Introduction to Multiple Linear Regression

Author: Nicholas G Reich

This material is part of the **statsTeachR** project

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Outline

Introduction to multiple regression

- Many variables in a model
- Adjusted R^2

Multiple regression

- Simple linear regression: Bivariate - two variables: y and x
- Multiple linear regression: Multiple variables: y and $x_1, x_2, \cdots$

Weights of books

	weight (g)	volume (cm^3)	cover
1	800	885	hc
2	950	1016	hc
3	1050	1125	hc
4	350	239	hc
5	750	701	hc
6	600	641	hc
7	1075	1228	hc
8	250	412	pb
9	700	953	pb
10	650	929	pb
11	975	1492	pb
12	350	419	pb
13	950	1010	pb
14	425	595	pb
15	725	1034	pb

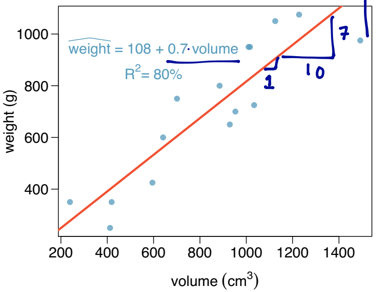
From: Maindonald, J.H. and Braun, W.J. (2nd ed., 2007) “Data Analysis and Graphics Using R”

Weights of books (cont.)

$\widehat{weight}_{20} = 108 + \frac{7 \cdot 20}{14} = 122$

$\widehat{weight}_{10} = 108 + 7 = 115$

The scatterplot shows the relationship between weights and volumes of books as well as the regression output. Which of the below is correct?



(a) Weights of 80% of the books can be predicted accurately using this model.

(b) We would expect a book that is 10 cm^3 bigger than another ~~expected~~ to weigh 7 g more.

(c) The correlation between weight and volume is $R = 0.80^2 = 0.64$.

(d) The model underestimates the weight of the book with the highest volume.

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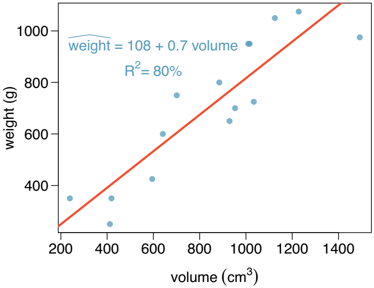


(a) Weights of 80% of the books can be predicted accurately using this model.

(b) *We would expect a book that is 10 cm^3 bigger than another expected to weigh 7 g more.*

(c) The correlation between weight and volume is $R = 0.80^2 = 0.64$.

(d) The model underestimates the weight of the book with the highest volume.

Modeling weights of books using volume

somewhat abbreviated output...

β_0

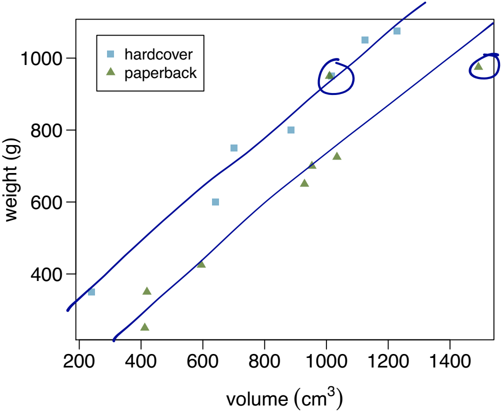
```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 107.67931   88.37758   1.218    0.245
volume        0.70864    0.09746   7.271 6.26e-06
```

β_1

```
Residual standard error: 123.9 on 13 degrees of freedom
Multiple R-squared: 0.8026,Adjusted R-squared: 0.7875
F-statistic: 52.87 on 1 and 13 DF,  p-value: 6.262e-06
```

Weights of hardcover and paperback books

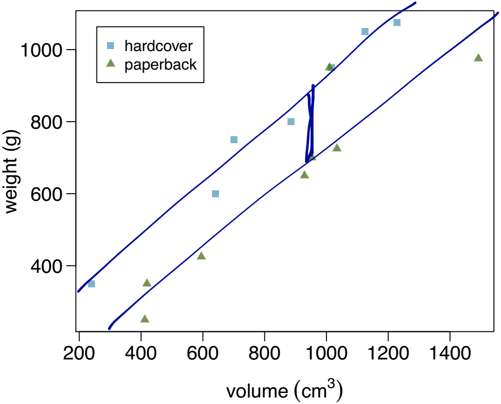
Can you identify a trend in the relationship between volume and weight of hardcover and paperback books?



Weights of hardcover and paperback books

Can you identify a trend in the relationship between volume and weight of hardcover and paperback books?

Paperbacks generally weigh less than hardcover books after controlling for the book's volume.



Modeling weights of books using volume and cover type

$$Y = \beta_0 + \beta_1 \cdot V + \beta_2 \cdot PB + \varepsilon$$

1 , if PB
0 , if HC => REFERENCE LEVEL

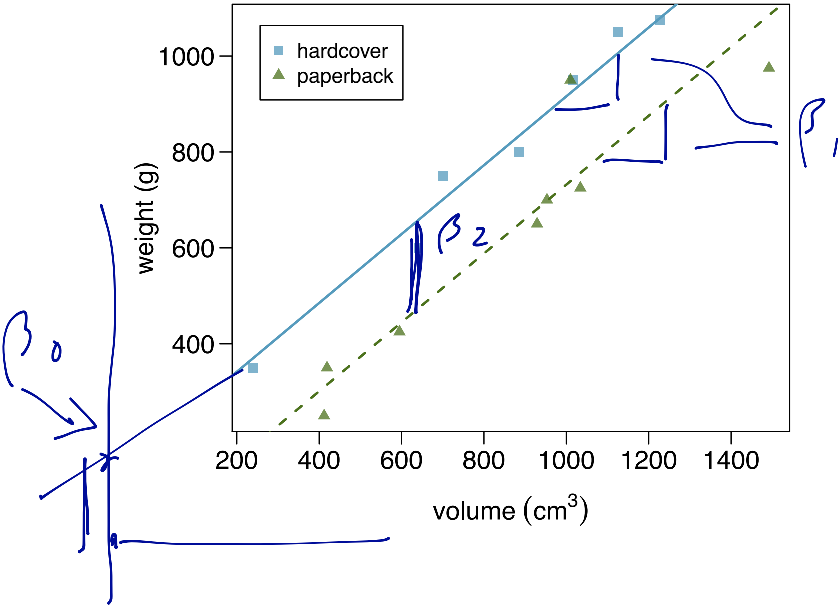
```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)  197.96284   59.19274   3.344 0.005841 **
volume         0.71795    0.06153  11.669  6.6e-08 ***
cover:pb    -184.04727   40.49420  -4.545 0.000672 ***

Residual standard error: 78.2 on 12 degrees of freedom
Multiple R-squared: 0.9275,Adjusted R-squared: 0.9154
F-statistic: 76.73 on 2 and 12 DF,  p-value: 1.455e-07
```

Handwritten labels beside the estimates: β_0 (Intercept), β_1 (volume), β_2 (cover:pb).

HC : $Y = \beta_0 + \beta_1 \cdot V$

PB : $Y = (\beta_0 + \beta_2) + \beta_1 \cdot V$

Visualising the linear model



Determining the reference level

Based on the regression output below, which level of `cover` is the reference level? Note that `pb`: paperback.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.9628	59.1927	3.34	0.0058
volume	0.7180	0.0615	11.67	0.0000
cover:pb	-184.0473	40.4942	-4.55	0.0007

(a) paperback

(b) hardcover

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(a) paperback

(b) *hardcover*

Determining the reference level

Which of the below correctly describes the roles of variables in this regression model?

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(a) **response**: weight; **explanatory**: volume, paperback cover

(b) **response**: weight; **explanatory**: volume, hardcover cover

(c) **response**: volume; **explanatory**: weight, cover type

(d) **response**: weight; **explanatory**: volume, cover type

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(d) **response**: *weight;* **explanatory**: *volume, cover type*

Linear model

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
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$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05\ cover : pb$$

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1. For *hardcover* books: plug in *0* for `cover`

$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05 \times 0$$

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2. For *paperback* books: plug in *1* for `cover`

$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05 \times 1$$

Linear model

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(Intercept)	197.96	59.19	3.34	0.01
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$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05\ cover : pb$$

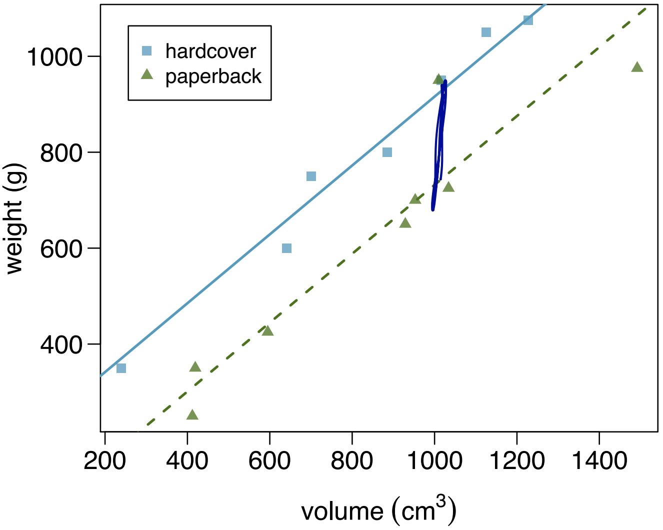
1. For *hardcover* books: plug in *0* for `cover`

$$\begin{aligned}\widehat{weight} &= 197.96 + 0.72\ volume - 184.05 \times 0 \\ &= 197.96 + 0.72\ volume\end{aligned}$$

2. For *paperback* books: plug in *1* for `cover`

$$\begin{aligned}\widehat{weight} &= 197.96 + 0.72\ volume - 184.05 \times 1 \\ &= 13.91 + 0.72\ volume\end{aligned}$$

Visualising the linear model



Interpretation of the regression coefficients

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
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- *Slope of volume:* All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.

Interpretation of the regression coefficients

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- *Slope of volume:* All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.
- *Slope of cover:* All else held constant, the model predicts that paperback books weigh 184 grams lower than hardcover books.

Interpretation of the regression coefficients

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- *Slope of volume:* All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.
- *Slope of cover:* All else held constant, the model predicts that paperback books weigh 184 grams lower than hardcover books.
- *Intercept:* Hardcover books with no volume are expected on average to weigh 198 grams.

Interpretation of the regression coefficients

	Estimate	Std. Error	t value	Pr(>|t|)
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- *Slope of volume:* All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.
- *Slope of cover:* All else held constant, the model predicts that paperback books weigh 184 grams lower than hardcover books.
- *Intercept:* Hardcover books with no volume are expected on average to weigh 198 grams.
  - Obviously, the intercept does not make sense in context. It only serves to adjust the height of the line.

Prediction

Which of the following is the correct calculation for the predicted weight of a paperback book that is 600 cm^3?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
cover:pb	-184.05	40.49	-4.55	0.00

(a) 197.96 + 0.72 * 600 - 184.05 * 1

(b) 184.05 + 0.72 * 600 - 197.96 * 1

(c) 197.96 + 0.72 * 600 - 184.05 * 0

(d) 197.96 + 0.72 * 1 - 184.05 * 600

Prediction

Which of the following is the correct calculation for the predicted weight of a paperback book that is 600 cm^3?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
cover:pb	-184.05	40.49	-4.55	0.00

(a) *197.96 + 0.72 * 600 - 184.05 * 1* $=$ 445.91 *grams*

(b) 184.05 + 0.72 * 600 - 197.96 * 1

(c) 197.96 + 0.72 * 600 - 184.05 * 0

(d) 197.96 + 0.72 * 1 - 184.05 * 600

Another example: Modeling kid's test scores

Predicting cognitive test scores of 434 three- and four-year-old children using characteristics of their mothers. Data are from a survey of adult American women and their children - a subsample from the National Longitudinal Survey of Youth.

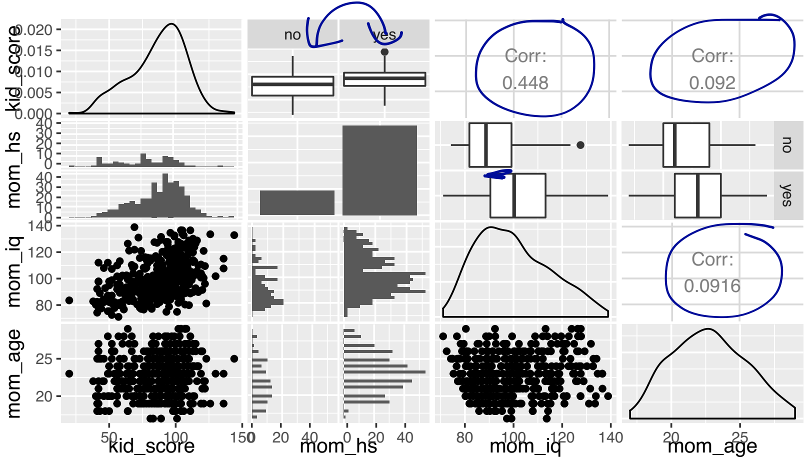
```
library(rstanarm)
data("kidiq")
head(kidiq)

##   kid_score mom_hs    mom_iq mom_age
## 1        65      1 121.11753      27
## 2        98      1  89.36188      25
## 3        85      1 115.44316      27
## 4        83      1  99.44964      25
## 5       115      1  92.74571      27
## 6        98      0 107.90184      18
```

Gelman, Hill. *Data Analysis Using Regression and Multilevel/Hierarchical Models*. (2007) Cambridge University Press.

Exploratory analysis

```r
library(GGally)
kidiq$mom_hs <- factor(kidiq$mom_hs, levels=c(0,1), labels=c("no", "yes"))
ggpairs(kidiq)
```



What is a reasonable model

In generic model syntax

Kidscore ~ mom_hs + mom_age + mom_iq

In regression formula syntax

$$Y = \beta_0 +$$

Interpreting the slope

What is the correct interpretation of the slope for mom's IQ?

```r
fm <- lm(kid_score ~ mom_hs + mom_iq + mom_age, data=kidiq)
round(summary(fm)$coef, 3)
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   20.985      9.130   2.298    0.022
## mom_hsyes      5.647      2.258   2.501    0.013
## mom_iq         0.563      0.061   9.276    0.000
## mom_age        0.225      0.331   0.680    0.497
```

, kids with mothers whose IQs are one point higher tend to score on average 0.56 points higher.

Interpreting the slope

What is the correct interpretation of the intercept?

```
round(summary(fm)$coef, 3)

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```

Kids whose moms haven't gone to HS, whose moms have an IQ of 0, and who are 0 yrs old are expected on average to score 20.98. Obviously, the intercept does not make any sense in context.

Interpreting the slope

What is the correct interpretation of the slope for `mom_hs`?

```
round(summary(fm)$coef, 3)
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   20.985      9.130   2.298    0.022
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```

All else being equal, kids whose moms graduated from high school are estimated to score than

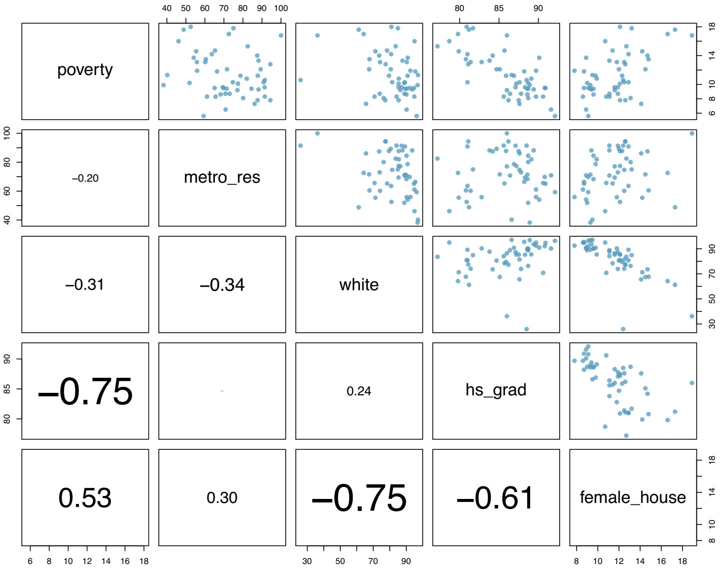
those whose moms did not work.

Modeling poverty

Description: Data for 3083 counties in the United States, including variables for demographic, financial, education, and other characteristics.
Source: Census website.

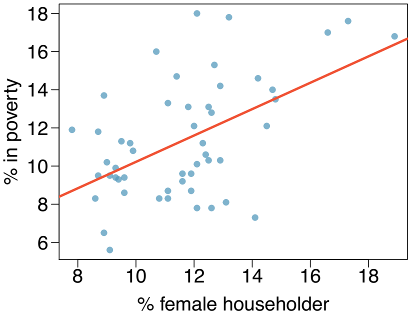
- FIPS: FIPS code.
- poverty: Percent below poverty level (2006-2010).
- pop2010: 2010 county population.
- female_house: Percent of population that lives in a female-owned house (2010).
- metro_res: Percent of population living in metropolitan area.
- hs_grad: Percent of population that is a high school graduate (2006-2010).
- ...

Modeling poverty



Predicting poverty using % female householder

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.31	1.90	1.74	0.09
female_house	0.69	0.16	4.32	0.00



$R = 0.53$

$R^2 = 0.53^2 = 0.28$

Another look at R^2

R^2 can be calculated in three ways:

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1. square the correlation coefficient of x and y (how we have been calculating it)

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$$R^2 = \frac{\text{explained variability in } y}{\text{total variability in } y}$$

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1. square the correlation coefficient of x and y (how we have been calculating it)
2. square the correlation coefficient of y and $\hat{y}$
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$$R^2 = \frac{\text{explained variability in } y}{\text{total variability in } y}$$

Using *ANOVA* we can calculate the explained variability and total variability in y.

Sum of squares

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.68	0.00
Residuals	49	347.68	7.10		
Total	50	480.25			

Sum of squares

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female_house	1	132.57	132.57	18.68	0.00
Residuals	49	347.68	7.10		
Total	50	480.25			

Sum of squares of y: $SS_{Total} \quad = \quad \sum (y - \bar{y})^2 = 480.25$ → *total variability*

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Total	50	480.25			

$$\text{Sum of squares of } y\text{: } SS_{Total} = \sum (y - \bar{y})^2 = 480.25 \rightarrow \textit{total variability}$$

$$\text{Sum of squares of residuals: } SS_{Error} = \sum e_i^2 = 347.68 \rightarrow \textit{unexplained variability}$$

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$$\begin{aligned}
\text{Sum of squares of } y\text{: } SS_{Total} &= \sum (y - \bar{y})^2 = 480.25 \rightarrow \textit{total variability} \\
\text{Sum of squares of residuals: } SS_{Error} &= \sum e_i^2 = 347.68 \rightarrow \textit{unexplained variability} \\
\text{Sum of squares of } x\text{: } SS_{Model} &= SS_{Total} - SS_{Error} \rightarrow \textit{explained variability} \\
&= 480.25 - 347.68 = 132.57
\end{aligned}$$

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\text{Sum of squares of } x\text{: } SS_{Model} &= SS_{Total} - SS_{Error} \rightarrow \textit{explained variability} \\
&= 480.25 - 347.68 = 132.57
\end{aligned}
$$

$$
R^2 = \frac{\text{explained variability}}{\text{total variability}} = \frac{132.57}{480.25} = 0.28 \checkmark
$$

Why bother?

Why bother with another approach for calculating R^2 when we had a perfectly good way to calculate it as the correlation coefficient squared?

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Why bother with another approach for calculating R^2 when we had a perfectly good way to calculate it as the correlation coefficient squared?

- *For single-predictor linear regression, having three ways to calculate the same value may seem like overkill.*
- *However, in multiple linear regression, we can't calculate R^2 as the square of the correlation between x and y because we have multiple xs.*
- *And next we'll learn another measure of explained variability, adjusted R^2, that requires the use of the third approach, ratio of explained and unexplained variability.*

Predicting poverty using % female hh + % white

Linear model:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2.58	5.78	-0.45	0.66
female_house	0.89	0.24	3.67	0.00
white	0.04	0.04	1.08	0.29

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.74	0.00
white	1	8.21	8.21	1.16	0.29
Residuals	48	339.47	7.07		
Total	50	480.25			

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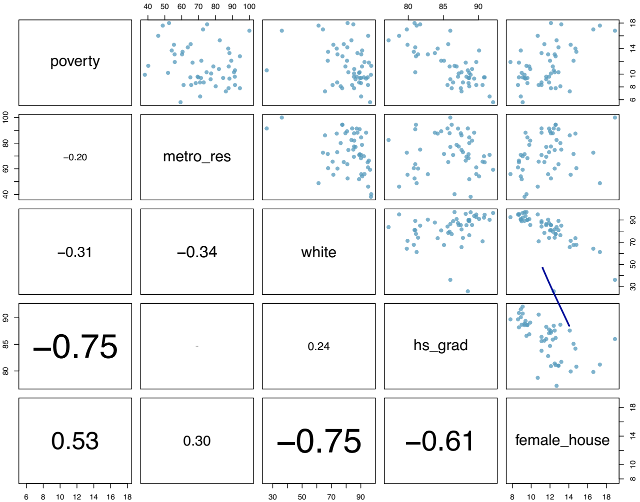
white	1	8.21	8.21	1.16	0.29
Residuals	48	339.47	7.07		
Total	50	480.25			

$$R^2 = \frac{\text{explained variability}}{\text{total variability}} = \frac{132.57 + 8.21}{480.25} = 0.29$$

Does adding the variable `white` to the model add valuable information that wasn't provided by `female_house`?



R^2 vs. adjusted R^2

	R^2	Adjusted R^2
Model 1 (Single-predictor)	0.28	0.26
Model 2 (Multiple)	0.29	0.26

R^2 vs. adjusted R^2

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- When <u>any</u> variable is added to the model R^2 increases.

R^2 vs. adjusted R^2

	R^2	Adjusted R^2
Model 1 (Single-predictor)	0.28	0.26
Model 2 (Multiple)	0.29	0.26

- When <u>any</u> variable is added to the model R^2 increases.
- But if the added variable doesn't really provide any new information, or is completely unrelated, adjusted R^2 does not increase.

Adjusted R^2

Adjusted R^2

$$R^2_{adj} = 1 - \left(\frac{SS_{Error}}{SS_{Total}} \times \frac{n-1}{n-p-1} \right)$$

where n is the number of cases and p is the number of predictors (explanatory variables) in the model.

- Because p is never negative, R^2_{adj} will always be smaller than R^2.
- R^2_{adj} applies a penalty for the number of predictors included in the model.
- Therefore, we choose models with higher R^2_{adj} over others.

Calculate adjusted R^2

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.74	0.0001
white	1	8.21	8.21	1.16	0.2868
Residuals	48	339.47	7.07		
Total	50	480.25			

$$R^2_{adj} = 1 - \left(\frac{SS_{Error}}{SS_{Total}} \times \frac{n-1}{n-p-1} \right)$$

Calculate adjusted R^2

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.74	0.0001
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Residuals	48	339.47	7.07		
Total	50	480.25			

$$
\begin{aligned}
R^2_{adj} &= 1 - \left(\frac{SS_{Error}}{SS_{Total}} \times \frac{n-1}{n-p-1} \right) \\
&= 1 - \left(\frac{339.47}{480.25} \times \frac{51-1}{51-2-1} \right)
\end{aligned}
$$

Calculate adjusted R^2

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.74	0.0001
white	1	8.21	8.21	1.16	0.2868
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$$
\begin{aligned}
R^2_{adj} &= 1 - \left(\frac{SS_{Error}}{SS_{Total}} \times \frac{n-1}{n-p-1} \right) \\
&= 1 - \left(\frac{339.47}{480.25} \times \frac{51-1}{51-2-1} \right) \\
&= 1 - \left(\frac{339.47}{480.25} \times \frac{50}{48} \right)
\end{aligned}
$$

Calculate adjusted R^2

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
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&= 1 - \left(\frac{339.47}{480.25} \times \frac{50}{48} \right) \\
&= 1 - 0.74
\end{aligned}
$$

Calculate adjusted R^2

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
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$$
\begin{aligned}
R^2_{adj} &= 1 - \left(\frac{SS_{Error}}{SS_{Total}} \times \frac{n-1}{n-p-1} \right) \\
&= 1 - \left(\frac{339.47}{480.25} \times \frac{51-1}{51-2-1} \right) \\
&= 1 - \left(\frac{339.47}{480.25} \times \frac{50}{48} \right) \\
&= 1 - 0.74 \\
&= 0.26
\end{aligned}
$$

On your own

Play around with some regression models and ANOVAS.

```
load(url("http://www.openintro.org/stat/data/cc.RData"))
dim(countyComplete)
colnames(countyComplete)
fm <- lm(poverty ~ white_not_hispanic + female, data=countyComplete)
anova(fm)
```

Note: the actual dataset has slightly different variable names than those in the slides. More details on the dataset can be found here https://www.openintro.org/stat/data/?data=cc.