Empirical Exercise on Structural Estimation

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Abstract

This document briefly describes what are structural models in Economics. It distinguishes structural and non-structural estimation as approaches that enable to answer different research questions. Also, it discusses parametric and non-parametric assumptions as auxiliary means that help to recover parameters that enable to answer these research questions. The document builds on Keane et al. (2011). The difference between this document and that paper is that this document intends to be more concrete and schematic about how the modeling approaches behave, it skips the least algebra steps in all the derivations, and it provides practical examples. Also, it provides extra exercises that teach basic tools of Computational Econometrics.

1 Introduction

The objective of this document is to build on Keane et al. (2011) and clarify Structural Estimation methods. The focus is on the estimation of Discrete Choice Dynamic Programming (DCDP) models. In particular, the idea is to (i) illustrate the basic ideas and concepts; (ii) provide examples.

1.1 DCDP as an Extension of the Static Discrete Choice Framework

DCDP models are a natural generalization of static discrete choice models. They share the latent variable specification. To illustrate this, consider a binary choice model in which an economic agent, i, makes a choice at each discrete period t. $\mathcal{I}$ indexes individuals and $\mathcal{T}$ time. She has two alternatives: $d_{it} \in \{0, 1\}$. A latent variable, v^*_{it}, which is the difference in the expected payoffs between the choices $d_{it} = 1$ and $d_{it} = 0$, determines the outcome. Specifically, if v^*_{it} is greater than certain threshold, the agent chooses $d_{it} = 1$. Without loss of generality, the threshold is normalized to zero. Thus, $d_{it} = 1$ iff $v_{it}^* \geq 0$ and $d_{it} = 0$ otherwise.

Exercise 1.1 *(Identification of the Probit Model) Model a bivariate, static, discrete choice through a Probit model. The convention is that in this model the unobserved variable is normally distributed. (i) Show that you can normalize the threshold that defines the agent's decision without loss of generality; (ii) why is the normalization without loss of generality?; (iii) what other normalization can you make without loss of generality in your Probit model?; (iv) what does this normalization implies with respect to the distribution of the unobserved variable? (v) are you able to identify all the parameters of the model?; (vi) how does identification and the normalizations relate to each other? Hint: think of the scalar and spatial identification issues that the structure of a Probit model generates.*

Exercise 1.2 *(Computational Econometrics: Warm-up) Solve the exercise "Warmup.pdf" posted on the web site. The objective of this is for you to go from the basics (i.e., installing Python in your computer and setting up the function maximizer) to an exercise in which you can maximize a likelihood function.*

Exercise 1.3 *(Estimation of the Probit Model) From Exercise 1.1 you have clear the setup of the Probit model. Make sure you know what the correct parametric assumptions are and what can you identify in the model. Simulate all the data necessary to estimate a Probit model. The instructions are loose in purpose because we want to evaluate your ability to create the data and estimate the model from scratch. Hint: simulate a single independent variable. This is sufficient for the purposes of this exercise.*

In general, the latent variable is a function of three variables: (i) $\tilde{D_{it}}$, a vector of the history of past choices (i.e., $d_{i\tau}, \tau = 0, \ldots, t-1$); (ii) $\tilde{X_{it}}$, a vector of contemporaneous and lagged values of J variables (i.e., $X_{ij\tau}, j = 1, \ldots, J; \tau = 0, \ldots, t-1$); (iii) $\tilde{\epsilon_{it}}$, a vector of contemporaneous and lagged unobserved variables (i.e., $\epsilon_{i\tau}, \tau = 0, \ldots, t-1$). Thus, the general decision rule of the agent is:

$$d_{it} = \begin{cases} 1 \text{ if } v_{it}^{*}\left(\tilde{D_{it}}, \tilde{X_{it}}, \tilde{\epsilon_{it}}\right) \ \geq 0 \\ 0 \text{ if } v_{it}^{*}\left(\tilde{D_{it}}, \tilde{X_{it}}, \tilde{\epsilon_{it}}\right) < 0. \end{cases} \tag{1}$$

Any binary choice model is a special cases of this formulation, no matter if they are static or dynamic. The model is dynamic if agents are forward looking and either $v_{it}^{*}(\cdot)$ contains past choices, $\tilde{D_{it}}$, or unobserved variables in $\tilde{\epsilon_{it}}$ that are serially correlated. The model is static if (i) agents are myopic so that even when they accumulate information on past decisions or past unobserved variables they do not take them into account; (ii) agents are forward looking but there is no link between present and past decisions and unobserved variables.

Remark 1.4 *In this context, forward looking refers to the behavior in which agents consider how their present decisions affect their future welfare. The exact way in which they form the expectations on how their welfare is affected is a modeling decision that the researcher makes.*

Exercise 1.5 *The last paragraphs clarify that there is a general framework to think of either static or dynamic binary choice models. Argue that this can be generalized for multivariate models. Write down a general framework for the multivariate case that encompasses static and dynamic models. Specify conditions under which the model is either static or dynamic.*

This document follows Keane et al. (2011) and argues that there are three broad research goals in the estimation of DCDP models:

1. Test a prediction of the theory: how an observed variable in v_{it}^{*} affects d_{it}.

2. Determine the effect of an endogenous shift: how a change in $\tilde{D_{it}}$ or $\tilde{X_{it}}$ affects d_{it}.

3. Determine the effect of an exogenous shift: how a change in something not in $\tilde{D_{it}}$ or $\tilde{X_{it}}$ affects d_{it}.

The objective is to answer these questions *caeteris paribus*. *Caeteris paribus* in this context not only means that the "rest" of the variables are held fixed. It implies that the the unobserved variables are also held fixed and that their joint distribution is not altered.[1] Different modeling

[1] See Heckman and Pinto (2013) for a discussion on what "fixing" in Economics means and how it differs from "conditioning" in Statistics.

decisions and estimation approaches enable to attain these research goals to different extents (see Section 2).

2 Model Classifications, Estimation Strategies, and Research Goals

In this section we consider the static model in Keane et al. (2011) to illustrate how different modeling approaches and estimation strategies enable to attain the research goals in Section 1.1.

2.1 Woman's Labor Force Participation

Consider the following model of the labor force participation of a married woman. The model is unitary and the couple's i utility at time t is

$$U_{it} = U\left(c_{it}, 1 - d_{it}; n_{it}(1 - d_{it}), \kappa_{it}(1 - d_{it}), \epsilon_{it}(1 - d_{it})\right) \tag{2}$$

where c_{it} is consumption, d_{it} is an indicator of the woman's labor supply (1 if she works and 0 if she does not), n_{it} is the number of young children that the couple has, κ_{it} and ϵ_{it} are observed and unobserved factors that shift the couple's valuation of home production. Actually, t corresponds to the couple's marriage duration. The utility function satisfies standard concavity and Inada conditions.

The wife receives a wage offer w_{it} in each period t and the husband, who works every period, receives an income y_{it}. If the wife works, the family needs to pay child care, π for each child in each period. Hence, the budget constraint is

$$c_{it} = y_{it} + w_{it}d_{it} - \pi n_{it}d_{it}. \tag{3}$$

In this simple model, a wage function determines the wage offer that women receive:

$$w_{it} = w(z_{it}, \eta_{it}) \tag{4}$$

where z_{it} are observed and η_{it} unobserved factors. By assumption, ϵ_{it}, η_{it} are not serially correlated between each other.

Exercise 2.1 *Is this model static or dynamic?*

Exercise 2.2 *What are the variables that you expect to find in z_{it}?*

Exercise 2.3 *Why is no serial correlation between ϵ_{it}, η_{it} a relevant assumption? Is it a technical or an economic assumption? Is it realistic? Hint: go ahead and answer the reminder of the exercise and then come back to this question.*

This structure, actually, is enough to describe the problem through a decision rule that a latent variable dictates, as in (1). Specifically, substitute (3),(4) into (2) and note that

$$d_{it} = \begin{cases} 1 \text{ if } v_{it}^{*}\left(y_{it}, z_{it}, n_{it}, \kappa_{it}, \epsilon_{it}, \eta_{it}\right) \ \geq 0 \\ 0 \text{ if } v_{it}^{*}\left(y_{it}, z_{it}, n_{it}, \kappa_{it}, \epsilon_{it}, \eta_{it}\right) < 0 \end{cases} \tag{5}$$

where $v_{it}^{*}\left(y_{it}, z_{it}, n_{it}, \kappa_{it}, \epsilon_{it}, \eta_{it}\right) \equiv U_{it}^{1} - U_{it}^{0}$ and

$$\begin{aligned} U_{it}^{1} &= U(y_{it} + w_{it}(z_{it}, \eta_{it}) - \pi n_{it}, 0) & (6) \\ U_{it}^{0} &= U(y_{it}, 1; n_{it}, \kappa_{it}, \epsilon_{it}). & (7) \end{aligned}$$

Definition 2.4 *(The State Space)*

1. *Household State Space:* $\Omega_{it} = \{y_{it}, z_{it}, n_{it}, \kappa_{it}, \epsilon_{it}, \eta_{it}\}$.

2. *Observed Household State Space:* $\Omega_{it}^{-} = \{y_{it}, z_{it}, n_{it}, \kappa_{it}\}$.

3. *The set of values of the unobserved variables that makes a household with observed state space Ω_{it}^{-} choose $d_{it} = 1$:* $S\left(\Omega_{it}^{-}\right) = \{\epsilon_{it}, \eta_{it} : v^{*}\left(\epsilon_{it}, \eta_{it}; \Omega_{it}^{-}\right) \geq 0\}$.

This enables to write

$$\begin{aligned} \Pr\left(d_{it} = 1 | \Omega_{it}^{-}\right) &= \int\limits_{S\left(\Omega_{it}^{-}\right)} dF_{\epsilon,\eta|y,\kappa,z,n} \\ &= G\left(y_{it}, z_{it}, n_{it}, \kappa_{it}\right). \end{aligned} \tag{8}$$

Obviously, $\Pr\left(d_{it} = 0 | \Omega_{it}^{-}\right) = 1 - \Pr\left(d_{it} = 1 | \Omega_{it}^{-}\right)$. The main components of $G\left(y, \kappa, z, n\right)$ are

$U(\cdot), w(\cdot), F_{\epsilon,\eta,y,\kappa,z,n}$, which conform the *structure* or the *set of primitives* of the model. Consider the following definitions of estimation approaches and auxiliary assumptions.

Definition 2.5 *(Estimation Approaches)*

1. *Structural (S): it recovers some or all of the parameters of that define the structure of the model.*

2. *Non-Structural (NS): it recovers $G(\cdot)$.*

Definition 2.6 *(Auxiliary Assumptions for Identification)*

1. *Parametric (P): assumes parametric forms about the structure of the model or about $G(\cdot)$.*

2. *Non-Parametric (NP): it does not impose parametric forms on either the structure or $G(\cdot)$.*

The combination of the initial approaches and the two auxiliary assumptions for identifications leads to a total of four possible estimation approaches: (i) S-P; (ii) S-NP; (iii) NS-P; (iv) NS-NP. The relevant question to ask is which of the estimation approaches enable to attain the research goals in Section 1.1.

Exercise 2.7 *(The Joint Distribution of Observed and Unobserved Variables) Give a sufficient condition on the joint distribution of observed and unobserved variables to attain each of the research goals in Section 1.1. Hint: Think if you can ask the questions implied by the research goals without an assumption about the relation between the unobserved variables that affect preferences and the wage function and the observed variables. Then, make a simple assumption about the joint distribution of the observed and unobserved variables.*

This model enables to illustrate how different estimation approaches help to attain the different research goals in Section 1.1. Consider the following examples:

1. Goal 1: from (2) note that an increase in wage increases the utility the household has if the woman works and does not affect the utility the household has when the woman does not work. Then, a test of the theory is to analyze if the probability of woman's employment is increasing in wage.

2. Goal 2: take the derivative of $G(\cdot)$, the probability of woman's employment, with respect to any of the state variables.

3. Goal 3: take the derivative of $G(\cdot)$, the probability of woman's employment, with respect to a variable that is outside the model (e.g., π, the per-child cost of child-care).

Exercise 2.8 *(Estimation Approaches and Research Goals) What research goals can you attain with the estimation approaches NP-NS, P-NS, P-NS. Be as formal as possible. Hint: think of the different effects that you are able to identify.*

2.2 Estimation of a Parametric, Structural Model

In this exercise you will take a S-P approach to estimate the model in Section 2.1. Of course, there are many variations of parametric assumptions that you can impose. We guide you and you estimate. Various exercises lead to the final answer.

Assumption 2.9 *(Utility and Wage Functions and the Joint Distribution of Unobserved Variables) The utility function is:*

$$U_{it} = c_{it} + \alpha_{it}(1 - d_{it}) \tag{9}$$

where $\alpha_{it} = \beta_\kappa \kappa_{it} + \beta_n n_{it} + \epsilon_{it}$ *and* β_κ, β_n *are scalars. The wage function is:*

$$w_{it} = z_{it}\gamma + \eta_{it}. \tag{10}$$

The distribution of unobserved variables is

$$f\left(\epsilon_{it}, \eta_{it}\right) \sim \mathcal{N}\left(0, \Lambda\right) \tag{11}$$

where $\begin{pmatrix} \sigma_\epsilon^2 & \cdot \\ \sigma_{\epsilon,\eta} & \sigma_\eta^2 \end{pmatrix}$.

Exercise 2.10 *(Wage Normality) Is it odd to model the shock to wages as normal? Why is it useful?*

Exercise 2.11 *(The State Space) Define* Ω_{it} *and* Ω_{it}^{-} *for this problem.*

Exercise 2.12 *(Latent Variable Function) Use Assumption 2.9 to write down the latent variable function. First define* U_{it}^1 *and* U_{it}^0*. Your latent function should be a function of* $\xi_{it} \equiv \eta_{it} - \epsilon_{it}$ *and* $\xi_{it}^* \left(\Omega_{it}^-\right) \equiv z_{it}\gamma - (\pi\beta_n) - \kappa_{it}\beta_\kappa$*. Use this notation for the rest of the problem.*

Exercise 2.13 *(Individual and Sample Likelihood Function) Write down the individual likelihood that individual* i *at time* t *contributes to the sample likelihood function. Write down the sample likelihood function.*

Exercise 2.14 *(Estimands and Identification) What is the set of parameters that you want to estimate? Are all of these parameters identified? Hint: read Heckman (1979).*

Exercise 2.15 *(Simulation) Simulate a strongly balanced data set with* $N = 1000$ *observations and* $T = 6$*. Use the following parameters:* $\beta_\kappa = 0.5, \beta_n = 0.2, \sigma_\epsilon = 1, \pi = 0.2, \gamma_1 = 0.8, \sigma_\eta = 0.2, \sigma_{\epsilon\eta} = 0.3$*. Assume that* $y_{it} \overset{iid}{\sim} \mathcal{U}(0, 10)$*.* κ_{it}*,* z_{it} *and* n_{it} *are time invariant. In particular,* $\kappa_i, z_i \overset{iid}{\sim} \mathcal{U}(0, 5)$*, and* n_i *follows a discrete uniform distribution and* $n_i \in \{0, 1, 2, 3\}$*. Use the Numpy random package in Python and set the seed to zero.*

Exercise 2.16 *(Estimation) Estimate the parameters of the model by ML. Compare your results with the parameters in Exercise 2.15. Hint: if the BFGS algorithm does not work use the Nelder-Mead algorithm.*

Exercise 2.17 *What research goals are you able to attain with this approach?*

Exercise 2.18 *Think of three policy questions that you can address with each of estimation of the model you just did. You need to link each question to each research goal in Section 1.1.*

Exercise 2.19 *What are you able to learn from each estimation approach? Is any estimation approach better than the other? Why?*

3 Discrete Choice Dynamic Programming

Consider the dynamic version of the model in Section 2.1. The utility function, the budget constraint, and the distribution of the unobserved variables are the same. The dynamics of the model come through the wage process. In particular, w_{it} increases with work experience, h_{it}. This equals the total number of periods that the woman in household i accumulates in all the periods previous to t:

$$h_{it} = \sum_{\tau=1}^{t-1} d_{i\tau}, \tag{12}$$

where $h_{i1} = 0$ for simplicity. The wage function is

$$w_{it} = z_i \gamma_1 + \gamma_2 h_{it} + \eta_{it}. \tag{13}$$

For simplicity, the variables κ_{it}, n_{it}, z_{it} are non-stochastic and time invariant.

Exercise 3.1 *(Dynamic Programming Set-up) Write down the household's problem for each period t. Let δ be the discount factor, Ω_{it} the state space, and Ω_{it}^{-} the observed state space. Write down the elements in $\Omega_{it}, \Omega_{it}^{-}$.*

Answer:

The problem is

$$\max_{d_{it}} \mathbb{E} \left\{ \sum_{\tau=t}^{T} \delta^{\tau-t} \left[U_{i\tau}^{1} d_{i\tau} + U_{i\tau}^{0} \left(1 - d_{i\tau}\right) \right] \middle| \Omega_{it} \right\} \tag{14}$$

where

$$U_{i\tau}^{1} = y_{i\tau} + z_i \gamma_1 + \gamma_2 h_{i\tau} + \eta_{i\tau} - \pi n_i \tag{15}$$

$$U_{i\tau}^{0} = y_{i\tau} + \beta_\kappa \kappa_i + \beta_n n_i + \epsilon_{i\tau} \tag{16}$$

$$\Omega_{it} = \{y_{it}, z_i, n_i, \kappa_i, \epsilon_{it}, \eta_{it}, h_{it}\} \tag{17}$$

$$\Omega_{it}^{-} = \{y_{it}, z_i, n_i, \kappa_i, h_{it}\} \tag{18}$$

Exercise 3.2 *(Bellman Equation) Write down the recursive formulation of the household's problem.*

Exercise 3.3 *(Solution) Solve the dynamic problem of the household for for an arbitrary time horizon, T. Hint: use a backward recursion.*

3.1 Simulation and Estimation

Exercise 3.4 *(Likelihood) What is the individual likelihood of household i at time t? What is the sample likelihood across all periods?*

Exercise 3.5 *(Simulation) Simulate a balanced data set with $N = 1000$ observations and $T = 6$. Use the same parameters as in the static model in Section 2. For the parameters that are exclusive of the dynamic model use the following: $\gamma_2 = 0.9, \delta = 0.85$. Set the experience of every woman to zero in $t = 1$. Save the data in a ".csv" file.*

Exercise 3.6 *(Estimation) Estimate the parameters of the model by ML. Compare your results with the parameters in Exercise 3.5.*

References

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