Exercise 5

ECON / MATH C103 - Mathematical Economics
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due March 7, 4:59pm

Helpful Material:
  - Last week's lecture notes.

Exercise 1: Consider a set up with n agents and $k \geq 1$ identical objects. The designer can decide whether agent i receives an object $x_i = 1$ or does not receive an object $x_i = 0$. Each agent can receive at most one object. Agents have quasi-linear preferences of the form

$$u_i((x, t_i), \theta_i) = x_i \theta_i - t_i,$$

where $\theta_i \geq 0$ is agent i's value for an object. Each agent's value for an object θ_i is drawn independently from the uniform distribution on $[1, 10]$.

    Economic Example: As an interpretation of this exercise consider for example k cell-phone carriers who want to by spectrum from the government. The government is interested in allocating the spectrum efficiently, and conditional on efficient allocation wants to raise as much revenue as possible.

(a) Derive the efficient allocation rule. (4pts)

(b) Characterize all efficient, dominant strategy incentive compatible (DIC), direct mechanisms. (4pts)

(c) Derive the Pivot mechanism. (4pts)

(d) Derive the DIC direct mechanisms that maximizes revenue in the over the set of all efficient mechanisms. (4pts)

Exercise 2: Suppose there are two units of a good that the social planner can distribute among three agents. Each agent has quasilinear utility over the outcome and her transfer to the planner, where her value over units is described below.

	zero units	one unit	two units
Agent 1	0	20	20
Agent 2	0	0	25
Agent 3	0	10	20

For example, Agent 3's utility from receiving one unit and paying 8 dollars is $10 - 8 = 2$, and Agent 2's utility from receiving both units and paying 18 dollars is $25 - 18 = 7$.

(a) Which allocations are Pareto efficient if **no** transfers between the agents are allowed.

(b) Derive the efficient allocation when transfers are allowed.

(c) Find the transfer for each agent in the pivot mechanism.