Exercise 3

ECON / MATH C103 - Mathematical Economics
Philipp Strack

due Tue Feb 7, 4:59pm

Each sub-exercise is weighted equally.

Helpful Material:
 - Last week's lecture notes.

Exercise 1: (12 points) Consider a general mechanism a design problem with a single agent. The agent has a single dimensional type $\theta \in [\underline{\theta}, \overline{\theta}] \triangleq \Theta \subseteq \mathbb{R}$. Types are distributed according to $F : \Theta \to [0,1]$, where the support of F equals Θ. The principal chooses an allocation $a \in A$, where we do not make any assumption on A. For example the allocation might or might not involve monetary transfers. The agent's utility is given by $u : A \times \Theta \to \mathbb{R}$ and we assume that u is uniformly Lipschitz continuous in $\theta \in \Theta$. Prove that the outcome

$$a : \theta \to A$$

of *any* mechanism satisfies

$$u(a(\theta), \theta) = u(a(\underline{\theta}), \underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_\theta(a(s), s)\mathrm{d}s\,.$$

Exercise 2: (16 points) Let the set of physical allocations X be single dimensional $X = [0, \bar{x}] \subset \mathbb{R}$. Assume quasilinear preferences described by the utility function

$$u((x,t), \theta) = \sqrt{x}\,\theta - t$$

and a single dimensional type $\theta \in [\underline{\theta}, \overline{\theta}] \triangleq \Theta \subseteq \mathbb{R}$, distributed according to $F : \Theta \to [0,1]$, with full support, and density $f = F'$.

   (a) Characterize the set if incentive compatible direct mechanisms.

(b) Characterize the set of incentive compatible mechanism which satisfy the participation con-
strained.

(c) What is the information rent an agent of type θ receives? Explain in your own words the
economic meaning of this information rent.

(d) Derive the maximal expected revenue which can be generated in a mechanism which imple-
ments the physical allocation $x : \Theta \to [0, \bar{x}]$ and satisfies the participation constrained.