Discussion 4

CUNY MSDS DATA 605

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C26 † Page 443

Book: Beezer: A First Course in Linear Algebra

Exercise

Verify that the function below is a linear transformation.

$$T : P_2 \to C^2, T\left(a + bx + cx^2\right) = \begin{pmatrix} 2a - b \\ b + c \end{pmatrix}$$

Solution

For this, we need to verify the two defining conditions of a linear transformation; that is:

- 1. $T(x + y) = T(x) + T(y)$
- 2. $T(\alpha x) = \alpha T(x)$

1. First let's work on **(1)** $T(x + y) = T(x) + T(y)$.

$$T(x + y) = T\left((a_1 + b_1 x + c_1 x^2) + (a_2 + b_2 x + c_2 x^2)\right)$$

$$T(x + y) = T\left((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2\right)$$

Let's define as follows:

$a = a_1 + a_2$

$b = b_1 + b_2$

$c = c_1 + c_2$

By making a substitution; we have as follows:

$$T(x + y) = T\left((a) + (b)x + (c)x^2\right)$$

And by our given problem, we have as follows:

$$T\left(a + bx + cx^2\right) = \begin{pmatrix} 2a - b \\ b + c \end{pmatrix}$$

And by substituting back we have:

$$T(x+y) = \begin{pmatrix} 2(a_1+a_2)-(b_1+b_2) \\ (b_1+b_2)+(c_1+c_2) \end{pmatrix}$$

$$T(x+y) = \begin{pmatrix} (2a_1+2a_2)-(b_1+b_2) \\ (b_1+b_2)+(c_1+c_2) \end{pmatrix}$$

$$T(x+y) = \begin{pmatrix} (2a_1-b_1)+(2a_2-b_2) \\ (b_1+c_1)+(b_2+c_2) \end{pmatrix}$$

$$T(x+y) = \begin{pmatrix} (2a_1-b_1) \\ (b_1+c_1) \end{pmatrix} + \begin{pmatrix} (2a_2-b_2) \\ (b_2+c_2) \end{pmatrix}$$

$$T(x+y) = \begin{pmatrix} 2a_1-b_1 \\ b_1+c_1 \end{pmatrix} + \begin{pmatrix} 2a_2-b_2 \\ b_2+c_2 \end{pmatrix}$$

$$T(x+y) = T\left(a_1+b_1x+c_1x^2\right) + T\left(a_2+b_2x+c_2x^2\right)$$

$$T(x+y) = T(x)+T(y)$$

This proof the first part of the definition is met.

2. Now, I will focus on **(2)** $T(\alpha x) = \alpha T(x)$.

$$T(\alpha x) = T\left(\alpha(a_1+b_1x+c_1x^2)\right)$$

$$T(\alpha x) = T\left(\alpha a_1+\alpha b_1x+\alpha c_1x^2\right)$$

Let's define as follows:

$a = \alpha a_1$

$b = \alpha b_1$

$c = \alpha c_1$

And by making a substitution we get:

$$T(\alpha x) = T\left(a+bx+cx^2\right)$$

Given our original equation, we have as follows:

$$T(\alpha x) = \begin{pmatrix} 2a-b \\ b+c \end{pmatrix}$$

By replacing back our previous definitions, we get as follows:

$$T(\alpha x) = \begin{pmatrix} 2\alpha a_1-\alpha b_1 \\ \alpha b_1+\alpha c_1 \end{pmatrix}$$

$$T(\alpha x) = \begin{pmatrix} \alpha(2a_1-b_1) \\ \alpha(b_1+c_1) \end{pmatrix}$$

$$T(\alpha x) = \alpha \begin{pmatrix} 2a_1 - b_1 \\ b_1 + c_1 \end{pmatrix}$$

$$T(\alpha x) = \alpha T\left(a + bx + cx^2\right)$$

$$T(\alpha x) = \alpha T(x)$$

This demonstrates that the second equation is met.

Conclusion:

$$T : P_2 \to C^2, T\left(a + bx + cx^2\right) = \begin{pmatrix} 2a - b \\ b + c \end{pmatrix}$$

The above function is a linear transformation since the two defining equations are met.