test can weigh evidence for/against these two settings.

Little's H_0 : The data is MCAR

Low p-values suggest that the data are MAR; high p-values suggest they are MCAR.

```
test <- BaylorEdPsych::LittleMCAR(titanic[,c("pclass", "survived", "age", "sex"
## this could take a while
test$p.value
## [1] 0</pre>
```

Types of analyses for missing data

Analysis strategies (in rough order of desirability, low to high)

- MCAR only: Complete case a.k.a. "listwise deletion"
- Ad-hoc methods (e.g. mean imputation)
- Weighting methods
- MAR: Likelihood-based approaches (e.g. EM algorithm)
- MAR: Multiple Imputation (many flavors)
- MAR: Bayesian methods

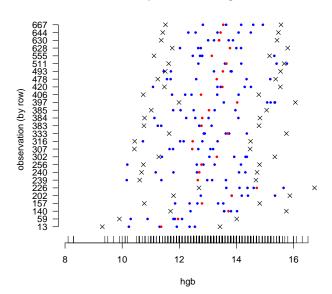
Multiple imputation

General approach

- For each missingness pattern, a model is built to use the available covariates to estimate the missing covariates.
- Random samples are taken from the predictive distribution to create multiple "complete" datasets.
- Typically, 10-15 datasets is seen as being sufficient.
- Coefficient and SE estimates are combined across datasets.

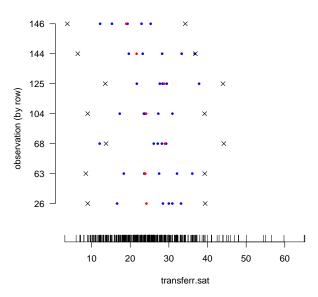
Multiple imputation: example

imputed data for hgb



Multiple imputation: example

imputed data for transferr.sat



Multiple imputation results

Regression coefficients from five imputed data sets

Data set	Estimated parameter	b_{θ}	b_I	\boldsymbol{b}_2	\boldsymbol{b}_3	b_4	\boldsymbol{b}_5
1	Coefficient	-11.535	-2.780	1.029	031	-0.359	0.572
	Variance	43.204	3.323	0.013	0.013	0.013	0.012
2	Coefficient	-11.501	-4.149	1.040	-0.093	-0.583	0.876
	Variance	40.488	2.680	0.010	0.009	0.009	0.007
3	Coefficient	-10.141	-5.038	0.766	0.123	-0.252	0.625
	Variance.	42.055	3.301	0.010	0.010	0.010	0.009
4	Coefficient	-11.533	-6.920	0.870	0.084	-0.458	0.815
	Variance	28.751	1.796	0.081	0.007	0.007	0.007
5	Coefficient	-14.586	-1.115	0.718	0.050	-0.373	0.814
	Variance	32.856	2.362	0.009	0.009	0.009	0.008
	Mean b_i	-11.859	-4.000	0.885	0.027	-0.405	0.740
	Mean $Var_{\cdot}(\overline{W})$	37.471	2.692	0.025	0.010	0.010	0.009
	Var. of $b_i(B)$	2.682	4.859	0.022	0.008	0.015	0.018
	T						
	\sqrt{T}	40.69	8.523	0.051	0.020	0.028	0.031
	t	6.379	2.919	0.226	0.141	0.167	0.176
	,	-1.859	-1.370	3.916*	0.191	2.425*	4.204*

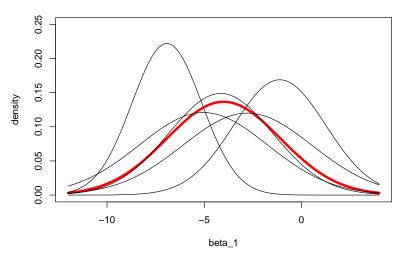
^{*} p < .05 "Var." refers to the squared standard error of the coefficient.

DC Howell, Treatment of Missing Data - Part II.

Multiple imputation results

The final estimated sampling distribution for each β is an average of the sampling distributions from each imputed dataset.

sampling distributions for imputed datasets



Multiple imputation software

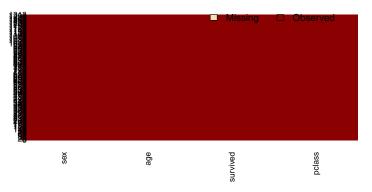
There are two commonly used implementations of multiple imputation in R:

- MICE: http://www.stefvanbuuren.nl/mi/
- To be used together: Amelia (runs the MI) and Zelig (fits models to, among other things, MI datasets): http://gking.harvard.edu/amelia and http://zeligproject.org/

Multiple imputation for titanic data

```
t2 <- titanic[,c("pclass", "survived", "age", "sex")]
imp_titanic <- amelia(x = t2, m = 10, noms=c("sex", "pclass"))
missmap(imp_titanic$imputations$imp1)</pre>
```

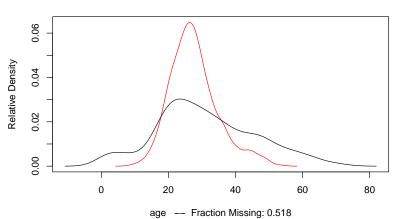
Missingness Map



Multiple imputation for titanic data

plot(imp_titanic, which.vars = "age")

Observed and Imputed values of age



Multiple imputation for titanic data

```
t2 <- t2[complete.cases(t2),] ## only include complete cases
m_full <- glm(survived~sex+age+pclass, data=t2, family=binomial)
summary(m_full)$coef

## Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.52216290 0.471007573 9.601041 7.914121e-22
## sexmale -3.08670894 0.241062738 -12.804588 1.545447e-37
## age -0.04930858 0.008732002 -5.646882 1.633840e-08
## pclass2nd -1.49522913 0.281986441 -5.302486 1.142363e-07
## pclass3rd -2.84127142 0.338897350 -8.383870 5.121522e-17
```

```
library(Zelig)
m_imp <- zelig(survived~sex+age+pclass, model="logit", data=imp_titanic)</pre>
```

```
## Model: Combined Imputations
## Estimate Std.Error z value Pr(>|z|)
## (Intercept) 3.92513 0.396870 9.890 0.000e+00 ***
## sexmale -2.52819 0.166904 -15.148 0.000e+00 ***
## age -0.04712 0.007589 -6.210 5.305e-10 ***
## pclass2nd -1.39975 0.236773 -5.912 3.384e-09 ***
## pclass3rd -2.88690 0.252901 -11.415 0.000e+00 ***
```

Best practices

Hard to argue with an approach that does the following:

- quantify the completeness of covariate data
- present and discuss patterns of or reasons for missing data
- provide details about your approach for handling missing data

Proposed guidelines for reporting missing covariate data (Burton and Altman 2004)

Bonus: ROC for Titanic data

```
library(ROCR)
pred <- prediction(predict(m_full, type="response"), t2$survived)
perf <- performance(pred, measure = "tpr", x.measure = "fpr")
plot(perf)</pre>
```

