Dynamic Factor Models

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Econ 722

Survey Articles on Dynamic Factor Models

Stock & Watson (2010)

Best general overview of dynamic factor models and applications.

Bai & Ng (2008)

Comprehensive review of large-sample results for high-dimensional factor models estimated via PCA.

Stock & Watson (2006)

Handbook chapter on forecasting with many predictors. One section is devoted to dynamic factor models.

Breitung & Eickmeyer (2006)

Brief overview with an application to Euro-area business cycles.

Why Factor Models?

- 1. Factors could be intrinsically interesting if they arise from a theoretical model (e.g. Financial Economics)
- 2. Many variables without running out of degrees of freedom
 - More information could improve forecasts/macro analysis
 - Mimic central banks "looking at everything"
- Eliminate measurement error and idiosyncratic shocks to provide more reliable information for policy
- 4. "Remain Agnostic about the Structure of the Economy"
 - Advantages over SVARs: don't have to choose variables to control degrees of freedom, and can allow fewer underlying shocks than variables.

Last Time: Classical Factor Analysis Model

$$X_{t} = \mu + \Lambda Z_{t} + \epsilon_{t}$$
(N×1)

$$\left[\begin{array}{c} Z_t \\ \epsilon_t \end{array}\right] \stackrel{\textit{iid}}{\sim} \mathcal{N} \left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} \textit{I}_k & 0 \\ 0 & \Psi \end{array}\right]\right)$$

 $\Lambda = matrix$ of factor loadings

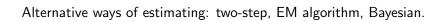
 $\Psi = \text{diagonal matrix of idiosyncratic variances}.$

Adding Some Dynamics

Some (Confusing) Terminology

Static vs Dynamic, Exact vs Approximate

PCA stuff, when and why it works, what the basic conditions are, and when it fails.



Choosing the Number of Factors

Onatski paper: no one in the class listed it as a preference! Bai & Ng (2002).

What Can We Do with Factors?

Among other possibilities:

- 1. Use them as Instrumental Variables
- 2. Use them to construct Forecasts
- 3. Use them to "Augment" a VAR

Factors as Instruments – Bai & Ng (2010)

Endogenous Regressors x_t

$$y_t = x_t' \beta + \epsilon_t$$
 $E[x_t \epsilon_t] \neq 0$

Unobserved Variables F_t are Strong IVs

$$x_t = \Psi' F_t + u_t$$
 $E[F_t \epsilon_t] = 0$

Observe Large Panel (z_{1t}, \ldots, z_{Nt})

$$z_{it} = \lambda_i' F_t + e_{it}$$

Factors as Instruments – Bai & Ng (2010)

$$y_t = x_t' \beta + \epsilon_t, \qquad x_t = \Psi' F_t + u_t, \qquad z_{it} = \lambda_i' F_t + e_{it}$$

Procedure

- 1. Calculate the PCs of Z
- 2. Calculate \widetilde{F}_t using the first r PCs of Z
- 3. Use \widetilde{F}_t in place of F_t for IV estimation

Main Result

Under certain assumptions, as $(N, T) \to \infty$ "estimation and inference can proceed as though F_t were known." The resulting estimator is consistent and asymptotically normal.

Forecasting with Dynamic Factors

Big literature on this and we'll talk more about it next time, but give an overview here.

Factors as Instruments – Bai & Ng (2010)

Why Might This be Helpful?

- 1. Avoid many instruments bias
- 2. Avoid bias from irrelevant instruments
- 3. Allow more observed instruments z_{it} than sample size T
- 4. Provided that $\sqrt{T}/N \to 0$, all of the observed instruments z_{it} can be *endogenous* as long as F_t is exogenous

FAVARs – Bernanke, Boivin & Eliasz (2005)

Two Problems with Structural VARs

- 1. Number of parameters is *quadratic* in the number of variables. Unrestricted VAR infeasible unless T is large relative to N.
 - You've studied one solution to this problem already this semester: Bayesian Estimation with informative priors
- To keep estimation tractable we typically use a small number of variables, but then the VAR innovations "might not span the space of structural shocks."

FAVARs - Bernanke, Boivin & Eliasz (2005)

Factor-Augmented VAR Model

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t$$

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$$

 $Y_{t}=$ observable variables that "drive dynamics of the economy" $_{(M imes 1)}^{(M imes 1)}$

 $F_t = \text{Small } \# \text{ of unobserved factors: "additional information"} _{(\mathcal{K} imes 1)}$

 $F_t = \mathsf{Large} \ \# \ \mathsf{of} \ \mathsf{observed} \ \text{``informational time series''} \ ({\scriptscriptstyle \mathsf{N}}{\scriptscriptstyle imes}1)$

FAVARs – Bernanke, Boivin & Eliasz (2005)

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \qquad X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$$

Consider Two Estimation Procedures

- 1. Two-step Procedure:
 - ▶ Estimate space spanned by factors using first K + M PCs of X
 - Estimate VAR with \hat{F}_t in place of F_t
- 2. Full Bayes (Gibbs Sampler)

Empirical Application

Additional information contained in FVAR is "important to properly identify the monetary transmission mechanism."