

# Consumption & Earnings Dynamics II

*Econ 350, The University of Chicago*

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- ① Consumption and Labor Supply with Partial Insurance: an Analytical Framework by Jonathan Heathcote, Kjetil Storesletten, & Giovanni L. Violante.
- ② Consumption Inequality and Family Labor Supply by Richard Blundell, Luigi Pistaferri, & Itay Saporta-Eksten.

# Research Questions of the First Paper

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- ① How effectively can households smooth idiosyncratic wage fluctuations via private insurance arrangements and labor supply adjustments?
- ② To what degree has the four-decade-long rise in wage dispersion passed through to inequality in consumption and hours worked?
- ③ What is the role uninsurable life-cycles shocks to wages relative to initial heterogeneity in skills and preferences in accounting for observed inequality?

- ① What is the link between wage inequality and consumption inequality?

*Novel feature: a life cycle model incorporates household consumption and family labor supply decision.*

*Focus on the importance of family labor supply as an insurance mechanism to wage shocks.*

## **Consumption and Labor Supply with Partial Insurance: an Analytical Framework**

*Jonathan Heathcote*

*Kjetil Storesletten*

*Giovanni L. Violante*

- ▶ Yaari perpetual youth model: agents born at age 0 and survive from age  $a$  to age  $a + 1$  with constant probability  $\delta < 1$ .
- ▶ Lifetime utility for agent born in cohort birth year  $b$  is given by

$$\mathbb{E}_b = \sum_{t=b}^{\infty} (\beta\delta)^{t-b} u(c_t, h_t; \varphi).$$

- ▶ The period utility is

$$u(c_t, h_t; \varphi) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \exp(\varphi) \frac{h_t^{1+\sigma}}{1+\sigma} \quad (1)$$

- ▶ The preference "weight"  $\varphi$  captures the strength of an individual's aversion to work: its distribution for the cohort with birth year  $t$  is denoted by  $F_{\varphi t}$  (with cohort specific variance  $v_{\varphi t}$ ).
- ▶ This source of heterogeneity helps explaining the observed cross-sectional joint distribution over wages, hours, and consumption.

- ▶ The population in the economy is partitioned into "islands", where each island contains a continuum of individuals.
- ▶ Agents face two types of orthogonal, labor productivity shocks
  - ① At the individual level: uncorrelated across island members and denoted by  $\varepsilon_t$ .
  - ② At island level: uncorrelated across islands and denoted by  $\alpha_t$ .
- ▶ Individual labor productivity,  $w_t$  has the following log process

$$\log w_t = \alpha_t + \varepsilon_t. \quad (2)$$



- Island-level component

$$\alpha_t = \alpha_{t-1} + \omega_t. \quad (3)$$

- Individual-level component

$$\varepsilon_t = \varkappa_t + \theta_t \quad (4)$$

$$\varkappa_t = \varkappa_{t-1} + \eta_t \quad (5)$$

where  $\theta_t$  is an i.i.d. transitory component ;  $\varkappa_t$  is the permanent component and follows a unit root process.

- Agents who enter the labor market at age  $a = 0$  in year  $t$  draw initial realizations from cohort specific distributions  $F_{\alpha^0 t}, F_{\varkappa^0 t}$ . The initial draws  $\alpha^0, \varkappa^0, \varphi$  are uncorrelated.

- ▶ One final good; CRS technology; consumption and labor are traded in a perfectly competitive economy; wages are equal to individual productivities.
- ▶ Progressive tax system. Following Benabou (2002), an individual with gross labor income  $y_t = w_t h_t$  receives disposable post-government earnings given by

$$\tilde{y}_t = \lambda (y_t)^{1-\tau}. \quad (6)$$

- ▶ All assets in the economy are in zero net supply; the asset market is competitive; agents are endowed with zero initial wealth.
- ▶ Trade structure:
  - ① Initial draws happen before markets open.
  - ② Island-location is defined; each island is characterized by an ex-ante unknown sequence  $\{\omega_t\}_{t=b+1}^{\infty}$  that applies to all island members.
  - ③ Within island, agents trade a complete set of insurance contracts: at every period  $t \geq b$  agents can purchase contracts indexed to  $s_{t+1} = (\omega_{t+1}, \eta_{t+1}, \theta_{t+1})$ .
  - ④ Across island, agents only trade insurance contracts indexed to their individual-level shocks  $(\eta_{t+1}, \theta_{t+1})$ . Inter-island contracts contingent on the realization of the island-level shock  $\omega_{t+1}$  are ruled out.

► Insurance:

- 1 Insurance contracts incorporate mortality risk (when state  $s_{t+1}$  is insured against and realized the contracts pay  $\frac{1}{\delta}$  if the agents survives and 0 if she dies) .

- Information: agents take as given the sequences of distributions  $\{F_{\varphi t}, F_{\alpha^0 t}, F_{\alpha^1 t}, F_{\omega t}, F_{\eta t}, F_{\theta t}\}$ , i.e. they have perfect foresight over future wage distributions.

# Model: Agent's Problem

- Let  $s^t = (s_b, s_{b+1}, \dots, s_t)$  denote the individual history of the shocks for an agent from birth year  $b$  up to date  $t$ , where

$$s_j = \begin{cases} (b, \varphi, \alpha^0, \varkappa^0, \theta_b) & \in \mathbb{S}_b = \mathbb{N} \times \mathbb{R}^4 & j = b \\ (\omega_j, \eta_j, \theta_j) & \in \mathbb{S} = \mathbb{R}^3 & j \neq b \end{cases} \quad (7)$$

with  $s^t = \mathbb{S}_b \times \mathbb{S}^{t-b}$ .

- Also, define  $z_{t+1} = (\eta_{t+1}, \theta_{t+1}) \in Z \subseteq \mathbb{Z} = \mathbb{R}^2$ .

- The budget constraint is

$$\lambda [w_t(s^t) h_t(s^t)]^{1-\tau} + d(s^t) - c_t(s^t) = \int_{\mathcal{S}} Q_t(\cdot) B_t(\cdot) ds_{t+1} + \int_{\mathcal{Z}} Q_t^*(\cdot) B_t^*(\cdot) dz_{t+1} \quad (8)$$

where

$$d(s^t) = \delta^{-1} \left[ B_t(s_t; s^{t-1}) + B_t^*(z_t; s^{t-1}) \right]. \quad (9)$$

- There exists a competitive equilibrium in which

$$B_t^*(Z, s^t) = 0 \forall Z, s^t \quad (10)$$

$$\log c_t = -(1 - \tau) \hat{\varphi} + (1 - \tau) \left( \frac{1 + \hat{\sigma}}{\hat{\sigma} + \gamma} \right) \alpha_t + C_t^a$$

$$\log h_t = -\hat{\varphi} + \left( \frac{1 - \gamma}{\hat{\sigma} + \gamma} \right) \alpha_t + \frac{1}{\hat{\sigma}} \varepsilon_t + H_t^a.$$

# Model: Income vs. Substitution Effects

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- ▶ A higher  $\varphi$  reduces labor: higher distaste for work. This transmits in a straightforward way to earnings and consumption.
- ▶ Hours worked are increasing in the insurable component: and its response to  $\varepsilon_t$  is given by  $\frac{1}{\sigma}$ . (perfect insurance rules out income effect)
- ▶ Hours worked ambiguously change after an uninsurable shock: if  $\gamma > 1$  the income effect dominates and hours worked fall after an increase in  $\alpha_t$ . The converse happens when  $\gamma < 1$ .



# Model: Competitive Equilibrium (contd)

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- ▶ This equilibrium implies zero private insurance against shocks  $\alpha_t$ : this shocks are common to all island members and there is no trade across islands. (*uninsurable*)
- ▶ There is perfect insurance against shocks to  $\varepsilon_t$ : this shocks are "washed-out" within islands (*insurable*).

- ▶ Usually, incomplete markets models do not admit an analytical solution (need to use numerical methods). This paper retains tractability because:
  - ① Individual wealth is a redundant variable.
  - ② Agents have access to perfect private insurance against some shocks.
- ▶ Why does this help?
  - ① Full-insurance within island implies that within-island allocations can be derived by solving a planner problem with a weighted function defined by the initial asset positions for all agents, subject to an island-level budget constraint (and then appeal to the Second Theorem of Welfare Economics and support this allocation with a particular vector of prices).
  - ② The inter-island wealth distribution does not show up in the allocations because it remains degenerate at zero.

- ▶ The island is a technical structure that allows agents to
  - ① Trade an unrestricted set of insurance claims: perfect insurance against  $\varepsilon_t$  is possible.
  - ② Have no inter-island trade: agents cannot pool the island-level risk.
- ▶ This is what defines the "partial insurability" that characterizes the model.
- ▶ Some permanent shocks are perfectly insured: "excess of smoothing" (contradicting the permanent income hypothesis as observed ). This is actually a feature of the data which Krueger (2006), Attanasio and Pavoni (2006), and others have modeled through incomplete "partial insurance models".

- ▶ This theoretical framework has as an important forebear Constantinides and Duffie (CD, 1996).
- ▶ CD model an environment in which no-trade equilibrium exists when:
  - ❶ Income process is multiplicative and  $I(1)$ .
  - ❷ Innovations are drawn from a common distribution.
  - ❸ Preferences are in the power utility class.
  - ❹ Assets are in zero net supply.
- ▶ Implications: no risk sharing/each individual consumes her own initial endowment.

- ▶ This model extends the CD environment to a richer framework:
  - ① Primitive exogenous stochastic process is over hourly wages and includes a transitory component beyond the unit root.
  - ② Allow for progressive taxation: distinguish between government provided insurance and private insurance.
  - ③ Agents are heterogeneous with respect to work distaste.
  - ④ Risks are privately insurable within islands, so the version of "no trade" applies across islands rather across individuals.
- ▶ One interpretation of this model is that the CD holds at island-level: there is no risk sharing between islands, although there is private insurance at the individual level. Again: this gives rise to partial insurance.

# Summary of Notation



- ▶ Before going on with the identification strategy, consider the following summary of notation:
  - ▶  $c_t, h_t$  endogenous variables: consumption and hours worked.
  - ▶  $y_t, w_t$  earnings and wage related by  $y_t = w_t h_t$ .
  - ▶  $\delta, \tau$  are the parameters set outside of the model: probability of dying and tax rate.
  - ▶  $\gamma, \sigma$  EIS: consumption and hours worked.
  - ▶  $\alpha_t$  island level component of the shock (only permanent with shock  $\omega_t$ ).
  - ▶  $\varepsilon_t$  individual level component of the shock (permanent  $\varkappa_t$  with shock  $\eta_t$  + transitory  $\theta_t$ ).
  - ▶  $\frac{1}{\sigma} \equiv \frac{1-\tau}{\sigma+\tau}, \hat{\varphi} \equiv \varphi / (\sigma + \gamma + \tau(1 - \gamma))$  tax adjusted parameters.
  - ▶  $v_{\mu h}, v_{\mu y}, v_{\mu c}$  variances of the measurement error (stationary).
  - ▶  $\hat{v}_{\varphi t}, v_{\alpha 0 t}, v_{\varkappa 0 t}, v_{\theta t}, v_{\omega t}, v_{\eta t}$ : variance of shock

# Identification and Estimation Method

- ▶ The baseline scenario of "data needs" is an unbalanced panel on wages and hours (e.g., PSID) and a repeated cross section on wages, hours, and consumption (e.g., CEX).
- ▶ The authors show that the parameters  $\{\sigma, \gamma, v_{\mu h}, v_{\mu y}, v_{\mu c}\}$  as well as the sequences  $\left\{ \hat{v}_{\varphi t}, v_{\alpha^0 t} \right\}_{t=1}^T, \{v_{\varkappa^0 t}, v_{\theta t}\}_{t=1}^T, \{v_{\omega t}\}_{t=2}^T, \{v_{\eta t}\}_{t=2}^T$  are identified. Also, the identify  $v_{\eta T} + v_{\theta T}$  and  $v_{\varkappa^0 T} + v_{\theta T}$ .
- ▶ Other variations of identification are considered due to data constraints (CEX and PSID differ in time availability, PSID becomes biannual, etc.).
- ▶ The authors show that with an external measure of measurement error in earnings,  $v_{\mu y}$ , all the parameters could be identified without using consumption data.

# Identification and Estimation Method (contd 1)

- ▶ We discuss the baseline Identification.
- ▶ Use within-cohort "macro" moments to identify

$$\hat{\sigma}, \gamma, \{v_{\omega t}\}_{t=2}^T, \{v_{\eta t} + \Delta v_{\theta t}\}_{t=2}^T.$$

- ▶  $v_{\omega t} = \Delta cov_t^a \left( \log \hat{w}, \log \hat{c} \right)^2 / \Delta var_t^a \left( \log \hat{c} \right)$

- ▶  $v_{\eta t} + \Delta v_{\theta t} = \Delta var_t^a \left( \log \hat{w} \right)$

- ▶  $\Delta var_t^a \left( \log \hat{h} \right) =$   

$$\left[ \Delta cov_t^a \left( \log \hat{h}, \log \hat{c} \right) / \Delta cov_t^a \left( \log \hat{w}, \log \hat{c} \right) \right]^2 v_{\omega t} +$$

$$\frac{1}{\hat{\sigma}^2} (v_{\eta t} + \Delta v_{\theta t}) \text{ to pick-up } \hat{\sigma}.$$

- ▶  $\Delta cov_t^a \left( \log \hat{h}, \log \hat{c} \right) / \Delta cov_t^a \left( \log \hat{w}, \log \hat{c} \right) = \frac{1-\gamma}{\hat{\sigma}+\gamma}$  to pick-up  $\gamma$ .



# Identification and Estimation Method (contd 2)

- ▶ Use the difference between the dispersion in growth rates ("micro moments") and the growth rates of dispersion ("macro" moments) to identify  $\{v_{\theta t}\}_{t=1}^T$ :
  - ▶ 
$$\begin{aligned} & cov_t^a \left( \Delta \log \hat{w}, \Delta \log \hat{c} \right) + var_t^a \left( \Delta \log \hat{h} \right) - \\ & \Delta cov_t^a \left( \log \hat{w}, \log \hat{c} \right) - \Delta var_t^a \left( \log \hat{h} \right) = 2 \left( 1 + \hat{\sigma} \right) \frac{1}{\sigma^2 v_{\theta t}} \end{aligned}$$
- ▶ Combine  $\{v_{\theta t}\}_{t=1}^T$  with  $\{v_{\eta t} + \Delta v_{\theta t}\}_{t=2}^T$  to identify  $\{v_{\eta t}\}_{t=2}^T$ , substitute  $v_{\theta T-1}$  into  $(v_{\eta T} + \Delta v_{\theta T})$  and from the first step identify  $(v_{\eta T} + v_{\theta T})$ .
- ▶ Use initial covariances to obtain  $\left\{ \hat{v}_{\varphi t}, v_{\alpha^0 t} \right\}_{t=1}^T$ :
  - ▶ 
$$cov_t^0 \left( \log \hat{w}, \log \hat{c} \right) = (1 - \tau) \left( 1 + \hat{\sigma} \right) / \left( \hat{\sigma} + \gamma \right) v_{\alpha^0 t}.$$

► ...

$$\begin{aligned} \text{► } cov_t^0 \left( \log \hat{h}, \log \hat{c} \right) = \\ (1 - \tau) v_{\varphi t}^{\hat{}} + (1 - \tau) \left( 1 + \hat{\sigma} \right) (1 - \gamma) / \left( \hat{\sigma} + \gamma \right)^2 v_{\alpha 0 t}. \end{aligned}$$

► Use initial variances to obtain  $\{v_{\alpha 0 t}\}_{t=1}^{T-1}$  and  $(v_{\alpha 0 T} + v_{\theta T})$ :

$$\begin{aligned} \text{► } cov_t^0 \left( \log \hat{h}, \log \hat{w} \right) + var_t^0 \left( \log \hat{h} \right) = v_{\varphi t}^{\hat{}} + \\ (1 - \gamma) \left( 1 + \hat{\sigma} \right) / \left( \hat{\sigma} + \gamma \right)^2 v_{\alpha 0 t} + \left( 1 + \hat{\sigma} \right) / \hat{\sigma}^2 v_{\alpha 0 t} + v_{\theta t}. \end{aligned}$$

► And finally use

$cov_t^0 \left( \log \hat{h}, \log \hat{w} \right), var_t^0 \left( \log \hat{w} \right), var_t^0 \left( \log \hat{c} \right)$  to identify the measurement error parameters.

# Identification and Estimation Method (contd 4)



- ▶ Use male population between ages 25 and 59 who work at least 260 hours in the year (avoid selection problems).
- ▶ Regress all variables against year dummies, quartic in age, and (for consumption) household composition dummies.
- ▶ The authors minimize the weighted sum of differences between each moment in the data and its empirical counterpart.
- ▶ They pick the moments according to an identity weighting matrix and use block-bootstrap at the household level to estimate standard errors.
- ▶ Parameters set outside the model:  $\delta = .996$  to match the annualized US data; regress  $\tilde{y}_t = \lambda (y_t)^{1-\tau}$  in logs to get a consistent estimate of  $(1 - \tau)$ .

# Results

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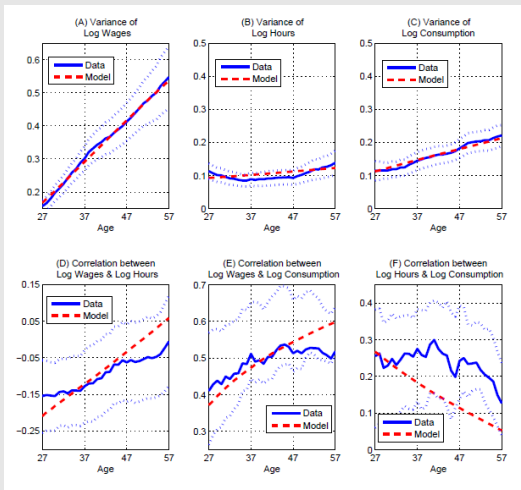
Table 1: Baseline Parameter Estimates

Preference Elasticities			Life-Cycle Shocks		
$\sigma$	$\gamma$		$\overline{v_{\omega}}$	$\overline{v_{\eta}}$	$\overline{v_{\theta}}$
1.686	1.555		0.0065	0.0040	0.042
[1.581, 1.871]	[1.502, 1.641]		[0.002, 0.013]	[0.001, 0.008]	[0.034, 0.050]
Initial Heterogeneity			Measurement Error		
$\overline{v_{\alpha^0}}$	$\overline{v_{\kappa^0}}$	$\overline{v_{\varphi}}$	$v_{\mu y}$	$v_{\mu h}$	$v_{\mu c}$
0.109	0.040	0.517	0.000	0.035	0.053
[0.093, 0.123]	[0.019, 0.063]	[0.448, 0.625]	[0.000, 0.000]	[0.034, 0.036]	[0.051, 0.055]

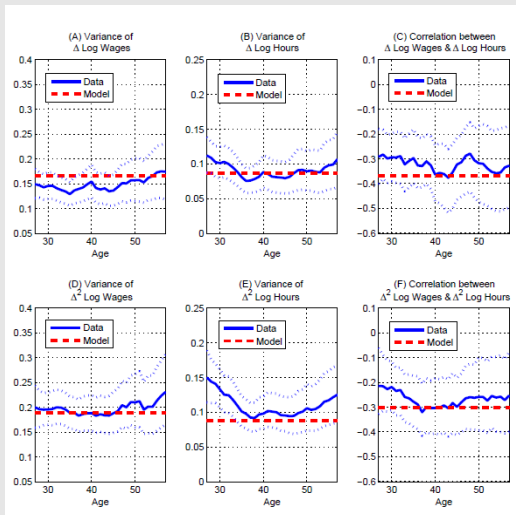
Notes: Bars denote sample averages. 90-10 bootstrapped confidence intervals are shown in parentheses.

- ▶ Implied Frisch elasticity of  $\frac{1}{\hat{\sigma}} = .38$ , in the ballpark of the literature (Keane, 2011)
- ▶ 38% of permanent life-cycle wage innovations are insurable; 30% of wage variation at labor market entry is insurable.
- ▶ Transitory shocks are more variable.
- ▶ Figures 1 and 2 show that the model-implied moments almost always lie within the 90 – 10 confidence intervals around the empirical moments.

# Results (contd 2)



# Results (contd 4)





- ▶ The US and the model-simulated data have the following features:
  - ① The variance of log wages increases by around 35 log points, approximately linearly, between ages 35 – 57.
  - ② The variance of log consumption grows much less, 10 log points over the life cycle: a significant share (38%) of permanent shocks are insurable.
- ▶ Figure 3 depicts the evolution of insurable and uninsurable wage dispersions over time.

# Results (contd 6)

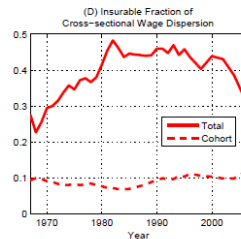
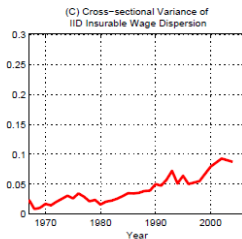
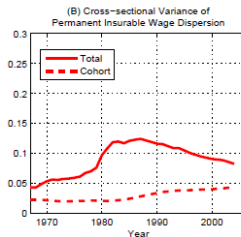
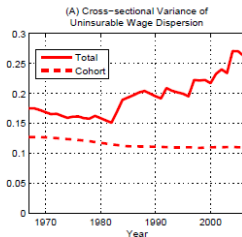


Table 3: Decomposition of Cross-Sectional Inequality

	Total Variance	Percent Contribution to Total Variance					
		Initial Heterogeneity			Life-Cycle Shocks		Measurement
		Prefs.	Unins.	Ins.	Unins.	Ins.	Error
$var(\log \hat{w})$	0.351	0.0	32.0	8.3	21.7	28.2	9.9
$var(\log \hat{h})$	0.107	46.6	1.9	4.0	1.3	13.5	32.8
$var(\log \hat{y})$	0.432	11.1	22.8	10.1	13.1	42.9	0.0
$var(\log \hat{c})$	0.159	16.9	29.5	0.0	20.0	0.0	33.5

Table 4: Decomposition of Changes in Inequality over Time

	Total Growth 2002-6 vs. 1967-71	Percent Contribution of		
		$v_{\varepsilon}$	$v_{\alpha}$	$v_{\hat{\varphi}}$
$\Delta var(\log \hat{w})$	0.150	65.4	34.6	0.0
$\Delta var(\log \hat{h})$	0.020	70.7	4.6	24.8
$\Delta var(\log \hat{y})$	0.230	80.9	17.0	2.1
$\Delta var(\log \hat{c})$	0.024	0.0	88.9	11.1

# Given these results... what's next?

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- ▶ Incorporate to the discussion simultaneously:
  - ❶ Labor decisions at the household level: give rise to insurance via family labor supply.
  - ❷ External sources of insurance.
  - ❸ Self insurance via asset accumulation.

## Consumption Inequality and Family Labor Supply

*Richard Blundell*

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# Model

- ▶ Set up a model that allows for three potential sources of smoothing:
  - ❶ Self-insurance through credit markets.
  - ❷ Family labor supply (hours of work can be adjusted along with, or alternatively to, spending on goods in response to shocks to economic resources).
  - ❸ External sources of insurance (from help received by networks of relatives and friends to social insurance such as UB).
- ▶ Framework:
  - ❶ Life-cycle setup in which two individuals (husband & wife) make unitary decisions about household consumption and their individual labor supply.
  - ❷ There is uncertainty about offered market wages.
  - ❸ Allow for partial insurance through assets accumulation; heterogeneous Frisch elasticities; non-separability of consumption and leisure; differences between extensive and intensive margins of labor supply.

# Wage Process

- ▶ Permanent-transitory type wage process for each earner (Permanent component evolves as a unit root process).
- ▶ The log real wage of individual  $j = 1, 2$  in household  $i = 1, \dots, N$  at time  $t = 1, \dots, T$  is

$$\begin{aligned}\log W_{i,j,t} &= x'_{i,j,t} \beta_W^j + F_{i,j,t} + u_{i,j,t} \\ F_{i,j,t} &= F_{i,j,t-1} + v_{i,j,t}\end{aligned}\tag{11}$$

where  $x'_{i,j,t}$  are characteristics that influence wages and are known to the household;  $u_{i,j,t}$  and  $v_{i,j,t}$  are the transitory and permanent shocks, respectively.

- ▶ This process adequately fits the data, although it is far from controversial (superior information and heterogeneous growth issues are studied in related literature).



- The "across-time" and "across-shocks" moments are given by

$$\mathbb{E} (u_{i,j,t}, u_{i,j,t-s}) = \begin{cases} \sigma_{u_j}^2 & j = k, s = 0 \\ \sigma_{u_j u_k} & j \neq k, s = 0 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$\mathbb{E} (v_{i,j,t}, v_{i,j,t-s}) = \begin{cases} \sigma_{v_j}^2 & j = k, s = 0 \\ \sigma_{v_j v_k} & j \neq k, s = 0 \\ 0 & \text{otherwise} \end{cases}$$

- The process is time-invariant, serially uncorrelated, correlated across spouses (direction is theoretically ambiguous: assortative matching vs. negatively correlated job selection).

$$\max_{\{C_{i,t+s}, H_{i,1,t+s}, H_{i,2,t+s}\}_{s=0}^{T-t}} \mathbb{E}_t \sum_{s=0}^{T-t} u_{t+s} \left( \begin{array}{c} C_{i,t+s}, H_{i,1,t+s}, H_{i,2,t+s}; \\ z_{i,t+s}, z_{i,1,t+s}, z_{i,2,t+s} \end{array} \right) \quad (13)$$

s.t.

$$A_{i,t+1} = (1 + r) (A_{i,t} + H_{i,1,t} W_{i,1,t} + H_{i,2,t} W_{i,2,t} - C_{i,t}). \quad (14)$$

- ▶  $z_{i,t+s}, z_{i,1,t+s}, z_{i,2,t+s}$  are per-period utility shifters which are household and spouse specific, respectively.
- ▶ This general setup enables to consider the additive and non-additive separabilities.

- ▶ Not found in general under this specification (only under strong assumptions about utility). Solution: use approximation methods.
- ▶ Use Taylor approximation to the log-linearized forms of the f.o.c.s and the intertemporal budget constraint.
- ▶ Found this for the cases of additive separability, non-additive separability, measurement error, and non-linear taxation.

# Add. Separability: Euler Eqs. and the BC

The system that described the solution in log difference of the relevant variables is

$$\begin{bmatrix} \Delta c_{i,t} \\ \Delta y_{i,1,t} \\ \Delta y_{i,2,t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\ 0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2} \end{bmatrix} \begin{bmatrix} \Delta u_{i,1,t} \\ \Delta u_{i,2,t} \\ v_{i,1,t} \\ v_{i,2,t} \end{bmatrix} \quad (15)$$

## Add. Separability: Euler Eqs. and the BC (contd 1)



where

$$\begin{aligned}\kappa_{c,v_j} &= -\frac{\eta_{c,p} (1 - \pi_{i,t}) s_{i,j,y} (1 - \eta_{h_j,w_j})}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}^-} \\ \kappa_{y_j,u_j} &= 1 + \eta_{h_j,w_j} \\ \kappa_{y_j,v_j} &= 1 + \eta_{h_j,w_j} \left( 1 - \frac{(1 - \pi_{i,t}) s_{i,j,y} (1 - \eta_{h_j,w_j})}{\eta_{c,p} + \eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}^-} \right) \\ \kappa_{y_j,v_{-j}} &= -\frac{\eta_{h_j,w_j} (1 - \pi_{i,t}) s_{i,-j,y} (1 - \eta_{h_{-j},w_{-j}})}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}^-}\end{aligned}\tag{16}$$

## Add. Separability: Euler Eqs. and the BC (contd 2)

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and

$$\begin{aligned}\eta_{c,p} &= -\frac{u_c}{u_{cc}} \frac{1}{c} \\ &> 0 \\ \eta_{h_j, w_j} &= \frac{g_{H_j}^j}{g_{H_j H_j}^j} \frac{1}{H_j} \\ &> 0\end{aligned}$$

are the EIS for consumption and labor supply of earner  $j$ , respectively.

## Add. Separability: Euler Eqs. and the BC (contd 3)



and

$$\begin{aligned}\pi_{i,t} &= \frac{Assets_{i,t}}{Assets_{i,t} + HumanWealth_{i,t}} \\ s_{i,j,y} &= \frac{HumanWealth_{i,j,t}}{HumanWealth_{i,t}} \\ \eta_{h,w}^- &= \sum_{j=1}^2 s_{i,j,t} \eta_{h_j,w_j}\end{aligned}$$

are the partial insurance coefficient, the share of earner  $j$ 's human wealth over family human wealth, and the household's weighted average of the EIS of labor supply of the two earners.

# Insurance above Self-Insurance

- ▶ Households have access to multiple external sources of insurance.
- ▶ This is hard to credibly model: myriad of external insurance channel.
- ▶ Subsume this mechanism into one parameter,  $\beta$ , that captures the proportion of this insurance.
- ▶ The share  $\pi_{i,t}$  is multiplied by  $(1 - \beta)$  when it appears.
- ▶ The parameter measures all consumption insurance that remains after accounting for the "self insurance" represented by asset accumulation and labor supply.
- ▶  $\beta > 0$ : external insurance;  $\beta = 0$  no external insurance;  $\beta < 0$  (consumption over-responds to shocks given illiquid forms of asset accumulation/transaction costs exceeding benefits from smoothing).



- ▶ Before discussing identification consider the following summary of parameters
  - ▶  $(\sigma_{v_1}^2, \sigma_{v_2}^2, \sigma_{u_1}^2, \sigma_{u_2}^2, \sigma_{v_1 v_2}, \sigma_{u_1 u_2})$ , variances of the permanent and transitory components of the shocks and respective within household covariances (stationary).
  - ▶  $(\pi, s, \beta)$ , insurance parameters.
  - ▶  $(\eta_{h_1 w_1}, \eta_{h_2 w_2}, \eta_{c p}, \eta_{h_1 w_2}, \eta_{h_2 w_1}, \eta_{c w_1}, \eta_{c w_2}, \eta_{h_1 p}, \eta_{h_2 p})$ , direct and crossed Frisch elasticities of consumptions and hours worked by each individual in the household.

- Identification of the wage parameters is obtained through a strategy followed by Meghir and Pistaferri (2004) that is based on the equation that represents the wage growth of earner  $j$  at time  $t$ ,  $\Delta w_{j,t} = \Delta u_{j,t} + v_{j,t}$ :

$$\sigma_{u_j}^2 = -\mathbb{E}(\Delta w_{j,t}, \Delta w_{j,t+1}) \quad (17)$$

$$\sigma_{v_j}^2 = \mathbb{E}(\Delta w_{j,t} (\Delta w_{j,t+1} + \Delta w_{j,t} - \Delta w_{j,t-1}))$$

$$\sigma_{u_1 u_2} = -\mathbb{E}(\Delta w_{1,t}, \Delta w_{2,t+1})$$

$$\sigma_{v_1 v_2} = \mathbb{E}(\Delta w_{1,t} (\Delta w_{2,t+1} + \Delta w_{2,t} - \Delta w_{2,t-1}))$$

- ▶ Identification of the rest of the parameters comes from the cross moments of the wage growth, the earnings growth, and the use of the symmetry of the Frisch substitution matrix.
- ▶ Identification of  $\pi, s$  comes strictly from the data.
- ▶ The authors use the PSID data for the periods 1999-2009.
- ▶ The authors generalize the model above to allow for measurement error in consumption, wages, and earnings and identification is still possible.
- ▶ Roughly speaking, the estimation is based on the use of multiple moments by the method of GMM (with an identity matrix as the weighting scheme)
- ▶ Inference is based on block bootstrap (i.e., re-samples are taken at the individual level).



- ▶ The model induces interior solutions for labor supply for both spouses.
- ▶ A major concern when modeling labor supply is endogenous selection into work: need to distinguish between extensive and intensive margins.
- ▶ 93% of male is employed in the data: justification for using only employed male.
- ▶ 80% of the wives of this 93% of males is employed, which makes accounting for selection potentially important.
- ▶ Follow Low et al. (2010). Use state year dummies intended to capture labor market related policy changed at the state level and the presence of first and second mortgages (some evidence shows that female participation rises when households move into home ownership).

- ❶ Regress log differences of the relevant variables (consumption, wages and earnings) onto observable characteristics and construct first difference residuals for each of them.
- ❷ Estimate the wage variances and covariances using the second moments of the log first differences.
- ❸ Estimate the smoothing parameters using asset and (current and projected) earnings data.
- ❹ Estimate the preference parameters using the second moments for earnings and consumption conditioning in the results obtained in the previous steps.

# Results: Consumption and Labor Supply

Table 5: Parameters Estimates

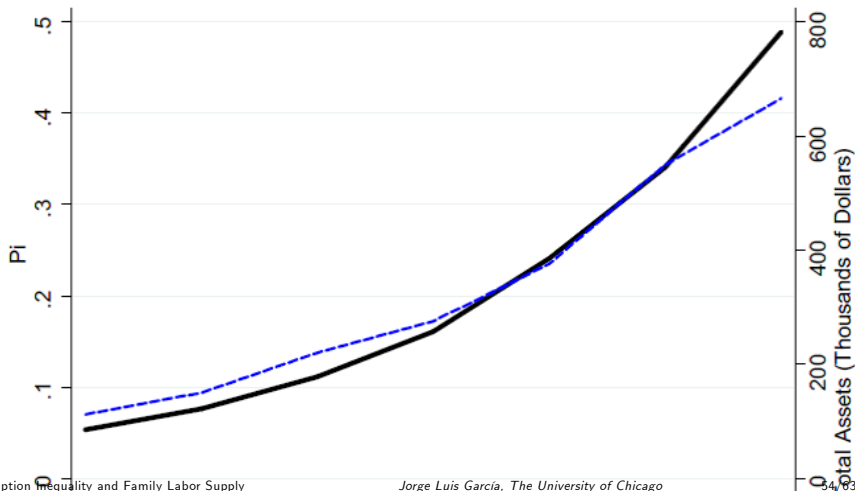
	(1)	(2)	(3)	(4)		(1)	(2)	(3)	(4)
	Additive separability	Non- separability	Non- Separability $\beta = 0$	Nonlinear taxes		Additive separability	Non- separability	Non- Separability $\beta = 0$	Nonlinear taxes
<i><math>\pi</math>, <math>\beta</math> and own elasticities:</i>					<i>Cross-elasticities:</i>				
$E(\pi)$	0.181 (0.008)	0.181 (0.008)	0.181 (0.008)	0.193 (0.008)	$\eta_{c,w_1}$	--	-0.141 (0.051)	-0.141 (0.052)	-0.186 (0.067)
$\beta$	0.741 (0.163)	-0.120 (0.298)	0	-0.140 (0.381)	$\eta_{h_1,p}$	--	0.082 (0.030)	0.082 (0.031)	0.108 (0.039)
$\eta_{c,p}$	0.201 (0.077)	0.437 (0.124)	0.448 (0.126)	0.495 (0.151)	$\eta_{c,w_2}$	--	-0.138 (0.139)	-0.158 (0.121)	-0.175 (0.146)
$\eta_{h_1,w_1}$	0.431 (0.097)	0.514 (0.150)	0.497 (0.150)	0.621 (0.176)	$\eta_{h_2,p}$	--	0.162 (0.166)	0.185 (0.145)	0.206 (0.176)
$\eta_{h_2,w_2}$	0.831 (0.133)	1.032 (0.265)	1.041 (0.275)	1.133 (0.289)	$\eta_{h_1,w_2}$	--	0.128 (0.052)	0.12 (0.064)	0.197 (0.061)
					$\eta_{h_2,w_1}$	--	0.258 (0.103)	0.242 (0.119)	0.397 (0.127)
Observ.	8,191	8,191	8,191	8,191					

Notes. Parameters estimated using GMM. Baseline sample is applied (see section 3.1 for details). Column 1 reports the estimates for the additive separable case. Column 2 allows for non-separability of hours of the two earners and for non-separability of hours and consumption. Column 3 is similar to column 2, restricting for no insurance over and above savings ( $\beta = 0$ ). Column 4 is similar to column 2 but allowing for non-linear taxes (see section 5.4). Standard errors are bootstrapped.

# Results: Smoothing Parameters



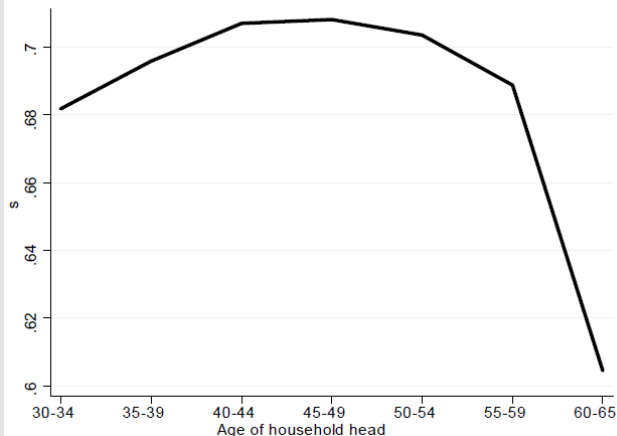
Figure 1:  $\pi$  by Age of Head of Household



# Results: Smoothing Parameters (contd 1)



Figure 2:  $s$  by Age of Head of Household





## Results: Smoothing Parameters (contd 2)

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► Figure 1...

- ❶ Self-insurance due to asset accumulation is negligible at the beginning of the life-cycle.
- ❷ Asset accumulation and the decline of human capital due to shortening of time horizon imply and increase in  $\pi_{i,t}$  as time goes by, and hence households are able to smooth permanent wage shocks better overtime.

► Figure 2...

- ❶ Life-cycle evolution if the distribution of earnings power within the household.
- ❷ On average, the husband commands about 2/3 of the total of the total household human wealth.
- ❸ The peak is, presumably, due to fertility decisions.

# Results: Wage Variances



Table 4: Variance Estimates

Sample			All
Males	Trans.	$\sigma_{u_1}^2$	0.033 (0.008)
	Perm.	$\sigma_{v_1}^2$	0.032 (0.005)
Females	Trans.	$\sigma_{u_2}^2$	0.012 (0.006)
	Perm.	$\sigma_{v_2}^2$	0.043 (0.005)
Correlation of shocks	Trans.	$\sigma_{u_1, u_2}$	0.244 (0.164)
	Perm.	$\sigma_{v_1, v_2}$	0.113 (0.082)
Observations			8,191

Notes: Wage process parameters estimated using GMM.  
Baseline sample is applied (see section 3.1 for details).  
Standard errors are bootstrapped.

# Results: Wage Variances (contd 1)

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- ① "Wage instability" defined as variance of the transitory component (Gottschalk & Moffitt, 2008) are larger for males (larger influence of turnover, etc.).
- ② The variance of the more structural component (permanent) is similar for men and women (higher for women maybe due to greater dispersion in returns to unobserved skills).
- ③ Both the transitory and permanent components are positively correlated. This could evidence assortative matching.

► Separability case:

- ① EIS is estimated .2, implying a relative risk aversion coefficient of around 5 (fairly high but in the ballpark of previous estimates in this literature).
- ② Frisch elasticity of men labor supply is estimated to be .4, which ranges in values given by MaCurdy (1981), Altonji's (1986), and Keane (2011).
- ③ Frisch elasticity of women labor supply is estimated to be .8, in comparison of the estimate of 1 obtained by MaCurdy and Heckman (1980) and consistent with high values found in this literature surveyed by Keane (2011).
- ④ Implausible value for  $\beta = .74$ , which implies a very large amount of "external" insurance over and above self-insurance. This value implies an excessive degree of consumption smoothing.

## ► Non-Separability

- ❶ Additive separability is strongly rejected: four individual parameters particular to this model are significantly different than zero.
- ❷ Complementarity of husband and wife leisure.
- ❸ Both leisures (husband and wife) are complements with consumption.
- ❹  $\beta$  is not significant in this case, which gives the author another reason to have this model as their preferred.

- ▶ The mute response to consumption to wage shocks may be due to wage changes not being shocks.
- ▶ The authors follow Cuhna et al. (2005) and test if future wage predicts current consumption growth to test for advance "information".
  - ▶ They compute  $\mathbb{E}(\Delta c_{i,t}, \Delta w_{i,j,t+\tau})$  for and test their joint significance. They cannot reject the null of zero correlation.
- ▶ Evidence for complementarity between leisure and consumption has been rare. Authors like Aguiar and Hurst (2005) find substitution among these goods.

- ▶ However, the "usual" evidence comes from studying the relationship between changes in consumption and large changes in hours, often associated with unemployment, retirement, etc. (extensive margin).
- ▶ This paper focuses in the relationship between changes in consumption and small changes in hours (vastly employed sub-sample).
- ▶ The authors also extend the model to include non-linear progressive taxation. This does not affect qualitatively the results and has "small" impacts (Frisch elasticities are typically larger due to the feedback effect of taxes).

# Discussion: (contd 2)



Figure 4: Decomposition of consumption smoothing by age

