Problem Set # 6

Econ 722

1. In this question you will derive the simplest possible version of the FIC. Consider a linear regression model with two scalar regressors x and z

$$y_t = \theta x_t + \gamma z_t + \epsilon_t$$

where $\{(x_t, z_t, \epsilon_t)\}_{t=1}^T \sim$ iid with means $(0,0,0)$ and variances $(\sigma_x^2, \sigma_z^2, \sigma_\epsilon^2)$. The target parameter is the mean response at a *particular* covariate level (x^*, z^*). In other words we have $\mu(\theta, \gamma) = \theta x^* + \gamma z^*$ where (x^*, z^*) are fixed constants.

   (a) Derive the FIC for this problem, where our goal is to choose between the *full* model, which carries out OLS estimation using both x and z, and the *narrow model* which carries out OLS estimation using x only. This corresponds to the restriction $\gamma = 0$, so we consider a DGP in which $\gamma_T = \delta/\sqrt{T}$. *None of the other parameters of the DGP vary with sample size.* The easiest way to proceed is directly from the formulas for the OLS estimators rather than via the results in Claeskens & Hjort (2003). Be sure to explain your asymptotic arguments.

   (b) Compare the FIC decision rule for this problem to those of the AIC, BIC, Mallow's C_p, and the t-test of the null hypothesis $H\colon \gamma = 0$ at the $\alpha \times 100\%$ level. Comment on any relationships you uncover.

2. **Don't Start This Question Yet: I'm going to add some details to simplify it once I hear back from Lorenzo...** In this question you will replicate and slighly extend the simulation studies from Hansen (2005), which is available in the shared Dropbox Folder for the course. The true DGP is

$$y_t = \alpha y_{t-1} + \epsilon_t - \gamma \epsilon_{t-1} \qquad \epsilon_t \sim \text{iid } N(0,1)$$

$$(y_t = \alpha y_{t-1} + \epsilon_t + \gamma \epsilon_{t-1} \qquad \epsilon_t \sim \text{iid } N(0,1))$$

and your target parameter is the mth impulse response: $\theta_m = (\alpha - \gamma)\alpha^{m-1}(\theta_m = (\alpha+\gamma)\alpha^{m-1})$. Your task is the identify the AR order k^* that minimizes $E[(\widehat{\theta}_m(k) - \theta_m)^2]$ for a given horizon m, where $\widehat{\theta}_m(k)$ denotes the estimated impulse response based on a fitted AR(k) model. For all of your simulations, use a sample size of $T = 200$ and as many simulation replications as your machine can handle. Consider all AR models from order 0 up to 12.

(a) Write a function that approximates $E[(\widehat{\theta}_m(k) - \theta_m)^2]$ by simulation for *fixed values* of α, γ, m, and k.

(b) Using your function from part (a), replicate Table 1 from the paper.

(c) In some regions of the parameter space, the best AR model varies substantially depending on the horizon of interest. Write a function that takes arguments α, γ and plots RMSE against k for all $k = 0, 1, \ldots, 12$ and $m = 1, \ldots 6$. To be clear, your plot should have k on the horizontal axis, RMSE on the vertical axis and include 6 curves, each corresponding to a different impulse response horizon. The point is to see *how much better* the best AR model is compared to the others. Test out your plotting function on various combinations of α, γ. Try to find one combination of parameter values for which the differences are large and another for which the differences are small.

(d) Write code to replicate Tables 2 and 3 of the paper.

(e) Repeat the preceding part, but compare FIC to BIC rather than AIC. Comment on your results.