Exercise 2

ECON / MATH C103 - Mathematical Economics
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due Tue Jan 31, 4:59pm

Please raise questions, in the office hours, via email or at bcourses:

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email: `mathematicaleconomicsc103@gmail.com`
bcourses: `http://bcourses.berkeley.edu`.

Each sub-exercise is weighted equally.

Helpful Material:
- Last week's lecture notes.

Exercise 1 (Optimal Taxation)**:** Consider an economy with a single worker. Denote by $\theta \in [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+, 0 \leq \underline{\theta} < \overline{\theta} < \infty$ the worker's ability.

There are two type of jobs, high and low intensity jobs denoted by $0 < l < h$. High intensity jobs pay a high wage w_h greater than the wage paid in low intensity jobs $w_l < w_h$, but also induce a higher cost of effort than low intensity jobs. We denote the type of job the worker has by $x \in \{l, h\}$.

The workers utility $u((x,t),\theta)$, depending on the taxes he pays $t \in \mathbb{R}$, the type of job he holds x, as well as his ability θ is given by

$$u((x,t),\theta) \triangleq (w_x - t) - \frac{x}{\theta}.$$

We assume that the workers ability θ is his private information and not known to the principal. The principal can choose which type of job x a worker takes and how many takes he has to pay.

(a) Define what a mechanism is in this context.

(b) Define what a *direct* mechanism is in this context.

(c) Define incentive compatibility.

(d) Prove that x has to be increasing in the ability θ in every IC direct mechanism.

(e) Characterize the transfer in every IC direct mechanism.

(f) Characterize the set of outcomes (x,t) that can be implemented in any mechanisms.

(g) Describe an implementation of an outcome that can be implemented in some mechanism, as an *indirect* mechanism where an agent's tax t, is only a function of his labor income w_x and the agent can decide himself between high and low intensity jobs.

(h) Define the participation constrained formally and discuss it's economic meaning in the context of taxation. Does it make sense to impose it here?

Now assume that the low intensity job, does not entail any labor cost $l = 0$, and leads to no labor income $w_l = 0$ (ie unemployment). Furthermore, restrict attention to mechanisms where the agent can not make payments which are higher than his labor income $t \leq w_x$. Assume that ability θ is distributed according to some distribution $F : [\underline{\theta}, \overline{\theta}] \to [0,1]$ with strictly positive density f.

(i) Derive the structure of the direct mechanism which maximizes the expected revenue collected from taxes $\mathbb{E}[t]$.

(j) Suppose, that the wage would be given by $w_x = 4$ and the disutility of work $h = 1$. Furthermore, assume θ uniformly distributed on $[0,1]$, i.e. $F(\theta) = \theta$. With which abilities would the agent choose the high intensity job if taxes were zero $t = 0$ and he could decide freely. What is the probability that he picks the high intensity job?

(k) Derive a tax revenue maximizing mechanism for $w_x = 4, h = 1, F(\theta) = \theta$. With which abilities will the agent choose the high intensity job in this mechanism? What is the probability that he picks the high intensity job?

Remark: An interesting remark about the above exercise is that we can interpret the random ability θ of a single agent as representing a continuum of agents with different abilities distributed according to F. The model thus approximates a situation with many workers.