# Exercise 4

ECON / MATH C103 - Mathematical Economics
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**due Tue Feb 14, 4:59pm**

Each sub-exercise is weighted equally.

**Helpful Material:**
- Last week's lecture notes.

**Exercise 1:** (28 points) There are $n$ agents, each agent's willingness to pay is given by $\theta_i \geq 0$. There is a single object, we set $x_i = 1$ if agent $i$ gets the object and $x_i = 0$ otherwise. Each agent $i$'s utility function is quasi-linear and given by

$$u((x_i, t_i), \theta_i) = x_i \theta_i - t_i .$$

Assume furthermore, that $\theta_i$ are identically and independently distributed (iid) according to some distribution $F : [0, \overline{\theta}] \to [0,1]$ with strictly positive density $f > 0$ and bounded support $\overline{\theta} < \infty$.

Consider the following *second price auction* or Vickrey auction: Each agent submits a message (called bid) $m_i \in M_i = \mathbb{R}_+$. The agent who submitted the highest bid receives the object and pays the *second highest bid*, all other players make no payments and receive nothing.

(a) Describe the second price auction formally as a mechanism, by stating $(M, (x,t))$.

(b) Let $m_{-i}$ denote the vector of bids submitted the agents $j \neq i$. Derive the bid(s) which maximizes agent $i's$ utility given $m_{-i}$ as a function of his type $\theta_i$.

(c) Does there exists a strategy which maximizes agent $i's$ utility for any $m_{-i}$. If your answer is yes derive it, if no argue why not.

(d) Does the second price auction have a unique dominant strategy equilibrium? If yes, what is the expected revenue in equilibrium?

Consider the following strategy player 1 always bids $\overline{\theta}$ and all other players always bid 0.

(e) Is this strategy vector a dominant strategy equilibrium?

(f) Is this strategy vector a Bayes Nash equilibrium?

(g) What is the expected revenue in this strategy profile?

**Exercise 2:** (20 points)

(a) (12 points) Prove formally (!) that every dominant strategy equilibrium is also a Bayes Nash equilibrium.

(b) (8 points) Provide an example of a mechanism which has no dominant strategy equilibrium.