Discussion 3

CUNY MSDS DATA 605

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Book: Beezer: A First Course in Linear Algebra

Exercise

Find the characteristic polynomial of the matrix A.

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix}$$

Solution

A characteristic polynomial of square matrix A is defined as $f_A(\lambda) = \det(A - \lambda I)$ where I is the identity matrix of same dimension as A.

$$f_A(\lambda) = \det(A - \lambda I)$$

First, I will calculate $(A - \lambda I)$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\lambda & 2 & 1 & 0 \\ 1 & -\lambda & 1 & 0 \\ 2 & 1 & 1-\lambda & 0 \\ 3 & 1 & 0 & 1-\lambda \end{pmatrix}$$

Now, I will proceed to find determinant $(A - \lambda I)$.

$$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 2 & 1 & 0 \\ 1 & -\lambda & 1 & 0 \\ 2 & 1 & 1-\lambda & 0 \\ 3 & 1 & 0 & 1-\lambda \end{pmatrix}$$

By reducing the matrix to row echelon form, have :

$$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 2 & 1 & 0 \\ 0 & \frac{-\lambda-5}{-\lambda+1} & -\frac{3}{-\lambda+1} & -\lambda+1 \\ 0 & 0 & \frac{2(2\lambda+3)}{\lambda+5} & -\frac{-\lambda^3+2\lambda^2+\lambda-2}{-\lambda-5} \\ 0 & 0 & 0 & -\frac{\lambda^4-3\lambda^3-2\lambda^2+2\lambda+2}{2(2\lambda+3)} \end{pmatrix}$$

Since the determinant of the matrix equals the diagonal product of the matrix.

$$\det(A-\lambda I) = (1-\lambda)\,\frac{-\lambda-5}{-\lambda+1}\cdot\frac{2\,(2\lambda+3)}{\lambda+5}\left(-\frac{\lambda^4-3\lambda^3-2\lambda^2+2\lambda+2}{2\,(2\lambda+3)}\right)$$

By simpliying the above expression, we obtain as follows:

$$\det(A-\lambda I) = \lambda^4-3\lambda^3-2\lambda^2+2\lambda+2$$

Since $f_A(\lambda) = \det(A-\lambda I)$

The answer will be:

$$f_A(\lambda) = \lambda^4-3\lambda^3-2\lambda^2+2\lambda+2$$

Solving in R

- Defining Matrix

```
# https://www.rdocumentation.org/packages/pracma/versions/1.9.9/topics/charpoly

A <- matrix(data = c(1,2,1,0,
                     1,0,1,0,
                     2,1,1,0,
                     3,1,0,1), ncol=4, byrow=TRUE)
```

- For this I will employ the charpoly function from the pracma library in R.

```
charpoly(A, info = FALSE)
```

```
## [1]  1 -3 -2  2  2
```

To interpret the answer, it will be as follows:

$$f_A(\lambda) = \lambda^4-3\lambda^3-2\lambda^2+2\lambda+2$$