

Econ C103 Problem Set 9

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Exercise 1

- (a) Suppose the mechanism did condition on the cardinal preferences of the agents. Take allocation a to be the agent's most preferred alternative, so $u_i(a) \geq u_i(\hat{a})$ for $\hat{a} \in A$. To be incentive-compatible, the allocation must be monotone. So the probability that the mechanism achieves allocation a must be increasing in $m_i(a)$, the utility that the agent *reports* for allocation a . Then, since the agent does not pay a transfer that depends on their report, it is preferable for the agent to report $m_i(a) \rightarrow \infty$. So the mechanism is not incentive compatible, since $m_i(a) \neq u_i(a)$.

If instead the mechanism conditioned only on the ordinal preferences of the agent, the allocation would only change if the agent reported $m_i(a) < m_i(\hat{a})$ for some $\hat{a} \in A$ (because this implies that $a \prec_i \hat{a}$). But then, since the probability of allocation a must be increasing in $m_i(a)$, this means the allocation a occurs less often, and the agent is worse off. Then the agent is indifferent between any report $m_i(a) \geq u_i(a)$, thus the mechanism is dominant-strategy incentive compatible.

So, to be dominant-strategy incentive compatible, a mechanism must condition only on the ordinal preferences of the agents. Note that this is necessary but not sufficient for dominant-strategy incentive compatibility.

- (b) The set of all possible preferences is

$$\{\alpha \prec \beta \prec \gamma, \alpha \prec \gamma \prec \beta, \beta \prec \alpha \prec \gamma, \beta \prec \gamma \prec \alpha, \gamma \prec \alpha \prec \beta, \gamma \prec \beta \prec \alpha\}.$$

- (c) Given $\alpha < \beta < \gamma$, the only single-peaked preferences are

$$\{\alpha \prec \beta \prec \gamma, \alpha \prec \gamma \prec \beta, \gamma \prec \alpha \prec \beta, \gamma \prec \beta \prec \alpha\}.$$

- (d) No, agent 3's preferences are not single-peaked. In the median voting mechanism, allocation β is implemented.
- (e) Yes, if agent 3 reports α , then the allocation will be α , which the agent prefers to β .