Discussion 9

CUNY MSDS DATA 605

Duubar Villalobos Jimenez mydvtech@gmail.com

March 28, 2018

1 † Page 338

Book: Grinstead: Introduction to Probability

Exercise

Let S_{100} be the number of heads that turn up in 100 tosses of a fair coin. Use the Central Limit Theorem to estimate the below exercises:

Preamble

The second fundamental theorem of probability is the **Central Limit Theorem**. This theorem says that if S_n is the sum of n mutually independent random variables, then the distribution function of S_n is well-approximated by a certain type of continuous function known as a **normal density function**, which is given by the formula

$$f_{\mu,\sigma}(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

Also, from the above information provided, we know as follows:

$n = 100$

$p = 0.5$

$\mu = np$

$\mu = 100 \times 0.5 = 50$

$\sigma^2 = np(1-p)$

$\sigma^2 = 100 \times 0.5 \times (1 - 0.5)$

$\sigma = \sqrt{100 \times 0.5 \times (1 - 0.5)}$

$\sigma = 5$

Also, The Central Limit for Bernoulli Trials says as follows:

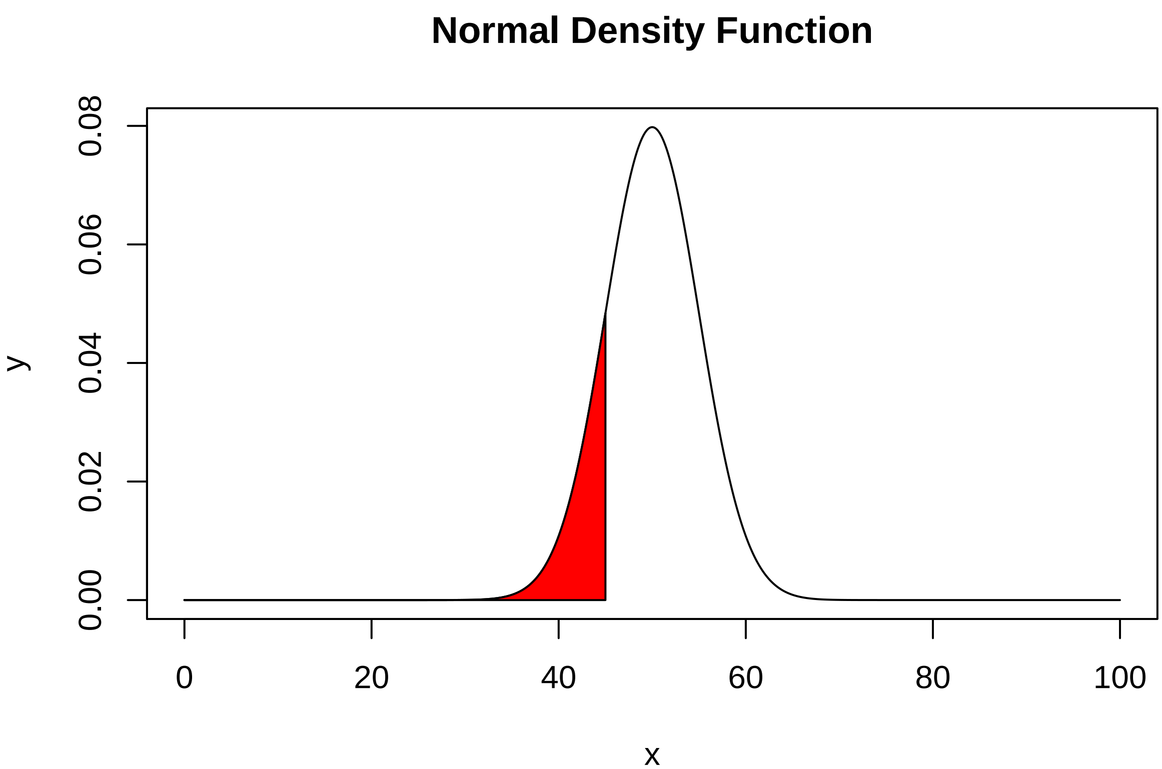
Let S_n be the number of successes in n Bernoulli trials with probability p for success, and let a and b be two fixed real numbers. Then

$$lim_{n\to\infty} P\left(a \le S_n \le b\right) = \int_a^b f_{\mu,\sigma}(x)dx$$

For demonstration purposes, I will write a small function that find it's approximation based on the Central Limit Theorem using the **normal density function** only.

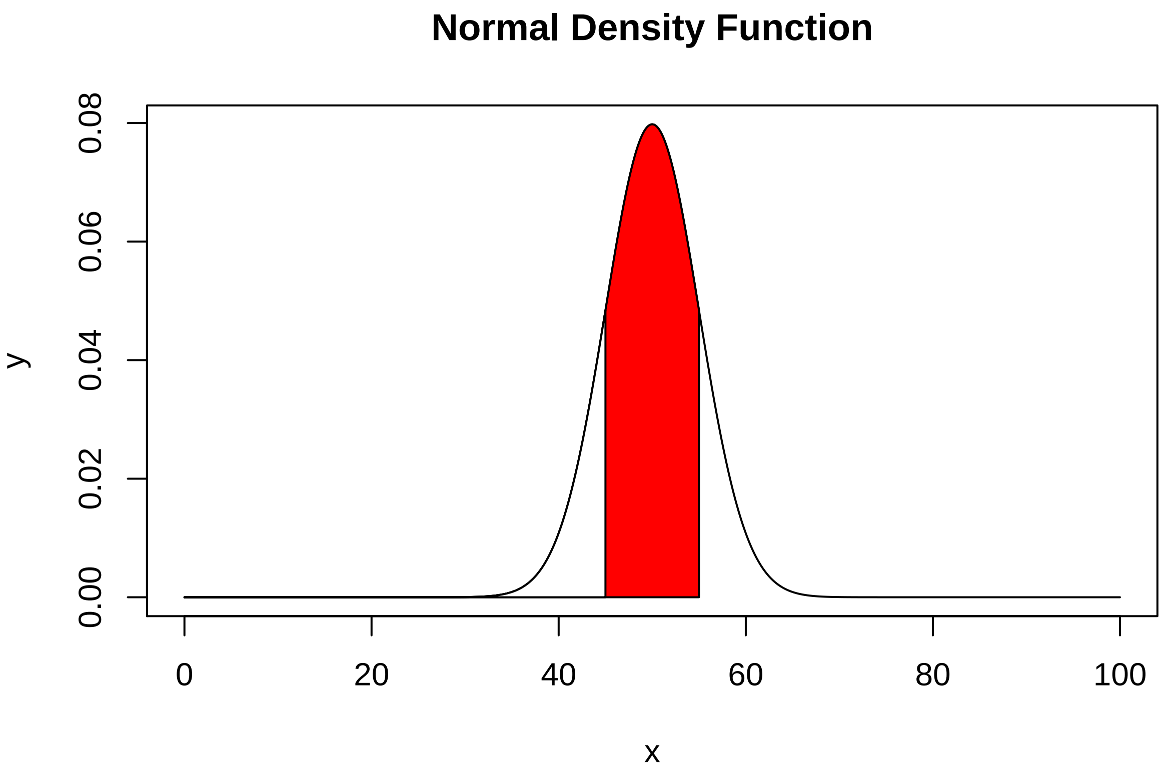
```r
# This can also be calculated by using >>>>>   pnorm(b, mu, sigma)   <<<<<
CLT <- function(a, b){
  mu <- 50
  sigma <- 5
  ndf <- function(x){exp(1)^(-(x - 50)^2 / (2 * 5^2)) / (5 * sqrt(2 * pi))}
  p_ndf <- integrate(ndf, lower = a, upper = b)
  return(p_ndf$value)
}
```

(a) P ($S_{100} \leq 45$).



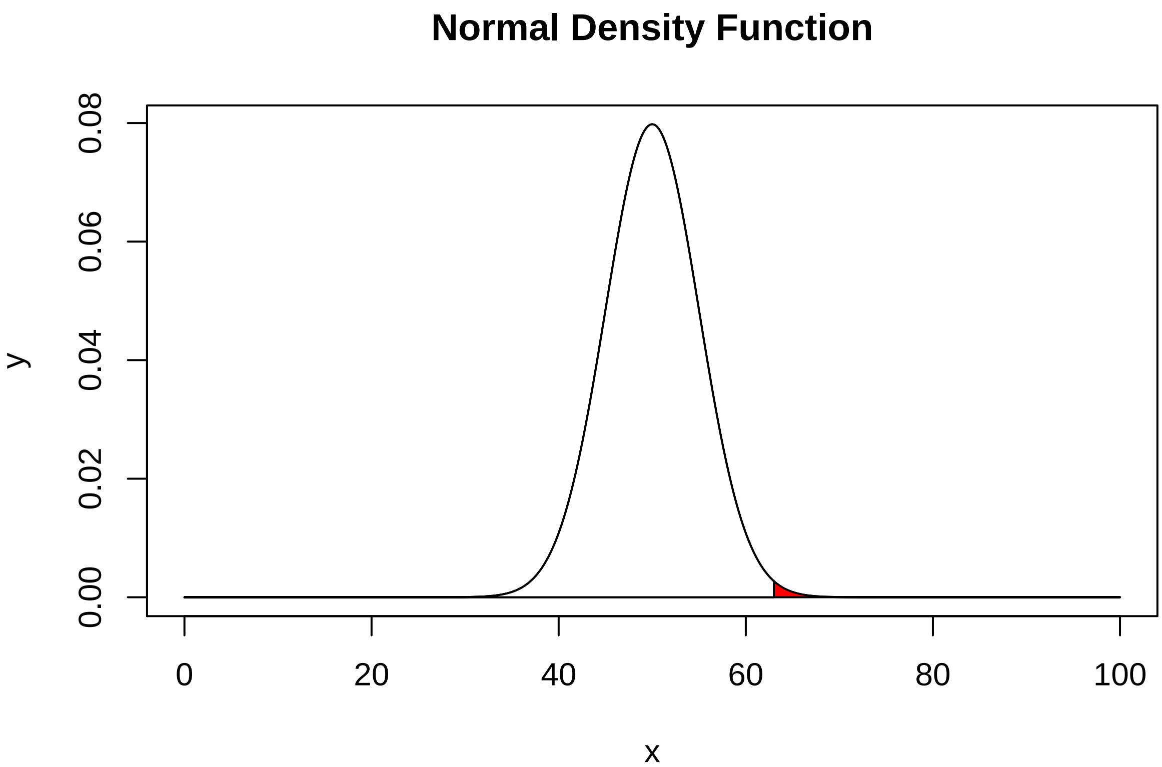
The probability will be 0.1587 or 15.87%.

(b) $\mathbf{P}\ (45 < S_{100} < 55)$.



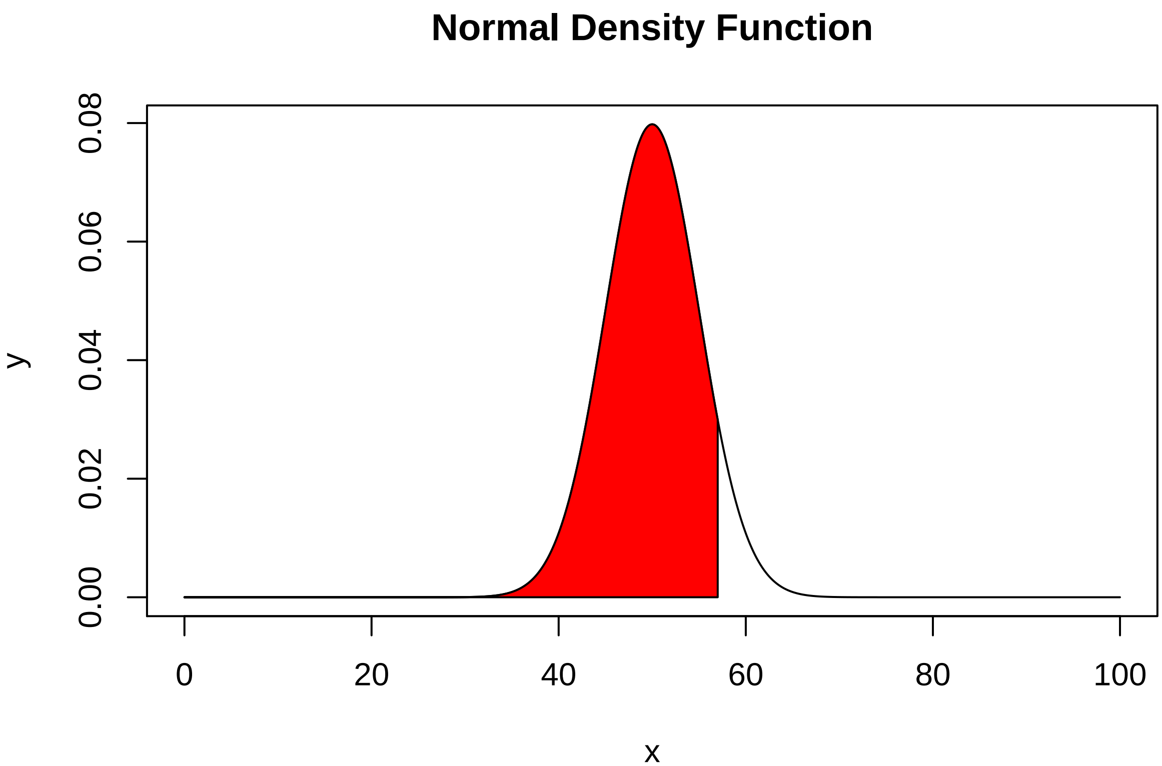
The probability will be 0.6827 or 68.27%.

(c) P $(S_{100} > 63)$**.**



The probability will be 0.0047 or 0.47%.

(d) P ($S_{100} < 57$).



The probability will be 0.9192 or 91.92%.