# Dynamic Factor Models

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# Survey Articles on Dynamic Factor Models

### Stock & Watson (2010)

Best general overview of dynamic factor models and applications.

### Bai & Ng (2008)

Comprehensive review of large-sample results for high-dimensional factor models estimated via PCA.

### Stock & Watson (2006)

Handbook chapter on forecasting with many predictors. One section is devoted to dynamic factor models.

### Breitung & Eickmeyer (2006)

Brief overview with an application to Euro-area business cycles.

# Why Factor Models?

1. Factors could be intrinsically interesting if they arise from a theoretical model (e.g. Financial Economics)
2. Many variables without running out of degrees of freedom
   - More information could improve forecasts/macro analysis
   - Mimic central banks "looking at everything"
3. Eliminate measurement error and idiosyncratic shocks to provide more reliable information for policy
4. "Remain Agnostic about the Structure of the Economy"
   - Advantages over SVARs: don't have to choose variables to control degrees of freedom, and can allow fewer underlying shocks than variables.

# Last Time: Classical Factor Analysis Model

$$\underset{(N\times1)}{X_t} = \mu + \Lambda \underset{(k\times1)}{Z_t} + \epsilon_t$$

$$\begin{bmatrix} Z_t \\ \epsilon_t \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_k & 0 \\ 0 & \Psi \end{bmatrix} \right)$$

$\Lambda =$ matrix of factor loadings

$\Psi =$ diagonal matrix of idiosyncratic variances.

# Adding Some Dynamics

# Some (Confusing) Terminology

Static vs Dynamic, Exact vs Approximate

PCA stuff, when and why it works, what the basic conditions are, and when it fails.

Alternative ways of estimating: two-step, EM algorithm, Bayesian.

# Choosing the Number of Factors

Onatski paper: no one in the class listed it as a preference! Bai & Ng (2002).

# What Can We Do with Factors?

Among other possibilities:

1. Use them as Instrumental Variables
2. Use them to construct Forecasts
3. Use them to "Augment" a VAR

# Factors as Instruments – Bai & Ng (2010)

Endogenous Regressors $x_t$

$$y_t = x_t'\beta + \epsilon_t \qquad E[x_t\epsilon_t] \neq 0$$

Unobserved Variables $F_t$ are Strong IVs

$$\underset{(k\times 1)}{x_t} = \Psi' \underset{(r\times 1)}{F_t} + u_t \qquad E[F_t\epsilon_t] = 0$$

Observe Large Panel $(z_{1t}, \ldots, z_{Nt})$

$$z_{it} = \lambda_i' F_t + e_{it}$$

# Factors as Instruments – Bai & Ng (2010)

$$\boxed{y_t = x_t'\beta + \epsilon_t, \qquad x_t = \Psi' F_t + u_t, \qquad z_{it} = \lambda_i' F_t + e_{it}}$$

## Procedure

1. Calculate the PCs of Z
2. Calculate $\widetilde{F}_t$ using the first $r$ PCs of Z
3. Use $\widetilde{F}_t$ in place of $F_t$ for IV estimation

## Main Result

Under certain assumptions, as $(N, T) \to \infty$ "estimation and inference can proceed as though $F_t$ were known." The resulting estimator is consistent and asymptotically normal.

# Forecasting with Dynamic Factors

Big literature on this and we'll talk more about it next time, but give an overview here.

# Factors as Instruments – Bai & Ng (2010)

## Why Might This be Helpful?

1. Avoid many instruments bias
2. Avoid bias from irrelevant instruments
3. Allow more observed instruments $z_{it}$ than sample size $T$
4. Provided that $\sqrt{T}/N \to 0$, all of the observed instruments $z_{it}$ can be *endogenous* as long as $F_t$ is exogenous

# FAVARs – Bernanke, Boivin & Eliasz (2005)

## Two Problems with Structural VARs

1. Number of parameters is *quadratic* in the number of variables. Unrestricted VAR infeasible unless $T$ is large relative to $N$.
   - You've studied one solution to this problem already this semester: Bayesian Estimation with informative priors
2. To keep estimation tractable we typically use a small number of variables, but then the VAR innovations "might not span the space of structural shocks."

# FAVARs – Bernanke, Boivin & Eliasz (2005)

## Factor-Augmented VAR Model

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t$$

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$$

$\underset{(M \times 1)}{Y_t}$ = observable variables that "drive dynamics of the economy"

$\underset{(K \times 1)}{F_t}$ = Small # of unobserved factors: "additional information"

$\underset{(N \times 1)}{F_t}$ = Large # of observed "informational time series"

# FAVARs – Bernanke, Boivin & Eliasz (2005)

$$\boxed{\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \qquad\qquad X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t}$$

## Consider Two Estimation Procedures

1. Two-step Procedure:
   - Estimate space spanned by factors using first $K + M$ PCs of $X$
   - Estimate VAR with $\widehat{F}_t$ in place of $F_t$
2. Full Bayes (Gibbs Sampler)

## Empirical Application

Additional information contained in FVAR is "important to properly identify the monetary transmission mechanism."