Discussion 5

CUNY MSDS DATA 605

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Book: Grinstead: Introduction to Probability

Exercise

Consider the bet that all three dice will turn up sixes at least once in n rolls of three dice. Calculate $f(n)$, the probability of at least one triple-six when three dice are rolled n times. Determine the smallest value of n necessary for a favorable bet that a triple-six will occur when three dice are rolled n times.

(DeMoivre would say it should be about $216 \cdot log(2) = 149.7$ and so would answer 150—see Exercise 1.2.17. Do you agree with him?)

Solution

For this, we could approach it in two different forms.

- The probability of wining by having one six in one dice is as follows:

For this, let's say X will be the action of rolling the dice.

Probability of Wining $= P(X = 6) = \frac{1}{6}$

OR

Probability of Wining = 1 - Probability of loosing.

Probability of Wining $= 1 - P(X \neq 6) = 1 - \frac{5}{6}$

From here, we can deduct as follows:

- The probability of wining by having three sixes in one roll is as follows:

$P(X_1 = 6) = \frac{1}{6}$ and

$P(X_2 = 6) = \frac{1}{6}$ and

$P(X_3 = 6) = \frac{1}{6}$

This means as follows:

$P(X_1 = 6 \cap X_2 = 6 \cap X_2 = 6) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{6^3}$

OR

$1 - P(X_1 \neq 6) = 1 - \frac{5}{6}$ and

$1 - P(X_2 \neq 6) = 1 - \frac{5}{6}$ and

$1 - P(X_3 \neq 6) = 1 - \frac{5}{6}$

From here we can also deduct that the probability of winning can be as follows:

Probability of wining $= (1 - P(X_1 \neq 6)) \cdot (1 - P(X_2 \neq 6)) \cdot (1 - P(X_3 \neq 6))$

Probability of wining $= \left(1 - \frac{5}{6}\right) \cdot \left(1 - \frac{5}{6}\right) \cdot \left(1 - \frac{5}{6}\right)$

Probability of wining $= \left(1 - \frac{5}{6}\right)^3$

Probability of wining $= \left(\frac{6}{6} - \frac{5}{6}\right)^3$

Probability of wining $= \left(\frac{1}{6}\right)^3$

Probability of wining $= \frac{1}{6^3}$

By creating a table of rolls, we can deduct as follows:

```
##   n_roll pX1 pX2 pX3 pXWin3_6 pCumXWin3_6
## 1      1 1/6 1/6 1/6    1/6^3      1 /6^3
## 2      2 1/6 1/6 1/6    1/6^3      2 /6^3
## 3      3 1/6 1/6 1/6    1/6^3      3 /6^3
## 4      4 1/6 1/6 1/6    1/6^3      4 /6^3
## 5      5 1/6 1/6 1/6    1/6^3      5 /6^3
## 6      6 1/6 1/6 1/6    1/6^3      6 /6^3
## 7    ... 1/6 1/6 1/6    1/6^3         ...
## 8      n 1/6 1/6 1/6    1/6^3       n/6^3
```

Based on the above table our probability of winning function will be as follows:

$$f(n) = \frac{n}{6^3}$$

And we want to know for which n, $f(n) > \frac{1}{2}$?

for this, we solve the inequation as follows:

$$\frac{n}{6^3} > \frac{1}{2}$$

$$n > \frac{6^3}{2}$$

$n > 108$

Based on that; the smallest value of n necessary for a favorable bet that a triple-six will occur when three dice are rolled n times will be 109 times.

Programing in R

```
f <- function(n){
  f <- n / (6 ^ 3)
  return(f)
}
```

Let's generate a table of probabilities

```
p <- function(n){
  n_roll <- c()
  fn <- c()
  for (i in 1:n) {
    n_roll[i] <- i
    fn[i] <- f(i)
  }
```

```r
  fn <- data.frame(n_roll, fn)
  return(fn)
}
```

Let's see our table for $n = 109$

```r
n <- 109
pn <- p(n)
pn[n,]
```

```
##     n_roll        fn
## 109    109 0.5046296
```

Table of probabilities:

```r
pn
```

```
##     n_roll          fn
## 1        1 0.004629630
## 2        2 0.009259259
## 3        3 0.013888889
## 4        4 0.018518519
## 5        5 0.023148148
## 6        6 0.027777778
## 7        7 0.032407407
## 8        8 0.037037037
## 9        9 0.041666667
## 10      10 0.046296296
## 11      11 0.050925926
## 12      12 0.055555556
## 13      13 0.060185185
## 14      14 0.064814815
## 15      15 0.069444444
## 16      16 0.074074074
## 17      17 0.078703704
## 18      18 0.083333333
## 19      19 0.087962963
## 20      20 0.092592593
## 21      21 0.097222222
## 22      22 0.101851852
## 23      23 0.106481481
## 24      24 0.111111111
## 25      25 0.115740741
## 26      26 0.120370370
## 27      27 0.125000000
## 28      28 0.129629630
## 29      29 0.134259259
## 30      30 0.138888889
## 31      31 0.143518519
## 32      32 0.148148148
## 33      33 0.152777778
## 34      34 0.157407407
## 35      35 0.162037037
## 36      36 0.166666667
## 37      37 0.171296296
## 38      38 0.175925926
```

```
## 39        39 0.180555556
## 40        40 0.185185185
## 41        41 0.189814815
## 42        42 0.194444444
## 43        43 0.199074074
## 44        44 0.203703704
## 45        45 0.208333333
## 46        46 0.212962963
## 47        47 0.217592593
## 48        48 0.222222222
## 49        49 0.226851852
## 50        50 0.231481481
## 51        51 0.236111111
## 52        52 0.240740741
## 53        53 0.245370370
## 54        54 0.250000000
## 55        55 0.254629630
## 56        56 0.259259259
## 57        57 0.263888889
## 58        58 0.268518519
## 59        59 0.273148148
## 60        60 0.277777778
## 61        61 0.282407407
## 62        62 0.287037037
## 63        63 0.291666667
## 64        64 0.296296296
## 65        65 0.300925926
## 66        66 0.305555556
## 67        67 0.310185185
## 68        68 0.314814815
## 69        69 0.319444444
## 70        70 0.324074074
## 71        71 0.328703704
## 72        72 0.333333333
## 73        73 0.337962963
## 74        74 0.342592593
## 75        75 0.347222222
## 76        76 0.351851852
## 77        77 0.356481481
## 78        78 0.361111111
## 79        79 0.365740741
## 80        80 0.370370370
## 81        81 0.375000000
## 82        82 0.379629630
## 83        83 0.384259259
## 84        84 0.388888889
## 85        85 0.393518519
## 86        86 0.398148148
## 87        87 0.402777778
## 88        88 0.407407407
## 89        89 0.412037037
## 90        90 0.416666667
## 91        91 0.421296296
## 92        92 0.425925926
```

```
## 93      93 0.430555556
## 94      94 0.435185185
## 95      95 0.439814815
## 96      96 0.444444444
## 97      97 0.449074074
## 98      98 0.453703704
## 99      99 0.458333333
## 100    100 0.462962963
## 101    101 0.467592593
## 102    102 0.472222222
## 103    103 0.476851852
## 104    104 0.481481481
## 105    105 0.486111111
## 106    106 0.490740741
## 107    107 0.495370370
## 108    108 0.500000000
## 109    109 0.504629630
```

Just for fun

Let's generate random picks for 3 dice 100000 times and see if at least one of those rolls get three sixes on the same "roll" before the 109 roll.

```
n <- 109 # Minimum n required to get 3 sixes on the same roll

roll_3_dice <- function(n){
  n_roll <- c()
  X1 <- c()
  X2 <- c()
  X3 <- c()
  pXWin3_6 <- c()
  Win <- c()
  for (i in 1:n){
     n_roll[i] <- i
     X1[i] <- as.numeric(round(runif(1, 1, 6),0))
     X2[i] <- as.numeric(round(runif(1, 1, 6),0))
     X3[i] <- as.numeric(round(runif(1, 1, 6),0))
     pXWin3_6[i] <- as.numeric(X1[i]) + as.numeric(X2[i]) + as.numeric(X3[i]) # Maximum addition will b
     if (pXWin3_6[i] == as.numeric(18)) {
        Win[i] <- "YES"
     }
     else {
       Win[i] <- "NO"
     }
  }
  pTable <- data.frame(n_roll, X1, X2, X3, pXWin3_6, Win)
  return(pTable)
}
```

Rolling 3 dice 100000 times.

```
results <- roll_3_dice(100000)
subset(results, Win == "YES")
```

```
##       n_roll X1 X2 X3 pXWin3_6 Win
```

```
## 73          73  6  6  6         18  YES
## 2405      2405  6  6  6         18  YES
## 3236      3236  6  6  6         18  YES
## 4436      4436  6  6  6         18  YES
## 4838      4838  6  6  6         18  YES
## 5390      5390  6  6  6         18  YES
## 7735      7735  6  6  6         18  YES
## 9197      9197  6  6  6         18  YES
## 9228      9228  6  6  6         18  YES
## 9954      9954  6  6  6         18  YES
## 10452   10452  6  6  6         18  YES
## 11827   11827  6  6  6         18  YES
## 12663   12663  6  6  6         18  YES
## 12721   12721  6  6  6         18  YES
## 13638   13638  6  6  6         18  YES
## 14295   14295  6  6  6         18  YES
## 14552   14552  6  6  6         18  YES
## 16808   16808  6  6  6         18  YES
## 17600   17600  6  6  6         18  YES
## 18855   18855  6  6  6         18  YES
## 19874   19874  6  6  6         18  YES
## 20615   20615  6  6  6         18  YES
## 21061   21061  6  6  6         18  YES
## 23586   23586  6  6  6         18  YES
## 24709   24709  6  6  6         18  YES
## 25482   25482  6  6  6         18  YES
## 27073   27073  6  6  6         18  YES
## 27389   27389  6  6  6         18  YES
## 28691   28691  6  6  6         18  YES
## 30881   30881  6  6  6         18  YES
## 30891   30891  6  6  6         18  YES
## 32031   32031  6  6  6         18  YES
## 32302   32302  6  6  6         18  YES
## 33850   33850  6  6  6         18  YES
## 34294   34294  6  6  6         18  YES
## 35185   35185  6  6  6         18  YES
## 38648   38648  6  6  6         18  YES
## 38666   38666  6  6  6         18  YES
## 39179   39179  6  6  6         18  YES
## 40079   40079  6  6  6         18  YES
## 40383   40383  6  6  6         18  YES
## 40607   40607  6  6  6         18  YES
## 43507   43507  6  6  6         18  YES
## 46145   46145  6  6  6         18  YES
## 47351   47351  6  6  6         18  YES
## 50739   50739  6  6  6         18  YES
## 51001   51001  6  6  6         18  YES
## 51608   51608  6  6  6         18  YES
## 51897   51897  6  6  6         18  YES
## 52613   52613  6  6  6         18  YES
## 53198   53198  6  6  6         18  YES
## 53403   53403  6  6  6         18  YES
## 54432   54432  6  6  6         18  YES
## 54668   54668  6  6  6         18  YES
```

```
## 55216  55216  6  6  6        18  YES
## 56322  56322  6  6  6        18  YES
## 57022  57022  6  6  6        18  YES
## 58904  58904  6  6  6        18  YES
## 60002  60002  6  6  6        18  YES
## 61067  61067  6  6  6        18  YES
## 61086  61086  6  6  6        18  YES
## 62232  62232  6  6  6        18  YES
## 63007  63007  6  6  6        18  YES
## 64288  64288  6  6  6        18  YES
## 68868  68868  6  6  6        18  YES
## 70781  70781  6  6  6        18  YES
## 73255  73255  6  6  6        18  YES
## 75052  75052  6  6  6        18  YES
## 75345  75345  6  6  6        18  YES
## 75636  75636  6  6  6        18  YES
## 76401  76401  6  6  6        18  YES
## 77405  77405  6  6  6        18  YES
## 77600  77600  6  6  6        18  YES
## 77819  77819  6  6  6        18  YES
## 78822  78822  6  6  6        18  YES
## 79020  79020  6  6  6        18  YES
## 79123  79123  6  6  6        18  YES
## 81051  81051  6  6  6        18  YES
## 81792  81792  6  6  6        18  YES
## 83084  83084  6  6  6        18  YES
## 83235  83235  6  6  6        18  YES
## 83879  83879  6  6  6        18  YES
## 84720  84720  6  6  6        18  YES
## 85949  85949  6  6  6        18  YES
## 85982  85982  6  6  6        18  YES
## 88053  88053  6  6  6        18  YES
## 89650  89650  6  6  6        18  YES
## 90965  90965  6  6  6        18  YES
## 91592  91592  6  6  6        18  YES
## 91765  91765  6  6  6        18  YES
## 92458  92458  6  6  6        18  YES
## 94529  94529  6  6  6        18  YES
## 95083  95083  6  6  6        18  YES
## 95760  95760  6  6  6        18  YES
## 96720  96720  6  6  6        18  YES
## 99492  99492  6  6  6        18  YES
```

Let me know what you think.