Econ C103 Problem Set 5

Sahil Chinoy

February 28, 2017

Exercise 1

(a) Each agent $i \in \{1, 2\}$ has type indicated by the value of the object that they have at the beginning of the game, $\theta_i \in \Theta = [0, 1]$, and can send a message from $M_i = \{0, 1\}$ indicating whether they want to exchange objects. If both agents choose $m_i = 1$, then the trade takes place, which we denote with $x = 1$. So the allocation rule is

$$x(m) = \begin{cases} 0 & \text{if } m_1 = 0 \text{ or } m_2 = 0 \\ 1 & \text{if } m_1 = m_2 = 1 \end{cases}$$

and each agent's utility is

$$u_i(x, \theta_i) = \begin{cases} \theta_i & \text{if } x = 0 \\ \theta_{-i} & \text{if } x = 1 \end{cases}.$$

(b) Trading will never be Pareto efficient, so the only Pareto efficient outcome is the original allocation, $x = 0$. If $\theta_1 \neq \theta_2$, trading would make one agent worse off, since both agents assign the same value to the objects. If $\theta_1 = \theta_2$, then trading won't change either agent's utility and thus won't make either agent strictly better off.

Either allocation ($x = 0$ or 1) maximizes the sum of the agents' utilities, which is always $\theta_1 + \theta_2$.

(c) The best response of each agent depends on the message the other agent sends and the other agent's type. If $m_{-i} = 0$, then trade will never take place, so it doesn't matter what message the agent chooses; we will pick $m_i = 0$. Then the best response for agent i is

$$\sigma_i(m_{-i}, \theta) = \begin{cases} 0 & \text{if } m_{-i} = 0, \text{ or } m_{-i} = 1 \text{ and } \theta_i \geq \theta_{-i} \\ 1 & \text{if } m_{-i} = 1 \text{ and } \theta_i < \theta_{-i} \end{cases}.$$

Thus the agent's best response depends on m_{-i} and θ_{-i}. This means that there is no dominant strategy equilibrium.

(d) Given that the types θ are uniformly distributed on $[0, 1]$, each agent initially expects the other agent's type to be $\mathbb{E}[\theta_{-i}] = 0.5$. This means it is rational to trade, i.e. send the message $m_i = 1$, only if $\theta_i < 0.5$. But then the expected payoff from the trade for the *other* agent is $\mathbb{E}[\theta_{-i} | m_{-i} = 1] = 0.25$, and if the first agent expects the other agent to play optimally, their expected payoff is now $\mathbb{E}[\theta_{-i} | m_{-i} = 1] = 0.125$, and so on. So trading will never be optimal.

Sending the message $m_i = 0$ is a Bayes-Nash equilibrium, however, because given that every agent expects the other agents to play $\sigma_i(\theta_i) = 0$, a trade can never occur, so it is never in anyone's interest to send the message $m_i = 1$ as it will not change the outcome. Formally, $\forall m_i \; \mathbb{E}[u_i(x(\sigma_i, \sigma_{-i}), \theta_i) | \theta_i] = \theta_i = \mathbb{E}[u_i(x(m_i, \sigma_{-i}), \theta_i) | \theta_i]$. So $\sigma_i(\theta_i) = 0$ is a Bayes-Nash equilibrium.