3 LDA

Logistic regression involves direction modeling $P(Y = k|X = x)$ using the logistic function for the case of two response classes. We now consider a less direct approach.

Idea:

Why do we need another method when we have logistic regression?

1.

2.

3.

3.1 Bayes' Theorem for Classification

Suppose we wish to classify an observation into one of K classes, where $K \geq 2$.

π_k

$f_k(x)$

$P(Y = k|X = x)$

In general, estimating π_k is easy if we have a random sample of Y's from the population.

Estimating $f_k(x)$ is more difficult unless we assume some particular forms.

3.2 p = 1

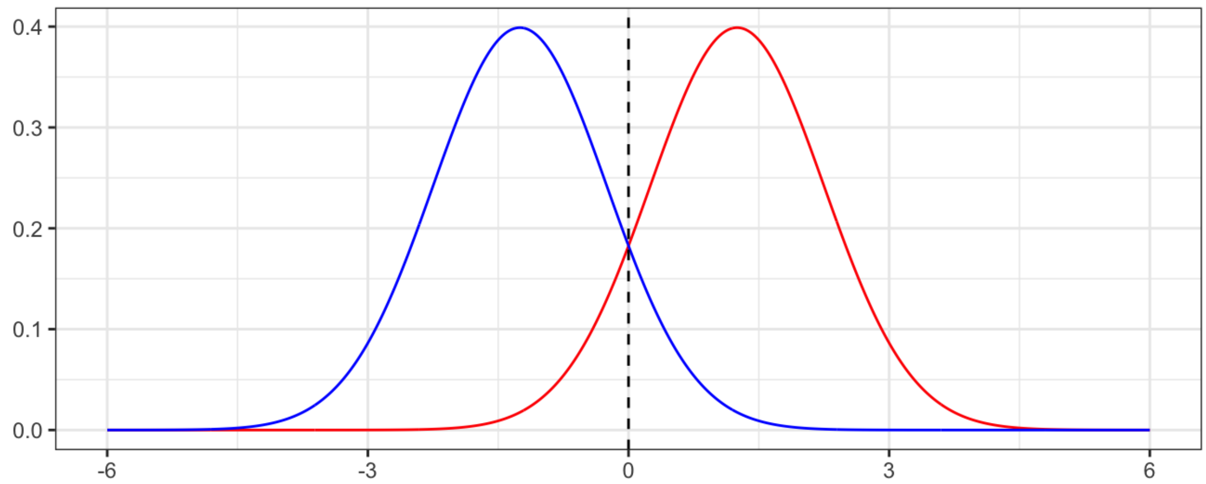
Let's (for now) assume we only have 1 predictor. We would like to obtain an estimate for $f_k(x)$ that we can plug into our formula to estimate $p_k(x)$. We will then classify an observation to the class for which $\hat{p}_k(x)$ is greatest.

Suppose we assume that $f_k(x)$ is normal. In the one-dimensional setting, the normal density takes the form

Plugging this into our formula to estimate $p_k(x)$,

We then assign an observation $X = x$ to the class which makes $p_k(x)$ tthe largest. This is equivalent to

Example 3.1 Let $K = 2$ and $\pi_1 = \pi_2$. When does the Bayes classifier assign an observation to class 1?

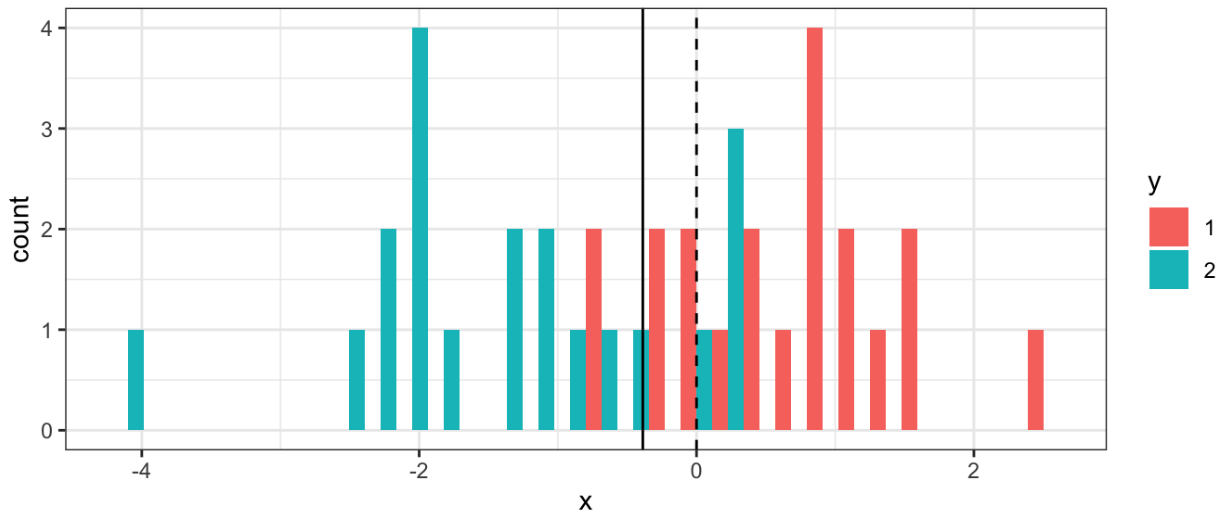


In practice, even if we are certain of our assumption that X is drawn from a Gaussian distribution within each class, we still have to estimate the parameters $\mu_1, \ldots, \mu_K, \pi_1, \ldots, \pi_K, \sigma^2$.

The *linear discriminant analysis* (LDA) method approximated the Bayes classifier by plugging estimates in for π_k, μ_k, σ^2.

Sometimes we have knowledge of class membership probabilities $\pi_1, \ldots, \pi_K$ that can be used directly. If we do not, LDA estimates π_k using the proportion of training observations that belong to the kth class.

The LDA classifier assignes an observation $X = x$ to the class with the highest value of




```
##    pred
## y       1     2
##   1 18966  1034
##   2  3855 16145
```

The LDA test error rate is approximately 12.22% while the Bayes classifier error rate is approximately 10.52%.

The LDA classifier results from assuming that the observations within each class come from a normal distribution with a class-specific mean vector and a common variance σ^2 and plugging estimates for these parameters into the Bayes classifier.