Summary: Asymmetric Forecast Densities for U.S. Macroeconomic Variables from a Gaussian Copula Model of Cross-Sectional and Serial Dependence
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Many existing reduced-form macroeconomic time-series models assume symmetric Gaussian distribution. However, we can easily show that Gaussian distributions are unrealistic for U.S. macro variables: notably, output growth, inflation, unemployment, and the interest rate. The authors introduce a parsimonious version of Gaussian Copula Model. The authors show that their model capture cross-sectional and serial dependence of macro variables. This helps the authors to better forecast macro variables.

1 Literature Review and Author's Contribution

Patton (2006), Rodriguez 2007 and Patton 2012 used Copulas model to capture cross-sectional dependence in multivariate time series. Domma et al. (2009), Beare (2010) and Smith et al. (2010) used Copulas model to capture serial dependence in univariate case. However, none of the papers used Copulas model to capture both cross-sectional and serial dependence and authors' contribution lie in this intersection.

2 Model

Consider a random vector $\mathbf{y}_t = (y_{1,t}, ..., y_{m,t})'$ and $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_T)'$. Let us set the joint CDF of $\mathbf{y}$ be $F(\mathbf{y})$ and the joint PDF of $\mathbf{y}$ be $f(\mathbf{y})$. Then, Sklar's Theorem states that there a copula function C that satisfies:

$$F(\mathbf{y}) = C(\mathbf{u})$$

where $\mathbf{u} = F(\mathbf{y})$ and $\mathbf{u}$'s marginal distributions are uniform distribution. Let us define $c(\mathbf{u}) = \frac{\partial}{\partial \mathbf{u}} C(\mathbf{u})$ and call it copular density. Although there are many options to choose for $c(\mathbf{u})$, to be able to nest Gaussian VAR case, authors decide to use Gaussian copula density defined as:

$$c(\mathbf{u}) = |\Omega|^{-1} \exp\left\{-\frac{1}{2}\mathbf{w}'(\Omega^{-1} - I_N)\mathbf{w}\right\}$$

where $\mathbf{w} = \Phi^{-1}(\mathbf{u})$, Φ is standard normal CDF, and $\mathbf{w} \sim N(0, \Omega)$. Please note that $\mathbf{y}$, $\mathbf{u}$ and $\mathbf{w}$ are $mT \times 1$ vector. Then, $\mathbf{y}$'s PDF is

$$f(\mathbf{y}) = \frac{\partial}{\partial \mathbf{y}} F(\mathbf{y}) = c(\mathbf{u}) \prod_{t=1}^{T} \prod_{j=1}^{m} f(y_{i,t})$$

If $f(y_{i,t})$ is restricted to be Gaussian with time invariant means and variance, then $\mathbf{y}$ is multivariate Gaussian with correlation Ω. Thus, this framework nests Gaussian VAR.

The authors' primary interest is when $f(y_{i,t})$ are not Gaussian yet they still have Gaussian copular density. Even in this case, the dependence structure is still determined by the correlation structure of the latent stationary $VAR(p)$. In order to make estimation tractable, they set $p = 4$, i.e. serial correlations are set to 0 when the lag is greater than 4. This would make $\hat{\Omega}$ very sparse matrix, thus relatively easy to estimate.

3 Results and Conclusion

For $f(y_{i,t})$, authors try two non-Gaussian distribution: 1) skew t distribution (SKT) and 2) empirical density function (EDF). Authors compare SKT/EDF to two different forecasting models: 1) Bayesian VAR with a Minnesota prior (BVAR)and 2) Bayesian VAR with a Minnesota prior and stochastic volatility (BVAR-SV). They test the goodness of the forecasting by looking at two measures: root-mean square of error and continuous ranked probability score.

Based on these measures, they found that EDF's performance is dismal due to small sample problem. SKT's performance is almost equivalent to BVAR for the entire sample. However, SKT performs better than BVAR for the subsample that corresponds to Great Recession because the asymmetric and heavy-tailed predictive distributions were more prevalent during the Great Recession. Sadly, authors acknowledge that BVAR-SV model outperforms the SKT overall in all aspects mainly because BVAR-SV takes the second moments into consideration whereas SKT does not. Authors left it as a future exercise.