The Properties of the Exponential Distribution

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Synopsis

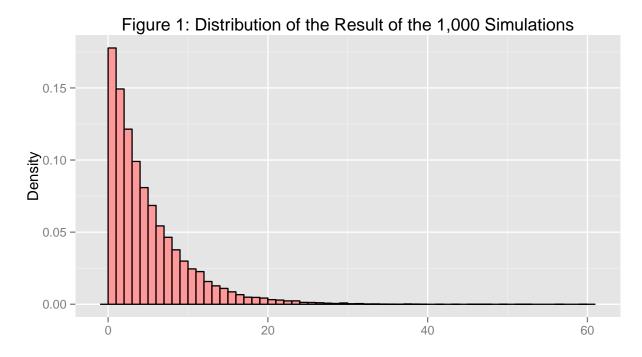
To explore the properties of exponential distributions we conducted a simulation. In each simulation we randomly selected 40 exponential distributions and average them. The simulation was run 1,000 times. Once simulated we explored the properties of the sample.

Simulation of Exponential Distribution

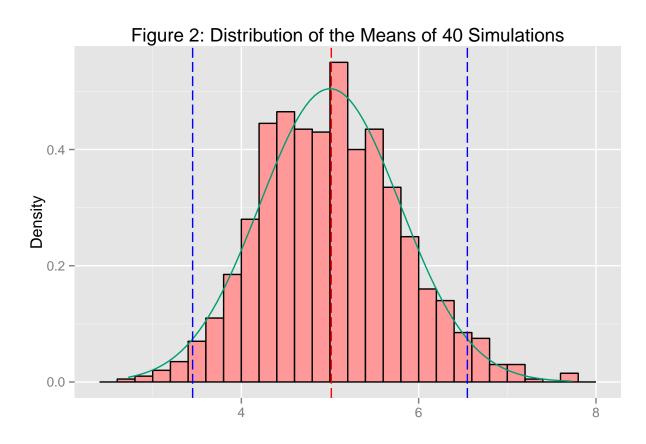
```
library(dplyr)
## Simulation Parameters
lambda \leftarrow 0.2
simulations <- 1000
number.of.exponentials.in.a.mean <- 40
## Objects to hold the results
sim.results <- mean.results <- data.frame(value=numeric())</pre>
## Simulation loop
for(i in 1:simulations) {
  ## Run the Simulation
  simulation <- rexp(number.of.exponentials.in.a.mean, lambda)</pre>
  ## Load simulation results into sim.results data frame
  sim.results <- rbind_list(sim.results, data.frame(value = simulation))</pre>
  ## Compute the mean of the simulations and load it into the mean.results data frame
  mean.results <- rbind_list(mean.results, data.frame(value = mean(simulation)))</pre>
}
```

Results

Now that we have completed the simulation let's visualize the results. First let's explore the distribution of the randomly selected observations:



We observe the simulation is indeed an exponential distribution. Now let's look at the distribution of the mean of 40 exponentials:



The following observations can be made about exponential distributions:

- The average (shown as the red line in figure 2) is 5.0147 which is a close approximation of the theoretical mean of 5.
- The variance of the simulation is 24.9286 which is again a close appoximation of the theoretical variance of 25.
- The standard deviation of the simulation is 4.9929 which is close to 5.
- We observe that the distribution of the means generally fits in the green normal curve.
- We also observe that 94.6% of the means fall within the 95% confidence interval (the blue dashed lines in figure 2).