



College Enrollment and Dropout  
*Stepping Stone and Option Value*

Econ 350, Jorge L. García

This draft: February 19, 2014

# Outline

---



1 Ozdagli & Trachter, 2011

2 Trachter, 2014

- ▶ On the Distribution of College Dropouts: Wealth and Uninsurable Idiosyncratic Risk
- ▶ 2011
- ▶ Ali K. Ozdagli (FRB-Boston) and Nicholas Trachter (FRB-Richmond)
- ▶ R&R JOLE

- ▶ Dynamic model of the decision to pursue college
  - ① Students' uncertainty: about future income stream due to unobserved scholastic ability
  - ② Expectations reevaluation: on success in college after matriculation and after taking exams
- ▶ Findings (only theoretical)
  - ① Poorer students are
    - ▶ less likely to graduate
    - ▶ likely to dropout sooner
- ▶ Interesting feature: no need to introduce credit constraints

- ▶ The authors motivate their work claiming that inequality perpetuates and exacerbates as follows in the U.S.:
  - ① Large fraction of every cohort that enrolls in colleges drops out
  - ② High concentration of dropouts among students from lower-income families
  - ③ Students from low-income families drop out earlier than students from high-income families
  - ④ Less low-income individuals graduate from college
  - ⑤ High return to education

**Table 1** Dropout rates and mean time before dropping out by socioeconomic status of family

	Socio. status <sup>a</sup>	% that drop	Mean tenure in college <sup>b</sup>	st. dev. of tenure
NLSY97	Low	28.98	2.31	1.65
	High	13.40	3.14	1.73
NLSY79	Low	62.5	2.78	1.6
	High	26.96	3.94	1.9
NLS-72	Low	65.6	2.02	1.29
	High	52.86	2.69	1.63

<sup>a</sup> For the NLSY79 and NLSY97 we constructed the measure of socioeconomic status through the income level of the family prior to the respondent's enrollment in college. We choose the deciles so as to match the distribution of socioeconomic status of the NLS-72. <sup>b</sup> We only have the length of the tenure in college for a sub-sample of the population.

**Table 2** Dropout rates of low- vs. high-income students

	tenure between						
	0 and 1 years	1 and 2 years	2 and 3 years	3 and 4 years	4 and 5 years	5 and 6 years	6 and 7 years
NLSY97	1.76	1.65	1.57	0.91	0.37	0.69	0.37
NLSY79	3.28	1.31	1.61	0.8	0.35	0.23	0.4
NLS-72	1.49	1.17	0.94	0.38	0.52	0.19	0.47

Each number in the table represents the dropout rate of low-income students as a share of the total dropout rate of low-income students divided by the yearly dropout rate of high-income students as a share of the total dropout rate of high-income students.

**Table 3** Marginal and percentage effect of socioeconomic status on dropout probability

		$dF/dx$	std. error	% effect	$p$	N
NLSY97	Low SES	0.0265	0.0266	12.78	0.02	1948
	High SES	-0.0901	0.0187	-43.48		
NLSY79	Low SES	0.1576	0.063	41.38	0.03	635
	High SES	-0.0588	0.0468	-15.44		
NLS-72	Low SES	0.0256	0.0298	4.27	0.00	2705
	High SES	-0.138	0.0211	-23.03		

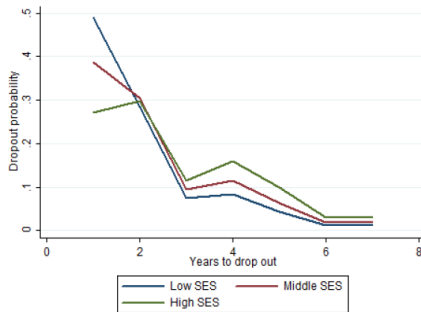
The results are obtained from running logit regressions on the dropout probability based on the socioeconomic status of the student's family and a set of controls. The complete regression results and description of control variables can be found in [Table 5](#), [Table 6](#), and [Table 7](#) in [Appendix E](#).  $N$  is the number of observations and  $p$  is the p-value of the  $\chi^2$  test that compares the effect of high and low socio-economic status on dropout probability.



# Motivation, contd 4

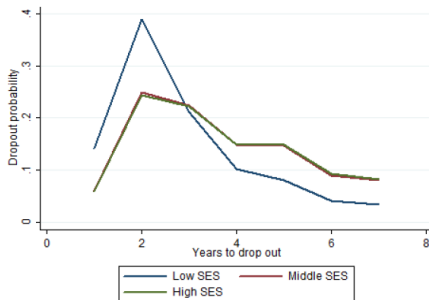


Figure 3 NLS-72: Predicted time to drop out



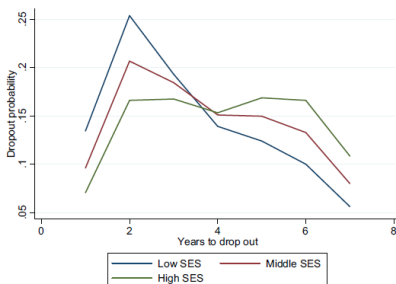
The figure plots the predicted probabilities of dropping out in a given time interval for students with different socioeconomic statuses, conditioning on the average characteristics of students who dropped out. To compute the probabilities we run an ordered logit regression that involves the probability of dropping out in a given year as a function of the socioeconomic status variable plus a set of controls. The complete regression results can be found in [Table 9](#) in [Appendix E](#).

Figure 2 NLSY79: Predicted time to drop out



The figure plots the predicted probabilities of dropping out in a given time interval for students with different socioeconomic statuses, conditioning on the average characteristics of students who dropped out. To compute the probabilities we run an ordered logit regression that involves the probability of dropping out in a given year as a function of the socioeconomic status variable plus a set of controls. The complete regression results can be found in [Table 9](#) in [Appendix E](#).

Figure 1 NLSY97: Predicted time to drop out



The figure plots the predicted probabilities of dropping out in a given time interval for students with different socioeconomic statuses, conditioning on the average characteristics of students who dropped out. To compute the probabilities we run an ordered logit regression that involves the probability of dropping out in a given year as a function of the socioeconomic status variable plus a set of controls. The complete regression results can be found in [Table 8](#) in [Appendix E](#).

- ▶ Based on Miao and Wang (2007): framework of entrepreneurial learning and analysis
- ▶ Add relevant ingredients of dropout decision:
  - ❶ Wage profile: depends on experience and college graduation (and corresponding interaction)
  - ❷ Include information unfolding through learning about unobserved ability in college

- ▶ The author argues that credit constraints and learning about risk are not fundamental
  - ▶ Credit constrains: (i) 29% of students from the richest families dropout (NLSY79); (ii) rising house prices lead to higher graduation rates, especially among low-income families (NLSY97)
  - ▶ Learning about stochastic taste: no relation with wealth
- ▶ Decides to model information unfolding through learning about ability

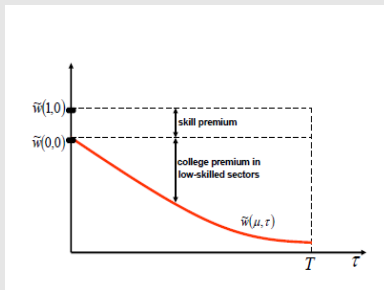
# Model, Primitives

- ▶ Continuous time;  $t \in [0, \infty]$
- ▶ At  $t = 0$ 
  - ❶ Initial endowment,  $x(0) \equiv x_0$
  - ❷ Unobserved ability to acquire human capital,  $\mu \in \{0, 1\}$
  - ❸ Prior on ability,  $\Pr(\mu = 1 | t = 0) = p(0) \equiv p_0$
  - ❹ Either enrolled in college (full-time) or working (in low-skilled or high-skilled sector)
  - ❺ High-skill sector only hires high-skilled workers with college degrees
- ▶ Work: absorbing state
- ▶ Wage function:

$$\tilde{w}(\mu, \tau) \begin{cases} w(\tau) & , \tau > 0 \\ w_0 & , \tau = 0, \mu = 0 \\ w_1 & , \tau = 0, \mu = 1 \end{cases} \quad (1)$$

with  $\tau = T - t$ ,  $w_0 \equiv w(0) < w_1$ ;  $\tau_0 > \tau_1 \Leftrightarrow w_1 > w_0$ .

Figure 4. Wage Function





- ▶  $T$ , time of graduation
- ▶  $c$ , consumption
- ▶  $a$ , per-unit of time cost of college
- ▶  $\rho$ , discount rate;  $r$ , interest rate
- ▶  $\gamma$ , CRRA parameter
- ▶  $\lambda_1$ , probability of getting an excellent grade for high ability student
- ▶  $\lambda_0$ , probability of getting a failing grade for low ability student
  - ▶ Interpret  $\lambda_1, \lambda_0$  as speed of learning
  - ▶ In a continuous (Brownian motion) setting this is analogue to having two volatility parameters



# Model, Primitives contd 2



**Table 4** Probability of receiving different grades based on student's type

	Fail	Pass	Excellent
$\mu = 0$	$\lambda_0 dt$	$1 - \lambda_0 dt$	0
$\mu = 1$	0	$1 - \lambda_1 dt$	$\lambda_1 dt$

► Evolution of wealth

$$\dot{x} = \begin{cases} rx + \tilde{w}(\mu, \tau) - c & , \text{if working} \\ rx - a - c & , \text{if enrolled in college} \end{cases} \quad (2)$$

# Model, Student's Problem (Sequential)

---



$$\max_{c(t)} \mathbb{E} \left\{ \int_0^{\infty} \exp^{-\rho t} \frac{c(t)^{1-\gamma}}{1-\gamma} | p_0, x_0 \right\} \quad (3)$$

s.t. (2) holds

# Model, Student's Problem (Recursive)

---



- ▶  $J(x, p, \tau)$ , student's with current wealth  $x$ , prior on ability  $p$ ,  $\tau$  time before graduation value function
- ▶  $V(x, \mu, \tau)$ , with current wealth  $x$ , type  $\mu$ ,  $\tau$  time before graduation value function

# Model, Worker's Value Function



- ▶ Instantaneous utility derived: consumption + change in wealth

$$\rho V(x, \mu, \tau) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + V_x(x, \mu, \tau)\dot{x} \quad (4)$$

- ▶ First order condition:

$$c^{-\gamma} = V_x(\cdot) \quad (5)$$

- ▶ Define  $W(\mu, \tau)$  as the present value of earnings, let  $A$  be a constant in terms of  $\gamma, \rho$ , and rearrange to get

$$V(x, \mu, \tau) = A (r [x + W(\mu, \tau)])^{1-\gamma} \quad (6)$$

- ▶ Note that this implies that the fact that wages are constant is relatively easy to assume away by changing  $W(\cdot)$  in 6

# Model, Student's Value Function with Known Types



- ▶ Focus on  $\mu = 1$
- ▶ For  $\mu = 0$  we wait for the results (want to guarantee that  $J(x, 0, \tau) = V(x, 0, \tau)$ )
- ▶ Same principle leads to

$$\rho J(x, 1, \tau) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + J_x(x, 1, \tau)\dot{x} + J_\tau(x, 1, \tau)\dot{\tau} \quad (7)$$

with  $J(x, 1, 0) = V(x, 1, 0)$

- ▶ First order condition

$$c^{1-\gamma} = J_x(\cdot) \quad (8)$$

- ▶ Solve to get

$$J(x, 1, \tau) = A \left[ r \left( x + \exp^{r\tau} W(1, 0) - a \frac{1 - \exp^{r\tau}}{r} \right) \right]^{1-\gamma} \quad (9)$$

## Student's Value Function with Unknown Types

### ► Difficulties

- ❶ Wage upon graduation depends on true ability
- ❷ New information arrives after each exam
- ❸ Some students drop out

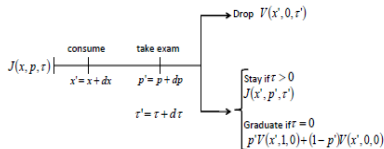
### ► Prior's evolution

$$p(t + dt) = \begin{cases} 0 & , \text{ fails} \\ 1 & , \text{ excellent grade (10)} \\ \frac{p(t)[1 - \lambda_1 dt]}{p(t)[1 - \lambda_1 dt] + (1 - p(t))[1 - \lambda_0 dt]} & , \text{ otherwise} \end{cases}$$

- If the student does not fail or has an excellent grade then

$$\dot{p} = -(\lambda_1 - \lambda_0)p(1 - p) \quad (11)$$

Figure 5 Timeline



A student starts the current period with wealth level  $x$ , prior  $p$ , and remaining time in college  $\tau$ . At the beginning of the period a student chooses her consumption level and thus provides the new value for wealth  $x'$ . Before the end of the period the student takes an exam used to produce the posterior  $p'$  and reduces the time left to graduation to  $\tau'$ . At the beginning of next period the student chooses between dropping out or remaining as a student (or graduation is  $\tau' = 0$ ).





- ▶ Same principle leads to

$$\begin{aligned}\rho J(x, p, \tau) &= \max_c \frac{c^{1-\gamma}}{1-\gamma} + J_x(x, p, \tau)\dot{x} + J_p(x, p, \tau)\dot{p} \\ &+ J_\tau(x, p, \tau)\dot{\tau} + \lambda_1 p [J(x, 1, \tau) - J(x, p, \tau)] \\ &+ \lambda_0 (1-p) [V(x, 0, \tau) - J(x, p, \tau)]\end{aligned}\quad (12)$$

- ▶ Define  $p^*(x, \tau)$  as the threshold such that if  $p \leq p^*(x, \tau)$  the student drops out college



### ► Terminal conditions

#### ① Terminal Condition

$$J(x, p, 0) = pV(x, 1, 0) + (1 - p)V(x, 0, 0) \quad (13)$$

#### ② Value Matching Condition

$$J(x, p^*(\cdot), \tau) = V(x, 0, \tau) \quad (14)$$

#### ③ Smooth Pasting Conditions

$$J_p(x, p^*(\cdot), \tau) = 0 \quad (15)$$

$$J_x(x, p^*(\cdot), \tau) = V_x(x, 0, \tau) \quad (16)$$

$$J_\tau(x, p^*(\cdot), \tau) = V_\tau(x, 0, \tau) \quad (17)$$



- Use FOC to obtain

$$p^*(x, \tau) = \frac{a + rW(0, \tau) + W_\tau(0, \tau)}{\lambda_1} \frac{V_x(x, 0, \tau)}{J(x, 1, \tau) - V(x, 0, \tau)} \quad (18)$$

## ► Lemma 1

- Assume  $r \exp^{-r\tau} W(1, 0) - a(1 - \exp^{-r\tau}) \geq rW(1, \tau)$
- A student of type  $\mu = 1$  with current state  $(x, \tau)$  chooses to remain in college until  $\tau = 0$
- Intuition: graduation premium needs to be high enough for high-ability types to remain in college

## ► Lemma 2

- Assume  $a + rW(0, \tau) + W_\tau(0, \tau) > 0$
- A student of type  $\mu = 0$  immediately drops college
- Intuition: if the per-unit marginal cost of attending college is positive, the low-skilled type student drops college immediately
- Implication:  $J(x, 0, \tau) = V(x, 0, \tau)$

► Result 1

- Let the assumptions in Lemmas 1 and 2 hold
- Students with a greater endowment have a lower value of  $p^*$ , i.e. the belief threshold for which they drop college is lower

$$\frac{\partial p^*(x, \tau)}{\partial x} < 0 \quad (19)$$

- Let  $\tau^*$  denote the time to graduation at the moment individual joins the workforce
  - $\tau^* = T$ , joins workforce immediately
  - $\tau^* = 0$ , joins workforce with college degree

# Results, Unknown Types contd 1

## ► Proposition 1

- Consider two endowments  $x_0^i, x_0^j$  with  $x_0^i > x_0^j$

$$\forall \bar{\tau} \in \mathbb{R}_{++}, \Pr\{\tau^* \leq \bar{\tau} | x_0^i, p_0, \mu\} = \Pr\{\tau^* \leq \bar{\tau} | x_0^j, p_0, \mu\} \quad (20)$$

- Meaning: given a skill level,  $\mu$ , and a initial belief,  $p_0$ , richer students drop out later (and have longer expected tenures!)
- Intuition:
  - Cannot diversify uncertainty related to the outcomes of college education
  - College is riskier than the risk-free asset!
  - CCRA: risk aversion is decreasing in wealth
  - Students with more wealth invest more in riskier projects
- Corollary: once condition on ability type and prior belief richer students are more likely to graduate from college
- Robust? Results hold for HARA utility iff the risk aversion parameter is decreasing in wealth

# Outline

---



1 Ozdagli & Trachter, 2011

2 Trachter, 2014

- ▶ Stepping Stone and Option Value in a Model of Post-Secondary Education
- ▶ 2014
- ▶ Nicholas Trachter (FRB-Richmond)
- ▶ R&R QE



- ▶ Dynamic model of the student's decision to switch from a 2-year (community) college to a 4-year college
  - ① Learn about uncertain educational outcomes
  - ② Drop out or transfer to more rewarding schools
  - ③ Carry a fraction of the accumulated human capital
- ▶ Community college as a “stepping stone” which provides
  - ① Learning
  - ② Transferable human capital
- ▶ Analogous to Jovanovic and Nyarko (1997), workers move up in the work ladder once they acquire skills
- ▶ Main findings:
  - ① Option value (of transferring/dropping) explains a large part of the returns to school enrollment
  - ② Enrollment in community college is driven by the option to “transfer up”
  - ③ The value of the stepping stone itself is small

- Based on facts of the NLS-72

Table 1: Educational transitions	
Type of school	Value
Vocational School (enrollment share: 0.09)	
Fraction that drops out	0.88
Fraction that graduates	0.06
Fraction that transfers to Ac. two-year c.	0.03
Fraction that transfers to four-year c.	0.03
Academic two-year colleges (enrollment share: 0.15)	
Fraction that drops out	0.59
Fraction that graduates	0.05
Fraction that transfers to Voc. school	0.04
Fraction that transfers to four-year c.	0.32
four-year colleges (enrollment share: 0.25)	
Fraction that drops out	0.41
Fraction that graduates	0.56
Fraction that transfers to Voc. school	0.03
Fraction that transfers to Ac. two-year c.	0.02

- Interesting fact: 56% of students who transfer from community colleges to colleges graduate

# Motivation, contd 1



Table 2: Summary statistics for ability measures conditional on enrollment

	Work	V.S.	Ac. two-year C.	four-year C.
Male	0.49 (0.5)	0.4 (0.5)	0.54 (0.5)	0.52 (0.5)
Black	0.11 (0.3)	0.13 (0.3)	0.08 (0.3)	0.10 (0.3)
Socioeconomic status				
Low	0.42 (0.5)	0.29 (0.5)	0.19 (0.4)	0.17 (0.4)
Medium	0.51 (0.5)	0.58 (0.5)	0.55 (0.5)	0.41 (0.5)
High	0.08 (0.27)	0.12 (0.33)	0.25 (0.43)	0.43 (0.49)
Education of father				
< than HS completion	0.52 (0.5)	0.39 (0.5)	0.29 (0.4)	0.21 (0.4)
HS completion	0.32 (0.5)	0.40 (0.5)	0.34 (0.5)	0.29 (0.4)
four-year college drop out	0.11 (0.3)	0.14 (0.3)	0.21 (0.4)	0.19 (0.4)
four-year college graduate	0.05 (0.2)	0.06 (0.2)	0.15 (0.4)	0.31 (0.5)
Rank	0.50 (0.3)	0.43 (0.3)	0.39 (0.3)	0.27 (0.2)

Rank=rank in high school class. Socio-Status: Socioeconomic status of family at moment of high school graduation.

- ▶ Why is community college a stepping stone?
  - ❶ Provides a less risky investment (than college) to learn about abilities
- ▶ Although empirically (almost) irrelevant, the model also allows for “bandit steps”
  - ▶ Enroll in the hardest step and move down the ladder
  - ▶ Johnson (1978), Miller (1984)

# Model, Primitives

- ▶ Discrete time, infinite horizon;  $t \in [0, \infty]$
- ▶  $\gamma$ , CARA coefficient;  $r$ , interest/discount rate
- ▶ At  $t = 0$ 
  - ① Initial level of assets,  $a_0$
  - ② Unobserved ability to acquire human capital,  $\mu \in \{0, 1\}$
  - ③ Prior on ability,  $\Pr(\mu = 1|t = 0) = p(0) \equiv p_0$
- ▶ The agent either
  - ① Works,  $W$
  - ② Four-year college,  $C$
  - ③ Two-year college,  $A$
- ▶ School credits,  $s$
- ▶ Time before graduation tied to credit accumulation:  $T^C > T^A$
- ▶ Evolution of credits tied to signals,  $\eta \in \mathcal{H}$
- ▶  $\eta \sim f_i(\eta|\mu)$  for  $i = A, C$  satisfies the MLRP
- ▶ Per-unit of time tuition:  $\tau^C > \tau^A$

# Model, Primitives contd 1

- ▶ Working: absorbing state
- ▶ Wage function:

$$h(GS, i, \mu) = \begin{cases} h^w & , GS = 0 \\ h^i(\mu) & , GS = 1 \end{cases} \quad (21)$$

with  $GS = 1$ (graduation);  $h^i(1) \geq h^i(0)$  for  $i = C, A$  and  $h^C(\mu) \geq h^A(\mu)$  for  $\mu = 0, 1$

- ▶ Evolution of assets

$$a_{t+1} = \begin{cases} (1+r)a_t + h(GS, i, \mu) - c_t & , \text{if working} \\ (1+r)a_t - \tau^i - c_t & , \text{if enrolled in } i \end{cases} \quad (22)$$

- ▶ No borrowing constraints!



- Evolution of credits

$$s' = s + \tilde{\Omega}(\eta, s) \quad (23)$$

with

$$\tilde{\Omega}(\eta, s) = \begin{cases} \Omega(\eta) & , s < T^i \\ 0 & , s \geq T^i \end{cases} \quad (24)$$

- Accumulation of credit is only a function of current signal
- Current signal has positive correlation with ability (MLRP)
- High-grade students accumulate at least as much credits as low-grade student:  $\Omega(\eta_1) \geq \Omega(\eta_2) \Leftrightarrow \eta_1 \geq \eta_2$



- Credits map ... for  $i \neq j$  and  $I = [A, C]$

$$\theta^i(s) : \mathbb{R}_+ \times I \rightarrow [0, T^j] \quad (25)$$



# Model, Student's Problem (Sequential)

---



- Choose  $c(t)$  and either to enroll, dropout, or transfer in  $A, C$  to maximize

$$\mathbb{E} \left\{ \sum_{t=0}^{\infty} \frac{\exp^{-\gamma c_t} - 1}{-\gamma} \middle| p_0, a_0 \right\} \quad (26)$$

s.t. (22) holds

- CARA utility: order of risky projects is independent of financial wealth

# Model, Worker's Value Function



- ... defined as

$$W(a; h) = \max_{a', c} = \frac{\exp^{-\gamma c_t} - 1}{-\gamma} + \frac{1}{1+r} W(a'; h) \quad (27)$$

with  $a' = (1 + r)a + h - c$

- Use the FOC and solve to get

$$W(a; h(GS, i, \mu)) = -\frac{1+r}{\gamma r} \exp^{-\gamma(ra+h(GS, i, \mu))} + \frac{1+r}{\gamma r} \quad (28)$$

- Beliefs (use Baye's rule to obtain posterior)

$$\begin{aligned} p' &\equiv b(\eta; p) \\ &= \frac{1}{1 + \frac{f_i(\eta|\mu=0)}{f_i(\eta|\mu=1)} \frac{1-p}{p}} \end{aligned} \quad (29)$$

- Expected “governing” CDF

$$H_i(\eta, p) = pF_i(\eta|\mu = 1) + (1 - p)F_i(\eta|\mu = 0) \quad (30)$$

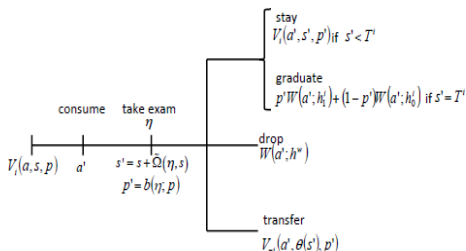
► ... defined as

$$\begin{aligned} V_i(a, s, p) &= \max_{a', c} \frac{\exp^{-\gamma c_t} - 1}{-\gamma} \\ &+ \frac{1}{1+r} \int_{\eta \in \mathcal{H}} \tilde{V}_i(a', s', p') dH_i(\eta, p) \end{aligned} \quad (31)$$

$$\begin{aligned} \tilde{V}_i(a', s', p') &= \max \{ W(a'; h^w), \mathbf{1}(s' < T^i) V_i(a', s', p') \\ &+ (1 - \mathbf{1}(s' < T^i)) p' W(a'; h^i(1)) \\ &+ (1 - \mathbf{1}(s' < T^i)) (1 - p') W(a'; h^i(0)) \\ &, V_j(a', \theta^i(s'), p') \} \end{aligned} \quad (32)$$

with  $i, j = A, C, i \neq j$

Figure 1: Timeline



At the beginning of the period,  $V_i(a, s, p)$  denotes the value of enrollment at institution type  $i$  with wealth level  $a$ , accumulated credits  $s$  and prior  $p$ . The student chooses her consumption level producing the accumulated wealth for next period  $a'$ . Later, she takes an exam, receiving grade  $\eta$ . This produces a re-evaluation of beliefs  $p' = b(\eta, p)$  and accumulation of credits  $s' = s + \bar{\Omega}(\eta, s)$ . The student then chooses between staying in institution type  $i$  (or graduating if enough credits were accumulated), transferring or dropping out.

- ▶ Restrictions on
  - ▶ Expected wages upon graduation
  - ▶ Low-skill -high-skill wage differential
  - ▶ Learning technology
  - ▶ Tuition costs
- ▶ ...enable to generate the desired pattern:
  - ▶ Low ability individuals: better-off joining the workforce, if enrolled better-off in community college
  - ▶ High ability individuals: better-off joining college, if not better-off in community college than in workforce
- ▶ Community colleges are learning mechanism:
  - ❶ Pessimistic agents join the workforce
  - ❷ Optimistic agents enroll in college
  - ❸ Average join community colleges
- ▶ “A pessimist is a well informed optimist” Mario Benedetti

- ▶ Students who transfer from community colleges to colleges usually do so after the first year
- ▶ Students who transfer from colleges to community colleges spend for time studying after high school, on average

$$\theta^i(s) = \begin{cases} \theta^i s, & \theta^i s < T^j \\ T^j, & \theta^i s \geq T^j \end{cases} \quad (33)$$

with  $\theta^A = \frac{1}{2}$  and  $\theta^C = 0$

- Signals are grades
  - ❶ Update beliefs
  - ❷ Generate credit accumulation
  - ❸ Fail(F), N(Neutral), Excel(E)

$$\Omega(\eta) = \begin{cases} 0 & , \eta = F \\ 1 & , \eta = N \\ 1 & , \eta = E \end{cases} \quad (34)$$



- ▶  $q_{\mu}^i(\eta)$ : probability of each grade
  - ▶  $q_1^A(F) = q_0^C(E) = 0$
- ▶ Prior: estimate Ordered Probit on initial educational choice and construct

$$\begin{aligned} p_0 &= \frac{1}{1 + \exp^{-X'\beta - \varepsilon}} \\ \varepsilon &\sim \mathcal{N}(0, 1) \end{aligned} \tag{35}$$

# Calibration, Time Periods and Interest Rates

---



- ▶ Time is measures in quarters:  $T^A = 8, T^C = 16$
- ▶ Quarterly risk free interest rate .45
- ▶  $h^w$  is the numeraire (mean wage in 1985: \$17,740.63)
- ▶  $\tau^A = .1152; \tau^C = .3205$
- ▶  $\gamma = 8; (\gamma c = \sigma)$

- ▶ Use Conditional Indirect Inference to calibrate the remaining parameters matching enrollment moments
  - ▶ Long-run wage (4 parameters)
  - ▶ Probability of dropout, transfer, and graduation (6 parameters)
  - ▶
- ▶ Claims over-identification as follows: “educational histories greatly differ across students in the sample and I have 3,462, the model is over-identified”
- ▶ Calibration-Estimation satisfies MPLR and the structure necessary for the desired schooling pattern
- ▶ Transition probabilities implies by the model resemble the data

Table 3: Calibrated wage differentials

		estimate
Academic two-year colleges		
low ability	$\frac{h^A(0)-h^w}{h^w}$	0.03
high ability	$\frac{h^A(1)-h^w}{h^w}$	0.08
Four-year colleges		
low ability	$\frac{h^C(0)-h^w}{h^w}$	0.04
high ability	$\frac{h^C(1)-h^w}{h^w}$	0.22

Table 4: Calibrated learning parameters

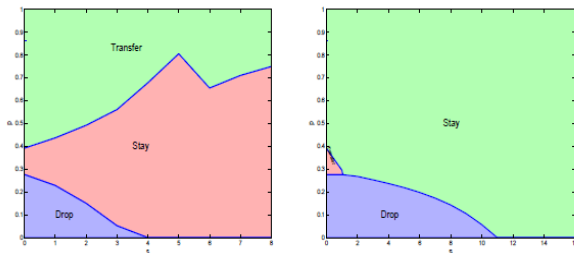
	Academic two-year colleges		Four-year colleges	
	low ability $\mu = 0$	high ability $\mu = 1$	low ability $\mu = 0$	high ability $\mu = 1$
Fail	0.23	0*	0.2275	0.01
Neutral	0.695	0.87	0.7725	0.81
Excel	0.075	0.13	0*	0.18

\* by assumption.

Table 5: Average value of educational histories in data and model

	Data	Model
% of High-school graduates who join Workforce	59.4	56.9
enroll in $A$	15.2	15.69
% of those initially enrolled in $A$ that drop at $A$ ( $1^{st}$ spell)	63.2	48.63
transfer from $A$ to $C$ ( $1^{st}$ spell)	32.2	43.79
% of those initially enrolled in $C$ that drop at $C$ ( $1^{st}$ spell)	40.9	42.1
graduate at $C$ ( $1^{st}$ spell)	57.1	57.79
Mean Wage Differential for graduates from $A$	0.045	0.045
graduates from $C$	0.212	0.148
Moments to Discipline Priors		
FOCs from Ordered Probit (for $\beta$ )		YES
distribution of $X$		YES

Figure 2: Regions at academic two-year and four-year colleges



Left panel: academic two-year colleges. Right panel: four-year colleges. The dropout and transfer thresholds at both academic two- and four-year colleges are as a function of amount of accumulated credits  $s$ .

- ▶ Compute the value of
  - ❶ Let  $\gamma \rightarrow 0$  (risk neutrality)
  - ❷ Rule out transfer option
  - ❸ Rule out transfer and dropout options
- ▶ Compute the value added of each component through decomposition of enrollment returns,  $R_i(p)$

$$\begin{aligned} R_i(p) &= \frac{\Delta_i(p)}{\frac{1+r}{r} h^w} \\ W(a; h^w) &= V_i(a - \Delta_i(p), 0, p) \end{aligned} \quad (36)$$



- Decompose return as value added from: transfer option + dropout option + enrollment

$$\begin{aligned} R_i^{E+D+T} &= (R_i^{E+D+T} - R_i^{E+D}) \\ &+ (R_i^{E+D} - R_i^E) + R_i^E \end{aligned} \quad (37)$$

# Counter-factual Analysis, Results



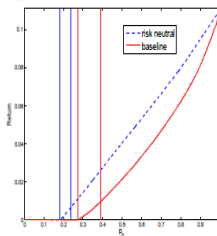
Table 6: Risk aversion and average measured returns

	ac. two-year c.	four-year c.
Baseline; $\gamma = 8$	0.4 %	3.1 %
Risk neutral; $\gamma \rightarrow 0$	0.3 %	3.4 %

# Counter-factual Analysis, Results contd 1



Figure 3: Risk aversion and individual returns



The vertical lines define the indifference belief for enrollment between work and academic two-year colleges and between academic two-year colleges and four-year colleges.

# Counter-factual Analysis, Results contd 2



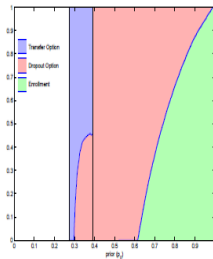
Table 7: Risk aversion and enrollment

	workforce	ac. two-year c.	four-year c.
Baseline; $\gamma = 8$	56.9%	15.7%	27.4%
Risk neutral; $\gamma \rightarrow 0$	39.8%	10.4%	49.8%

# Counter-factual Analysis, Results contd 1



Figure 4: Decomposition of Returns



The vertical lines define the indifference belief for enrollment between work and academic two-year colleges and between academic two-year colleges and four-year colleges.

# Counter-factual Analysis, Results contd 2



Table 8: Average value added of each option

	ac. two-year c.	four-year c.
Enrollment option	0%	13%
Dropout option	31%	87%
Transfer option	69%	0%