Discussion 2

CUNY MSDS DATA 605

Duubar Villalobos Jimenez

February 5, 2018

C27 † Page 353

Book: Beezer: A First Course in Linear Algebra

Exercise

Doing the computations by hand, find the determinant of the matrix A.

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & -1 & 1 \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Solution

First, I will reduce the matrix as follows:

Swap matrix rows: $R_1 \leftrightarrow R_2$

$$= \begin{pmatrix} 2 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Cancel leading coefficient in row R_2 by performing $R_2 \leftarrow R_2 - \frac{1}{2} \cdot R_1$

$$= \begin{pmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Cancel leading coefficient in row R_3 by performing $R_3 \leftarrow R_3 - 1 \cdot R_1$

$$= \begin{pmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & \frac{3}{2} & \frac{1}{2} \\ 0 & -1 & 4 & -1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Cancel leading coefficient in row R_4 by performing $R_4 \leftarrow R_4 - \frac{1}{2} \cdot R_1$

$$= \begin{pmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & \frac{3}{2} & \frac{1}{2} \\ 0 & -1 & 4 & -1 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Cancel leading coefficient in row R_3 by performing $R_3 \leftarrow R_3 - 1 \cdot R_2$

$$= \begin{pmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Cancel leading coefficient in row R_4 by performing $R_4 \leftarrow R_4 - \frac{1}{5} \cdot R_3$

$$= \begin{pmatrix} 2 & 2 & -1 & 1 \\ 0 & -1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & \frac{5}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & \frac{4}{5} \end{pmatrix}$$

Since the determinant of the matrix equals the diagonal product of the matrix:

$$det(A) = 2 \left(-1\right) \frac{5}{2} \cdot \frac{4}{5}$$

$$det(A) = -4$$

Now, since I have interchanged two rows, it negate the determinant, therefore multiply the result by $(-1)^1$.

$$det(A) = (-1)^1 (-4) = 4$$

Solving in R

- Defining Matrix

```
A <- matrix(data = c(1,0,1,1,
                     2,2,-1,1,
                     2,1,3,0,
                     1,1,0,1), ncol=4, byrow=TRUE)
```

- Finding determinant in R

```
det(A)
```

```
## [1] 4
```