College Enrollment and Dropout

Stepping Stone and Option Value

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Outline

1. Ozdagli & Trachter, 2011
2. Trachter, 2014

Paper

- On the Distribution of College Dropouts: Wealth and Uninsurable Idiosyncratic Risk
- 2011
- Ali K. Ozdagli (FRB-Boston) and Nicholas Trachter (FRB-Richmond)
- R&R JOLE

Overview

- Dynamic model of the decision to pursue college
  1. Students' uncertainty: about future income stream due to unobserved scholastic ability
  2. Expectations reevaluation: on success in college after matriculation and after taking exams
- Findings (only theoretical)
  1. Poorer students are
     - less likely to graduate
     - likely to dropout sooner
- Interesting feature: no need to introduce credit constraints

Motivation

- The authors motivate their work claiming that inequality perpetuates and exacerbates as follows:
  1. Large fraction of every cohort that enrolls in for year U.S. colleges drops out
  2. High concentration of dropouts among students from lower-income families
  3. Students from low-income families drop out earlier than students from high-income families
  4. Less low-income individuals graduate from college
  5. High return to education

Motivation, contd 1

Table 1 Dropout rates and mean time before dropping out by socioeconomic status of family

	Socio. status[a]	% that drop	Mean tenure in college[b]	st. dev. of tenure
NLSY97	Low	28.98	2.31	1.65
	High	13.40	3.14	1.73
NLSY79	Low	62.5	2.78	1.6
	High	26.96	3.94	1.9
NLS-72	Low	65.6	2.02	1.29
	High	52.86	2.69	1.63

[a] For the NLSY79 and NLSY97 we constructed the measure of socioeconomic status thorugh the income level of the family prior to the respondent's enrollment in college. We choose the deciles so as to match the distribution of socioeconomic status of the NLS-72. [b] We only have the length of the tenure in college for a sub-sample of the population.

Motivation, contd 2

Table 2 Dropout rates of low- vs. high-income students

	tenure between						
	0 and 1 years	1 and 2 years	2 and 3 years	3 and 4 years	4 and 5 years	5 and 6 years	6 and 7 years
NLSY97	1.76	1.65	1.57	0.91	0.37	0.69	0.37
NLSY79	3.28	1.31	1.61	0.8	0.35	0.23	0.4
NLS-72	1.49	1.17	0.94	0.38	0.52	0.19	0.47

Each number in the table represents the dropout rate of low-income students as a share of the total dropout rate of low-income students divided by the yearly dropout rate of high-income students as a share of the total dropout rate of high-income students.

Motivation, contd 3

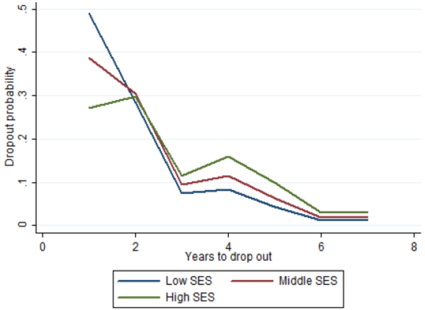
Table 3 Marginal and percentage effect of socioeconomic status on dropout probability

		dF/dx	std. error	% effect	p	N
NLSY97	Low SES	0.0265	0.0266	12.78	0.02	1948
	High SES	-0.0901	0.0187	-43.48		
NLSY79	Low SES	0.1576	0.063	41.38	0.03	635
	High SES	-0.0588	0.0468	-15.44		
NLS-72	Low SES	0.0256	0.0298	4.27	0.00	2705
	High SES	-0.138	0.0211	-23.03		

The results are obtained from running logit regressions on the dropout probability based on the socioeconomic status of the student's family and a set of controls. The complete regression results and description of control variables can be found in Table 5, Table 6, and Table 7 in Appendix E. N is the number of observations and p is the p-value of the χ^2 test that compares the effect of high and low socio-economic status on dropout probability.

Motivation, contd 4

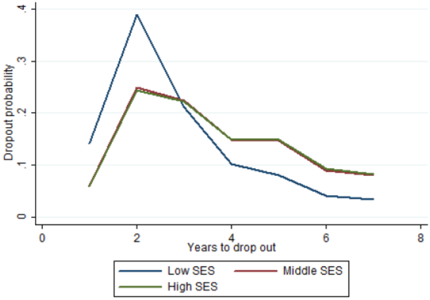
Figure 3 NLS-72: Predicted time to drop out



The figure plots the predicted probabilities of dropping out in a given time interval for students with different socioeconomic statuses, conditioning on the average characteristics of students who dropped out. To compute the probabilities we run an ordered logit regression that involves the probability of dropping out in a given year as a function of the socioeconomic status variable plus a set of controls. The complete regression results can be found in Table 9 in Appendix E.

Motivation, contd 5

Figure 2 NLSY79: Predicted time to drop out



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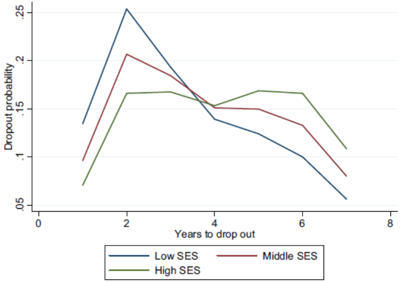
Motivation, contd 6

Figure 1 NLSY97: Predicted time to drop out



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Model, Basics

- Based on Miao and Wang (2007): framework of entrepreneurial learning and analysis
- Add relevant ingredients of dropout decision:
  1. Wage profile: depends on experience and college graduation (and corresponding interaction
  2. Include information unfolding through learning about unobserved ability in college

Model, Information

- The author argues that credit constraints and learning about risk are not fundamental
  - Credit constrains: (i) 29% of students from the richest families dropout (NLSY79); (ii) rising house prices lead to higher graduation rates, especially among low-income families (NLSY97)
  - Learning about stochastic taste: no relation with wealth
- Decides to model information unfolding through learning about ability

Model, Primitives

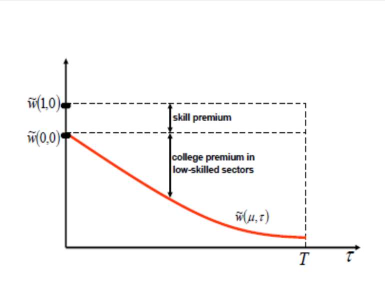
- Continuous time, finite horizon; $t \in [0, T]$
- At $t = 0$
  1. Initial endowment, $x(0) \equiv x_0$
  2. Unobserved ability to acquire human capital, $\mu \in \{0, 1\}$
  3. Prior on ability, $\Pr(\mu = 1 | t = 0) = p(0) \equiv p_0$
  4. Either enrolled in college (full-time) or working (in low-skilled or high-skilled sector)
  5. High-skill sector only hires high-skilled workers with college degrees
  6. Work: absorbing state
- Wage function:

$$\tilde{w}(\mu, \tau) \begin{cases} w(\tau) & , \tau > 0 \\ w_0 & , \tau = 0, \mu = 0 \\ w_1 & , \tau = 0, \mu = 1 \end{cases} \tag{1}$$

with $\tau = T - t, w_0 \equiv w(0) < w_1$; $\tau_0 > \tau_1 \Leftrightarrow w_1 > w_0$.

Model, Primitives contd 1

Figure 4. Wage Function



Model, Primitives contd 2

- c, consumption
- a, per-unit of time cost of college
- ρ, discount factor; r, interest rate
- γ, CRRA parameter
- λ_1, probability of getting an excellent grade for high ability student
- λ_0, probability of getting a failing grade for low ability student
  - Interpret λ_1, λ_0 as speed of learning
  - In a continuous (Brownian motion) setting this is analogue to having two volatility parameters

Model, Primitives contd 2

Table 4 Probability of receiving different grades based on student's type

	Fail	Pass	Excellent
$\mu = 0$	$\lambda_0 dt$	$1 - \lambda_0 dt$	0
$\mu = 1$	0	$1 - \lambda_1 dt$	$\lambda_1 dt$

Model, Primitives contd 3

- Evolution of wealth

$$\dot{x} = \begin{cases} rx + \tilde{w}(\mu, \tau) - c & \text{, if working} \\ rx - a - c & \text{, if enrolled in college} \end{cases} \tag{2}$$

Model, Student's Problem (Sequential)

$$\max_{c(t)} \mathbb{E}\left\{ \int_0^\infty \exp^{-\rho t} \frac{c(t)^{1-\gamma}}{1-\gamma} | p_0, x_0 \right\} \tag{3}$$

s.t. (2) holds

Model, Student's Problem (Recursive)

- $J(x, p, \tau)$, student's with current wealth x, prior on ability p, τ time before graduation value function
- $V(x, \mu, \tau)$, with current wealth x, type μ, τ time before graduation value function

Model, Worker's Value Function

- Instantaneous utility derived: consumption + change in wealth

$$\rho V(x,\mu,\tau) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + V_x(x,\mu,\tau)\dot{x} \tag{4}$$

- First order condition:

$$c^{-\gamma} = V_x(\cdot) \tag{5}$$

- Define $W(\mu,\tau)$ as the present value of earnings, let A be a constant in terms of γ, ρ, and rearrange to get

$$V(x,\mu,\tau) = A\left(r\left[x + W(\mu,\tau)\right]\right)^{1-\gamma} \tag{6}$$

- Note that this implies that the fact that wages are constant is relatively easy to assume away by changing $W(\cdot)$ in 6

Model, Student's Value Function with Known Types

- Focus on $\mu = 1$
- For $\mu = 0$ we wait for the results (want to guarantee that $J(x, 0, \tau) = V(x, 0, \tau)$)
- Same principle leads to

$$\rho J(x, 1, \tau) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + J_x(x, 1, \tau)\dot{x} + J_\tau(x, 1, \tau)\dot{\tau} \tag{7}$$

with $J(x, 1, 0) = J(x, 1, 0)$

- First order condition

$$c^{1-\gamma} = J(x, 1, \tau) \tag{8}$$

- Solve to get

$$J(x, 1, \tau) = A\left[r\left(x + \exp^{r\tau} W(1, 0) - a\frac{1 - \exp^{r\tau}}{r}\right)\right]^{1-\gamma} \tag{9}$$

Student's Value Function with Unknown Types

- Difficulties
  1. Wage upon graduation depends on true ability
  2. New information arrives after each exam
  3. Some students drop out
- Prior's evolution

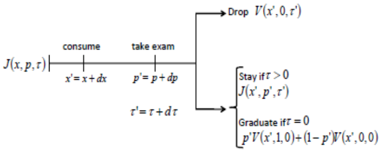
$$p(t+dt) = \begin{cases} 0 & \text{, fails} \\ 1 & \text{, excellent grade} \\ \frac{p(t)[1-\lambda_1 dt]}{p(t)[1-\lambda_1 dt]+(1-p(t))[1-\lambda_0 dt]} & \text{, otherwise} \end{cases} \quad (10)$$

- If the student does not fail or has an excellent grade then

$$\dot{p} = -(\lambda_1 - \lambda_0)\, p(1-p) \quad (11)$$

Model, Time-line

Figure 5 Timeline



A student starts the current period with wealth level x, prior p, and remaining time in college τ. At the beginning of the period a student chooses her consumption level and thus provides the new value for wealth x'. Before the end of the period the student takes an exam used to produce the posterior p' and reduces the time left to graduation to τ'. At the beginning of next period the student chooses between dropping out or remaining as a student (or graduation is $\tau'=0$).

- Same principle leads to

$$\begin{aligned}\rho J(x,p,\tau) &= \max_c \frac{c^{1-\gamma}}{1-\gamma} + J_x(x,p,\tau)\dot{x} + J_p(x,p,\tau)\dot{p} \\ &+ J_\tau(x,p,\tau)\dot{\tau} + \lambda_1 p\left[J(x,1,\tau) - J(x,p,\tau)\right] \\ &+ \lambda_0(1-p)\left[V(x,0,\tau) - J(x,p,\tau)\right] \qquad (12)\end{aligned}$$

- Define $p^*(x,\tau)$ as the threshold such that if $p \leq p^*(x,\tau)$ the student drops out college

Student's Value Function with Unknown Types, contd 1

- Terminal conditions
  1. Terminal Condition
     $$J(x,p,0) = pV(x,1,0) + (1-p)V(x,0,0) \tag{13}$$
  2. Value Matching Condition
     $$J(x,p^*(\cdot),\tau) = V(x,\tau,0) \tag{14}$$
  3. Smooth Pasting Conditions
     $$\begin{aligned} J_p(x,p^*(\cdot),\tau) &= 0 & (15)\\ J_x(x,p^*(\cdot),\tau) &= V_x(x,\tau,0) & (16)\\ J_\tau(x,p^*(\cdot),\tau) &= V_\tau(x,\tau,0) & (17) \end{aligned}$$

- Use FOC to obtain

$$p^*(x,\tau) = \frac{a + rW(0,\tau) + W_\tau(0,\tau)}{\lambda_1} \frac{V_x(x,0,\tau)}{J(x,1,\tau) - V(x,0,\tau)} \tag{18}$$

Results, Known Types

- Lemma 1
  - Assume $r \exp^{-r\tau} W(1,0) - a(1 - \exp^{-r\tau}) \geq rW(1,\tau)$
  - A student of type $\mu = 1$ with current state (x,τ) chooses to remain in college until $\tau = 0$
  - Intuition: graduation premium needs to be high enough for high-ability types to remain in college
- Lemma 2
  - Assume $a + rW(0,\tau) + W_\tau(0,\tau) > 0$
  - A student of type $\mu = 0$ immediately drops college
  - Intuition: if the per-unit marginal cost of attending college is positive, the low-skilled type student drops college immediately
  - Implication: $J(x,0,\tau) = V(x,0,\tau)$

Results, Unknown Types

- Result 1
  - Let the assumptions in Lemmas 1 and 2 hold
  - Students with a greater endowment have a lower value of p^*, i.e. the belief threshold for which they drop college is lower

$$\frac{\partial p^*(x,\tau)}{\partial x} < 0 \qquad (19)$$

- Let τ^* denote the time to graduation at the moment individual joins the workforce
  - $\tau^* = T$, joins workforce immediately
  - $\tau^* = 0$, joins workforce with college degree

Results, Unknown Types contd 1

- Proposition 1
  - Consider two endowments x_0^i, x_0^j with $x_0^i > x_0^j$

$$\forall\ \bar{\tau} \in \mathbb{R}_{++}, \Pr\{\tau^* \leq \bar{\tau} | x_0^i, p_0, \mu\} = \Pr\{\tau^* \leq \bar{\tau} | x_0^j, p_0, \mu\} \tag{20}$$

  - Intuition: given a skill level. μ, and a initial belief, p_0, richer students drop out later (and have longer expected tenures!)
  - Corollary: once condition on ability type and prior belief richer students are more likely to graduate from college

Outline

1. Ozdagli & Trachter, 2011
2. Trachter, 2014