Problem Set 3
Econ 350, Winter 2014
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Due January 30, 2014

1. (35 pts.) Derive the Ben Porath and Generalized Ben Porath model of earnings. (You may use the notes posted at the class website.) Specifically:

   (a) Derive the costate multiplier for each model (general case). Give a clear economic interpretation of the level and the change in the multiplier over the life cycle and the components of the multiplier. Why is it accurate to call the dynamic multiplier the "shadow price" or value per unit of human capital? See also the excerpt from Arrow and Kurz (1970, pp. 33–38) in the technical supplement.

   (b) For each model, derive the optimal level of schooling and the path of post-school investments.

   (c) How do the Ben Porath and Generalized Ben Porath models explain the "return to experience" in the Mincer model?

   (d) For the case where the only input is time and the human capital production function is

   $$\dot{H} = A(IH)^{\alpha}; \quad A > 0; 0 < \alpha < 1; \text{ and } \alpha = \frac{1}{2}, \qquad H_0 > 0,$$

   where $H_0$ is the initial condition ($I$ is time; $H$ is human capital) and $\dot{H}$ is its time rate of change, what is the implied autoregression for earnings? Consider the time series representation of the Huggett et al. (2011, AER) model.

   (e) How well does the Ben-Porath model explain the evolution of life cycle earnings? (Also, examine the paper by Polachek et al., 2013) The life cycle schooling decision?

   (f) Compare the estimates of the Ben-Porath model with those of the Generalized Ben-Porath model. (Hint: See the survey in Browning et al., 1999.)

(g) Interpret the Ben-Porath model as a framework for life cycle occupational choice (Hint: see Rosen, 1972, Journal of Human Resources).

2. (20 pts.) Compare the "learning by doing" model with the "on the job training" model. (Add labor supply to both models.)

   (i) What is the (marginal utility of wealth compensated) effect of an increase in the current wage on investment in the period of the wage increase in each model? What is the effect of an increase in current wages on wage growth? On labor supply in each model?

   (ii) Would either model suggest poverty traps for welfare programs?

   (iii) Which model better explains the Mincer earnings equation (the relationship between work experience and wages)?

   (Hint: See Cossa et al., 2003, "*Learning-by-Doing versus On-the-Job Training: Using Variation Induced by the EITC to Distinguish between Models of Skill Formation.*")

3. (15 pts.) Consider the following model of human capital accumulation due to Sheshinski (see the notes on Sheshinski models in the technical supplement on the reading list).

   | | |
   |---|---|
   | $H(\tau)$ | is human capital (a scalar) at age $\tau$ |
   | $I(\tau)$ | is time invested at age $\tau$ $(0 \leq I(\tau) \leq 1)$ |
   | $\sigma$ | is rate of depreciation of human capital |
   | $R$ | is reward per unit human capital. It is a constant for all $\tau$. |
   | $r$ | is the interest rate (constant) |
   | $T$ | is the length of life (finite) |

   Agent seeks to maximize

   $$\int_0^T e^{-r\tau} R(1 - I(\tau)) H(\tau) \, d\tau$$

   subject to

   $$\dot{H}(\tau) = AI(\tau) - \sigma$$

   The Hamiltonian is

   $$\mathcal{H} = e^{-rt} R(1 - I(\tau)) H(\tau) + \mu(\tau)(AI(\tau) - \sigma H(\tau)),$$

   where $\mu(\tau)$ is the "co-state multiplier" (i.e., marginal value of a unit of human capital).

(a) For this model derive the optimal investment policy, the implied life cycle earnings profile, and the life cycle profile of human capital. Take two cases: $\sigma = 0$ and $\sigma > 0$.

(b) The case where $I(\tau) = 1$ is interpreted as full-time schooling (no earnings). Ignore tuition costs. For the case $\sigma = 0$, derive the earnings function in terms of schooling and experience. How does it compare with Mincer's model?

(c) For the case of $\sigma > 0$, derive the multiplier $\mu(\tau)$. Is it possible for $\mu(\tau) \uparrow \tau$ over the entire life cycle $(0, T)$? Over segments of the life cycle? Give an economic interpretation of how this might arise and show how agents might cycle between full-time schooling and full-time work, then return to school, then go to work, etc.

(Hint: See the notes "*Modified Sheshinski Specification.*")

4. (35 pts.) Summarize to the best of your knowledge "the effects" of schooling and parental inputs on child test scores and adult outcomes. The papers by Almond and Currie (2011); Caucutt and Lochner (2012); Cunha and Heckman (2007, 2008, 2009); Cunha et al. (2010); Del Boca et al. (2013); Heckman and Mosso (2014); Todd and Wolpin (2003, 2007) on the reading list should be digested and the following questions answered.

   (a) What is an educational production function? How does it relate to the technology of skill formation of Cunha et al.?

   (b) How can the technology be identified? Discuss the available data, possible sources of identification, and the ones used in the literature.

   (c) How would you model the role of parental investment and parental inputs? (Distinguish parenting from environmental variables.)

   (d) If any monotonic function of a test score is still a test score, what is the meaning of the Todd-Wolpin estimates? How could you fix the scale of output? Of the coefficients that they estimate? Discuss how Cunha et al. address these questions, and whether or not it affects the estimates (consider both linear and nonlinear cases).

   (e) How attractive are linear production function specifications?

   (f) It is sometimes said that "teachers matter" but "teacher characteristics" do not. How does this match up with the evidence in Todd-Wolpin?

(g) How important are parents for producing test scores?

(h) What do we know about the response of parents to schooling inputs? To early preschool inputs?

5. (25 pts.) What is the evidence or the importance of credit constraints in forming child outcomes? (Define the various constraints studied in the literature, discussed in Lochner and Monge-Naranjo (2012), and how they are measured.) Hint: Examine the papers in Heckman and Mosso (2014).

6. (40 pts.) Consider a social planner (or a parent) deciding how much to invest in two children $A$ and $B$ born with different endowments $\theta_0^A < \theta_0^B$. Write the technology of skill formation as

$$\theta_1 = F^{(1)}(\theta_0, I_1)$$
$$\theta_2 = F^{(2)}(\theta_0, I_2)$$

Each function is strictly concave in $I$.

Assume that $F_{1,2}^{(2)} > 0$, but $F_{1,2}^{(1)}$ may be of either sign. The technology is common across $A$ and $B$. There is a fixed budget $E$ which can be allocated between the two children. Parents seek to maximize total second period output: $\theta_2^A + \theta_2^B$.

A. For the first case initially work out the allocation in the two cases

(I) $F^{(1)} = A_1 \theta_0^{\alpha_1} I_1^{\beta_1}$
$\quad F^{(2)} = A_2 \theta_0^{\alpha_2} I_2^{\beta_2}$, and

(II) $F^{(1)} = A_1(\gamma_1 \theta_0^{\phi_1} + (1 - \gamma_1)I_1^{\phi_1})^{\frac{\rho_1}{\phi_1}}$
$\quad F^{(2)} = A_2(\gamma_2 \theta_0^{\phi_2} + (1 - \gamma_2)I_1^{\phi_2})^{\frac{\rho_2}{\phi_2}}$

B. Then consider the general cases:

(i) $F_{1,2}^{(1)} > 0$ and $F_{1,2}^{(2)} > 0$, and

(ii) $F_{1,2}^{(1)} < 0$ and $F_{1,2}^{(2)} > 0$

For an optimum do you need the concavity of $F^{(1)}$ and $F^{(2)}$ in $\theta$?

C. Formulate the comparative statics of the problem where

$$\theta_0^A = \tau \theta_0^B, \qquad \tau \leq 1$$

and for the two cases in B. derive results for

$$\frac{\partial I_1^j}{\partial \tau} \text{ and } \frac{\partial I_2^j}{\partial \tau}, \qquad j \in \{A, B\}$$

and

$$\frac{\partial I_1^j}{\partial P_1} \text{ and } \frac{\partial I_2^j}{\partial P_1}, \qquad j \in \{A, B\}$$

7. (30 pts.) Read the Becker-Tomes 1986 paper on the reading list. Answer the following questions about it.

   (a) Derive their proposition about negative effects of grandparents' income on child outcomes. How robust is this conclusion?

   (b) What is the evidence on family altruism? (Define altruism and paternalism.) On the importance of credit constraints? (Hint: review the papers by Lochner and Monge-Naranjo on the reading list and the papers by Behrman, Pollak, and Taubman and their survey of the evidence.) How do you test for altruism? How does altruism differ from other social preferences (see Almlund, Duckworth, Heckman, and Kautz, 2011, Section 6)?

   (c) How would you modify their model to add parental influences on child preferences?

   (d) Consider the critique of the Becker-Tomes model in Goldberger (1989). In what sense does the Becker-Tomes model improve on a Galton-style intergenerational inheritance model?

   (e) How does the Becker-Tomes model explain the evidence by Mazumder (2005) and others, summarized by Black and Devereux (2011) on the reading list, that in the U.S. the correlation between the influence of the father's earnings and the son's is over .65, while in Denmark it is .07?

   (f) Compare and contrast the predictions of the Becker-Tomes model with the model of Cunha and Heckman (2007).

   (g) How accurately does the Solon (2004) model, reviewed in Black and Devereux, capture the Becker-Tomes model?

(h) In what ways do Seshadri and Lee (2012) expand on the Becker-Tomes model? Consider their value added.

(i) In what way do Heckman and Mosso (2014) extend the Becker-Tomes framework?

# Relevant Papers

Almlund, M., A. Duckworth, J. J. Heckman, and T. Kautz (2011). Personality psychology and economics. In E. A. Hanushek, S. Machin, and L. Wößmann (Eds.), *Handbook of the Economics of Education*, Volume 4, pp. 1–181. Amsterdam: Elsevier.

Almond, D. and J. Currie (2011). Human capital development before age five. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, Volume 4B, Chapter 15, pp. 1315–1486. North Holland: Elsevier.

Arrow, K. J. and M. Kurz (1970). *Public investment, the rate of return, and optimal fiscal policy*. Baltimore: Published for Resources for the Future by the Johns Hopkins Press.

Becker, G. S. and N. Tomes (1986, July). Human capital and the rise and fall of families. *Journal of Labor Economics 4*(3, Part 2), S1–S39.

Black, S. E. and P. J. Devereux (2011). Recent developments in intergenerational mobility. In O. Ashenfelter and D. Card (Eds.), *Handbooks in Economics Handbook of Labor Economics, Vol 4B*, Volume 4, Part B of *Handbook of Labor Economics*, pp. 1487 – 1541. Elsevier.

Browning, M., L. P. Hansen, and J. J. Heckman (1999, December). Micro data and general equilibrium models. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1A, Chapter 8, pp. 543–633. Elsevier.

Caucutt, E. M. and L. J. Lochner (2012). Early and late human capital investments, borrowing constraints, and the family. Working Paper.

Cunha, F. and J. J. Heckman (2007, May). The technology of skill formation. *American Economic Review 97*(2), 31–47.

Cunha, F. and J. J. Heckman (2008, Fall). Formulating, identifying and estimating the technology of cognitive and noncognitive skill formation. *Journal of Human Resources 43*(4), 738–782.

Cunha, F. and J. J. Heckman (2009, April). The economics and psychology of inequality and human development. *Journal of the European Economic Association 7*(2–3), 320–364. Presented as the Marshall Lecture, European Economics Association, Milan, Italy, August 29, 2008.

Cunha, F., J. J. Heckman, and S. M. Schennach (2010, May). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica 78*(3), 883–931.

Del Boca, D., C. J. Flinn, and M. Wiswall (2013). Household choices and child development. Unpublished manuscript, New York University. Forthcoming *Review of Economic Studies*.

Goldberger, A. S. (1989). Economic and mechanical models of intergenerational transmission. *American Economic Review 79*(3), 504–513.

Heckman, J. J. and S. Mosso (2014). The economics of human development and social mobility. Forthcoming, *Annual Reviews of Economics*.

Huggett, M., G. Ventura, and A. Yaron (2011). Sources of lifetime inequality. *American Economic Review 101*(7), 2923–2954.

Lochner, L. J. and A. Monge-Naranjo (2012). Credit constraints in education. Forthcoming, *Annual Reviews in Economics*.

Mazumder, B. (2005). Fortunate sons: New estimates of intergenerational mobility in the United States using Social Security earnings data. *Review of Economics and Statistics 87*(2), 235–255.

Polachek, S. W., T. Das, and R. Thamma-Apiroam (2013). Heterogeneity in the production of human capital. IZA Discussion Paper.

Rosen, S. (1972). Learning and experience in the labor market. *The Journal of Human Resources 7*(3), 326–342.

Seshadri, A. and S. Y. Lee (2012, October). Nature and nurture in the transmission of economic status.

Solon, G. (2004). A model of intergenerational mobility variation over time and place. In M. Corak (Ed.), *Generational income mobility in North America and Europe*, Chapter 2, pp. 38–47. Cambridge: Cambridge University Press.

Todd, P. E. and K. I. Wolpin (2003, February). On the specification and estimation of the production function for cognitive achievement. *Economic Journal 113*(485), F3–F33.

Todd, P. E. and K. I. Wolpin (2007, Winter). The production of cognitive achievement in children: Home, school, and racial test score gaps. *Journal of Human Capital 1*(1), 91–136.