Exercise 1

ECON / MATH C103 - Mathematical Economics
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due Tue Jan 24, 4:59pm

Please raise questions, in the office hours, via email or at bcourses:

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bcourses: `http://bcourses.berkeley.edu`.

Each sub-exercise (a,b,..) is weighted equally and gives 4 points.

Helpful Material:
- De la Fuente 2000, Chapter 5.2 page 200 ff.
- Krishna 2009, Appendix A page 253 ff. and Appendix C page 265 ff.

Exercise 1 (Random Variables)**:** Let x be a random variable distributed according to the absolutely continuous cumulative distribution function (CDF) $F : \mathbb{R} \to [0,1]$, with density $f : \mathbb{R} \to \mathbb{R}_+$.

(a) What is the expected value and the variance of x?

(b) Show that for every bounded, differentiable function $v : \mathbb{R} \to \mathbb{R}$ the following equality holds (hint: use integration by parts)

$$\int_{\mathbb{R}} v(x) f(x) \mathrm{d}x = \int_{\mathbb{R}} v'(x)(1 - F(x)) \mathrm{d}x + \lim_{x \to -\infty} v(x)\,.$$

(c) Suppose x_1 and x_2 are independently drawn from F. Derive the distribution of

$$y_1 \triangleq \max\{x_1, x_2\} \text{ and } y_2 \triangleq \min\{x_1, x_2\}\,.$$

(d) Derive the expected value of the maximum y_1, and the minimum y_2.

(e) Derive the expected value of the maximum y_1 conditional on it being above a constant threshold $z \in \mathbb{R}$,

$$\mathbb{E}[y_1 \mid y_1 \geq z]\,.$$

(f) Derive the expectation of y_1 conditional on y_2 being equal to $z \in \mathbb{R}$,

$$\mathbb{E}[y_1 \mid y_2 = z] \,.$$

Exercise 2 (Optimization and Maximizers)**:** Let $w : X \times \Theta \to \mathbb{R}$, where $X \triangleq [\underline{x}, \overline{x}], \Theta \triangleq [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$ are compact bounded intervals. Assume that w is differentiable in both arguments and strictly concave in the first.

(a) For all $\theta \in \Theta$ let

$$m(\theta) \triangleq \arg\max_{x \in X} w(x, \theta) \,.$$

What is the derivative of m?

(b) When is $m(\theta)$ increasing (decreasing) in θ for all $\theta \in \Theta$?

(c) For all $\theta \in \Theta$ let

$$v(\theta) \triangleq \max_{x \in X} w(x, \theta) \,.$$

Denote by w_θ the partial derivative of w with respect to the second argument. Show that

$$v'(\theta) = w_\theta(m(\theta), \theta) \,.$$

References

De la Fuente, A. (2000). *Mathematical methods and models for economists*. Cambridge University Press.

Krishna, V. (2009). *Auction theory*. Academic press.