Dynamic Factor Models

Francis J. DiTraglia

University of Pennsylvania

Econ 722

Survey Articles on Dynamic Factor Models

Stock & Watson (2010)

Best general overview of dynamic factor models and applications.

Bai & Ng (2008)

Comprehensive review of large-sample results for high-dimensional factor models estimated via PCA.

Stock & Watson (2006)

Handbook chapter on forecasting with many predictors. One section is devoted to dynamic factor models.

Breitung & Eickmeyer (2006)

Brief overview with an application to Euro-area business cycles.

Why Factor Models?

1. Factors could be intrinsically interesting if they arise from a theoretical model (e.g. Financial Economics)
2. Many variables without running out of degrees of freedom
   - More information could improve forecasts/macro analysis
   - Mimic central banks "looking at everything"
3. Eliminate measurement error and idiosyncratic shocks to provide more reliable information for policy
4. "Remain Agnostic about the Structure of the Economy"
   - Advantages over SVARs: don't have to choose variables to control degrees of freedom, and can allow fewer underlying shocks than variables.

Last Time: Classical Factor Analysis Model

I've eliminated the mean μ and renamed the factor F_t

$$\underset{(N\times 1)}{X_t} = \Lambda \underset{(k\times 1)}{F_t} + \epsilon_t$$

$$\begin{bmatrix} F_t \\ \epsilon_t \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_k & 0 \\ 0 & \Psi \end{bmatrix} \right)$$

$\Lambda =$ matrix of factor loadings

$\Psi =$ diagonal matrix of idiosyncratic variances.

Adding Time-Dependence

$$
\begin{aligned}
\underset{(N\times 1)}{X_t} &= \Lambda \underset{(k\times 1)}{F_t} + \epsilon_t \\
\underset{(k\times 1)}{F_t} &= A_1 F_{t-1} + \ldots + A_p F_{t-p} + u_t \\
\begin{bmatrix} u_t \\ \epsilon_t \end{bmatrix} &\overset{iid}{\sim} \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_k & 0 \\ 0 & \Psi \end{bmatrix} \right)
\end{aligned}
$$

Some Terminology

Static	X_t depends only on F_t
Dynamic	X_t depends on lags of F_t as well
Exact	Ψ is diagonal and ϵ_t independent over time
Approximate	Some cross-sectional & temporal dependence in ϵ_t

The model I wrote down on the previous slide is sometimes called an "exact, static factor model" even though it has dynamics! I'll still call it a dynamic factor model...

Editorial: This is all a bit confused...

1. The difference between "static" and "dynamic" is unclear
   - Can write "dynamic" model as a static one with more factors!
   - Static representation involves "different" factors, but we may not care: are the factors "real" or just a data summary?
2. Not really possible to allow cross-sectional dependence in ϵ_t
   - Unless the off-diagonal elements of Ψ are close to zero we can't tell them apart from the common factors
   - "Approximate" factor models basically assume conditions under which the off-diagonal elements of Ψ are negligible
   - Similarly, time series dependence in ϵ_t can't be very strong (stationary ARMA is ok)

Methods of Estimation for Dynamic Factor Models

1. Likelihood-based
   (a) Fully Bayesian Estimation
   (b) EM-algorithm + Kalman Filter
2. "Nonparametric"

PCA stuff, when and why it works, what the basic conditions are, and when it fails.

Alternative ways of estimating: two-step, EM algorithm, Bayesian.

Choosing the Number of Factors

Onatski paper: no one in the class listed it as a preference! Bai & Ng (2002).

What Can We Do with Factors?

Among other possibilities:

1. Use them as Instrumental Variables
2. Use them to construct Forecasts
3. Use them to "Augment" a VAR

Factors as Instruments – Bai & Ng (2010)

Endogenous Regressors x_t

$$y_t = x_t'\beta + \epsilon_t \qquad E[x_t\epsilon_t] \neq 0$$

Unobserved Variables F_t are Strong IVs

$$\underset{(k\times 1)}{x_t} = \Psi' \underset{(r\times 1)}{F_t} + u_t \qquad E[F_t\epsilon_t] = 0$$

Observe Large Panel $(z_{1t}, \ldots, z_{Nt})$

$$z_{it} = \lambda_i' F_t + e_{it}$$

Factors as Instruments – Bai & Ng (2010)

$$\boxed{y_t = x_t'\beta + \epsilon_t, \qquad x_t = \Psi' F_t + u_t, \qquad z_{it} = \lambda_i' F_t + e_{it}}$$

Procedure

1. Calculate the PCs of Z
2. Calculate $\widetilde{F}_t$ using the first r PCs of Z
3. Use $\widetilde{F}_t$ in place of F_t for IV estimation

Main Result

Under certain assumptions, as $(N, T) \to \infty$ "estimation and inference can proceed as though F_t were known." The resulting estimator is consistent and asymptotically normal.

Forecasting with Dynamic Factors

- Similar to Principal Components Regression (PCR)
- Estimate factors $\widehat{F}_t$ from a large number of regressors X_t
- Run a regression to forecast y_t using $\widehat{F}_t$ rather than X_t

We'll talk about this more next time and compare to other high-dimensional forecasting procedures.

Factors as Instruments – Bai & Ng (2010)

Why Might This be Helpful?

1. Avoid many instruments bias
2. Avoid bias from irrelevant instruments
3. Allow more observed instruments z_{it} than sample size T
4. Provided that $\sqrt{T}/N \to 0$, all of the observed instruments z_{it} can be *endogenous* as long as F_t is exogenous

FAVARs – Bernanke, Boivin & Eliasz (2005)

Two Problems with Structural VARs

1. Number of parameters is *quadratic* in the number of variables. Unrestricted VAR infeasible unless T is large relative to N.
   - You've studied one solution to this problem already this semester: Bayesian Estimation with informative priors
2. To keep estimation tractable we typically use a small number of variables, but then the VAR innovations "might not span the space of structural shocks."

FAVARs – Bernanke, Boivin & Eliasz (2005)

Factor-Augmented VAR Model

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t$$

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$$

$\underset{(M \times 1)}{Y_t}$ = observable variables that "drive dynamics of the economy"

$\underset{(K \times 1)}{F_t}$ = Small # of unobserved factors: "additional information"

$\underset{(N \times 1)}{F_t}$ = Large # of observed "informational time series"

FAVARs – Bernanke, Boivin & Eliasz (2005)

$$\boxed{\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \qquad X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t}$$

Consider Two Estimation Procedures

1. Two-step Procedure:
   - Estimate space spanned by factors using first $K + M$ PCs of X
   - Estimate VAR with $\widehat{F}_t$ in place of F_t
2. Full Bayes (Gibbs Sampler)

Empirical Application

Additional information contained in FVAR is "important to properly identify the monetary transmission mechanism."